

Max Routes Coverage: A Heuristic Wavelength Converters Placement Algorithm on WDM Optical Networks

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Abstract

In this paper, we consider the sparse placement of full-range wavelength converters on circuit-switched WDM optical networks. There are two problems to be considered: i) optimally placing a *given* number of full-range wavelength converters onto the network to minimize the connection blocking probability; ii) determining the minimal number of converters whose optimal placement achieves a blocking probability *sufficiently close* to that obtained in the same network with full conversion. A heuristic wavelength converter placement algorithm, the so-called Max Routes Coverage, which maximizes the routes coverage ratio (RCR) is presented in this paper to solve the first problem. The RCR metric, is then used to solve the second problem in polynomial time.

Index Terms

optical converter placement, network optimization

I. INTRODUCTION

Due to the very large capacity of optical fiber, backbone networks are expected to be using wavelength-routed all optical networks. In such networks, with circuit-switching, connections between source and destination nodes are established by setting optical virtual paths, called light-paths, using the so-called wavelength division multiplexing (WDM) technique. Setting up end-to-end light-paths without the intervention of optical to electronic to optical conversion is subject to wavelength continuity. When no continuous wavelengths exist, the connection is blocked. The connection blocking probability is defined as the ratio of the number of connections blocked during a period of

time to the number of connections arriving in the same period of time. To relax the stringent wavelength continuity constraint and mitigate its negative effects on the blocking probability, recent research has focused on equipping network nodes with wavelength converters which are capable of translating one wavelength into another. However, due to the high cost of such converters, only sparse conversion, where a few nodes in the network have conversion capability, is of particular interest. Previous results indicate that networks with sparse wavelength conversion can achieve a similar blocking performance as optical networks with full wavelength conversion, if the wavelength converters are placed appropriately [1].

There are two types of wavelength converters: limited-range converters and full-range converters. The former only allows conversion of wavelengths from an to a given limited subset of wavelengths, the latter allows any input wavelength to be converted into any other. We only consider the sparse placement of full-range converters in this paper. Much work has been done on the sparse placement of full-range wavelength converters [2]–[7].

One important issue that needs to be considered in this problem is that the blocking probability does not decrease linearly with the number of wavelength converters, even if the wavelength converters are optimally placed. In other words, the gain of placing wavelength converters converges to zero when the number of wavelength converters increases. Therefore finding the minimal number of wavelength converters which achieves nearly the same performance as a fully converted network, which is named here “pseudo optimal number”, is important from the point of view of minimizing cost. Intuitively, a brute-force approach would estimate the blocking probability of each placement of i converters in the network according to a given wavelength converter placement algorithm, while varying i from zero to the number of nodes in the network. However, in general, this approach is too time consuming as the time of obtaining the blocking probability by simulation is long. For most intelligent routing algorithms that depend on the load of the nodes, the blocking probability is not product form, therefore it is often obtained by simulations rather than analytically. Moreover, in the cases with simple routing algorithms where it can be obtained in product form, placing wavelength converters does not usually improve the blocking probability much: the difference of the blocking probability from a fully converted network to a non converted network lies often within one order of magnitude. In view of this complexity, in this paper we propose a new metric – viz., the routes coverage ratio (RCR), and show that the blocking probability converges when the RCR converges. An algorithm which searches for a pseudo optimal number of converters for given routing and wavelength assignment (RWA) algorithms and a given wavelength converter placement algorithm is then proposed. As the problem of maximizing the routes coverage ratio is proved to be NP complete, one heuristic algorithm, Max Routes Coverage, is proposed to achieve nearly maximal routes coverage ratio and its performance is compared to other intelligent wavelength converter placement algorithms under different routing algorithms.

The remainder of this paper is organized as follows. Definition of route coverage ratio, Max Routes Coverage heuristic and the pseudo optimal number of converters search algorithm are described in Section IV. The related work on wavelength converter placement algorithms are listed in Section III. A brief review of some RWA algorithms used in the performance evaluation section is given in Section II. Simulation results are discussed in Section V. Finally we conclude the paper in Section VI.

II. ROUTING AND WAVELENGTH ASSIGNMENT ALGORITHMS

In this section we review several routing and wavelength assignment algorithms that we use in our experiments to test the effectiveness of our heuristics.

