Joint Rate-and-Power Allocation for Multi-channel Spectrum Sharing Networks

Yuan WU
Department of Electronic and Computer Engineering
Hong Kong University of Science and Technology
ecewuy@ust.hk

Danny H. K. TSANG
Department of Electronic and Computer Engineering
Hong Kong University of Science and Technology
eetsang@ece.ust.hk

ABSTRACT

In this abstract, we propose a study on joint rate-and-power allocation problem for multi-channel spectrum sharing networks (SSNs). We formulate this cross-layer optimization problem as a non-cooperative potential game G_{JRPA} in which each user has a coupled two-tuple strategy, i.e., simultaneous rate and multi-channel power allocations. A multi-objective cost function is designed to represent user's awareness of both QoS provisioning and power saving. Using the game-theoretic formulation, we investigate the properties of Nash equilibrium (N.E.) for our G_{JRPA} model, including its existence, and properties of QoS provisioning as well as power saving. Furthermore, a layered structure is derived by applying Lagrangian dual decomposition to G_{JRPA} and a distributed algorithm is proposed to find the N.E. via this structure.

1. INTRODUCTION

The fast development of wireless communication systems with heterogeneous Quality of Service (QoS) requirements and various device capabilities entails a great demand for dynamic access mechanism to the valuable spectrum resource in a flexible, efficient and economical manner. Among all kinds of dynamic spectrum access (DSA) mechanisms[1], spectrum sharing which allows different users freely access to some common spectrum under their mutual interference limited, has attracted a lot of research interests during the recent years. A typical example for this sharing mechanism is the horizontal coexistence of Wi-Fi and Bluetooth systems in ISM band without harmful mutual interference.

Although gaining advantages of easy and flexible implementation, naive design of the spectrum sharing mechanism will result in inefficient spectrum utilization for the whole network, e.g., users are designed to blindly maximize their own transmission rates. Motivated by this consideration, we propose to study the joint rate-and-power allocation problem for multi-channel spectrum sharing networks (SSNs) with both QoS provisioning and power saving. We formulate this cross-layer optimization problem as a non-cooperative game G_{JRPA} where each user simultaneously arrange its rate and power allocations in order to achieve its target QoS as exactly as possible and minimize the power consumption at the same time. Specifically, we describe the SSNs scenario and game-theoretic formulation G_{JRPA} in Section 2. We discuss the properties of N.E. for G_{JRPA} in Section 3. A layered structure is derived by applying dual decomposition to G_{JRPA} in Section 4, and a distributed algorithm is proposed to find the N.E. via this structure. We conclude this abstract in Section 5 and point out our future work.

[1]Three categories of DSA mechanisms have been classified at present including: *Exclusive usage mechanism*, *Hierarchical access mechanism*, and *Spectrum sharing mechanism*.

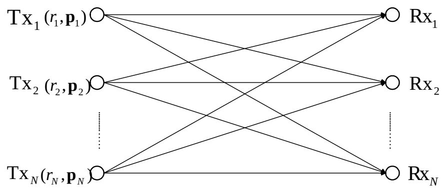

Figure 1: Network Model. N pairs share a common set of K channels.

2. NETWORK MODEL AND GAME THEORETIC FORMULATION

We consider a multi-channel SSN as shown in Figure 1, where a set of transceiver pairs[2] $\mathcal{N} = \{1, 2, ..., N\}$ share a common part of spectrum B equally divided into a set of $\mathcal{K} = \{1, 2, ..., K\}$ channels. Each pair $i \in \mathcal{N}$ consists of a transmitter TX_i and an intended receiver RX_i. We assume that each pair i has a target QoS represented by target data rate R_i^{tar} at link layer, and pair i aims to achieve its R_i^{tar} as exactly as possible and minimize the power allocation at the same time. Due to the co-channel interference among pairs, we formulate this multiuser joint rate-and-power allocation problem as a non-cooperative game with the formal description given as follows:

$$G_{JRPA} = \{\mathcal{N}, \{\chi_i\}_{i\in\mathcal{N}}, \{\Psi_i\}_{i\in\mathcal{N}}\} \quad (1)$$

where $\mathcal{N}$ denotes the set of coexisting pairs. χ_i and Ψ_i represent pair i's strategy space and cost function respectively.

