ON PRODUCTION FUNCTIONS, TECHNICAL PROGRESS, AND TIME TRENDS

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17 May, 2005

We would like to thank Franklin M. Fisher, Colin Hargreaves, Lutz Kilian, JSL. McCombie, the participants of the session on Quantitative Applications at the Western Economic Association International 71st Annual Conference (San Francisco, June 28 - July 2, 1996), and the participants of the Economics Seminars at the Hong Kong University of Science and Technology, the University of Hong Kong, and Universidad Carlos III de Madrid for their comments and suggestions. The usual disclaimer applies.

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This paper extends the works of Fisher (1971), Simon (1979), and Nelson and Kang (1984) on econometric estimation of aggregate production functions. It explores through Monte Carlo simulations why the estimation of the aggregate Cobb-Douglas usually yields good fits, parameter estimates consistent with actual shares of capital and labor, and approximately constant returns to scale. Our results suggest that: (i) spuriousness is, at most, a secondary explanation for the high R^2 value found by many researchers; (ii) the Cobb-Douglas form with a linear time trend will work well for actual economies as long as the variation in the growth rates of the wage and profit rates does not exceed a certain threshold; (iii) the marginal productivity condition for labor will, in most cases, explain wages well; and (iv) successful Cobb-Douglas production function estimations carry no implications for such issues as technical progress, the degree of returns to scale, competition, and profit maximization.

Keywords: Accounting Identity; Cobb-Douglas Production Function; Random Walk; Spurious Regression; Time Trend; Total Factor Productivity Growth; Trend Stationary Process

JEL classification: C15, C22, E23, O47

1. Introduction

The aggregate Cobb-Douglas production function is perhaps the most ubiquitous production function form in the theoretical and empirical analyses of production and technical progress.[2] OLS estimation of the aggregate Cobb-Douglas production function with an exponential time trend accounting for the effect of exogenous, disembodied, and Hicks-neutral technical progress frequently yields the following "good" results:[3]

(i) a high R^2 value;

(ii) parameter values consistent with the shares received by capital and labor;

(iii) a coefficient of the time trend which provides a plausible rate of technical progress;

(iv) a sum of the exponents of capital and labor close to unity;

(v) a marginal product of labor which gives a good explanation of wages.

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2 See Brown (1966, pp.31-3) for a survey of early estimates of the Cobb-Douglas production function. Recent examples where there is an explicit use of aggregate production functions are Chen (1991), Mankiw et al. (1992), Chow (1993), Kim and Lau (1994), Bernard and Jones (1996), Black and Lynch (1996).

3 The notion of "good" results follows Lucas (1970) and Fisher (1971). The Cobb-Douglas production function has, in most cases, been estimated as a single equation using OLS, despite knowledge that the estimates are biased and inconsistent in a model of firm behavior (Marshak and Andrews 1944). Authors have invoked on numerous occasions Zellner et al. (1966), who proved that if we assume that a firm is operating under uncertainty and therefore can only maximize expected profits, then the estimation of the production function alone yields unbiased and consistent estimates.

The objective of this paper is to explore why the estimation of such a simple function often fits data so well and yields the results pointed out above.[4] This is a puzzling question in the light of the so-called aggregation problems. Economists are well aware of this issue, discussed at length, for example, by Fisher (1969, 1992). Fisher showed conclusively that an aggregate production function exists only under very stringent conditions, and that the analytic use of such aggregates as 'capital', 'output', 'labor' or 'investment', as though the production side of the economy could be treated as a single firm, is without sound foundation. At the empirical level, and in the context of the Cobb-Douglas, Fisher (1971) simulated an economy consisting of N industries, each of them characterized by a microeconomic Cobb-Douglas production function relating its homogeneous output to its homogeneous labor input and its own distinct machine stock, and purposefully violated the conditions for aggregation. Surprisingly, he found that the aggregate Cobb-Douglas production function worked well when labor's share was approximately constant. This led Fisher to conclude that "...the view that the constancy of labor's share is due to the presence of an aggregate Cobb-Douglas production function is mistaken. Causation runs the other way and the apparent success of aggregate production functions is due to the relative constancy of labor's share" (Fisher 1971, 306). Likewise, Garegnani (1970) proved that aggregate production functions can be derived theoretically only under conditions which neoclassical theory rejects (e.g., the labor theory of value). Therefore, aggregate production functions are

[4] This is not to say that on many occasions the Cobb-Douglas has given very poor results. Nevertheless, the good results must have appeared sufficiently enough; otherwise this form would

used today in theoretical work *only* as convenient aggregate parameterizations of the data, without implying that they are strictly true. And at the empirical level one should not expect the estimation of an aggregate production function to yield the good results it often does. But it is perhaps due to the paradox that such results appear in applied work, that (in empirical exercises) macroeconomists quite often proceed as if the aggregate production function represented the economy's aggregate technology. Thus, when they fit it econometrically, they go on to discuss the values of the parameters obtained, elasticity of substitution, returns to scale, or tests for the hypotheses of profit maximization and perfect competition, as if they were the equivalent to the micro notions (Chow 1993, Kim and Lau 1994, Pack 1994, Mankiw 1997, 103-7).[5]

An econometric issue that potentially explains some of the good results in production function estimations is the issue of spuriousness. Nelson and Kang (1984) and Durlauf and Phillips (1988) discussed the econometric implications of including a linear time trend as one of the right-hand-side variables in the regression of one difference stationary process (e.g., output) on one or more unrelated difference stationary processes (e.g., labor and capital). Using simulation analysis, Nelson and Kang (1984) showed that high R^2 values and significant coefficient t-values are due to spurious detrending. From the economic point of view, the trend is included as a measure of technological progress. From the econometric point of view, however, if

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have been discarded in the empirical literature.

5 Another strand of work uses the aggregate production function as the starting point for the calculation of total factor productivity growth (Young 1995). In this case the parameters of the production function are not estimated econometrically. The first-order conditions are assumed, and thus factor shares equal the elasticities.

outputs and inputs are difference stationary processes (DSP), the regression is spurious and should therefore be run in first differences. Only if all time series are trend stationary processes (TSP) is inclusion of a time trend econometrically acceptable.

Based on the work of Phelps Brown (1957), Simon and Levy (1963), Simon (1979) and Shaikh (1974, 1980) we advance a reason that explains good results in estimations of aggregate Cobb-Douglas production functions despite the aggregation problem and independent of the issue of spuriousness: if factor shares and the growth rates of the wage and profit rates are constant over time, then fitting a Cobb-Douglas production function with an exponential time trend is tantamount to estimating an accounting identity. This argument, and its implications for empirical analyses, is summarized in section 2.