The most commonly considered algorithm for routing is the so-called shortest path routing. In this algorithm, only the shortest path between a source and the destination (calculated using Dijkstra's algorithm) for each connection request is considered when checking wavelength availability. If a wavelength is not available the request is rejected. To further improve the performance over the shortest path routing algorithm, the so-called fixed alternate routing (FAR) pre-calculates and uses for routing a number k of nodes/edge-disjoint paths between the source and the destination (usually the k -shortest paths). These paths are ranked in a predetermined order and if no free wavelength is found after trying all the k paths in the predetermined order, the request is blocked.

An alternative to this class of static routing algorithms is a class of algorithms where routes are no longer pre-computed as their cost is based on dynamic metrics such as the link load. Least load routing (LLR) algorithm [8] which selects the least loaded path to route a request falls within this class. Conjoining LLR with FAR, that is, choosing the path from the predetermined path lists for each connection, achieves similar performance as the original LLR at a much lesser complexity. In [5], we proposed an LLR algorithm which takes into consideration the placement of wavelength converters. We briefly review this algorithm, the Least Load Routing with Min-Sum-Min (LLR-MSM), here.

A path p is defined as an ordered set of links $l = (u, v)$ starting at the source node and ending at the destination node. In an optical network with sparse wavelength conversion, we define a segment s on a path p , as an ordered subset of p starting at the source or at wavelength convertible node and ending at the destination or at the next wavelength convertible node along the path. The wavelength continuity constraint can be relaxed at the frontier between segments. An example of paths segmented by a wavelength converter WC is shown in Fig. 1.

LLR-MSM is a least load routing algorithm over a fixed number of alternate paths on route R , that is, k edge-disjoint shortest paths are predetermined for each route R (source-destination pair). These paths are edge disjoint to ensure the independence of the blocking along these paths. Define M_l as the number of fibers on link l , and A_{lj} as the number of fibers of link l on which wavelength j is already in use, then decompose the path from a

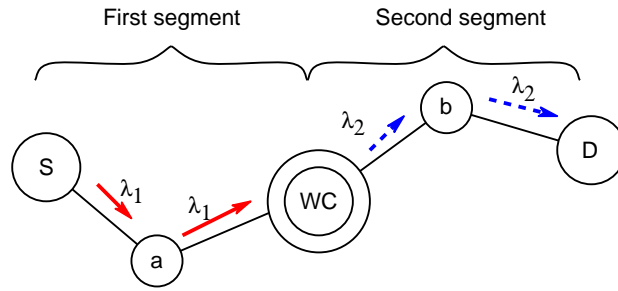


Fig. 1. Path segments separated by a wavelength converter

source to the destination into segments, then the cost of a segment is the sum of the available channels of all the wavelengths through the segment and the cost of a path is defined as the bottleneck cost of all segments on the path, that is the minimum cost of all the segments in the path. This can be written as

$$C(p(R)) = \min_{s \in p(R)} \sum_j \min_{l \in s} M_l - A_{lj}. \quad (1)$$

The LLR-MSM routing algorithm's chooses the available path with the minimal number of segments. When there exists multiple choice, the algorithm chooses the one with the maximal path cost (1).

Experimental results in [9] show that all smart wavelength assignment algorithms including the first-fit achieve similar blocking probability. First-fit sorts all the wavelengths into a list by a predetermined order, and chooses the first free wavelength along the path at the moment the request connection arrives. As we do not need to keep track of usage information of each wavelength and do not need global information to compute the wavelength utilization over the network, the scheme is quite simple. Therefore it maintains a good tradeoff between simplicity and efficiency and we adopt FF in our experiments.

III. RELATED WORK IN WAVELENGTH CONVERTER PLACEMENT ALGORITHMS

Many wavelength converter placement algorithms have been proposed in the past few years. The algorithms depend on the analytical model of a WDM optical network or on the mode of derivation of the blocking probability (e.g., by simulation), or some graph theoretic property of the network such as the degree of each node. The analytical model of WDM optical network depends on the networks's RWA algorithm. As far as we know, there is no perfect model, especial for network with dynamic RWA algorithms. Whereas obtaining blocking probability from simulation results is too time consuming. The algorithms based on network property obtain the placement by graph theoretic arguments.

