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Decorated membrane resonators as underground seismic wave barriers against high magnitude earthquakes

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Lock-in Amplifiers
up to 600 MHz

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ABSTRACT

We report a framework of underground barriers against seismic waves from high-magnitude earthquakes, which consist of the deep subwavelength decorated membrane resonators (DMR's) modeled as a type of artificial soil, the meta-soil, which is buried in the host soil. The meta-soil has the same elastic properties as the host soil but with a dynamic effective mass density that mimics the functionality of the DMR's mixed with the host soil. Metallic membrane DMR's with multiple working frequencies down to 7 Hz are experimentally demonstrated. Transmission attenuation exceeding 20 dB and absorption above 98% are numerically demonstrated for the 1 Hz Rayleigh waves using about 12 rows of underground meta-soil columns. The findings obtained from the meta-soil are validated by using discrete generic tuned mass dampers buried in the soil. A scaling law similar to the mass density law for acoustic waves that relates the meta-soil mass density to the wave frequency for a given transmission attenuation has been analytically derived and numerically verified. The main mechanism for the wave attenuation is the resonant enhancement of scattering and dissipation by the resonators in the meta-soil. The findings for attenuating the 1 Hz seismic waves serve as a benchmark and design platform for the broadband seismic wave barriers due to the following findings. (1) It is a representative frequency for high magnitude earthquakes. The practical lower frequency limit is about 0.5 Hz, so the dimension and weight of the dampers for 0.5 Hz will be comparable to that for 1 Hz, while the dampers for higher frequencies will be smaller in size and lighter in weight. (2) The length of the wells will be comparable to that of the 1 Hz ones and proportionally shorter for higher frequency waves. Our findings provide a viable solution that could fill the gap between the maximum earthquake resistance capability of most modern buildings and infrastructures (~seventh magnitude) and the need to resist the highest (~eighth) magnitude earthquakes recorded in human history, and remedy one of the main shortcomings in the literature on the local resonators for underground seismic wave barriers, which is the gross over-estimation of the seismic wave blocking power of the barriers due to the omission or unrealistic under-estimation of the intrinsic dissipation of the elastic components of the local resonators.

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I. INTRODUCTION

Earthquakes are one of the most devastating natural disasters to human lives and infrastructures. Numerous researches and developments have been carried out in the past few decades to mitigate the damage to buildings and other infrastructures when seismic waves reach and shake their foundations.¹ In recent years, researchers have also begun to pay attention to developing a means

to attenuate the seismic waves before they reach the foundations. Since many modern buildings are designed to resist earthquakes up to the seventh grade of the Richter scale, there is a real need to protect modern cities from stronger earthquakes. The vibration spectra of most earthquakes have several peaks at frequencies, which become lower for higher magnitude earthquakes.² Such low frequency seismic waves make the mitigation extremely difficult. One must admit that there is still a long way to go in human

struggle against earthquakes, which desperately calls for breakthrough technologies.

Phononic crystals^{3–8} and local resonator metamaterials^{9–20} offer a viable alternative to the early attempts of using simple structures such as ditches and wells to block the underground seismic waves.²¹ Periodically positioned hollow cylinders, which serve as individual scatterers for Bragg reflections, were dug into the ground to form a phononic crystal with the bandgap frequency around 50 Hz.^{4,5} These investigations, while providing a proof of concept for the viability of using phononic crystals to block the underground seismic waves, also reveal two major limitations of the approach. The first limitation is that the period of the crystals must be comparable to the wavelength of the seismic waves.¹ Therefore, the phononic crystal array must be several hundred meters in dimensions to be effective. The second limitation is that the wave attenuation by phononic crystals depends very much on the perfection of the crystals. A large and highly uniform ground is needed for the deployment of the crystal array. Any irregularities and uneven distribution of soil, rock, etc., could weaken the performance. Likewise, change of moisture in the soil by rain and drought could significantly change the wave speed and the gap frequency of the phononic crystals. Therefore, while it may be viable to use phononic crystals for lower magnitude earthquakes or other vibration sources at much higher frequency, they cannot be used for blocking the seismic waves around 1 Hz.

The local resonators acoustic metamaterials,^{10–20} which are essentially made of a mass block oscillator connected by an elastic spring to an anchor,⁹ offer another alternative. By its very nature, each individual local resonator in a resonator array essentially functions independently of the others, and their collective effect is due more to the cumulative contributions of individual ones than to the deliberate arrangement of their positions in order to facilitate Bragg reflections. Since the interference among the waves scattered by the resonators does not play an essential role, the geometric dimension of the resonator array could be drastically reduced. The resonant frequencies of the local resonators are the essential elements in determining the frequencies of the seismic waves to be attenuated. Arrays of local resonators similar in design to the coated spheres⁹ and essentially working like a mass-spring oscillator system were shown theoretically to have good and, in some cases, seemingly excellent attenuation of seismic waves. For example, arrays made of cylindrical tubes containing a resonator suspended by soft bearings,¹¹ iron spheres connected to concrete blocks via iron or rubber ligaments,¹² cross-shaped steel cavities or cylinders either being hollow or coated with rubber about 10 m in size,¹³ and cylindrical mass suspended by elastomeric springs within a concrete cage¹⁴ have been studied.

A down-scaled experimental investigation¹⁴ found qualitative agreement between the theoretical predictions and the experimental outcomes. To be noted in all these theoretical investigations was the neglect of dissipation in the elastic bearing materials, which essentially made these resonators nearly dissipation-free or having a very large Q -factor. The direct consequences of such neglect are twofold. First, the blocking power of the barriers is grossly overestimated. Second, a small disturbance to the anchor at the resonant frequency would be amplified several hundred times in the displacement of the oscillator. A 10 cm ground displacement from an

eighth magnitude earthquake would lead to over 10 m of displacement of the oscillator in a local resonator, which none of the designs reported so far would be able to accommodate.