The rest of the paper is organized as follows. Sections 3 extends the simulations of Nelson and Kang (1984), and discusses how far spuriousness can go to explain the good fits of production function estimations. Based on the arguments advanced in section 2, Section 4 extends Fisher's (1971) simulation work, and derives (through Monte Carlo simulations) the empirical conditions under which the estimated Cobb-Douglas production function yields good results. In particular, we answer the following question: how much do the two assumptions of constant factor shares and constant growth rates of the wage rate and the profit rate have to be relaxed for the regression fit and parameter estimates to no longer be "good". Section 5 discusses why and when the marginal product of labor will explain wages well. Section 6 summarizes the findings and concludes on the implications of good aggregate Cobb-Douglas production function estimation results.

2. Why do aggregate Cobb-Douglas production estimations often yield good results?

Notwithstanding that a spurious regression will induce an artificially high R^2 and high coefficient t-values in the regression of the aggregate Cobb-Douglas production function, spuriousness cannot account for the rest of the stylized facts mentioned above. This section shows that as long as factor shares *and* the growth rates of the wage and the profit rates are constant, the aggregate Cobb-Douglas production function with an exponential time trend is observationally equivalent to the national income accounting identity. The starting point is the accounting identity for national income, the sum of the wage bill plus the operating surplus

$$Q_t \equiv w_t L_t + r_t K_t \tag{1}$$

where Q, w, r, L, and K denote output (national income), wage rate, profit rate, employment, and stock of capital, respectively.[6] We can rewrite expression (1) in growth rates ("^" denotes the growth rate) as

$$\hat{Q}_t = a_t \hat{w}_t + (1 - a_t)\hat{r}_t + a_t \hat{L}_t + (1 - a_t)\hat{K}_t \quad , \tag{2}$$

[6] Expression (1) holds as an identity for every period. It states that national income equals the sum of the wage bill plus all types of profits. It does not assume constant returns to scale, or perfect competition. It holds in every type of market. The operating surplus (i.e., all profits on all

where $$\frac{w_t\, L_t}{Q_t} = a_t \qquad (3)$$

and $$\frac{r_t\, K_t}{Q_t} = 1 - a_t \quad , \qquad (4)$$

and where a_t and (1-a_t) are the labor and capital shares, respectively. A first assumption of constant factor shares over time, $a_t = a$, leads through integration of equation (2) with respect to time to

$$\ln(Q_t) = a\ln(w_t) + (1-a)\ln(r_t) + a\ln(L_t) + (1-a)\ln(K_t) + B \qquad (5)$$

where B is the constant of integration. Taking antilogarithms yields

$$Q_t = A_t\, L_t^{a}\, K_t^{1-a} \quad , \qquad \text{where} \quad A_t = e^{B}\, w_t^{a}\, r_t^{1-a} \ . \qquad (6)$$

To complete the derivation, let us hypothesize that wage and profit rates follow a simple function of time, and assume that their growth rates are constant, i.e., $\hat{r}_t = \hat{r}$ and $\hat{w}_t = \hat{w}$ (i.e., $r_t = r_0\, e^{\hat{r}t}$ and $w_t = w_0\, e^{\hat{w}t}$). Substitution into equation (5) leads to

$$\ln(Q_t) = j\ t + a\ \ln(L_t) + (1-a)\ \ln(K_t) + C \quad , \qquad (7)$$

where $$j = a\hat{w} + (1-a)\hat{r} \qquad (8a)$$

and $$C = a\ln(w_0) + (1-a)\ln(r_0) + B \qquad (8b)$$

Taking antilogarithms yields

$$Q_t = A\, e^{(a\hat{w} + (1-a)\hat{r})\ t}\, L_t^{a}\, K_t^{1-a} = A\, e^{jt}\, L_t^{a}\, K_t^{1-a} \qquad \text{where} \quad A = e^{B}\, w_0^{a}\, r_0^{1-a} \ . \qquad (9)$$

Equation (9) is the accounting identity rewritten under the assumptions of constant factor shares and constant growth rates of the wage and profit rates. This expression is

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types of capital goods) is written as the ex-post average profit rate (not the user cost of capital) times the stock of capital.

identical to the Cobb-Douglas production function with constant returns to scale and a 'neutral' time effect, the coefficient of which has been traditionally interpreted as total factor productivity growth. If the assumptions of constant factor shares and constant growth rates of the wage and profit rates are correct, the econometric estimation of the rewritten accounting identity in logarithms, that is, of

$$\ln(Q_t) = a + j\ t + g_1 \ln(L_t) + g_2 \ln(K_t) + u_t \tag{10}$$

where u_t is the error term must yield a perfect fit for being an identity (i.e., $R^2 = 1$ and $u_t=0$).[7] This result has several other implications for empirical analysis, namely: (i) The income identity is, of course, compatible with any aggregate production technology (or lack of it), but will give a perfect fit to a putative Cobb-Douglas production function so long as factor shares are constant. Factor shares can be constant for many reasons, such as a constant mark-up on unit labor costs or the Kaldorian theory of distribution, both of which do not depend on an underlying Cobb-Douglas production function. Should this be the case, the data will indicate that a Cobb-Douglas form is the correct one even though the true underlying technology is, for example, fixed coefficients. This result explains Fisher's (1971) simulation findings that the data gave a good fit to an aggregate Cobb-Douglas production function when factor shares were constant, even

[7] If the first assumption, i.e., constant factor shares, is relaxed, then a more flexible functional form is needed, such as the translog or the CES. These more complex forms can be derived from the accounting identity through a different set of assumptions about factor shares and the weighted average of the growth rates of wages and the profit rate. See McCombie (1987), McCombie and Dixon (1991), Felipe and McCombie (1997, 1998) for the derivation. The second assumption, the assumption of constant growth rates of wages and the profit rate, is necessary to generate the linear time trend. Its modification leads to the appearance of other types of trends. See Shaikh (1980) for an approximation to the weighted average of the growth rates of wages and the profit rate with sines and cosines.

