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Higher-Order Flux with Spherical Harmonics Transform for Vascular Analysis

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Abstract. In this paper, we present a novel flux-based method to robustly identify the vasculature structure in the angiography, where the curvilinear geometry is delineated by the higher-order tensor computed in the spherical frequency domain. We first modify the vesselness measurement derived from the oriented flux and introduce an antisymmetry measurement to generate the curvilinear responses. We then extend the responses to the cylindrical model and fit them into spherical harmonics transform to perform high-order tensor analysis, in which fiber orientation distribution function is utilized. A graphical framework based on the random walker is applied for vascular segmentation. It is experimentally demonstrated that the proposed method can achieve accurate and stable segmentation performance with various noise levels, demonstrating the proposed method can deliver reliable curvilinear structure responses.

Keywords: Oriented flux · Spherical harmonics · Fiber ODF

1 Introduction

Curvilinear structure segmentation is of importance in computer-aided diagnosis, which is a fundamental pre-processing step and the foundation of a range of applications in medical image analysis. One major category of the vascular network extraction methods is designed based on the order-2 tensor, targeting designing filters with high responses at the curvilinear structures. The Frangi filter [6] is one of the well-known vascular filters exploiting the second-order statistics of the image intensity in a multi-scale scenario. The Hessian matrix is constructed by an isotropic Gaussian function, which will limit the detection performance due to bifurcations, intensity inhomogeneity and changing of curvature in the clinical practice. Recently, [13] proposed a steerable curvilinear Gaussian filterbank to compute vascular connectivity map based on the Hessian matrix and Laplacian. Considering second-order based methods are susceptible to intensity fluctuation and adjacent structures, a flux-based method, which exploits

the first-order derivative, was presented [15]. Followed by the flux geometry, an oriented-flux descriptor was proposed [10], namely Optimal Oriented Flux (OOF), by projecting the gradient on a local sphere and computing the minimal inward flux. The order-2 tensor has limitations of orientation estimation and modeling abnormal cross-sections. Therefore, higher-order Cartesian tensor (HOT) has raised research interests in the community. For vascular analysis, [3] constructed HOTs by least-square estimation, while [14] generated the tensors directly by outer product. In the high angular resolution diffusion imaging (HARDI), HOT can be fit to HARDI data by spherical harmonics (SH). Comparing to HOT, the determination of the SH coefficients is more efficient, which is analogous to a Fourier decomposition restricted on a unit sphere. On the other hand, although HOT is hypersymmetry, the non-redundant components need to be decided manually, which is not convenient to extend the orders of the tensor.

In this paper, we first propose a gradient symmetry quantification based on a new vesselness measurement derived from OOF and an antisymmetry measurement based on the derivatives of the Gaussian function. Such quantification is then modified in a cylindrical scheme, where the cylindrical length is determined by the scale detected from the vesselness measurement and the size of the input volume. We then fit the curvilinear responses with spherical harmonics series using least-square estimation and estimate the fiber orientation distribution function as the final curvilinear descriptor of the proposed method. A random walker based graphical framework is applied to perform vascular segmentation.

2 Antisymmetry Optimal Oriented Flux

2.1 Background of Optimal Oriented Flux (OOF)

The optimal oriented flux (OOF) in [10] reflects the minimal amount of projected gradient going through a local sphere S_r in the 3D space. Given a gradient field $\mathbf{v}$ of an image I and a direction of interest $\hat{\rho}$, the projected flux is defined as,

$$f(\mathbf{x}; r, \hat{\rho}) = \frac{1}{4\pi r^2} \int_{\partial S_r} ((\mathbf{v}(\mathbf{x} + \mathbf{h}) \cdot \hat{\rho})\hat{\rho}) \cdot \hat{n} dA = \hat{\rho}^T \mathbf{Q}_{r,\mathbf{x}} \hat{\rho}, \quad (1)$$