The Total Outgoing Traffic(TOT) algorithm, proposed in [3], is a wavelength converter placement algorithm by network property. In TOT, at node v , the incoming traffic is defined as the sum of the loads on all routes which

start from v . The transit traffic is the sum of the loads on all routes which have v as an intermediate node. The total outgoing traffic at node v is defined as the sum of the incoming traffic and the transit traffic. After the TOT value at each node is calculated, the converters are placed sequentially at the nodes with the highest TOT values. It is shown that for a network with fixed path routing algorithm TOT works well. Genetic algorithms are also invoked to speed up the search for the optimal placement by simulation results in [5]. The algorithm works well with any type of RWA algorithm, because the genetic algorithm uses the blocking probability given by the simulation results as an input parameter. It is shown that genetic algorithm works better than TOT in the cases when the running time of the algorithm is long enough. The drawback of the genetic algorithm is that it is too time consuming. Branch and Bound method [6] and graph decomposition [7] are applied to accelerate the procedure of searching for the optimal placement for the network with fixed path routing. The blocking probability is obtained from an analytical model and the two algorithms are shown to be efficient compared with the exhaustive search. Although the algorithms are claimed to find the optimal placement quickly, they suffer the following drawbacks: i) the analytical model is just an approximation which sometimes deviate from the real blocking probability; ii) the algorithms are not suitable for networks with dynamic routing. In [4] a heuristic algorithm is proposed to place converters based on the concepts of the K-Minimum Dominating Set of the network's graph. The nodes in the K-MDS have the property that any node in the network is either in the K-MDS or it is at most K hops away from a node in the K-MDS. The converters are placed on the nodes of the K-MDS. With the network topology as the only consideration, the K-MDS algorithm ignores the impact of the traffic pattern, the routing and wavelength assignment algorithms upon the converter placement.

IV. MAX ROUTES COVERAGE PLACEMENT ALGORITHM

Given a node k , we say that route p from node i to node j is *covered* by k iff k is an intermediate node on p other than i or j . A single hop route is by definition covered by the empty set. Given a network, we denote \mathbf{R} to be the set of routes for all source and destination given the routing algorithm, and R^k to be the subset of routes in \mathbf{R} which have at least k hops. The subset of routes in \mathbf{R} which is covered by node s is denoted R_s . The subset of routes in \mathbf{R} which is covered by set S is denoted R_S . The set of single hop routes is denoted R_ϕ . The routes coverage ratio $RCR(S)$ of a subset S of nodes in the network, is defined as:

$$RCR(S) = \frac{\left| \bigcup_{s \in S} R_s \right|}{|R - R_\phi|}. \quad (2)$$

$RCR(R)$ equals 1 and $RCR(\phi)$ equals 0 according to the above definition. Hence the routes coverage ratio is normalized.

A. Max routes coverage heuristic algorithm

Intuitively, the more routes in the network covered by nodes with wavelength converters, the more conversion capacity is utilized, which results in a lower blocking probability for the same load. Thus, it is appropriate to place wavelength converters on those nodes that cover most routes such as to maximize routes coverage ratio. The problem of maximizing the routes coverage ratio is formally defined as follows:

Problem A: Given \mathbf{R} , the set of routes for all source-destination pairs as determined by the routing algorithm, and a number k of converters. Find the k converters placement that achieves the maximal routes coverage ratio.

Let $M = |\mathbf{R}|$ be the number of routes and N be the number of nodes in the network. Problem A can be reformulated into another Problem B:

Problem B: Given N finite sets $T_i, i = 1, 2, \dots, N$, of integers, $T_i \subseteq \{1, \dots, M\}$, find k different sets $T_{i_j}, j = 1, \dots, k$, out of these N sets such that $\left| \bigcup_{j=1}^k T_{i_j} \right| / \left| \bigcup_{i=1}^N T_i \right|$ is maximal.

Let us consider the following well known problem: given N finite sets $T_i, T_i \subseteq \{1, \dots, M\}, i = 1, \dots, N$, given a number k , is there k different sets $T_{i_j}, j = 1, 2, \dots, k$, out of these N sets whose union equals the full set? This is known to be a variant of *set cover*, which is known to be NP complete [10]. It is easy to prove that problem B is in NP and problem B can be reduced to the above problem in polynomial time. Therefore both problem A and problem B are NP complete.