though the micro-production functions were deliberately constructed to violate the rules of successful theoretical aggregation. In fact, Fisher concluded: "the point of our results...is not that an aggregate Cobb-Douglas fails to work well when labor's share ceases to be roughly constant, it is that an aggregate Cobb-Douglas will continue to work well so long as labor's share continues to be roughly constant, even though that rough constancy is not itself a consequence of the economy having a technology that is truly summarized by an aggregate Cobb-Douglas" (Fisher 1971, 307); (ii) φ, the proxy for the rate of technological progress, must equal the weighted average of the constant growth rates of the wage and profit rates, i.e., equation (8a); (iii) The estimates of γ_1 and γ_2 must equal the constant shares of labor and capital, i.e., a and (1-a). This implies that the values of the factor shares determine the values of the output elasticities in a statistical sense, rather than the other way around for economic reasons (such as factor shares are paid their marginal products under competitive conditions); (iv) γ_1 plus γ_2 must add up to 1, thus showing constant returns to scale. This indicates that when authors have found other than constant returns to scale using this functional form (see Walters 1963a in his discussion of Solow 1957), it simply means that factor shares were not constant enough, or that the growth rates of wage and profit rates were not (sufficiently) constant (or both), for the Cobb-Douglas to give a good approximation to the accounting identity. But the important aspect to remark is that all this is a result of the underlying accounting identity, and does not imply that in actual production returns to scale are constant. They might or might not be. The claim through the above derivation is that if factor shares and the growth rates of the wage and profit rates are constant, and one fits a Cobb-Douglas form, the parameters must add up to one. Thus, the empirical estimation of aggregate production functions can shed no light on the degree of returns to scale (for an extension of these results to other functional forms see Felipe and McCombie 1997, Felipe 1998). The implication is, therefore, that if one fits the wrong approximation to the accounting identity (in production function form) one could obtain parameters that add up to more than one (see Kim and Lau 1994), and Felipe 1998). As above, this does not imply increasing returns to scale (see the simulations below), and it cannot be taken as evidence that the production function has been refuted in any way; (v) The statistical properties of the series (i.e., type of underlying trend and the possibility of cointegration) are secondary issues.[8]

We conclude that it is not possible to test statistically and hence potentially refute the hypothesis that an economy can be represented by an aggregate production function. Estimating a Cobb-Douglas production function with a linear time trend can only be regarded as a test for the hypothesis that factor shares and the weighted average of the growth rates of the wage and profit rates are constant over time.

3. How far can spuriousness go to explain the good fits of the Cobb-Douglas production function?

[8] They affect the results, however, if one fits the wrong approximation to the identity. See simulations below. For a recent use of cointegration analysis in production functions see Otto and Voss (1996).

In this section we study the effects of the spurious regression phenomenon in the context of the production function. In particular, we carry out a simulation experiment with a view to analyzing how much of the high R^2 value observed in the estimation of the aggregate Cobb-Douglas function can be explained by spuriousness. Our simulations extend Nelson and Kang's (1984) regression equation to include three random walks. Nelson and Kang (1984, 78-80) ran the regression $Y_t = a + b\ t + g\ X_t + u_t$, "where $\{Y_t\}$ is a nonstationary variable such as output, $\{X_t\}$ is a nonstationary independent variable (or set of such variables) such as a production input, and $\{u_t\}$ is a sequence of disturbances. The role of time is to account for growth in Y not attributable to X, for example, the impact of technological change on output" (Nelson and Kang 1984, 78). In order to obtain a lower bound for the R^2 value, Nelson and Kang created Y and X as independent zero-drift random walks with unit variance Normal distributions, and then regressed Y on a constant, time and X. Across one thousand runs they obtained a mean R^2 value of 0.501, the time coefficient $\hat{b}$ was significant at a nominal 5% level in 83% of the runs, and the coefficient of X, $\hat{g}$, was significant at a nominal 5% (1%) level in 64% (55%) of the runs. These results led them to conclude that the "regression of one random walk on another, with time included to account for trend, is strongly subject to the spurious regression phenomenon." (Nelson and Kang 1984, 80).

The first set of results in Table 1 shows the estimation results in the case of three unrelated random walks.[9] In the second set the three random walks are related through

[9] The regressions here—in contrast to those of Nelson and Kang—are run in logarithms. Taking the logarithm of a negative value is avoided by choosing a sufficiently large positive first-period

the accounting-identity-link equations (1) and (3). The third set extends the results to the case where all variables are TSP. All statistics cover 1000 runs, where each run has 100 observations.

The first set of results constitutes the "lower bound" case for three unrelated random variables. Compared to Nelson and Kang's regression with one explanatory random walk, the R^2 value is up slightly at 0.5754 (from 0.501). All other results resemble those in Nelson and Kang's regression: there are clearly no constant returns to scale, parameter estimates are not close to any realistic factor share, and differencing leads to a negligible R^2 value.[10]

The second set of results is based on random walks generated according to the accounting identity (1) and the definition of the labor share (3). In this set and in the following we arbitrarily choose a value for the labor share of 0.6. Since the growth rates of the wage and profit rates in this set are not constant (w_t and r_t are random walks), the R^2 value is not equal to one and there is still some scope for spuriousness to improve the R^2 value (by 0.12) when going from the non-spurious regression in first differences (R^2 value of 0.7522) to the spurious regression in levels (R^2 value of 0.8735).[11]

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value of 1000 in combination with Nelson and Kang's N(0,1) shock. Running a regression in logarithms of one random walk on an unrelated second random walk with initial values 1000 we fully reproduce Nelson and Kang's results.

10 If all DSP processes include a drift of 0.5 the R^2 value reaches 0.9743 (dropping to 0.0201 once the regression is run in first differences), but otherwise there is no systematic difference to the version without drift.

11 We could have chosen a smaller shock or a larger initial value than the ones used here ($\varepsilon_t \sim N(0,1)$, $X_0 = 1000$) but leave these explorations to the next section; all that matters here is to show the existence of an effect of the accounting identity link.

The number of runs with significant t-values (at the 5% significance level) for the parameters of capital and labor is up compared to the previous set, and close to 1000. Given the existence of the accounting identity link, taking first differences does not reduce the significance of these parameters, unlike in Set 1.

Non-constant returns to scale in the estimation of a Cobb-Douglas production function can only occur when the labor share and the growth rates of the wage and profit rates are not (sufficiently) constant, or if measurement errors prevail. In Set 2 the growth rates of the wage and profit rates are not constant and constant returns to scale are rejected in 62.3% of all runs in levels; this percentage is lower than in the previous set where the accounting identity did not hold.[12] First differencing reduces this number to a more appropriate 5.5%. The effects of spuriousness are further visible in the regression in levels in two respects. First, the Durbin-Watson statistic is very low at 0.3221. Second, the parameter of time, β, traditionally interpreted as the rate of technical progress, is significantly different from zero at the 5% level in 819 runs, despite the fact that the mean is zero. (In the case of random walks, the weighted average of the growth rates of the wage and profit rates must be zero.)

Focusing on the overall fit and on whether the parameter estimates are close to the factor shares, the statistics obtained here (in levels) would be viewed as a "successful" production function estimation.

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[12] The mean sum of the parameters of labor and capital across 1000 runs is 0.985042 with a standard deviation of 0.276800, a maximum value of 2.097430 and a minimum value of 0.008175. After differencing, the mean sum of the parameters of labor and capital is 0.999553 with a standard deviation of 0.069087, a maximum value of 1.234804 and a minimum value of 0.763048.