where $\hat{n}$ is the outward unit normal of the spherical surface ∂S_r , $\mathbf{h}$ and dA are the position vector and infinitesimal area on ∂S_r . $\mathbf{Q}_{r,\mathbf{x}}$ is a matrix with the entry $q_{r,\mathbf{x}}^{i,j} = \int_{\partial S_r} v_i(\mathbf{x} + \mathbf{h}) n_j dA$ at i th row and j th column ($i, j \in \{1, 2, 3\}$). Finding the outward optima of Eq. 1 can be solved as a generalized eigenvalue problem of the order-2 tensor $\mathbf{Q}_{r,\mathbf{x}}$ s.t. $\|\hat{\rho}\| = 1$. Let $\lambda_i(\mathbf{x}; r)$ and $\omega_i(\mathbf{x}; r)$ be the eigenvalues and eigenvectors of $\mathbf{Q}_{r,\mathbf{x}}$ ($i \in \{1, 2, 3\}$). $\lambda_3(\cdot) \leq \lambda_2(\cdot) \ll \lambda_1(\cdot) \approx 0$ when $\mathbf{x}$ is on the centerline and the radius of S equals to the scale of the structure. $\omega_2(\cdot)$ and $\omega_3(\cdot)$ span the normal plane and $\omega_1(\cdot)$ represents the orientation of the structure at $\mathbf{x}$. The values of $q_{r,\mathbf{x}}^{i,j}$ can be easily acquired by just convolving the input image with a group of filters $\psi_{r,i,j}(\mathbf{x}) = (b_r * g_{\hat{a}_i, \hat{a}_j, \sigma})(\mathbf{x})$, where $b_r(\mathbf{x})$ is the spherical step function with values equal to $(4\pi r^2)^{-1}$ within the sphere $\|\mathbf{x}\| \leq r$

(0 otherwise) and $g_{\hat{a}_i, \hat{a}_j, \sigma}$ is the second-order derivative of Gaussian with the scale σ . The convolution can be calculated efficiently in the frequency domain, where multiplication will be used to replace convolution. Denote $\Psi_{r,i,j}(\mathbf{u})$ as the corresponding Fourier transform of $\psi_{r,i,j}$, where $\mathbf{u} = (u_1, u_2, u_3)^T$ is the position vector in the frequency domain. Then, in the frequency domain, we have,

$$\mathcal{F}\{q_{r,x}^{i,j}\} = \mathcal{F}\{\psi_{r,i,j} * I\} = \Psi_{r,i,j}(\mathbf{u}) \cdot \mathcal{F}\{I\} = \mathcal{F}\{b_r\} \cdot \mathcal{F}\{g_{\hat{a}_i, \hat{a}_j, \sigma}\} \cdot \mathcal{F}\{I\}. \quad (2)$$

Different from the responses computed in the original paper, the vesselness measurement we use are defined by,

$$\Lambda_{23}(\mathbf{x}; r) = \pi(\lambda_2^2 + \lambda_3^2)/r. \quad (3)$$

2.2 Gradient Symmetry Quantification

Inspired by [1], we propose a gradient symmetry measurement based on the derivatives of Gaussian function and Hankel transform. Combing the Hessian and the directional derivatives along the gradient of the image, define the gradient antisymmetry measurement as below,

$$I_{ww}(\mathbf{x}; r) = \frac{1}{r(I_x^2 + I_y^2 + I_z^2)} \sum_{i \in \{x, y, z\}} (I_x I_{ix} + I_y I_{iy} + I_z I_{iz}) I_i. \quad (4)$$

The second-order derivative in Eq. 4 can be obtained directly in OOF,

$$\mathcal{F}\{I_{ij}\} = \mathcal{F}\{\psi_{r,i,j} * I(x)\} = \Psi_{r,i,j}(\mathbf{u}) \cdot \mathcal{F}\{I\} \quad (i, j \in \{x, y, z\}), \quad (5)$$

$$\Psi_{r,i,j}(\mathbf{u}) = 4\pi r u_i u_j e^{-2(\pi \|\mathbf{u}\| \sigma)^2} \frac{1}{\|\mathbf{u}\|} (\cos(2\pi \|\mathbf{u}\| r) - \frac{\sin(2\pi \|\mathbf{u}\| r)}{2\pi \|\mathbf{u}\| r}). \quad (6)$$

