Tensile Failure of Short Fiber-Reinforced Composites

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ABSTRACT

Enhanced material properties of brittle matrix composites reinforced with unidirectional short fiber subjected to uniaxial tension have been investigated by using the energy approach of fracture mechanics, in which the bridging stress of the short fibers in the matrix crack is determined by the inclusion method. The total fracture resistance are calculated as the energy summation including interfacial debonding energy dissipation, fractional sliding work between the fibers and matrix, fiber and matrix strain energy increment. Chemical bond of the interfaces of fibers and matrix is also taken into account. After getting the total energy, the Griffith fracture criterion is applied to predict the bend over point(BOP) stress. The necessary conditions of fiber and matrix for the multiple cracking in a short fiber-reinforced composite are discussed and the critical fiber volume ratios for the strain hardening response are determined by the iteration method.

1. INTRODUCTION

Various types of short fibers have been incorporated into the composites to increase the tensile strength and toughness of concrete, due to their easy processing and low cost. The short fiber-reinforced composites manufactured by extrusion technology show a significant increase in both tensile strength and toughness as compared with the traditional concrete composites, due to the improvement of the interfacial bonding and fiber direction alignment by high shear and compression forces. They show also some properties similar to those of long fiber-reinforced composites, such as strain hardening behavior[1,2].

For short fiber-reinforced composites, in most cases, a crack initiates at one end of fibers. Then under the increasing external load, the crack grows. As soon as it meets the neighbour fibers, it will be bridged by these fibers. Hence, the effective embedded lengths of these bridge fibers can vary from 0 to $L_f/2$. The average effective embedded length can be consider as $L_f/4$.

In case, there is no fiber in a composite, i.e. plain matrix, by the Criffith criterion, the crack will propagate if the strain energy release rate G is larger than the matrix resistance. For quasi-brittle cement-based composites, the resistance is a function of the crack radius. Only the crack radius reaches a certain critical value a , and in the mean time the strain energy release rate exceeds a critical resistance G_c^M , the crack will grow across the structure to cause failure. This critical resistance together with the corresponding crack radius can be obtained by experiment[3]. The external stress can be determined as

$$\sigma \geq \sqrt{\frac{\pi E_m G_c^M}{4a(1-\nu_m^2)}} \quad (1)$$

Where E_m and ν_m are Young's modulus and Poisson ratio of matrix, respectively.

For fiber-reinforced composites, the total energy dissipation is increased, i.e. the new fracture resistance R is increased. Equation (1) can be rewritten as

$$\sigma \geq \sqrt{\frac{\pi E_c R}{4a(1-\nu_c^2)}} \quad (2)$$

Where E_c and ν_c are Young's modulus and Poisson ratio of composites, respectively. R is the total energy dissipation rate. It's a summation of resistance of plain matrix, energy consuming rate of fibers' sliding in the matrix, energy dissipation rate due to debonding of fibers and strain energy increment rate of fibers, i.e. $R = G_c^M + \Delta G_s + \Delta G_d + \Delta G_f$, where $\Delta G_s, \Delta G_d, \Delta G_f$ are energy consuming rate of fibers' sliding, energy dissipation rate of fibers' debonding and strain energy increment rate of fibers, respectively. Thus, from eqn.(2), as soon as the total fracture resistance is known, the external stress (bend over point stress) can be predicted.

Fig.1. An Element of an Infinite Aligned Short-fiber-reinforced Plate

2. SPACING BETWEEN SHORT FIBERS

In an infinite uniformly distributed short fiber-reinforced plate, a periodic linear distribution is assumed, i.e.

$$z = \frac{L_f}{b} x. \text{ An element of this plate is shown in Fig.1.}$$

A transverse strap plate is cut for analysis(dot line). By calculating the matrix volume and then the fiber volume, the following expression can be obtained.

$$\lambda = \frac{\pi d_f^2}{16V_f}(2-\phi) + \frac{d_f}{16V_f} \sqrt{\pi^2 d_f^2 (2-\phi)^2 + 32V_f \pi (2-2\phi + \phi^2)} \quad (3)$$

Where $\phi = \frac{\delta}{L_f}$. V_f is the fiber volume ratio. d_f and L_f are fiber radius and length respectively.

3. FRACTURE RESISTANCE

By inclusion method[4], the average bridging stress of a fiber, σ_T , can be obtained(Appendix I). Then, by the shear lag method, the stress in a fiber, σ_f , can be found.

When $0 \leq z \leq l_s$,

$$\sigma_f = \sigma_T - \frac{4}{d_f} \tau_f z \quad (4)$$

When $l_s < z \leq L_j / 4$,

$$\sigma_f = Ae^{\sqrt{k_4}z} + Be^{-\sqrt{k_4}z} + \frac{k_5}{k_4} \sigma_T \quad (5)$$

Where

$$k_4 = \frac{4k}{d_f E_f} (1 + \eta), \quad k_5 = \frac{4k}{d_f E_f} \eta$$

$$A = k_6 \sigma_T - k_7, \quad B = k_8 \sigma_T + k_9$$

$$k_6 = \frac{1 - \frac{k_5}{k_4} + \frac{k_5}{k_4} e^{\sqrt{k_4} L_j / 4} e^{-\sqrt{k_4} l_s}}{e^{\sqrt{k_4} l_s} - e^{\sqrt{k_4} L_j / 2} e^{-\sqrt{k_4} l_s}}, \quad k_7 = \frac{\frac{4}{d_f} \tau_f l_s}{e^{\sqrt{k_4} l_s} - e^{\sqrt{k_4} L_j / 2} e^{-\sqrt{k_4} l_s}}$$

$$k_8 = \frac{-(1 - \frac{k_5}{k_4}) e^{\sqrt{k_4} L_j / 2} - \frac{k_5}{k_4} e^{\sqrt{k_4} L_j / 4} e^{\sqrt{k_4} l_s}}{e^{\sqrt{k_4} l_s} - e^{\sqrt{k_4} L_j / 2} e^{-\sqrt{k_4} l_s}}, \quad k_9 = \frac{\frac{4}{d_f} \tau_f l_s e^{\sqrt{k_4} L_j / 2}}{e^{\sqrt{k_4} l_s} - e^{\sqrt{k_4} L_j / 2} e^{-\sqrt{k_4} l_s}}$$

τ_f is the frictional shear stress between fiber and matrix.