We thus propose a heuristic algorithm named Max Routes Coverage (MRC), that achieves nearly maximal routes coverage ratio. The MRC algorithm is shown in Algorithm 1. The algorithm is greedy as it places converters one by one, onto the node which covers the maximal number of routes each time, and to cover as many routes as possible, it deletes the routes that are already covered by a converter.

Algorithm 1 Max Routes Coverage algorithm

Notations.

\mathbf{R} set of routes

R_s set of routes covered by node s , $R_s \subseteq \mathbf{R}$

S set of nodes with converters

k number of converters

Nodes set of nodes in the network

BEGIN

$S \leftarrow \phi$

while $|S| < k$ **do**

 Find $j \in \mathbf{Nodes}$ such that $|R_j| \geq |R_s|, \forall s \in \mathbf{Nodes}$

$S \leftarrow S \cup \{j\}$

$R_s \leftarrow R_s \setminus R_j$ where $s \in \mathbf{Nodes}$

end while

END

In some specific scenarios, the RCR becomes one after placing only a subset of the converters. In such cases, the remaining converters are placed according to any other wavelength converter placement algorithm.

B. α -approximate pseudo optimal number search with routes coverage ratio

As mentioned before, when wavelength converters are appropriately placed and the number of wavelength converters are larger than a certain value, sparse placement of wavelength converters achieves similar performance to the full placement of wavelength converters. Therefore, we only need to place a sufficient number of wavelength converters into the network to approach the performance of a fully converted network. How to define this sufficient number? Denote by $Pb(i, RA, WA)$ the blocking probability obtained with the placement of i converters by algorithm WA under routing algorithm RA and by N the total number of nodes in the network. We say that n^* is the α -approximate pseudo optimal number of converters if

$$n^* = \arg \min_i (Pb(i, RA, WA) \leq Pb(N, RA, WA) * \alpha) \quad (3)$$

Since the blocking probability is not available in closed form, determining n^* incurs the penalty of long simulation times. As the RCR is a metric that partially determines the blocking performance of the wavelength converters placement we suggest to use the variation of RCR to indicate the variation of the blocking probability.

We denote the routes coverage ratio with the placement of i converters by algorithm WA with routing algorithm RA as $RCR(i, RA, WA)$. Through simulations we observe that the shape of $Pb(i, RA, WA)$ as a function of $i \in [0, N]$ is similar to the function $a + c * x^{-b}$ where x is a function of i . Routes coverage ratio function $RCR(i, RA, WA)$ on parameter i has a good property that it is normalized, and intuitively blocking probability decreases when routes coverage ratio increases. We therefore suggest to approximate the blocking probability by $\overline{Pb(i, RA, WA)} = a + c * (1 - RCR(i, RA, WA))^{-b}$. Furthermore, since for some particular values of i the blocking probability is readily available we let $\overline{Pb(i, RA, WA)} = Pb(i, RA, WA)$ for $i = 0, 1$ or N . This allows us to express parameters a, b and c as

$$\overline{Pb(i, RA, WA)} = Pb(N, RA, WA) + \quad (4)$$

$$(Pb(0, RA, WA) - Pb(N, RA, WA)) * (1 - RCR(i, RA, WA))^{-b}$$

$$b = \frac{\ln\left(\frac{Pb(1, RA, WA) - Pb(N, RA, WA)}{Pb(0, RA, WA) - Pb(N, RA, WA)}\right)}{\ln(1 - RCR(1, RA, WA))} \quad (5)$$

V. NUMERICAL RESULTS

In our experiments, we use the 14 nodes NSFnet and the 28 nodes US Long Haul network topologies. The network load is uniformly distributed and is set to 400 Erlangs. The number of wavelengths on each link is set to 40. Under the shortest path routing, the performance of maximal routes coverage algorithm is compared to TOT and K-MDS. Under the FAR and LLR-MSM algorithms, the performance of maximal routes coverage algorithm is compared with the K-MDS. The wavelength assignment algorithm is set to First-Fit in all cases.