And the first-order statistic parts can be acquired in a similar manner,

$$\mathcal{F}\{I_i\} = \mathcal{F}\{\Gamma_{r,i} * I(x)\} = \mathcal{F}\{b_r\} \cdot \mathcal{F}\{g_{i,\sigma}\} \cdot \mathcal{F}\{I\} \quad (i \in \{x, y, z\}), \quad (7)$$

$$\mathcal{F}\{\Gamma_{r,i}(\mathbf{u})\} = (-\frac{r}{\pi \|\mathbf{u}\|} (\cos(2\pi \|\mathbf{u}\| r) - \frac{\sin(2\pi \|\mathbf{u}\| r)}{2\pi \|\mathbf{u}\| r})) (-\mathbf{j} 2\pi u_i e^{-2(\pi \|\mathbf{u}\| \sigma)^2}), \quad (8)$$

where $\mathbf{j}$ is the imaginary unit in Eq. 8. Finally, the gradient symmetry quantification for spherical harmonics analysis is defined as,

$$\mathcal{M}(\mathbf{x}) = \max(0, \max_{r \in R} (\Lambda_{23}(\mathbf{x}; r)) - \gamma \max_{r \in R} (I_{ww}(\mathbf{x}; r))), \quad \gamma \in [0, 1], \quad (9)$$

where γ is the metric parameter and it was set to 0.5 in our experiments. Figure 1 provides an illustration of Λ_{23} , I_{ww} and $\mathcal{M}(\mathbf{x})$. It is clear that $\Lambda_{23} \gg I_{ww}$ inside the curvilinear structure and $\Lambda_{23} \ll I_{ww}$ at the object boundary. The second row exhibits the response maps of the same image disturbed by a high-level random noise generated by the operator Λ_{23} and $\mathcal{M}(\mathbf{x})$ respectively. It is obvious that combining with the gradient antisymmetric measurement, the new curvilinear descriptor can delineate the vascular structures better under noisy environment.

3 Spherical Harmonics Transform

We propose to construct the higher-order tensor in the frequency domain by spherical harmonics transform. The designed symmetry quantification in Sect. 2 is employed to mimic the diffusion signal required in HARDI. The higher-order tensor construction in the spherical frequency domain will be remarkably more efficient than that in the spatial domain [3, 16] since rank-1 decomposition is not required. We then focus on the spherical harmonics coefficients evaluated by solving a linear system for vessel segmentation. The proposed method is different from the Ring Pattern Detector in [12] where the high-order tensors are also constructed in the spatial domain using normalized gradient and the spherical harmonics coefficients are computed by the spherical harmonics defined with Cartesian coordinates. Our framework is actually an analog of fitting high-order tensors to HARDI data using spherical harmonics transform and focuses on calculating the spherical harmonics coefficients for vessel segmentation.

Fig. 1. An illustration for λ_{23} , I_{ww} and $\mathcal{M}(\mathbf{x})$. (a-c) 1st row: A synthetic tube, A_{23} and I_{ww} ; 2nd row: The same tube disturbed by random noise, A_{23} and $\mathcal{M}(\mathbf{x})$.

3.1 Fitting Curvilinear Responses with Spherical Harmonics Series

Any real function parameterized by the spherical coordinates (θ, ϕ) can be written in a Spherical Harmonics (SH) series,

$$x(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l c_l^m Y_l^m(\theta, \phi), \quad (10)$$

where l is the order, $Y_l^m(\theta, \phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta) e^{jm\phi}$ are the SH series and $P_l^m(\cdot)$ is a Legendre polynomial. In HARDI, the continuous function $x(\cdot)$ is replaced by the discrete measured diffusion signal $X(\mathbf{g}) = X(\theta, \phi)$ with the

number of SH: $R = \frac{(l+1)(l+2)}{2}$ for approximation. In the usual practice, a modified SH basis is employed to compute the coefficients c_l^m of the SH series [4], where $Y_j = \frac{\sqrt{2}}{2}((-1)^m Y_l^m + Y_L^{-m})$ if $-l \leq m < 0$; $Y_j = \frac{\sqrt{-2}}{2}((-1)^{m+1} Y_l^m + Y_L^{-m})$ if $0 < m \leq l$ and $Y_j = Y_l^0$ for $m = 0$, in which $j(l, m) = \frac{l^2+l+2}{2} + m$, l is even and $m = -l, \dots, 0, \dots, l$. With the above modification, the SH transform can be represented with only one summation and finite terms,