Hinted by the geophysical experiments, it was discovered that the local resonators on the soil surface, such as trees, could attenuate Rayleigh waves.¹⁵ Further theoretical studies on more systematic designs of pillar-like single resonant frequency¹⁶ and multiple resonant frequency¹⁷ structures indicated that they could effectively convert the surface Rayleigh waves into bulk-like shear waves and scatter them toward the underground abyss. Similar effects were found with thick columns of steel planted on the soil surface.^{18,19} A set of columns of different lengths arranged in a wedge could serve as a broadband wave converter that turns Rayleigh waves into bulk-like waves.²⁰ Similar effect could also be realized by a large slope,²² but the attenuation effect is rather limited.

So far, all the investigations based on local resonators are qualitative and for the purpose of proof of concept only, without considerations of realistic dissipation in the resonators. To move forward, we must carry out rigorous quantitative studies on seismic wave barriers using realistic materials and structural parameters. Accordingly, we propose four major criteria for the barriers to serve such purpose. (1) The attenuation frequency band should be in the frequency range between 1 Hz and 10 Hz; (2) Sufficient space must be made available for the oscillators in the local resonator devices to follow the ground displacement up to 10 cm in host soil; (3) The devices must remain effective and maintain close contact with the host soil during the entire earthquake; (4) Collectively, the devices should be able to form a barrier capable of up to 20 dB in transmission attenuation when necessary.

We now examine the works reported so far^{3–6,10–20} according to these criteria. While the first criterion could be met by simple upscaling of the dimensions and the oscillator mass of the local resonators to reach the resonant frequency of 1 Hz, the second criterion has been ignored in all the previous works on locally resonant metamaterials.^{10–20} As dissipation was neglected, the Q -factors of these resonators would be very large (at least ~ 100). As the ground displacement of 10 cm would be magnified by more than 100 times for the oscillator in a local resonator at resonance, none of the buried structures studied so far^{10–15} could accommodate such large displacement of the oscillator. The pillar-like resonators planted on the soil surface do not, in principle, have movement limitation above ground.^{16–20} However, the pillar resonators for 1 Hz are several meters in lateral dimension, over 10 m high, and weigh many tons. Such massive pillars could easily be detached from the soil base during the first few moments of ground motion and fall down, rendering significant functional reduction of the barriers before the arrival of the main shocks. As the structural elements of the seismic barriers in phononic crystals^{3–6} and most local resonators^{11–14,16–20} are inside massive and rigid enclosures, they could also be detached to the host soil after the first few moments of ground motion. Therefore, a practical barrier must maintain its intimate contact with the host soil throughout an earthquake. One solution is to use light weight ($\sim$ kg) and compact (less than a meter in every dimension) dampers as reported in this work. These miniature dampers buried in the host soil can easily follow the motion of the host soil and, therefore, maintain intimate contact with the soil throughout an earthquake.

Another shortcoming in ignoring the dissipation in the local resonators is the inevitable overestimations of their wave attenuation power. As will be shown in Sec. III, a row of local resonators, with nearly unlimited Q -factors, could attenuate waves with incredible power, up to 150 dB as in Ref. 11. Therefore, to meet the fourth criterion, one must use realistic dissipation parameters in the theoretical modeling to determine what is really needed to reduce the 1 Hz seismic waves by 20 dB.

In our recent paper, we reported folded metallic sheet resonators that are compact and lightweight and briefly investigated their seismic wave attenuation capability.²³ In this paper, which is a significantly extended version of the brief paper on the seismic wave attenuation phenomena, we present a comprehensive framework for the underground barriers that can block and almost totally absorb seismic waves from high-magnitude earthquakes. In Sec. II, we present experimental realization of a laboratory sample with the working frequency of 7 Hz is only tens of grams in total weight and $150 \times 15 \times 10$ mm in dimension, while the numerical simulations predict that the dampers with 1 Hz working frequency would be no more than five times larger in each dimension and in weight. The dampers buried in the host soil are homogenized into an artificial soil, the meta-soil, which has the same elastic properties as the host soil but with a dynamic effective mass density that mimics the damping functionality of the dampers buried in the host soil. In Sec. III, we present the analytic derivation of the seismic wave transmission and reflection by a simplified and idealized wall of such dampers buried underground, which are considered infinitely thin as compared to the seismic wavelength and infinitely deep as compared to the relevant Rayleigh wave depth profile from which we obtain the scaling law that relates the attenuation frequency with the dynamic mass of the local resonators for a given transmission attenuation, which is in the same form as the acoustic mass density law.²⁴ In Sec. IV, we present numerical simulations on a comprehensive collection of various deployment schemes of these resonators that could provide up to 20 dB in transmission attenuation and 0.98 in absorption of the 1 Hz seismic waves and validate the approach of approximating discrete dampers by the meta-soil continuum.