The third set of results shows what happens when the variables are created as TSP, all subject to the same size of (distinct) shocks and the same growth rate of an arbitrarily chosen 5%.[13] Since now all variables are TSP, the time trend is correctly included in the regression, and, unlike in the previous sets, the two regressions are not subject to the problems of spurious detrending. We report the results for the case that all variables are unrelated versus the case that the variables are linked in accordance with the accounting identity; the effects of imposing the accounting identity link are similar to those obtained for DSP variables. Once the accounting identity link has been instituted, R^2 improves from 0.684 (i.e., "lower bound" for the non-spurious regression) to 0.9530, constant returns to scale can no longer be rejected, as it must be, and most importantly, coefficient estimates turn highly significant and credible in terms of a factor share interpretation. The parameter of time is correctly estimated. It equals 0.6*5%+0.4*5%=0.05 (see equation (8a)) where 5% is the assumed value of the growth rate of the wage rate as well as that of the profit rate.

The results in the above three sets of regressions show that if the profit rate, wage rate, and labor are difference stationary processes, then spuriousness may explain part of the good fit of a production function estimation, but obscure some issues such as constant returns to scale. The accounting identity link in the simulations turns out to be the major force leading to a high R^2 value, independently of the issue of spuriousness,

[13] The initial value of each variable does not matter. We leave it at 1000. We have introduced a shock z_t to avoid the perfect multicollinearity that appears in case of a constant labor share and constant growth rates of wages and the profit rate. There is not a single country in which factor shares have remained completely constant over an extended period of time. That is why we do not obtain an R^2 equal to 1. On this see Felipe (1996).

and only the accounting identity link can explain the constant returns to scale and the accuracy of the estimated parameters.[14]

[14] To take into account the possibility of cointegration among the series we fitted an error correction model reparameterization of Set 2 in Table 1 with two lags in output and inputs. The results show an increase in the R^2 value (to 0.774) compared to the regression in first differences, but no change in the (long-run) parameters of capital and labor. These results are available upon request.

Table 1. How far can spuriousness go to explain the good fits of the Cobb-Douglas production function?

Statistic	Mean	Standard deviation	Number of runs (out of 1000, 5% signif. level)		Mean	Standard deviation	Number of runs (out of 1000, 5% signif. level)	
			with sign. t-val.	rejec-ting CRS			with sign. t-val.	rejec-ting CRS
	All variables are DSP created as $X_t = \mu + X_{t-1} + \varepsilon_t$; $\varepsilon_t \sim N(0,1)$							
	Spurious regression				Regression in first differences			
Sets 1 - 2	$\ln(Q_t) = a + b\ t + g_1 \ln(L_t) + g_2 \ln(K_t) + u_t$				$(\ln(Q_t) - \ln(Q_{t-1})) = b + g_1\ (\ln(L_t) - \ln(L_{t-1})) + g_2 (\ln(K_t) - \ln(K_{t-1})) + v_t$			
Set 1	Q, L, and K are R.W., and $Q_0 = K_0 = L_0 = 1000$, $\mu = 0$.							
R^2	0.575357	0.242049		902	0.020858	0.020051		1000
α	6.882530	3.993195	901					
S.e.(α)	1.004910	0.425451						
β	-0.000008	0.000120	771		0.000002	0.000101	48	
S.e.(β)	0.000016	0.000008			0.000100	0.000007		
γ_1	0.014979	0.396637	610		0.000754	0.107413	64	
S.e.(γ_1)	0.103367	0.041911			0.101153	0.010787		
γ_2	-0.011447	0.415785	627		-0.002655	0.099560	46	
S.e.(γ_2)	0.101244	0.039940			0.100753	0.010426		
D.W.	0.327604	0.141562			1.990242	0.203015		
Set 2	w, r, and L are R.W. with $w_0 = r_0 = L_0 = 1000$, $\mu = 0$. $Q_t = w_t L_t / a_t$, and $K_t = (Q_t - w_t L_t) / r_t$, where $a_t = a = 0.6$.							
R^2	0.873451	0.115429		623	0.752246	0.043172		55
α	7.654263	1.964840	997					
S.e.(α)	0.498156	0.198476						
β	-0.000003	0.000085	819		0.000001	0.000070	47	
S.e.(β)	0.000012	0.000006			0.000071	0.000005		
γ_1	0.500462	0.350018	857		0.500805	0.089568	999	
S.e.(γ_1)	0.087817	0.035798			0.087681	0.008813		
γ_2	0.494506	0.202831	964		0.498748	0.053252	1000	
S.e.(γ_2)	0.051409	0.021679			0.050449	0.005016		
D.W.	0.322063	0.140261			1.999682	0.198818		

Set 3	All TSP variables are created as X_t = 1000 exp(0.05 t + z_t) ; z_t ~ N(0,1) Regression: $\ln(Q_t) = a + b\ t + g_1 \ln(L_t) + g_2 \ln(K_t) + u_t$							
	Without accounting identity link (Q, L, and K are TSP)				With accounting identity link w,r, and L are TSP $Q_t = w_t L_t /0.6$, $K_t = (Q_t - w_t L_t)/r_t$.			
R^2	0.684621	0.044977		1000	0.952968	0.007881		60
α S.e.(α)	7.823030 1.157776	1.209966 0.119029	1000		8.610850 0.586601	0.603917 0.061528	1000	
β S.e.(β)	0.049195 0.008011	0.008299 0.000824	1000		0.049726 0.004375	0.004558 0.000444	1000	
γ_1 S.e.(γ_1)	0.004402 0.102048	0.105725 0.010554	52		0.502638 0.088039	0.090402 0.009167	1000	
γ_2 S.e.(γ_2)	0.008073 0.101539	0.102499 0.010273	52		0.499985 0.051274	0.051964 0.005434	1000	
D.W.	2.019948	0.196317			2.019756	0.193111		

"Mean" denotes the mean value across 1000 runs. "Standard deviation" denotes the sample standard deviation of the 1000 mean values. "Mean S.e." of a parameter denotes the mean of the 1000 standard errors. "Standard deviation" of the "S.e." of a parameter denotes the sample standard deviation of the 1000 standard errors. $\nu_t = u_t - u_{t-1}$.

4. How much do the two assumptions of constant factor shares and constant growth rates of the wage and profit rates have to be relaxed for the regression estimation results to no longer be "good?"