We design the multi-objective cost function Ψ_i to indicate each pair i's awareness of both QoS provisioning and power saving as follows:

$$\Psi_i(r_i, \mathbf{p}_i) = \alpha_i\left(\frac{r_i - R_i^{tar}}{R_i^{tar}}\right)^2 + \beta_i\left(\frac{\sum_{k\in\mathcal{K}} p_i^k}{P_i^{max}}\right) \quad (2)$$

where the two-tuple expression $(r_i, \mathbf{p_i})$ includes pair i's rate allocation r_i and power allocation vector $\mathbf{p}_i = (p_i^1, p_i^2, ..., p_i^K)$

[2]We use 'pair' and 'user' interchangeable in the following.

over the K channels. P_i^{max} denotes user i's power capacity. α_i and β_i represent pair i's weighting factors on QoS provisioning and power saving respectively.

In G_{JRPA}, due to the co-channel interference, each pair i's strategy space χ_i is coupled with all the other pairs. Specifically, the strategy space of pair i can be expressed as:

$$\chi_i(\mathbf{p}_{-i}) = \{(r_i, \mathbf{p}_i) \in \Omega_i^R \times \Omega_i^P | r_i \le \sum_{k\in\mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)\} \quad (3)$$

$\Omega_i^R = \{r_i \in \mathbb{R} | R_i^{min} \le r_i \le R_i^{max}\}$ is pair i's rate allocation range, where R_i^{min} and R_i^{max} denote the practical lower and upper bound of pair i's data rate. $\Omega_i^P = \{\mathbf{p}_i \in \mathbb{R}^K | \sum_{k\in\mathcal{K}} p_i^k \le P_i^{max}; pmin_i^k \le p_i^k \le pmax_i^k, \forall k \in \mathcal{K}\}$ is pair i's power allocation range, where $pmin_i^k$ and $pmax_i^k$ denote the practical lower and upper bound of pair i's power allocation on channel k. The constraint

$$r_i \le \sum_{k\in\mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k) \quad (4)$$

guarantees pair i's rate allocation r_i can't exceed the maximum achievable data rate according to its power allocation $\mathbf{p}_i$, given all the other pairs' power allocations $\mathbf{p}_{-i} = (\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_{i-1}, \mathbf{p}_{i+1}, ..., \mathbf{p}_N)$ fixed. Meanwhile, on each channel $k \in \mathcal{K}$, Φ_i^k has the channel capacity formula as:

$$\Phi_i^k(\mathbf{p}_i, \mathbf{p}_{-i}) = \frac{B}{K}\log(1 + \frac{p_i^k g_{ii}^k}{\eta_i^k}) \quad (5)$$

where $\eta_i^k = \sum_{j\ne i, j\in\mathcal{N}} p_j^k g_{ji}^k + n_i^k$ is the power of interference plus noise perceived by RX_i over channel k. g_{ji}^k is the channel gain from TX_j to RX_i over channel k[3]. n_i^k is the power of background noise of pair i over channel k. Based on (3), the whole strategy profile space χ for G_{JRPA} is:

$$\chi = \{(r_1, \mathbf{p}_1, r_2, \mathbf{p}_2, ..., r_N, \mathbf{p}_N) | (r_i, \mathbf{p}_i) \in \Omega_i^R \times \Omega_i^P, \forall i \in \mathcal{N}; \\ r_i \le \sum_{k\in\mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k), \forall i \in \mathcal{N}\}$$

Given $\mathbf{p}_{-i}$, each pair $i \in \mathcal{N}$ faces a joint rate and multi-channel power allocation problem as follows:

$$\min_{(r_i, \mathbf{p}_i)\in\chi(\mathbf{p}_{-i})} \Psi_i(r_i, \mathbf{p}_i) \quad (6)$$

According to the definition of Nash Equilibrium (N.E.), the equilibrium rate and power allocation profile $(\mathbf{r}^*, \mathbf{p}^*)$ for G_{JRPA} should satisfy the following equilibrium condition:

$$(r_i^*, \mathbf{p}_i^*) = \arg\min_{(r_i, \mathbf{p}_i)\in\chi(\mathbf{p}_{-i}^*)} \Psi(r_i, \mathbf{p}_i), \forall i \in \mathcal{N} \quad (7)$$

which guarantees that no single pair has the incentive to deviate from N.E. unilaterally.