II. UNDERGROUND DAMPERS

The tuned mass dampers (TMD's) were invented nearly 100 years ago to damp the vibration of a primary object at specific frequencies.^{25,26} A single-degree-of-freedom generic TMD consisting of an oscillator with mass m attached to a spring, a dashpot, and an anchor with mass m_A is shown in the left part of Fig. 1(a). The anchor is oscillating with the amplitude A and at the frequency ω due to the vibration of a primary object (not shown) the anchor is firmly attached to. The force exerted on the anchor by the spring and the dashpot is $F = -kx - i\gamma\omega x$, where x is the displacement of the oscillator relative to the anchor, k is the force constant of the spring, and γ is the viscous coefficient. The effective dynamic mass is defined as $\tilde{m} \equiv -\frac{F_{Ext}}{A\omega^2} = m_A - \frac{k+i\gamma\omega}{\omega^2} \frac{x}{A}$, where F_{Ext} is the force exerted from the primary object onto the anchor. It can be readily obtained by solving the equation of motion of the oscillator displacement x relative to the anchor given by $\omega^2 Am - kx - i\gamma\omega x = -m\omega^2 x$, where the term $\omega^2 Am$ is the inertia force due to the fact that the anchor is

oscillating. Then, using the conventional definition $\frac{\gamma}{m} \equiv \frac{\omega_0}{Q}$ and $k \equiv m\omega_0^2$, we have

$$\tilde{m} = m_A + \frac{\omega_0^2 + i\omega\omega_0/Q}{\omega_0^2 - \omega^2 + i\omega\omega_0/Q} m = m_A + f(\omega)m, \quad (1)$$

where the frequency response function is

$$f(\omega) \equiv \frac{\omega_0^2 + i\omega\omega_0/Q}{\omega_0^2 - \omega^2 + i\omega\omega_0/Q}. \quad (2)$$

For a composite damper with multiple oscillator mass m_n , damping frequencies ω_n , and Q -factors Q_n , the effective mass is given by

$$\tilde{m}(\omega) = \sum_{n=1}^N m_n f_n(\omega) = \sum_{n=1}^N \frac{(\omega_n^2 + i\omega\omega_n/Q_n)m_n}{\omega_n^2 - \omega^2 + i\omega\omega_n/Q_n}. \quad (3)$$

As has been shown in our earlier works,^{27,28} when attached to a primary structure via its anchor, the effective dynamic mass fully describes the dynamic functionality of the damper when the size of the damper can be ignored.

Decorated membrane resonators (DMR's)²⁹ can serve as compact and lightweight miniature dampers with single or multiple working frequencies.^{27,28,30} The photo of a typical one is shown in the right half of Fig. 1(a), which consists of a rubber membrane (purple in color) mounted on a rigid circular ring about 25 mm in diameter and decorated by a rigid metallic disk. The elastic membrane serves as both the very soft spring and the dissipating dashpot. Using membranes to provide the restoring forces, DMR's with resonant (working) frequency ω_0 around 100 Hz while having total weight about 1.5 g have been demonstrated.^{27,28,30} Furthermore, they can be effectively treated as equivalent TMD's for their damping effect on any primary object of sufficiently large size.^{27,28,30}

Due to the limited lifespan of the latex rubber membranes, DMR's with metallic membranes are preferred if they are to last for decades. The DMR-like dampers were reported in our previous work;²³ here, it is a laboratory sample containing a stainless steel sheet about 0.1 mm in thickness folded into a zigzag pattern as the replacement of the elastic rubber membrane. The width of each zig and zag is about 10 mm, and the length of the folded sheet in the loose state is about half of its original length of the flat sheet. Two M6-size nuts (2 g in mass) serve as the mass block of the damper. The rectangular frame serves as the anchor of the damper. The folded sheet greatly softens the restoring forces of the flat sheet, such that a block mass about 2 g can already lead to a resonant frequency near 10 Hz. Small amount of vacuum grease is applied to the sheet to increase dissipation. The photo of a prototype damper is shown in the inset of Fig. 1(b). The dimensions of the damper are $150 \times 15 \times 10$ mm.

The free vibration of the damper is measured by mounting its anchor on a force sensor. After an initial gentle knock on the block mass (the nuts), the time domain free vibration data are measured by the underneath force sensor and then Fourier transformed into the vibration spectra at various delayed moments. Figure 1(b) depicts four representative spectra at the moments of 0, 0.5, 0.7,

FIG. 1. (a) A generic TMD (left half) and the photo of a miniature DMR damper (right half); (b) The free vibration spectra of the damper at several moments after the initial excitation. The inset is the photo of a prototype damper. (c)–(f) are the amplitude decay at the four resonances of the prototype damper at 7.2 Hz, 9.5 Hz, 51.8 Hz, and 100.9 Hz, respectively. The Q-factor of each resonance obtained from the curve fitting is shown in each corresponding figure.

and 2.0 s, respectively, after the initial impact excitation is applied.³⁰ Four major resonances at 7.2, 9.5, 51.8, and 100.9 Hz can be clearly identified. The peak amplitudes of the four resonances at different delayed moments³⁰ are shown in the remaining four

figures after Fig. 1(b). By fitting the amplitude decay curves with a single exponent function, the decay time constant and then the Q-factor of each resonance is obtained. Of particular interest is the resonance at 9.5 Hz with a Q-factor as small as 30.5 and the one at

51.8 Hz with a Q -factor of 39. These experimental results strongly suggest that a damper of similar design with working frequency of 1 Hz and Q -factor of 10 is well within reach.

Using the design parameters extracted from the experimental results, we estimated via numerical simulations that a DMR-like damper with two 10 g mass blocks and a length around 300 mm for the folded steel sheet would reach the resonant frequency of 1 Hz. The required length of the folded sheet would be 560 mm if the mass block remains at 2 g. Both are much smaller than the wavelength of 1 Hz seismic waves, which is more than 100 m in soil. More studies are clearly needed to further explore and understand the properties of this type of dampers. In the remaining part of this work, we will address the more pressing issue of using the dampers we have investigated so far for the underground barriers against high magnitude earthquakes.