In this section we explore the argument advanced in section 2 as a possible explanation for the good results observed in fitting the Cobb-Douglas form. If factor shares and the growth rates of the wage and profit rates are perfectly constant, the Cobb-Douglas form is observationally equivalent to the accounting identity. However, paradoxically, if factor shares are perfectly constant, and wage and profit rates grow at perfectly constant rates, the result will be perfect multicollinearity, and one will not be able to obtain the OLS estimates (Felipe 1996). For this reason, the two assumptions are now relaxed step by step with the economy being simulated again using 1000 runs, each with 100 observations.[15]

Table 2 presents an overview over the three types of simulations conducted. In the first type of simulation, labor, as well as profit and wage rates, are DSP; in the second type these variables are TSP; and in the third type labor and wage rate are TSP, while the profit rate is DSP. Since the results of these three simulations do not differ systematically we focus in the following on the third one only.[16] Table 3 translates various shock sizes into characteristics of the time series of the labor share, profit rate

[15] Our simulations differ from Fisher's (1971) mainly in that he postulated Cobb-Douglas production functions at the micro-level, and wages equal to the marginal products. We directly generate aggregate values. We also look at a wider variety of DGPs.

[16] This case corresponds to the findings of Perron (1989).

growth, wage growth, and labor growth.[17] The most important results are reported in the graphs in the Appendix.

For each simulation four time series are created, namely, labor, labor share, profit rate, and wage rate. Output and capital are derived from these four time series through the identities. Throughout the simulations the standard deviation of the shock to labor, sd_4, is held at 0.01, with an imposed mean growth rate of 2.5% and an initial value L_0=1, yielding a mean of mean labor growth rates of 2.55% and a mean standard deviation of labor growth rates of 0.0209 (bottom Table 3).[18] As long as the resulting variation in the growth rate (reported in Table 3) appears realistic (i.e., as defined below, and shown in bold in Table 3), the combination of the size of mean growth with the size of the shock is irrelevant. The choice of initial value has no influence on growth rates and their variation.

The size of the other three shocks, namely the shock to the labor share, sd_1, to the profit rate, sd_2, and to the wage rate, sd_3, is then varied independently to assume a wide range of values. The DGP of the labor share is an AR(1) with drift 0.06 and first order autoregressive parameter 0.9.[19] The profit rate follows a DSP with initial value of unity and zero drift. The initial value matters only insofar as it influences the variation in the growth rate for a given shock. But as long as the resulting variation in the growth rate

17 Throughout all estimations that follow below we impose restrictions in our simulations. The labor share cannot exceed unity or fall below zero; the profit rate, wages, and employment cannot fall below zero. If any of them does, its value is redrawn. (The profit rate could theoretically be negative; however, this would be quite unusual.)

18 We have repeated all simulations with a standard deviation of the shock to labor of 0.1 and 0.001, and the results do not differ systematically.

19 This process generates a labor share series that looks credible.

(reported in Table 3) appears realistic, the combination of the initial value with the size of the shock does not matter.[20] The wage rate follows a trend stationary process with initial value unity and an imposed mean growth rate of 5% (the interpretation of the choice of initial value and growth rate is the same as above for labor).

How do we define for the purpose of the simulations what are realistic variations in both the labor share and in the growth rates of the wage and profit rates? Table 3 shows, for example, that standard deviations of the shock to the labor share of sd_1 = 0.01 and sd_1 = 0.005 translate into standard deviations of the labor share of 0.01994 and 0.010003, respectively. These values are viewed as realistic. A shock to the labor share of sd_1 = 0.01 implies that, if last period's value of the labor share was 0.6, this period's value will fall within the interval 0.56 to 0.64 with a probability of 95% (i.e., 0.6 $\pm$ two standard deviations). If sd_1 = 0.005, this period's labor share will be between 0.58 and 0.62 with the same probability. If, on the other hand, sd_1 = 0.1, the standard deviation of the labor share is 0.182977 which is too high to be realistic (i.e., this period's labor share would fall between 0.234 and 0.965 in 95% of all cases). Finally, in order to cover a wide range of values systematically, the standard deviations of the shocks to the profit rate (i.e., sd_2) and wage rate (i.e., sd_3) are varied following an exponential processes (see Table 3). For the growth rate of the profit rate, a mean standard deviation between 0.7% and 4.0% appears realistic (corresponding to a ±0.7

[20] Introduction of a drift would yield a growth rate different from zero, but since the growth rate would continuously decrease across 1000 observations, the objective of relaxing the assumption of constant growth rates by applying a shock of potentially equal size in each period would not be achievable.

and ±4.0 percentage point change in the growth rate between two periods), and for the growth rate of the wage rate between 1% and 2.7%. The corresponding shocks are $sd_2 = sd_3 = 0.0068$ and $sd_2 = sd_3 = 0.0183$ (i.e., $k_2 = k_3 = -5$ and $k_2 = k_3 = -4$, respectively. These values imply that the 'realistic' growth rate of the profit rate lies between -8% and +8% for $k_2 = -4$ with a probability of 95% (mean ± two standard deviations), and that the 'realistic' growth rate of the wage rate lies between -0.23% and 10.56% for $k_3 = -4$ with a probability of 95%. For $k_2 = -5$ and $k_3 = -5$, the corresponding growth rate ranges are from -1.4% to 1.4%, and from 3.13% to 7.13%, respectively.[21]

[21] These values are consistent with those of the NBER productivity database for U.S. aggregate manufacturing.

Table 2. How much do the two assumptions of constant factor shares and of constant growth rates of the wage and profit rates have to be relaxed for the regression estimation results to no longer be "good?"

Regression	$\ln(Q_t) = \alpha + \varphi\, t + \gamma_1 \ln(L_t) + \gamma_2 \ln(K_t) + u_t$
Accounting identity link	$Q_t = \frac{w_t L_t}{a_t}$ $\quad K_t = \frac{Q_t - w_t L_t}{r_t}$
Labor share process	$a_t = 0.06 + 0.9\, a_{t-1} + z_{1t}$, $z_{1t} \sim N(0, sd_1^2)$, $sd_1 = \{0.001, 0.01, 0.1\}$, and $a_0 = 0.6$.

All variables are DSP	**All variables are TSP**	**Mixed case**
$r_t = r_{t-1} + z_{2t}$ $z_{2t} \sim N(0, sd_2^2)$, $r_0 = 1$	$r_t = 1\, e^{0.01t + z_{2t}}$ $z_{2t} \sim N(0, sd_2^2)$, $r_0 = 1$	$r_t = r_{t-1} + z_{2t}$ $z_{2t} \sim N(0, sd_2^2)$, $r_0 = 1$
$w_t = w_{t-1} + z_{3t}$ $z_{3t} \sim N(0, sd_3^2)$, $w_0 = 1$	$w_t = 1\, e^{0.05t + z_{3t}}$ $z_{3t} \sim N(0, sd_3^2)$, $w_0 = 1$	$w_t = 1\, e^{0.05t + z_{3t}}$ $z_{3t} \sim N(0, sd_3^2)$, $w_0 = 1$
$L_t = L_{t-1} + z_{4t}$ $z_{4t} \sim N(0, sd_4^2)$ $sd_4 = 0.01$, $L_0 = 1$	$L_t = 1\, e^{0.025t + z_{4t}}$ $z_{4t} \sim N(0, sd_4^2)$ $sd_4 = 0.01$, $L_0 = 1$	$L_t = 1\, e^{0.025t + z_{4t}}$ $z_{4t} \sim N(0, sd_4^2)$ $sd_4 = 0.01$, $L_0 = 1$

For the case that all variables are DSP, the following regression is also run:
$\ln(Q_t) - \ln(Q_{t-1}) = \varphi + \gamma_1 (\ln(L_t) - \ln(L_{t-1})) + \gamma_2 (\ln(K_t) - \ln(K_{t-1})) + v_t$.