3. PROPERTIES OF N.E.

We characterize our G_{JRPA} as an exact potential game with potential function $\mathscr{F}(r_1, \mathbf{p}_1, ..., r_N, \mathbf{p}_N) = \sum_{i\in\mathcal{N}} \Psi_i(r_i, \mathbf{p}_i)$. Therefore, exploiting the desirable property of potential game, we describe the existence of N.E. for G_{JRPA} as follows:

Lemma 1[1]: There always exists a pure strategy N.E. for G_{JRPA} on the strategy profile space χ if $\alpha_i > 0, \beta_i \ge 0, \forall i \in \mathcal{N}$.

[3]We consider a relative static network in this abstract, i.e., the set of channel gains $g_{ji}^k, \forall i,j \in \mathcal{N}, \forall k \in \mathcal{K}$ keeps unchanged during the time interval of interest.

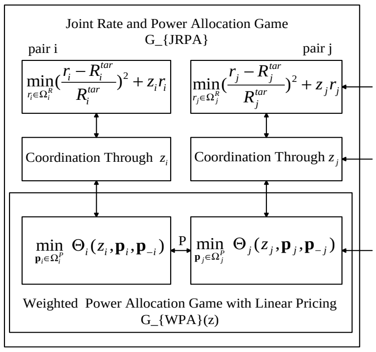

Figure 2: Layered Structure of G_{JRPA}

Furthermore, still based on the property of potential game, the uniqueness of N.E. for G_{JRPA} follows if the strategy profile space χ is a convex set. An another important property of G_{JRAP} is that after arriving at the N.E., each pair i can get a balance between QoS provisioning and power saving according to its value of $\frac{\alpha_i}{\beta_i}$. Specifically, no pair will get redundant data rate beyond its target and no power is wasted to get unwanted data rate. We describe these two points in the following lemma.

Lemma 2[1]: The following two properties always hold at the N.E. of G_{JRPA}: (a) $r_i^* = \sum_{k\in\mathcal{K}} \Phi_i^k(p_i^{k*}, \mathbf{p}_{-i}^{k*}), \forall i \in \mathcal{N}$; (b) $R_i^{min} \le r_i^* \le R_i^{tar}, \forall i \in \mathcal{N}$.

4. LAYERED STRUCTURE AND DISTRIBUTED ALGORITHM

The two-tuple strategy of coupled rate and power allocations (3) differs our G_{JRPA} from conventional game models, and is generally difficult to deal with when designing a distributed algorithm to approach the N.E.. Therefore, we apply Lagrangian dual decomposition to relax the constraint (4) and use the corresponding dual price z_i as a coordinator between rate allocation and power allocation for each pair i. Figure 2 shows the layered structure we get after decomposition. Specifically, each pair i's optimization problem (6) is vertically separated into two subproblems as follows:
(a) Rate allocation with QoS provisioning in link layer

$$r_i^*(z_i) = \arg\min_{r_i\in\Omega_i^R} \alpha_i(\frac{r_i - R_i^{tar}}{R_i^{tar}})^2 + z_i r_i \quad (8)$$

(b) Power allocation with linear pricing in PHY layer

$$\mathbf{p}_i^*(z_i, \mathbf{p}_{-i}) = \arg\min_{\mathbf{p}_i\in\Omega_i^P} \beta_i(\frac{\sum_{k\in\mathcal{K}} p_i^k}{P_i^{max}}) - z_i \sum_{k\in\mathcal{K}} \Phi_i^k(p_i^k, p_{-i}^k) \quad (9)$$

Subproblems (a) and (b) are connected by the dual problem:

$$\max_{z_i\ge 0} D_i(z_i) = \max_{z_i\ge 0} L_i(r_i^*(z_i), \mathbf{p}_i^*(z_i, \mathbf{p}_{-i}), z_i) \quad (10)$$

For any given $\mathbf{p}_{-i}$, strong duality exists between each pair i's primal problem (6) and dual problem (10). Therefore, the optimum solution for (6) can always be found by solving (10) without any performance loss.