While there are many possible schemes to deploy the dampers, here we mainly study the cases where the dampers are simply buried in the host soil in an ordered fashion. Each damper is placed in a rigid box and filled with viscous fluid if necessary to form a damper assembly. When buried in the host soil, each damper assembly will follow the motion of the host soil when seismic waves pass through, so that its wave damping and scattering functionalities will be fully maintained throughout the earthquake. Being deep subwavelength in size, each assembly can be approximated as a meta-particle with the effective dynamic mass in the form of Eq. (2) when it is moving with the host soil. Under long wavelength approximation, when many such meta-particles are mixed in the host soil, the soil becomes meta-soil that has approximately the same elastic properties as the host soil but with the dynamic mass density determined by the meta-particles. In the last part of Sec. IV, we will validate such approach by showing that the outcome obtained from meta-soil is consistent with the one from the corresponding discrete generic TMD's.

Among the three types of waves, namely, the body torsional waves, the Rayleigh waves, and Love waves, the body waves decay quickly over distance so the total area of severe damage caused by the body waves is much smaller than the Rayleigh and the Love waves, which are surface waves.^{31–35} Between the Rayleigh waves that reside in thick soil and the Love waves that reside in shallow soil above bed rock, the Rayleigh waves are more often encountered than Love waves because in geology thick soil cites are more common than shallow soil. Furthermore, once the means to attenuate Rayleigh waves are established, they could serve as a reference for the Love waves. At present, we are conducting studies on Love waves and a mixture of waves in more complicated and closer to real local geological settings with the mixture of soils, rocks, and caves.

III. THEORY FOR IDEAL META-SOIL SHEETS

To understand the essence of attenuating the seismic waves by the compact underground dampers, we first derive the analytic formula for the transmission and the reflection of Rayleigh waves by an infinitely large meta-soil sheet, which is fully compliant with the host soil and with a given dynamic areal mass density.

Consider a Rayleigh wave propagating along the horizontal X axis toward the meta-soil sheet parallel to the Y – Z plane and take Z as the vertical axis and Y as the lateral axis. The

displacement in the Z direction is $w = g(z)e^{ikx}$, where $g(z)$ is the depth profile of the Rayleigh wave amplitude obtained from the solutions of the coupled surface wave equations.¹ The expression for $g(z)$ can be found in Ref. 1. Omitting the time harmonic functions for the displacement fields, the total displacement on the incident side is $w_1 = w_I + w_R = g(z)e^{ikx} + rg(z)e^{-ikx}$, while the displacement on the transmit side is $w_2 = t \cdot g(z)e^{ikx}$. Here, t is the transmission coefficient and r is the reflection coefficient. The transmission attenuation is then $TL \equiv -20 \log(t)$.

The boundary conditions at the meta-soil sheet ($x = 0$) are (i) the displacements being continuous, i.e.,

$$1 + r = t, \quad (4)$$

and (ii) the Newton's second law for a unit area of the sheet,

$$G \frac{\partial w_1}{\partial x} \Big|_{x=0} - G \frac{\partial w_2}{\partial x} \Big|_{x=0} = -\tilde{m}\omega^2 w_2(0), \quad (5)$$

where G is the shear modulus and $\tilde{m}$ is the dynamic areal mass density of the meta-soil sheet with the static areal mass density m_0 and the response function given by Eq. (2). Solving Eqs. (4) and (5), we obtain

$$t = \frac{2}{2 + iK}, \quad (6)$$

$$r = \frac{iK}{2 + iK}, \quad (7)$$

where

$$K \equiv \frac{\tilde{m}\omega v}{G}. \quad (8)$$

Here, v is the wave velocity. It can be readily shown that the absorption $A \equiv 1 - |t|^2 - |r|^2 = 0$ if K is real, i.e., an inert block mass, such as a piece of rock, does not dissipate energy. Similar results can be obtained for the X -component of the Rayleigh waves when the shear modulus G is replaced by the Young's modulus Y in Eq. (8).

The above equations show that the wave blocking functionality of the meta-soil sheet is mainly from the dynamic areal mass density $\tilde{m}$. It is easily seen from Eq. (1) that at the resonant frequency ($\omega = \omega_0$), the dynamic mass is $\tilde{m} = (1 - iQ)m_0$, and $|\tilde{m}| \simeq Qm_0$ when Q is large, if the mass of the anchor is negligible. Therefore, without taking into account the realistic dissipation, the peak mass value of $\tilde{m}$ at resonance could be very large, leading to very low transmission even with small m_0 . Accordingly, the wave attenuation of the local resonators could be drastically overestimated if dissipation is ignored.

The factor $\tilde{m}\omega$ in Eq. (8) implies a scaling law for the dependence of the transmission attenuation of the meta-soil sheet on the wave frequency and the sheet mass density, as v and G are almost independent of the wave frequency. It shows that lower dynamic mass is required to provide the same attenuation for the higher frequency waves. For the same dynamic mass, the attenuation is higher for higher wave frequencies. Therefore, the higher frequency

portion of the seismic waves from the high magnitude earthquakes could readily be attenuated by lighter and smaller dampers with higher working frequencies or by the low frequency dampers if they also have higher working frequencies, such as the one shown in Fig. 1(c). In addition, as the wave speed v is proportional to $\sqrt{G}$, K is proportional to $1/\sqrt{G}$, i.e., softer ground with slower wave speed is preferred for the underground dampers to attenuate seismic waves.