$$X(\theta, \phi) = \sum_{j=1}^R c_j Y_j(\theta, \phi). \quad (11)$$

Define $X_{\mathbf{p}} = (X_{\mathbf{p}}(\theta_1, \phi_1), X_{\mathbf{p}}(\theta_2, \phi_2), \dots, X_{\mathbf{p}}(\theta_M, \phi_M))^T$ as the measured diffusivities at the voxel $\mathbf{p}$, where M is the number of sampled directions, $C_{\mathbf{p}} = (c_{1_{\mathbf{p}}}, c_{2_{\mathbf{p}}}, \dots, c_{R_{\mathbf{p}}})$ as the coefficients of SH series at $\mathbf{p}$ and $Y_{\mathbf{p}} = (\tilde{Y}_{\mathbf{p}}(1), \dots, \tilde{Y}_{\mathbf{p}}(M))^T$ as the SH series matrix at $\mathbf{p}$, where $\tilde{Y}_{\mathbf{p}}(i) = (Y_{1_{\mathbf{p}}}(\theta_i, \phi_i), \dots, Y_{R_{\mathbf{p}}}(\theta_i, \phi_i))$. The coefficients c_j can be obtained by least-square estimation: $C_{\mathbf{p}} = (Y_{\mathbf{p}}^T Y_{\mathbf{p}})^{-1} Y_{\mathbf{p}}^T X_{\mathbf{p}}$.

Consider the whole 3D volume with the size of $H \times W \times D$, the diffusivity matrix is denoted as X , which is a $M \times (HWD)$ matrix and the coefficient matrix of SH series is represented by C , which is a $R \times (HWD)$ matrix. Since the SH series are not related with the location $\mathbf{p}$ over the whole image, we have $Y = Y_{\mathbf{p}}$. Thus, the coefficients of each voxel can be computed simultaneously,

$$C = (Y^T Y + \beta L)^{-1} Y^T X, \quad (12)$$

where $L = \text{diag}(l_1^2(l_1+1)^2, \dots, l_R^2(l_R+1)^2)$ is the Laplace-Beltrami regularization term and l_j is the order corresponding to j th SH basis (e.g., $l_j = 0$ for $j = 1$).

To fit the curvilinear responses with SH series, we modify the gradient symmetry quantification in Sect. 2 with a cylindrical manner. Given a direction $\mathbf{g}_i := (\theta_i, \phi_i)$, the diffusivity gradient symmetry quantification is defined as,

$$\mathcal{M}(\mathbf{x}; \theta_i, \phi_i) = \mathcal{M}(\mathbf{x}; \mathbf{g}_i) = \sum_{l=0, \dots, h_{\mathbf{x}}} \mathcal{M}(\mathbf{x}_l), \quad \mathbf{x}_l = \mathbf{x} + \mathbf{g}_l l, \quad (13)$$

where $h_{\mathbf{x}} = f(\arg \max_{r \in R} A_{23}(\mathbf{x}; r))$ and $f(r) = \frac{\log(1+r) \log(HWD/3)}{2}$. The first term $\log(1+r)$ allows small curvilinear structures to have a larger $h_{\mathbf{x}}$ and the second term takes the size of the volume into consideration during traveling. Thus, the cylinder constructed for simulating the diffusion signal is adaptive.

3.2 Fiber Orientation Distribution Function

The vesselness response we propose is determined via the orientation distribution function (ODF) computed by the SH coefficients. In HARDI, diffusion ODF (dODF) is usually employed for fiber tracking, which is computed by multiplying the SH coefficients c_j with $2\pi P_{l_j}(0)$, where $P_{l_j}(\cdot)$ is a Legendre polynomial. Comparing to the dODF, fiber ODF (fODF) removes the smooth parts by spherical deconvolution transform and thus becomes sharper along the orientations of

fibers [5]. Since the smooth parts in dODF could be the disturbance for detecting the vessels especially on the bifurcations, we utilize fODF in our work,

$$\Psi_{sharp}(\theta, \phi) = \sum_{j=1}^R 2\pi P_{l_j}(0) \frac{c_j}{f_j} Y_j(\theta, \phi), \text{ where } f_j = 2\pi \int_{-1}^1 P_{l_j}(t) R(t) dt. \quad (14)$$