We note in (9) that each pair i's optimal power allocation $\mathbf{p}_i^*(z_i, \mathbf{p}_{-i})$ also depends on other pairs. Given a dual vector $\mathbf{z} = (z_1, z_2, ..., z_N)$, the PHY layer multi-channel power allocation problem can be considered as a non-cooperative

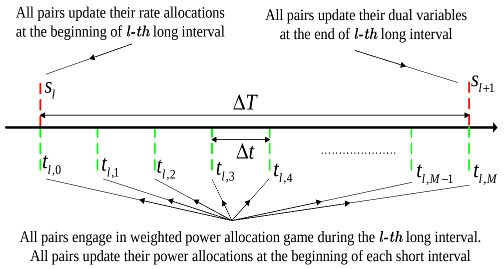

Figure 3: Rate, Power and Dual Updating in ΔT

game with linear pricing mechanism as follows:

$$G_{WPA}(\mathbf{z}) = \{\mathcal{N}, \{\Omega_i^P\}_{i \in \mathcal{N}}, \{\Theta_i(z_i)\}_{i \in \mathcal{N}}\} \quad (11)$$

In $G_{WPA}(\mathbf{z})$, each pair i has an independent strategy space only determined by its power allocation range Ω_i^P, and its cost function can be written as:

$$\Theta_i(z_i, \mathbf{p}_i, \mathbf{p}_{-i}) = \beta_i(\frac{\sum_{k \in \mathcal{K}} p_i^k}{P_i^{max}}) - z_i \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k) \quad (12)$$

where the achievable data rate $\sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k, \mathbf{p}_{-i}^k)$ is weighted by the dual price z_i to indicate its QoS awareness and a linear pricing mechanism $\beta_i \frac{\sum_{k \in \mathcal{K}} p_i^k}{P_i^{max}}$ is included to address its consideration on power saving. We describe the properties of N.E. for $G_{WPA}(\mathbf{z})$ as follows:

Lemma 3[1]: (a) There always exists a pure strategy N.E. for $G_{WPA}(\mathbf{z})$ on the strategy profile space $\Omega^P = \Omega_1^P \times \Omega_2^P \times ... \times \Omega_N^P$ for any given dual vector $\mathbf{z} = (z_1, z_2, ..., z_N) > \mathbf{0}$ and pricing vector $\beta = (\beta_1, \beta_2, ..., \beta_N) \geq \mathbf{0}$. (b) Furthermore, if the channel gains satisfy that $\rho(\mathcal{G}^k) < 1, \forall k \in \mathcal{K}$, where $\rho(.)$ denotes the spectrum norm and for each $k \in \mathcal{K}$, the N by N matrix $\mathcal{G}^k$ is defined as:

$$[\mathcal{G}^k]_{ij} = \begin{cases} \frac{g_{ji}^k}{g_{ii}^k}, & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}$$

then there exists a unique pure strategy N.E. on Ω^P for $G_{WPA}(\mathbf{z})$.

We propose a distributed algorithm to find the N.E. for G_{JRPA} based on the layered structure in Figure 2. To facilitate the proposed algorithm, we assume synchronization among all pairs and design two different time scales for rate and power updating. Specifically, let ΔT denote the long updating interval for both link layer rate allocation and dual updating. Meanwhile, let $\Delta t = \Delta T / M$ denote the short updating interval for PHY layer power allocation, where M is a sufficient large number such that for any given dual vector $\mathbf{z}$, the equilibrium power allocation for $G_{WPA}(\mathbf{z})$ can be reached before the end of ΔT. Figure 3 shows the procedures of rate, power allocation and dual updating in a single ΔT. Our algorithm is described in algorithm (**DRPA**).