IV. NUMERICAL SIMULATIONS

A. Simulation condition

We now proceed to the numerical simulations of the seismic wave attenuation by the meta-soil. The mass density of the host soil is $\rho = 2000 \text{ kg/m}^3$, Young's modulus is $E = 100 \text{ MPa}$, the shear modulus is $G = 100 \text{ MPa}$, and the Poisson ratio is $\nu = 0.3$. Given the parameters of the host soil, the Rayleigh wave velocity¹ is 128.6 m/s. The Rayleigh waves is numerically simulated using COMSOL Multiphysics in a rectangular segment of host soil 6000 m along the X axis, 400 m deep along the Z axis, and 10 m wide along the Y axis as shown in Fig. 2(a). The Z axis is perpendicular to the horizontal ground surface, with $z = 0$ being the ground surface coordinate. A line source parallel to the Y axis is placed on the ground surface at $x = -4000 \text{ m}$, as marked by the red arrow. The wave is propagating along the X axis. At both ends of the segment on the X axis are the non-reflection boundaries.

An infinite row of equally spaced and identical cylindrical columns (wells) perpendicular to the ground surface is dug into the host soil ground. Each well is filled with the meta-soil. The diameter of the wells is fixed at 5 m throughout the paper. The center-to-center separation between two adjacent wells is fixed at 10 m throughout the paper. As the well diameter and the separation are much shorter than the seismic wavelength at 1 Hz, the interference effect among the scattered waves by the wells is ignored, and the periodic boundary condition along the Y axis with the period of 10 m is imposed. The first row of wells will be at $x = 0$ as marked by the white arrow, if various types of wells are introduced in the simulations. Additional rows are placed behind the first row (to the right side in the figure) at 10-m interval when necessary. The meta-soil has the same Young's modulus, the shear modulus, and the Poisson ratio as the host soil, while its mass density is replaced by the dynamic effective mass density $\rho(\omega) = \rho_0 f(\omega)$, where $f(\omega)$ is given by Eq. (2). The quality factor is fixed at $Q = 10$ throughout the paper, and the resonant frequency ω_0 and the static mass density ρ_0 are the input parameters in the numerical simulations. Such Q -factor value is compatible to the DMR-like dampers reported in Sec. II.

The line source generates longitudinal, shear, and Rayleigh waves. Figure 2(b) depicts the wavefield in the X - Z plane without the wells. The color represents the amplitude of z -displacement. It can be seen that by the time the waves reach $x = 0$ after traveling by over 4000 m, the bulk waves spread out mostly into the underground abyss and become negligible when compared to the Rayleigh waves, which propagate only along the soil surface. The simulation results confirm that the surface waves maintain all the characteristics of Rayleigh waves,¹ such as the wave speed predicted

by the materials parameters of the soil, the X -to- Z vibration amplitude ratio, and the depth profile function $g(z)$.

Although high magnitude earthquakes contain some portion of energy at the frequencies higher than 1 Hz, we choose 1 Hz as a representative frequency for the design of the seismic wave barriers based on the following practical considerations. It is well known that lower frequency waves are more difficult to attenuate than the high frequency ones mainly for the following reasons. (1) The dampers for lower frequencies are generally heavier in weight and larger in size. Once the size and the weight of the dampers for the 1 Hz seismic waves are determined, they serve as a benchmark for the dampers with the working frequencies slightly below 1 Hz, say down to 0.5 Hz, and the upper limits in size and weight for the dampers with the working frequencies higher than 1 Hz. (2) Only shallower barrier wells are required to block the shorter wavelength waves. The wells for the 1 Hz waves set the upper limit for the well length and the number of rows of wells for a desired transmission attenuation value for the higher frequency waves. (3) According to the scaling law in Sec. III, the required mass density for a desired attenuation is inversely proportional to the wave frequency. In other words, it takes fewer dampers to provide the same attenuation to higher frequency waves than to lower frequency waves. Once the size and the weight for the 1 Hz dampers and the way to deploy them to reach the desired seismic wave attenuation are determined, the construction and the deployment of higher frequency dampers follows the same path absent of major technical hurdles. Therefore, the findings for the 1 Hz Rayleigh waves attenuation would serve as a benchmark for the design of the dampers and the construction of the broadband barriers against high magnitude earthquakes.

B. Case studies

1. Polarization dependence

Since our dampers are single degree of freedom, the effective mass density tensor for the meta-soil takes the form $\vec{\rho} = \begin{pmatrix} \rho_X(\omega) & 0 \\ 0 & \rho_Z(\omega) \end{pmatrix}$, as while the ground motion in the Y -direction is ignored for the Rayleigh waves. In the mass tensor, we take $\rho_Z(\omega) = \rho_0 f(\omega)$ and $\rho_X(\omega) = \rho_0 = 1000 \text{ kg/m}^3$ unless otherwise specified, where $f(\omega)$ is given by Eq. (2) with $f_0 = 1 \text{ Hz}$ and $Q = 10$ unless otherwise specified. Such value of inert mass density ρ_0 corresponds to the cases where about 1/8 of the space in the wells is occupied by steel oscillators. The mass density is anisotropic in that the TMD's in the wells only damp the vibration in the Z -direction. In the X -direction, the meta-soil behaves like the host soil. The top graph in Fig. 2(c) depicts the wave intensity distribution on the soil surface when a single row of vertical wells with length of 100 m are placed in front of the incoming Rayleigh waves at $f = 1 \text{ Hz}$. We find that the reflected waves and the transmitted waves maintain the Rayleigh wave characteristics, so unless specified otherwise, in the remaining part of the paper, the vertical motion on the soil surface is taken as the representative Rayleigh wave displacement. On the incident side of the wells, a standing wave pattern can be clearly seen, which is due to the reflection of the wells. Below the bottom of the wells are traces of scattered

FIG. 2. (a) Schematics of the soil segment for the numerical simulations. The upper part is the top view, and the lower part is the side view of the segment; (b) The color-coded wavefield generated by the line source. The color represents the amplitude of z -displacement. The red arrow marks the position of the line source. The white arrow marks the position of the first row of wells if they are introduced; (c) The Rayleigh wave intensity distribution at the soil surface for a row of wells filled with meta-soil for Z -direction ground vibration. (d) The Rayleigh wave intensity distribution at the soil surface for a row of empty wells (top), a row of wells filled with solid steel (middle), and a solid wall of steel (bottom).

waves propagating into the underground abyss. Using the formula $\frac{I_{Max}}{I_{Min}} = \frac{1+r}{1-r}$, the reflection coefficient estimated from the standing wave pattern is 0.16, while the transmission is 0.83, and the apparent absorption is 0.28, which includes the contribution from both the dissipation by the meta-soil and the scattered waves into the underground abyss.