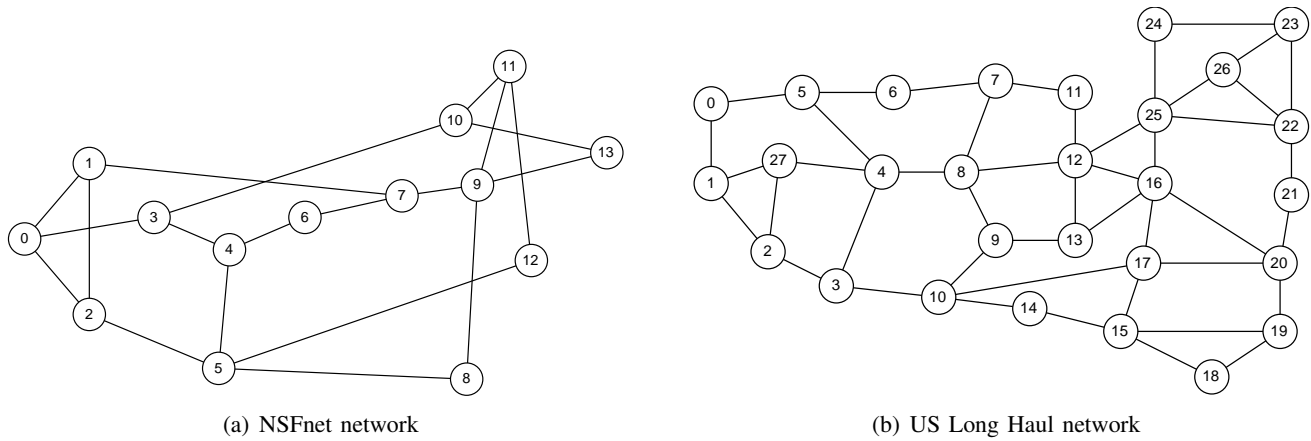


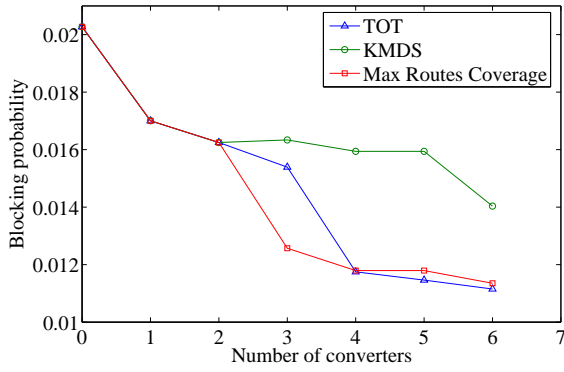
Fig. 2. Network topology

Fig. 3(a), 3(b), and 3(c) show the blocking probability under SP routing, FAR and LLR-MSM respectively for different placement algorithms. In all three examples MRC outperforms other applicable alternatives, however the major conclusion we draw from these figures is that under some topologies (such as the NSFnet), wavelength converters do not improve the performance significantly and that adopting an intelligent routing algorithm improves the performance dramatically.

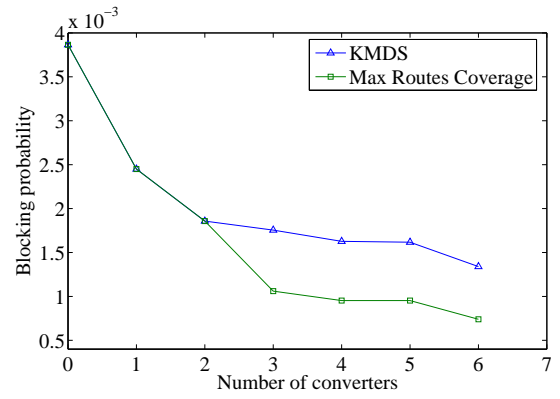
To investigate further this, we consider the same placement algorithms and routing algorithms using the US Long haul topology. Fig. 3(d), 3(e), and 3(f) show the blocking probability under SP routing, FAR and LLR-MSM respectively for different placement algorithms in this topology. Under the SP routing algorithm, the wavelength converters do not seem to improve the performance much, however, in conjunction with more intelligent routing algorithms, not only the routing algorithm improves the performance but also the wavelength converters; we can observe this clearly in the figures: LLR-MSM with no conversion is one order of magnitude better than SP. While LLR-MSM with 8 converters is one order of magnitude better than without converters.

In the following experiments, the placement of wavelength converters is obtained by MRC. We compute the semi-2-approximated pseudo optimal number by equation 3 where we substitute $\overline{Pb(i, RA, WA)}$ for $Pb(i, RA, WA)$.