Table 3. Characteristics of Simulated Variables

	Standard deviation of shock		Mean of means	Mean standard deviation	Redraws
	$a_t = 0.06 + 0.9\, a_{t-1} + z_{1t}$, $a_0 = 0.6$, $z_{1t} \sim N(0, sd_1^2)$				
Labor share		$sd_1 = 0$	0.600000	0.000000	0
		$sd_1 = 0.001$	0.600013	0.001998	0
		$sd_1 = 0.005$	**0.600045**	**0.010003**	0
		$sd_1 = 0.01$	**0.599955**	**0.019940**	0
		$sd_1 = 0.05$	0.598241	0.100183	0
		$sd_1 = 0.1$	0.582391	0.182977	1808
		$sd_1 = 1$	0.502730	0.283634	169530
	where the profit rate is created as: $r_t = r_{t-1} + z_{2t}$, $r_0 = 1$, $z_{2t} \sim N(0, sd_2^2)$, $sd_2 = e^{k_2}$				
Growth rate of profit rate	$k_2 = -11$	$sd_2 = 1.670*10^{-5}$	0.000000	0.000017	0
	$k_2 = -10$	$sd_2 = 4.540*10^{-5}$	0.000000	0.000045	0
	$k_2 = -9$	$sd_2 = 1.234*10^{-4}$	0.000000	0.000123	0
	$k_2 = -8$	$sd_2 = 3.355*10^{-4}$	-0.000001	0.000334	0
	$k_2 = -7$	$sd_2 = 9.119*10^{-4}$	-0.000006	0.000910	0
	$k_2 = -6$	$sd_2 = 0.002479$	0.000008	0.002484	0
	$k_2 = -5$	**$sd_2 = 0.006738$**	**0.000021**	**0.006816**	0
	$k_2 = -4$	**$sd_2 = 0.018316$**	**0.002658**	**0.039559**	121
	$k_2 = -3$	$sd_2 = 0.049787$	0.069520	0.573574	3741
	$k_2 = -2$	$sd_2 = 0.135335$	0.189191	1.629150	9015
	where the wage rate are created as: $w_t = 1\, e^{0.05t + z_{3t}}$, $z_{3t} \sim N(0, sd_3^2)$, $sd_3 = e^{k_3}$, $w_0 = 1$				
Growth rate of wage rate	$k_3 = -11$	$sd_3 = 1.670*10^{-5}$	0.051271	0.000025	
	$k_3 = -10$	$sd_3 = 4.540*10^{-5}$	0.051271	0.000067	
	$k_3 = -9$	$sd_3 = 1.234*10^{-4}$	0.051271	0.000183	
	$k_3 = -8$	$sd_3 = 3.355*10^{-4}$	0.051271	0.000498	
	$k_3 = -7$	$sd_3 = 9.119*10^{-4}$	0.051272	0.001361	
	$k_3 = -6$	$sd_3 = 0.002479$	0.051278	0.003683	
	$k_3 = -5$	**$sd_3 = 0.006738$**	**0.051317**	**0.009994**	
	$k_3 = -4$	**$sd_3 = 0.018316$**	**0.051629**	**0.027306**	
	$k_3 = -3$	$sd_3 = 0.049787$	0.053865	0.074314	
	$k_3 = -2$	$sd_3 = 0.135335$	0.071079	0.208469	

Growth rate of labor	where labor is created as: $L_t = 1\, e^{0.025t + z_{4t}}$, z_{4t} ~ $N(0, sd_4^2)$, $L_0=1$				
		sd_4 = 0.01	**0.025535**	**0.020939**	

"Redraws" denotes the number of times an observation had to be redrawn to avoid its value at any one point of time to exceed or fall below a certain limit. The labor share was forced to fall within the]0,1[interval. With an imposed mean labor share of 0.6, the upper limit is more likely to be reached than the lower limit; truncation is more likely to occur at the upper limit, and for large shocks the mean of means is therefore biased downward. Due to the truncation the mean standard deviation of the labor share series for large shocks is also biased downward. The profit rate was forced to be positive. For large shocks the mean of means is therefore biased upward, while the mean standard deviation is biased downward. Large shocks to the growth rate of a trend stationary process (wage rate and labor) lead to mean growth rates of the series exceeding the imposed growth rate due to the asymmetrical nature of the exponential function.

The graphs in the appendix report the effects of a certain variation in the growth rate of the profit rate and a certain variation in the growth rate of the wage rate on the regression results for a given shock to the labor share. Three different values for the standard deviation of the shock to the labor share, $sd_1 = 0.001$, $sd_1 = 0.01$, and $sd_1 = 0.1$ yield three sets of results. For each set of results, the coefficient, the standard error, and the number of runs with significant t-values for the three variables time, labor, and capital are reported, as well as the R^2 value, the Durbin-Watson statistic, and the number of runs rejecting constant returns to scale.

The first set of graphs, for the case of $sd_1 = 0.01$ (the value identified above as a realistic shock to the labor share), shows how for very small values of the standard deviations of the shock to the profit rate and of the shock to the wage rate (i.e., sd_2 and sd_3, respectively), the coefficients of time, labor, and capital are very accurately estimated with minimal standard errors and significant coefficients.[22]

Three observations as to what happens to the coefficients when the variation in the growth rate of the profit rate and/or in the growth rate of the wage rate increases are the following. First, once sd_3 exceeds a threshold level of $sd_3 = e^{-4}$ (i.e., a standard deviation of the variable of 2.73%) the coefficients of time and labor move towards zero rapidly as long as sd_2 remains smaller than e^{-4} (i.e., a standard deviation of 3.95%).[23] Second, once sd_2 exceeds a threshold level of $sd_2 = e^{-5}$ (i.e., a standard deviation of

[22] The expected coefficient of time equals the labor share times the mean growth rate of the wage rate, 0.05, plus the capital share times the mean growth rate of the profit rate, i.e., 0.6*0.05+0.4*0=0.03.

[23] Threshold refers to the value of the shock for which coefficient estimates begin deteriorating.