Next, we assigned half of the meta-soil to damp the X -direction ground motion. The mass density matrix then takes the form $\rho_Z(\omega) = \rho_X(\omega) = 0.5\rho_0[f(\omega) + 1]$. The transmission found is 0.90, higher than the case with all the meta-soil to damp the Z -direction vibrations. This is because the Z -component of the Rayleigh waves is significantly larger than the X -component, and the fact that X -component and the Z -component of a Rayleigh wave are strongly coupled. In the remaining part of the paper, we will use meta-soil and generic TMD's, which only damp the Z -direction vibrations.

2. Comparison with conventional means

For comparison, we have calculated three cases where conventional structures are used to block the seismic waves, namely, the case with the empty wells, the case of wells filled with solid steel, and the case of a continuous solid steel wall of 5 m in thickness across the wave front. The results are shown in the top, the middle, and the lower graphs in Fig. 2(d). The empty-well case has the highest transmission and little absorption. The transmission of both the steel-well case and the continuous steel wall case have comparable apparent absorption of $A = 0.09$, which is due to the waves scattered down to the abyss rather than really being absorbed, as there is no dissipation mechanism in the structures. The transmission for the empty wells is 0.99, i.e., the empty wells have almost no attenuation to the 1 Hz Rayleigh waves. The transmission for the steel wells is 0.95 and that for the continuous steel wall is 0.91. Both are less than

the wells with the meta-soil, showing clearly the meta-soil advantage over the conventional materials.

3. Well length dependence of transmission

Figure 3(a) depicts the well length dependence of t , r , and A for a single row of wells. All the scattering coefficients show signs of saturation for the well length beyond 100 m, indicating that the length of the wells should be comparable to the wavelength of the seismic waves. On the other hand, it is clear that a single row of wells, regardless of its length, can only attenuate the seismic wave amplitude by less than 20%, which is clearly inadequate for earthquake protection.

4. Mass density dependence of transmission

Figure 3(b) depicts the transmission of a single row of 300-m length wells with different mass density. A mass density factor of 5 in the figure means $\rho_Z(\omega) = 5\rho_0 f(\omega)$ in the dynamic mass density of the meta-soil, and so forth, while $f(\omega)$ remains the same as in Eq. (2) with the resonant frequency of 1 Hz and $Q = 10$. The green curve is the transmission of the 1 Hz seismic waves through the wells obtained by simulations. The red curve is obtained from Eq. (6) using the equivalent effective dynamic areal mass density, which agrees with the simulation results quite well. An important prediction of Eq. (6) is the scaling law that the transmission remains the same if the product of the mass density times the frequency remains constant. Indeed, when the frequency of the seismic waves and the resonant frequency of the meta-soil are both increased to 10 Hz, while reducing the mass density by a factor of 10, the transmission curve as a function of mass density factor obtained from simulations (the purple points) agree perfectly with the 1 Hz one. The scaling law obtained from the simple analytic

FIG. 3. (a) The well length dependence of the scattering coefficients at 1 Hz for the wells filled with the meta-soil with the nominal mass density ρ_0 . (b) The mass density factor dependence of the 1 Hz transmission coefficient of the wells 100 m in length (solid curves). The green curve is the simulation results for the wells filled with the meta-soil with the resonant frequency at 1 Hz and the mass density being ρ_0 multiplied by the mass density factor, while the red one is from Eq. (6) with the equivalent effective dynamic areal mass density. The purple disks are the simulation data points at 10 Hz wave frequency through the 10 Hz resonant frequency meta-soil with a factor of 10 reduction in the mass density as in the corresponding 1 Hz cases.

derivations is, therefore, verified by numerical simulations and could be conveniently applied to estimate the attenuation of seismic waves at other frequencies. In particular, the behavior at high frequencies confirms the expectations that the attenuation of higher frequencies seismic waves requires lower mass density or a fewer amount of dampers. The scaling law is the generalization of the acoustics mass density law²⁴ in the seismic wave domain. Our local resonators for seismic wave attenuation are essentially a dynamic mass effect, and their resonances result in mass magnification that leads to maximum scattering and energy dissipation.

5. Multiple rows of wells

We now investigate the seismic wave transmission attenuation by multiple rows of meta-soil wells. The interval between adjacent

rows is fixed at 10 m from center to center. Figure 4(b) depicts the transmission vs the number of rows of wells at different well length. The transmission attenuation increases with the increase of the row numbers without signs of saturation. For the shallow wells with length ranging from 20 to 100 m, the attenuation increases with the well length, while beyond 100 m in well length, the increase in attenuation slows down considerably. In particular, the difference in attenuation between the 150-m wells and the 200-m wells is rather insignificant. This is consistent with the results shown in Fig. 3(a). In terms of cost effectiveness, the 100-m wells are probably the most economical for the 1 Hz waves in the present case. The dashed curve in Fig. 4(b) is the analytic results of a single row of 100-m wells with the projected areal mass density as the corresponding number of rows of 100-m wells, i.e., a mass density of $10\rho_0$ corresponds to 10 wells with the standard mass density ρ_0 .