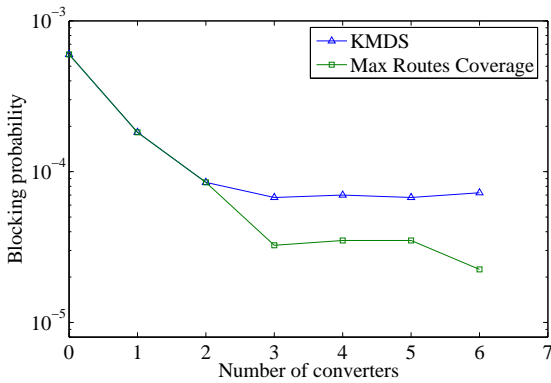
In the case when the ratio between the blocking probability of no placement of wavelength converters and full



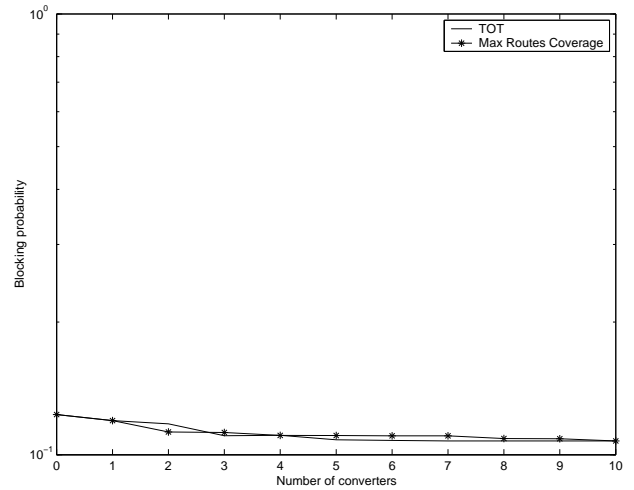
(a) Blocking probability for SP routing in NSFnet



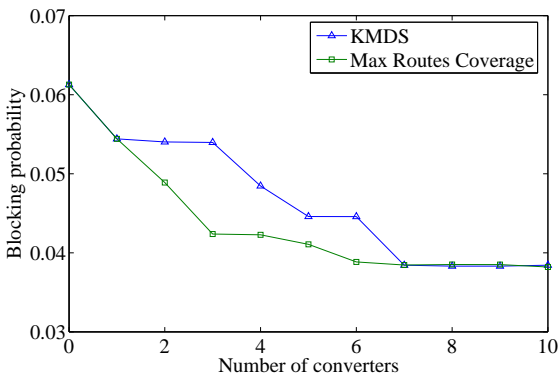
(b) Blocking probability for FAR in NSFnet



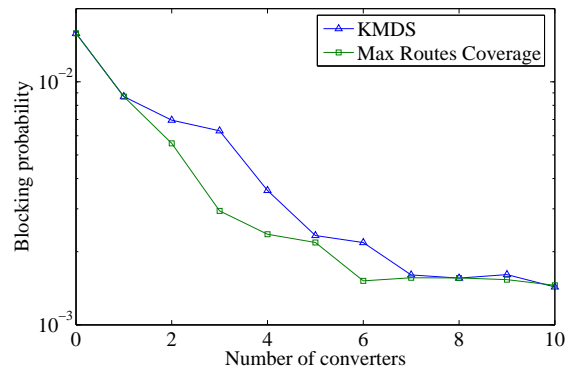
(c) Blocking probability for LLR-MSM in NSFnet



(d) Blocking probability for SP routing in US Long Haul



(e) Blocking probability for FAR in US Long Haul



(f) Blocking probability for LLR-MSM in US Long Haul

Fig. 3. Simulation results

placement of wavelength converters is less than 2, the 2-approximated pseudo optimal number is zero by definition. The ratios between the blocking probability of null placement of wavelength converters and full placement of wavelength converters for the NSFnet with SP routing, US Long Haul with shortest path routing and US Long

Haul with FAR are all less than zero. Thus we know their 2-approximated pseudo optimal number is 0 without computing the blocking probability.