0.68%), the coefficients of time and labor rise above their actual values as long as sd_3 remains smaller than e^{-3} (i.e., a standard deviation of 7.43%). And third, if sd_3 and sd_2 both exceed e^{-5} (i.e., standard deviations of 0.99% and 0.68%, respectively) the coefficients of time and labor may remain very accurate, although the standard errors increase very rapidly (i.e., high variation in the growth rate of the profit rate leads to a high coefficient, while a high variation in the growth rate of the wage rate leads to a low coefficient; the result in case of a mix of these two influences is a narrow range where the coefficients remain accurate.)

Throughout, the number of runs with significant coefficients of time and labor (in 1000 runs) drops off rapidly from 1000 once sd_3 exceeds e^{-4} independent of the size of sd_2. The coefficient of capital exhibits similar behavior, except that the dependence on sd_2 and sd_3 is reversed.[24] The R^2 value remains very high across all values of sd_2 and sd_3. It starts falling minimally once sd_2 or sd_3 exceeds e^{-5}; it experiences its biggest fall, to a minimum value of 0.9980, once $sd_2 = sd_3 = e^{-2}$ (i.e., a standard deviation of 162.91% for the growth rate of the profit rate and of 20.84% for the growth rate of the wage rate). The Durbin-Watson statistic appears to vary almost independently of sd_2 and reaches values close to two around $sd_3 = e^{-5}$. The null hypothesis of constant returns to scale can practically never be rejected.

[24] The only exception is the standard errors, which increase rapidly as either sd_2 or sd_3 exceeds e^{-6} but then drop off again once sd_2 or sd_3 exceeds e^{-3} —possibly due to the large technical impact of truncation (the profit rate cannot be negative) on the mean growth rate of the profit rate as well as its variation (see Table 3).

Comparing these results with the cases of $sd_1 = 0.001$ and $sd_1 = 0.1$ allows a generalization of the above observations and some additional conclusions.

1. The size of the shocks to the profit and wage rates relative to the size of the shock to the labor share matters; i.e., the individual threshold levels of sd_2 and sd_3 depend on sd_1. The larger sd_1, the larger may sd_2 and sd_3 be before parameter estimates deteriorate. In case of $sd_1 = 0.1$, the coefficients exhibit small (see the scale of the graph) unsystematic fluctuations around the actual values; however, the standard errors of the parameter estimates increase with sd_1.
2. The size of the shock to the profit rate relative to the size of the shock to the wage rate matters for all levels of the shock to the labor share, sd_1. If only one of the first two shocks exceeds a certain threshold level determined by sd_1, parameter estimates drop off or rise sharply, and standard errors increase; the number of runs with significant parameter estimates drops off depending only on the size of the shock corresponding to this time series (where the coefficient of time depends on sd_3), unless sd_1 is relatively large, in which case the number of runs with significant estimates of time and labor coefficients are low throughout. If sd_2 and sd_3 jointly exceed a certain threshold level determined by sd_1, the parameter estimates across a narrow range of sd_2 and sd_3 combinations may remain close to the actual values but their standard errors increase rapidly and the R^2 value drops.
3. Even when sd_2 and sd_3 exceed their threshold levels, the R^2 value still remains very high and close to unity across all levels of sd_1.

4. The Durbin-Watson statistic depends primarily on sd_3 with values close to two exactly at the threshold level of sd_3. As the threshold level of sd_3 changes with sd_1, so does the location of the ridge of Durbin-Watson statistics close to two.[25]

5. The number of runs rejecting constant returns to scale is independent of the variation in the labor share, and independent of the variation in the growth rate of the profit rate and in the growth rate of the wage rate; it hovers, correctly, around 50 runs out of 1000 runs under all shock combinations.

Overall, perhaps the most remarkable feature is that realistic combinations of sd_1, sd_2, and sd_3 are located right below the threshold levels of all three time series. Therefore a Cobb-Douglas production function "estimated" for a real economy (i.e., defined in terms of what we called 'realistic' variations in labor share, and growth rates of the wage and profit rates), is likely to display significant coefficient estimates close to the factor shares, approximately constant returns to scale, and a very high R^2 value.

5. Why does the marginal product of labor provide a good approximation of wages?

The final stylized fact referred to in the introduction is that the estimated marginal product of labor closely approximates the actual wage rate. We show through simulations how much factor shares have to vary for the marginal product of labor

[25] Good Durbin-Watson statistics throughout can be achieved by using an autoregressive parameter of 0.2 in the data generating process of the labor share, or by running the regression in

derived from the Cobb-Douglas production function not to approximate correctly the wage rate. [26]

The marginal productivity condition for labor derived from the Cobb-Douglas production function is

$$w_t = \frac{\partial Q_t}{\partial L_t} = \gamma_1 \frac{Q_t}{L_t} . \qquad (11)$$

On the other hand, from the definition of the labor share in equation (3), and assuming that it is constant, $\alpha_t = \alpha$, it follows directly that

$$w_t = \alpha \frac{Q_t}{L_t} . \qquad (12)$$

Expressions (11) and (12) are observationally equivalent. The latter, however, is an identity with no behavioral content. If factor shares are sufficiently constant, the marginal productivity condition derived from the Cobb-Douglas production function becomes a non-refutable hypothesis independent of the existence of perfect competition and profit maximization. In other words, if the estimation of (12) leads to poor results, it will simply imply that factor shares are not sufficiently constant, and it should not be taken as a rejection of the null hypothesis of profit maximization and competitive markets (see Felipe and McCombie 1997 and Felipe 1998).

Now we ask a similar question to the one posed in section 4 for the production function, namely, how much factor shares can vary for the marginal product of labor

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first differences.

[26] The marginal productivity condition is often used as an alternative for estimating the parameters of the production function (Walters 1963b). For recent uses of the first-order conditions in applied work see Chen (1991) and Kim and Lau (1994).

derived from the Cobb-Douglas production function to correctly approximate the wage rate. In this simulation, the wage rate and labor are taken to be TSP (as in the mixed case in Table 2). The standard deviation of the shock to the share is varied to assume the values 0.001, 0.01, 0.1, and 1, while the standard deviation of the shock to the wage rate and to labor is kept constant at 0.01 (see the previous section on the implications of these shock sizes on the characteristics of the time series).[27]

The results are reported in Table 4. The first two lines in each row in Table 4 would probably lead anyone to believe that the results can be taken as an empirical confirmation of the marginal productivity theory of distribution. Even the third simulation, with a parameter of 0.424278 and a R^2 value of 0.760064 looks convincing—despite the fact that in this case the variation in the share is already so large as to be no longer realistic under any circumstances. The conclusion therefore is that the neoclassical

[27] Neither the standard deviation of the shock to the wage rate nor the standard deviation of the shock to labor matter at all; we therefore report the results for the case when both are 0.01 (i.e., assume realistic values). Again, the initial values of the wage rate and labor are irrelevant; their growth rates matter only in so far as they combine with the size of the shock to yield a certain standard deviation of the time series.

marginal productivity condition can practically never be rejected; it is a non-falsifiable hypothesis.[28]

[28] In all four simulations the Durbin-Watson statistic is very poor. This is due to the DGP used. Nevertheless, here, as in the estimation of the production function, researchers mostly care about the parameter estimates and the fit.