FIG. 4. (a) The schematics of barriers with the same working frequency for rows (top) and with different working frequencies for different rows (bottom). (b) The row number dependence of the transmission attenuation at 1 Hz with various well lengths; (c) The color-coded wavefield for a single row of wells with 200 m length (top) and ten rows of wells with 20 m length (bottom). The color represents the amplitude of z -displacement; (d) The transmission attenuation spectra for several cases of meta-soil wells with different resonant frequencies and well arrangements.

The attenuation by a single row of 100-m wells with mass density $10\rho_0$ is significantly inferior to that of the 10 rows of 100-m wells with mass density ρ_0 , indicating that deploying the dampers in multiple rows of wells can provide higher attenuation than packing all the dampers closely in a single row, because multiple row of wells can reduce the diffraction leakage due to the finite well length.

Another interesting result indicated in Fig. 4(b) is that the attenuation of a single row of wells 200 m in length (~ 3 dB) is, in fact, lower than that of 10 rows of 20 m in length (~ 6 dB). This is expected, as the amplitude of the Rayleigh waves is the largest near the ground surface. For the same number of wells, if more dampers are placed near the surface instead of deeper underground, more wave energy could be dissipated. Besides the lower transmission coefficient, the reflection coefficient of the 10 rows of the 20-m wells is also very low ($r=0.02$) as compared to $r=0.16$ for the single row of 200-m wells. The wavefields of the two cases are shown in Fig. 4(c) where the color represents the amplitude of z -displacement. Significant energy flow (on the right side of the arrow) into the abyss can be seen in the latter case. Similar phenomenon was predicted in Ref. 19.

The transmission attenuation spectra of several other cases of multiple row of wells are depicted in Fig. 4(d). All the wells are 100 m in length. The red curve is the attenuation of ten rows of wells with the working frequency at 1 Hz. This barrier array is presented in the top half of Fig. 4(a) with $N=10$. The width at half height is ~ 0.1 Hz, which is expected for $Q=10$. The maximum attenuation of 16 dB occurs at 1 Hz. The green curve is the attenuation by ten rows of wells with meta-soil of different working frequency for different rows. This barrier array is presented in the bottom half of Fig. 4(a) with different colors. The working frequency ω_0 in the first row is 0.75 Hz and that of the last row is 1.20 Hz. The ones in between are in ascending order at 0.05 Hz increment. For the corresponding 20-row case, each pair of rows are of the same working frequency, and the working frequency of each of the ten pairs follows the same ascent order as the ten-row case. The 20 rows of wells with uniform working frequency of 1 Hz can provide more than 30 dB of transmission attenuation over a relatively narrow range, while the ones with distributed working frequency can provide broad band attenuation, but the maximum attenuation is only 14 dB. Therefore, dampers with multiple working frequencies are much desired to provide at least 20 dB of attenuation over a wide enough frequency range, and the DMR-like dampers with multiple mass blocks reported in Sec. II are good candidates.

6. Perfect absorbers

The above cases are good seismic wave barriers but reflect considerable amount of wave backward. Taking reference of the low reflection by the short wells, we study a tiered array of wells in a wedge formation as shown in the top half of Fig. 5(a) to achieve total absorption. The length of the first nine rows of wells, in consecutive order, range from 10 to 90 m at 10 m increments, while the last five rows are 100 m in length. All the wells are filled with meta-soil with the resonant frequency $f_0 = 1$ Hz. The reflection, transmission, and absorption coefficients are depicted in Fig. 5(b)

as the red, green, and blue curves. The reflection is below 0.04 throughout the frequency range of interest. The maximum transmission attenuation is 17.7 dB. The transmission attenuation would be over 20 dB if two more rows of 100-m wells are added at the back of the wedge. The maximum absorption is 0.98. The reflection, transmission, and absorption coefficients for the incident waves onto the opposite side (the back side) of the wedge are shown in Fig. 5(b) as the disks with the corresponding colors. The front and the back side transmission spectra are the same within computational errors. This is expected according to wave propagation reciprocity. The front and the back side reflection spectra are about the same except for the resonant frequency region from 0.88 Hz to 1.28 Hz. The maximum difference is 0.18. The wedge is, therefore, a good structure for large Willis coupling structures^{36–38} at 1 Hz.

To validate the concept of meta-soil, we use discrete fictive TMD's to replace the meta-soil in the wells, as shown in the bottom half of Fig. 5(a). Limited by the available computation resources, we replace a well segment 2 m in height and 5 m in diameter by a single generic TMD taking up the space. The block oscillator mass M_B is equal to the meta-soil mass in the segment with $M_B = \pi r^2 h \cdot \rho_0 = 3.9 \times 10^4$ kg. The equivalent stiffness of the spring is $k_s = M_B \omega_0^2$, and the equivalent viscosity of the dashpot is $\gamma_s = \frac{M_B \omega_0}{Q}$, where $Q=10$ and $\omega_0 = 6.28$ Hz. The anchor plate of the TMD is fixed on the wall of the well, and its mass is assumed to be negligible. With these parameters, the TMD has the same effective dynamic mass as the substituted meta-soil segment.

One issue to address is whether at resonance the local resonators are vibrating out of phase relative to the ground motion, which serves as the drive to the local resonators. This is verified by examining a generic TMD placed on the soil surface. The results show clearly that the motion of the mass block is 90° out of phase to the motion of the anchor, which corresponds to maximum dissipation of the soil vibration energy by the local resonator.