Table I shows the blocking probability and approximated blocking probability for the NSFnet with FAR for different number of wavelength converters. Both the 2-approximated pseudo optimal number (obtained with the exact blocking probability) and the semi-2-approximated pseudo optimal number (obtained with approximation (4)) are 3.

TABLE I
BLOCKING PROBABILITY VERSUS APPROXIMATED BLOCKING PROBABILITY WITH PLACEMENT BY MRC IN NSFNET WITH FAR

No. of wavelength converters	Blocking probability	Approximated blocking probability
0	0.00386478	0.00386478
1	0.00244954	0.00244954
2	0.00185715	0.00144774
3 ^{*,+}	0.00105982	0.00091099
4	0.00095229	0.00078196
5	0.00095229	0.00070705
6	0.00073986	0.00066864
14	0.00058487	0.00058487

* represents the 2-approximated pseudo optimal number.

+ represents the semi-2-approximated pseudo optimal number.

Table II shows the blocking probability and approximated blocking probability for the NSFnet with LLR-MSM routing for different number of wavelength converters. Both the 2-approximated pseudo optimal number and semi-2-approximated pseudo optimal number are 3.

TABLE II
BLOCKING PROBABILITY VERSUS APPROXIMATED BLOCKING PROBABILITY WITH PLACEMENT BY MRC IN NSFNET WITH LLR-MSM

No. of wavelength converters	Blocking probability	Approximated blocking probability
0	0.00059981	0.00059981
1	0.00018239	0.00018239
2	0.00008495	0.00005187
3 ^{*,+}	0.00003498	0.00002788
4	0.00003498	0.00002590
5	0.00003498	0.00002529
6	0.00002248	0.00002512
14	0.00002499	0.00002499

* represents the 2-approximated pseudo optimal number.

+ represents the semi-2-approximated pseudo optimal number.

Table III shows the blocking probability and approximated blocking probability for the US Long Haul with LLR-MSM routing for different numbers of wavelength converters. The 2-approximated pseudo optimal number is 4 whereas the semi-2-approximated pseudo optimal number is 5.

TABLE III
BLOCKING PROBABILITY VERSUS APPROXIMATED BLOCKING PROBABILITY WITH PLACEMENT BY MRC IN US LONG HAUL WITH LLR-MSM

No. of wavelength converters	Blocking probability	Approximated blocking probability
0	0.01583146	0.01583146
1	0.00869147	0.00869147
2	0.00557937	0.00465428
3	0.00346938	0.00319475
4*	0.00245686	0.00263561
5 ⁺	0.00173206	0.00222080
6	0.00163467	0.00194210
7	0.00151225	0.00171262
8	0.00151717	0.00159617
9	0.00140971	0.00150043
10	0.00142730	0.00143714
28	0.00123480	0.00123480

* represents the 2-approximated pseudo optimal number.

+ represents the semi-2-approximated pseudo optimal number.

These experiments show that the semi- α -approximated pseudo optimal number is very close to the α -approximated pseudo optimal number, and therefore the approximation is accurate enough and takes much less time than the exhaustive search.

Overall, it is well known that wavelength conversion reduces the blocking probability in WDM optical networks. However, the gain of wavelength conversion depends on the network topology, traffic load, placement of wavelength converters, number of wavelength converters in the network and routing algorithms. Several observations are made from Fig. 3(a), 3(b), 3(c), 3(d),3(e),3(f). First of all, the gain of wavelength conversion becomes insignificant when the number of wavelength converters in the network is more than the α -approximated pseudo optimal number n^* (when α is small). Secondly, we can observe that the gain of wavelength conversion becomes significant if the routing algorithm is smart.

VI. CONCLUSION

In this paper, we proposed a new wavelength converter placement algorithm, Max Routes Coverage. Experimental results show that Max Routes Coverage algorithm performs better in terms of blocking probability than other alternative wavelength converter placement algorithms such as the K-MDS under various routing algorithm. We also proposes to use the routes coverage ratio as a metric to determine the α -approximate pseudo optimal number of wavelength converters for a given routing algorithm and wavelength converter placement algorithm to achieve a given blocking. Such a number is important to achieve a tradeoff between the blocking probability and the cost of wavelength converters. It is shown that our algorithm for searching out this number is accurate and saves time. As

a conclusion, we argue that adopting a smart routing algorithm is more important than adopting a smart wavelength converter placement algorithm.

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