Table 4. How Large Can the Standard Deviation of the Shock to the Share Be For the Estimation of the Marginal Productivity Condition to Still Yield Good Results?

$w_t = c + a\frac{Q_t}{L_t}$ (the constant c is added for econometric purposes)

where:

$a_t = 0.06 + 0.9\ a_{t-1} + z_{1t}$,with $z_{1t} \sim N(0,sd_1^2)$, $a_0 = 0.6$, $sd_1 = \{0.001, 0.01, 0.1, 1\}$

$w_t = 1\ e^{0.05t + z_{2t}}$, with $z_{2t} \sim N(0,sd_2^2)$, $sd_2 = 0.01$

$L_t = 1\ e^{0.025t + z_{3t}}$, with $z_{3t} \sim N(0,sd_3^2)$, $sd_3 = 0.01$

$Q_t = \frac{w_t L_t}{a_t}$

Statistic	sd_1	Mean	Standard deviation	Number of runs with significant t-values (at 5% level)
R^2	0.001	0.999989	0.000007	
	0.01	0.998815	0.000732	
	0.1	0.760064	0.234196	
	1	0.158338	0.129084	
c	0.001	-0.001199	0.045439	183
	0.01	0.019231	0.452320	197
	0.1	6.148296	7.670019	471
	1	24.907112	4.236840	998
S.e. (c)	0.001	0.015838	0.004323	
	0.01	0.160885	0.045905	
	0.1	2.052867	0.846286	
	1	3.674911	0.207581	
a	0.001	0.600030	0.001887	1000
	0.01	0.599183	0.018148	1000
	0.1	0.424278	0.194016	969
	1	0.042210	0.047417	710
S.e. (a)	0.001	0.000198	0.000054	
	0.01	0.002004	0.000571	
	0.1	0.018262	0.007127	
	1	0.008054	0.005796	
D.W.	0.001	0.498595	0.252587	
	0.01	0.488398	0.245290	
	0.1	0.547243	0.256844	
	1	0.313011	0.253234	

For $sd_1 = 0.001$ the mean of means, the mean of standard deviations, and the number of redraws in this particular set of 1000 runs with 100 observations each were 0.599993, 0.002040, 1099.

For $sd_1 = 0.01$ these values were 0.600032, 0.020198, 1000.

For $sd_1 = 0.1$ these values were 0.578300, 0.183234, 2768.

For $sd_1 = 1$ these values were 0.499714, 0.283788, 170363.

6. Conclusions

This paper explored why production function estimations of the Cobb-Douglas type usually yield good fits, significant parameter values close to the actual factor shares, and approximately constant returns to scale, as well as why the marginal product of labor explains actual wages. The analysis yields the following conclusions:

(i) Spuriousness is at most a secondary explanation for high R^2 values and significant coefficient t-values, and cannot explain any of the other stylized facts.

(ii) The Cobb-Douglas production function with an exponential time trend is observationally equivalent to the national income accounting identity, provided factor shares and the growth rates of the wage and profit rates are constant. The two conditions of constant factor shares and constant growth rates of the wage and profit rates are necessary and sufficient for good results in production function estimations—the existence of an aggregate production function, the requirements for which are practically never met, is not necessary. Good estimation results of the accounting identity rewritten under the two assumptions carries no implications on the technology in use in the economy, such as on economies of scale, the elasticity of substitution, or technical progress.

(iii) Monte Carlo simulations show to what extent the above theoretical conclusions hold if the two assumptions are relaxed. In other words, how "constant" do factor shares and the growth rates of the wage rate and the profit rate have to be for the equation identical to the Cobb-Douglas production function to no longer yield "good" results?[29]

1. The R^2 value is very high and constant returns to scale tend to prevail independent of the size of the shocks to factor shares and to the growth rates of the wage and profit rates.
2. Results summarized in Table 5 show that coefficient estimates are reasonable in a variety of circumstances, even for large variations in the labor share and in the growth rates of wage and profit rates. Since the case of zero variation in the labor share (i.e., first column) is not realistic, we conclude that the only case in which estimates are poor occurs when the variation in the labor share is realistic combined with a larger-than-realistic variation in the growth rates of the wage and profit rates.

(iv) An equation identical to the marginal productivity condition derived from an aggregate Cobb-Douglas production function and profit maximization also follows from the accounting identity under the assumption of constant factor shares. Any realistic degree of relaxation in this assumption will always yield an excellent fit of the "marginal productivity condition" as well as sharp estimates of the relevant factor share. The marginal productivity condition thus is de facto non-falsifiable.

Overall, good results of production function estimations do not allow any conclusions concerning the underlying technology of the economy, the market structure,

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[29] The following observations also hold for the other two types of variable combinations reported in Table 2 (i.e., the cases where the profit rate, wage rate, and labor are either all difference

the distribution of value added between capital and labor, and returns to scale (and bad results do not imply refutation). Good results of Cobb-Douglas production function estimations only imply that factor shares and growth rates of the wage and profit rates are sufficiently constant in the economy under consideration. The results established in this paper are of relevance to much of today's macroeconomic practice. They suggest grave reservations about the use of aggregate production functions for estimation purposes, and the empirical analysis of technical change.

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stationary or all trend stationary variables).

Table 5. Coefficients of the Cobb-Douglas regression

Coefficient estimates (of time, labor, and capital) in

$$\ln(Q_t) = \alpha + \varphi\, t + \gamma_1 \ln(L_t) + \gamma_2 \ln(K_t) + u_t \quad \text{are ...}$$

		Variation in labor share		
		zero	*"realistic"*	*larger than realistic*
Variation in growth rates of the profit and wage rates	*zero*	Not obtainable*	Excellent	Good
	"realistic"	Poor	Good, close to threshold	Acceptable (large S.e.)
	larger than realistic	Bad	Poor	Still acceptable (large S.e.)

* Perfect Multicollinearity.

It is assumed here that the growth rates of wage and profit rates vary to the same degree with respect to what we defined as "realistic." If they vary to differing degrees, then the one with the higher variation determines the location in the table above. A very narrow range of combinations of variation in growth rates of the profit rate and of variation in growth rates of the wage rate exists which will *always* yield realistic coefficients of labor and capital, even in the lower left cells in this table, but at the expense of a high standard error (the lower the variation in the labor share, the narrower this range).

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