The transmission, reflection, and the absorption spectra for the front side incidence are shown in Fig. 5(c) as the red, green, and purple curves. The reflection spectra so obtained are much the same as the one obtained from the corresponding meta-soil case. Both show that the reflection is near zero around 1.12 Hz. The maximum transmission attenuation is 23.8 dB. The absorption maximum is 0.994. These results are consistent with the ones from the corresponding meta-soil. The fictive TMD's used in this case are probably too massive to satisfy criterion-4. Their relatively large size increases the local mass density mismatch that leads to a slight overestimation of the transmission attenuation than the meta-soil. Therefore, for more realistic investigations of deep subwavelength dampers the meta-soil approach is preferred.

Now, we examine the power dissipation by each fictive TMD. The color-coded dissipation map is shown in Fig. 5(d). Each TMD in the first row is dissipating about 100 KW of the incident wave energy flux. The seismic waves continue to penetrate and move around the bottom of the wedge, such that even the TMD's near the soil surface and the wedge bottom in the last row are still dissipating KW of power. Only the TMD's in the middle part of the last row remain almost dormant. As the total power dissipated by all the TMD's can now be computed, we can determine the wave energy scattered into the underground abyss. Figure 5(e) depicts

FIG. 5. (a) The schematics of the tiered-well configuration (top) and the generic TMD's (bottom). (b) The scattering coefficient spectra of the tiered-well configuration. The solid curves are for the scattering coefficients when the waves are incident onto the short well side (the front side), while the disks are the corresponding ones when the waves are incident from the back side. (c) The same coefficient spectra as in (b) but with the meta-soil replaced by the generic TMD's as schematically shown in the middle inset. (d) The color-coded dissipation power of each TMD where the color represents the logarithm of the power, the unit is Watt; (e) The input power (the red curve), the total dissipation power (the green curve), the transmitted power (the blue curve), and the reflected power (the purple curve) spectra for the same case as in (d).

the ratio of power of each process. At 1 Hz, over 91% of the incident wave energy is dissipated by the dampers, 2.7% is radiated as transmission and reflection waves, and 6% is scattered to abyss.

V. SUMMARY AND DISCUSSION

We have taken a major step beyond the proof of concept investigations of seismic wave attenuation by metamaterials. First, we have proposed four major criteria for the seismic barriers to attenuate the highest (eighth) magnitude earthquakes recorded in human history. Second, we have shown that all four criteria can be met by using the compact DMR-like local resonators using folded steel sheets to replace the membranes in conventional DMR's, which are further homogenized into meta-soil because of their deep subwavelength dimensions in the host soil. Using wells dug

into the ground filled with the meta-soil, we have demonstrated numerically a number of cases where sufficient transmission attenuation (20 dB or above) could be realized. We have also shown a tiered-well array in wedge form that could absorb over 98% of the incident Rayleigh waves, while providing 17.7 dB of transmission attenuation. With multiple frequency dampers, a seismic wave barrier array that satisfies all four criteria, including criterion-1 (broadband) and criterion-4 (20 dB attenuation), are within reach without major technical hurdles.

Although we have concentrated mostly on 1 Hz cases, combining with the design of the multiple frequency dampers as represented by Eq. (3) and the scaling law, there are no major technical hurdles in the realization of broadband barriers. The dampers can be scaled down in dimensions and weight in order to raise the working frequencies. Smaller dampers could be installed in the

spare space inside the boxes housing the 1 Hz dampers, such that one damper assembly could contain multiple dampers with multiple working frequency bands down to 1 Hz. The scaling law implies that lighter dampers are required for higher frequency seismic waves. Shorter wells and perhaps fewer rows of wells are needed for shorter wavelength waves to reach the same attenuation as the low frequency waves. Therefore, the dampers and the barriers targeting at 1 Hz actually serve as the design foundation and the size and weight limits for the higher frequency dampers and broad band seismic wave barriers.

Our work demonstrates that the main mechanism of seismic wave attenuation by the local resonators is the scattering by the resonant dynamic mass amplification and the absorption by the resonant dissipation. The deployment of the local resonators could be discontinuous and irregular at locations where large bedrock would provide natural obstacles. The combined effects of local resonators are mostly cumulative, so irregular soil conditions are not an essential issue. This is expected as each local resonator absorbs and scatters any type of seismic waves when excited, independent of the presence or absence of the others. The dampers in this work are readily applicable to block the body pressure and shear waves and Love waves, since the same dynamic mass mechanism applies to any wave polarization. For the Love waves in shallow soil above the bedrock, the length of the wells would probably extend all the way to the bedrock. For the protection of individual buildings, one can deploy the local resonators barrier completely surrounding its foundation with the maximum attenuation frequencies that match the first few resonant frequencies of the building. Such a barrier could block any type of seismic waves, including Rayleigh waves, Love waves, and pressure and the shear waves coming from any underground directions.

The multiple frequency dampers and the scaling law will also be particularly handy for designing the local resonators against higher frequency ground vibration excited by train or heavy vehicle movements. The prototype dampers reported in this work are undoubtedly primitive and much work is needed to improve the performance and functionalities. More comprehensive studies on this type of dampers are on-going and will be reported in due course. Other ingenious type locally resonant vibration damping devices can also be well fitted into our meta-soil framework as long as they can be represented by generic TMD's with deep subwavelength dimensions. While the means to block seismic waves in many specific geological settings still remain to be investigated, a viable means to protect infrastructures against eighth magnitude earthquakes is no longer a distant dream.

AUTHORS' CONTRIBUTIONS

F. X. and L. S. contributed equally to this article.

F.X. and L.S. carried out the numerical simulation work. L.Z. and X.H. supervised and guided F.X. for his work. Z.Y. wrote most of the manuscript, and supervised F.X. and L.S. L.Z., X.H., and Z.Y. had extensive and in depth discussions throughout the research.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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