The mechanism of fibers suspending the matrix crack growth may be attributed to three kinds of energy consumption: i) the work done against the frictional sliding, W_s ; ii) the surface energy consumed by the process of debonding, W_d ; iii) the strain energy increment in the fibers, W_f . All these three parts of energy can be calculated after the bridging stress of fiber, σ_T , and the stress in a fiber σ_f are obtained(Appendix II). The energy release rate can be defined by $\Delta G = \partial W / \partial (\pi a^2)$.

$$\Delta G_s = \frac{\partial W_s}{\partial (\pi a^2)} = \frac{\pi d_f \tau_f \sigma_T}{\lambda^2 E_f} l_s^2 - \frac{8 \pi \tau_f^2}{3 \lambda^2 E_f} (1 + \eta) l_s^3 \quad (6)$$

$$\Delta G_d = \frac{4 \pi d_f \gamma^* l_s}{\lambda^2} \quad (7)$$

$$\Delta G_f = \frac{\partial W_f}{\partial (\pi a^2)} = \frac{\pi d_f^2}{4 E_f \lambda^2} \xi \quad (8)$$

4. NECESSARY CONDITIONS FOR MULTIPLE CRACKING

For a plain matrix, there is no multiple cracking. The structure is fractured by a single crack. But for a fiber-reinforced composite, after the formation of the first transversal crack, the fibers can still carry the increasing external load until the fibers pulled out or broken. Thus, if the matrix strength is relative small comparing with the fiber strength. The multiple cracking could occur. The necessary conditions for the multiple cracking can be stated as

- i) The failure stress of fiber in the crack should be larger than the BOP stress,

$$\text{i.e.} \quad \sigma_f \geq \sigma^{BOP} \quad (9)$$

- ii) The bound strength of fiber should be great than the BOP force,

$$\text{i.e.} \quad \sum \left[\int_{l_i}^{L/4} \tau \pi d_f dl + \tau_f \pi d_f l_s \right] \geq P^{BOP} \quad (10)$$

- iii) The debonding length should be less than $L_f/4$,

$$\text{i.e.} \quad l_s < L_f/4 \quad (11)$$

- iv) The ultimate external stress should be larger than the BOP stress(This results in a critical fiber volume ratio for multiple cracking),

$$\text{i.e.} \quad \sigma^u > \sigma^{BOP} \quad (12)$$

For the steel fiber-reinforced concrete, Gopalaratnam and Shah [5] gave out an approximate formula for the maximum fiber debonding length

$$\text{Max}(l_s) = \frac{L_f}{4} - \frac{1}{\beta} \cosh^{-1} \sqrt{\tau_u / \tau_f} \quad (13)$$

where

$$\beta = \frac{\pi E_m}{(1 + v_m) A_f E_f \ln\left(\frac{1}{2} \sqrt{\frac{\pi}{V_f}}\right)} \quad (14)$$

5. RESULTS AND CONCLUSIONS

The influence of fiber volume fractions and fiber diameters on BOP stress and ultimate stress is shown in Fig.2. The BOP stresses are influenced little by the different fiber diameters(Fig.2a). But fiber diameters have large influence on the ultimate stresses of the composites(Fig.2b). This can be explained as if the fiber volume ratio is held, while the increasing of the fiber diameters, the spacing of fibers also get increased. This means the fibers become sparser and sparser in the composites. So, the bridging effects are degraded. The material parameters used here are from [6,7].

The mechanism of fiber-reinforced composites depends on many factors such as fiber and matrix properties, interfacial characterizations and processing methods. Based on the energy approach and the inclusion method, this paper develops a theoretical model to predict the BOP stress of the short-fiber-reinforced composites. It also gives out the conditions for multiple cracking. The model is simple to be applied to short fiber reinforced composites. In addition, this model can be extended the application to continuous fiber reinforced composites.

Fig.2 BOP Stresses(a) and Ultimate Stresses(b) of Various Fiber Volume Fractions and Diameters ($L_f=10\text{mm}$)

APPENDIX I. AVERAGE FIBER BRIDGING STRESS

Based on inclusion method, using Green's function[8], the average fiber bridging stress is given by

$$\sigma_T = k_1(1 - \alpha) \quad (15)$$

where $k_1 = \frac{E_c \pi c \epsilon_p}{2(1 - \nu_c^2) d_f}$; ϵ_p is the strain in matrix and can be replaced by the matrix failure strain determined by eqn(1). α is the bridging factor to characterize the crack opening in the bridged domain, $0 < \alpha < 1$.

Assume that all the fibers associated with the debonding inside the matrix crack have identical debonding length l_s and neglect the interaction between fibers. Through the microscopy analysis of the fiber pullout problem, Stang et al. [9] pointed out that the sliding distance l_s can be calculated by letting $\epsilon_m \approx \epsilon_f$ at $z=l_s$, see Fig.3.

$$l_s = k_2 \sigma_T \quad (16)$$

where $k_2 = \frac{d_f}{4\tau_f(1 + \eta)}$, $\eta = \frac{E_f V_f}{E_m V_m}$.

Fig.3 Debonding of the Fiber

α in eqn.(15) can be determined by the displacement relationship along the crack.