5. CONCLUSIONS AND FUTURE WORK

We propose a study on the joint rate-and-power allocation problem for SSNs with both QoS provisioning and power saving. We formulate this cross-layer optimization as a non-cooperative game where each user simultaneously arranges its rate and power allocations to achieve its target QoS as exactly as possible and minimize the power consumption at the same time. Our future work to continue this study include:

- study on the convergence performance of our proposed algorithm (**DRPA**), especially its robustness to the asynchronous impact and limited value of M for PHY layer power iteration.
- investigation in the individual user's intrinsic performance tradeoff between QoS provisioning and power saving under the condition of multiuser co-channel interference.
- analysis of the efficiency of our G_{JRPA} model and the price of anarchy induced by users' selfish behavior.
- study on the advantages of both QoS provisioning and power saving awareness in our G_{JRPA} model, compared to the conventional greedy transmission rate maximization model.

Distributed Rate and Power Allocation (DRPA)
Initialization Step:
Each pair $i \in \mathcal{N}$ initializes $r_i(s_0) \in \Omega_i^R$, $z_i(s_0) > 0$ and $\mathbf{p}_i(t_{0,0}) \in \Omega_i^P$, where we set $t_{0,0} = s_0$
Iteration Process:
(a) **Best Response for Rate Allocation:**
Given $z_i(s_l)$, each pair $i \in \mathcal{N}$ updates r_i at the beginning of l^{th} long updating index $s_l = s_0 + l * \Delta T, l = 0, 1, 2, ...,$ as follows:

$$r_i(s_l) = [R_i^{tar} - \frac{(R_i^{tar})^2}{2\alpha_i} z_i(s_l)]_{R_i^{min}}^{R_i^{max}} \quad (13)$$

(b) **Jacobian Iteration for Power Allocation:**
Given dual vector $\mathbf{z}(s_l)$, all pairs participate in $G_{WPA}(\mathbf{z}(s_l))$ during the l^{th} long updating interval. Specifically, during the l^{th} ΔT each pair $i \in \mathcal{N}$ updates $\mathbf{p}_i$ at the beginning of m^{th} short updating index $t_{l,m} = s_l + m * \Delta t, m = 0, 1, ..., M$ as follows: (Initialize $p_i^k(t_{l,0}) = p_i^k(t_{l-1,M}), \forall k \in \mathcal{K}$)

$$p_i^k(t_{l,m}) = [\frac{z_i(s_l)}{(\frac{\beta_i}{P_i^{max}}) + \lambda_i(t_{l,m})} - \frac{\eta_i^k(t_{l,m-1})}{g_{ii}^k}]_{pmin_i^k}^{pmax_i^k}, \forall k \in \mathcal{K} \quad (14)$$

and $\lambda_i(t_{l,m})$ is the optimal dual variable for pair i's power capacity constraint at the index $t_{l,m}$, which satisfies that:

$$\lambda_i(t_{l,m}) \geq 0, \ \ \lambda_i(t_{l,m})(P_i^{max} - \sum_{k \in \mathcal{K}} p_i^k(t_{l,m})) = 0, \forall i \in \mathcal{N}$$

(c) **Subgradient Updating for Dual Vector:**
Each pair $i \in \mathcal{N}$ updates z_i at the end of l^{th} long updating index as follows:

$$z_i(s_{l+1}) = [z_i(s_l) + \zeta_i(r_i(s_l) - \sum_{k \in \mathcal{K}} \Phi_i^k(p_i^k(t_{l,M}), \mathbf{p}_{-i}^k(t_{l,M})))]^+ \quad (15)$$

(d) **Repeat Steps (a), (b) and (c) until convergence**
End of Algorithm

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6. REFERENCES

[1] Y. Wu, D. H. K. TSANG, 'Efficient Spectrum Sharing with Balanced QoS Provisioning and Power Saving,' *Technical Report*, Mar. 2008.