$R(\cdot)$ is the dODF kernel, which exhibits a prolate tensor profile, and l_j is the even order corresponding to j th SH basis. The term $\frac{1}{f_j}$ is used to transform the dODF into fODF. In [5], analytical expressions of $R(\cdot)$ and f_j are given by,

$$R(t) := \frac{1}{Z} ((\frac{e_2}{e_1} - 1)t^2 + 1)^{\frac{1}{2}} \quad \text{and} \quad f_j = \frac{2\pi}{Z} \int_{-1}^1 P_{l_j}(t) ((\frac{e_2}{e_1} - 1)t^2 + 1)^{\frac{1}{2}} dt, \quad (15)$$

where $Z = \int_{-1}^1 ((\frac{e_2}{e_1} - 1)t^2 + 1)^{\frac{1}{2}} dt$ is the normalization term and the eigenvalues of the tensor profile are assumed to be $\{e_1, e_2, e_2\}$ ($e_1 \gg e_2$). In our computation, we use the order-2 tensor computed from OOF with the corresponding top- k largest $\mathcal{M}(\mathbf{x})$ values and set $e_1 = \sum(|\lambda_2| + |\lambda_3|)/k$, $e_2 = \sum(|\lambda_1|)/k$, where $k = (H * W * D)^{\frac{1}{3}}$. Then, the final curvilinear responses, namely Spherical Harmonics flux, are obtained by averaging summation among M fiber orientation distribution functions,

$$\Psi_f = \frac{1}{M} \sum_i^M \Psi_{sharp}(\theta_i, \phi_i). \quad (16)$$

3.3 Spherical Harmonics Connectivity Enhanced Random Walks

Random walks [7] is a segmentation framework formulated on a weighted graph. Given an undirected graph $G_o = (V_o, E_o)$ with vertices $v \in V_o$ and edges $e \subseteq E_o = V_o \times V_o$, denote w_{ij} as the weight of e_{ij} , v_b as the background seeds and v_s as the foreground seeds. Based on the random walks framework, we present a vasculature enhanced graphical framework with the Spherical Harmonics flux,

$$x = \arg \min_x \sum_{v_i \in V_o \setminus \{v_s, v_b\}} (\Psi_f(x_i)(x_i - 1)^2 + (1 - \Psi_f(x_i))(x_i - 0)^2) + \sum_{e_{ij} \in E_o} w_{ij} (x_i - x_j)^2, \quad \text{s.t.} \quad x(v_s) = 1, x(v_b) = 0, \quad (17)$$

where $x = (x_1, \dots, x_n)^T$ is the probability for a walker starting from a vertex j that first arrives at a foreground seed and $w_{ij} = e^{-\beta(g_i - g_j)^2}$, g_i is the intensity value at voxel i and β is the only parameter of random walks, which equals to 100 in all experiments. Define $SHG = (V_{shg}, E_{shg})$ with $V_{shg} = V_o \setminus \{v_s, v_b\}$ and $E_{shg} = V_{shg} \times \{v_s, v_b\}$, Eq. 17 can be rewritten as,

$$x = \arg \min_x \sum_{e_{ij} \in E_o} w_{ij} \cdot (x_i - x_j)^2 + \sum_{e_{ij} \in E_{shg}} s_{ij} \cdot (x_i - x_j)^2, \quad (18)$$

s.t. $x(v_s) = 1, x(v_b) = 0,$

where $s_{ij} = \Psi_f(x_i) \cdot x_j + (1 - \Psi_f(x_i)) \cdot (1 - x_j)$. With the construction of *SHG*, Eq. 18 can be solved efficiently and easily by the algorithm in [7].

4 Experiments

We have performed the validation on the synthetic images and clinical images. For the 1st group of synthetic experiments, the evaluation was performed on the synthetic helix-tube whose radii were from 1 to 4 voxels and the synthetic tree with radii from 2 to 4 voxels (Fig. 2). The intensities of the helix-tube varied from 0.5 on the top to 1 on the bottom. These two synthetic images were designed to simulate bifurcation challenges and shrinking problem for real vessel images. A subset of Vascusynth [8, 9] was used to perform the 2nd group of experiments, in which there were 12 synthetic data with distinct bifurcation numbers. Prior to performing segmentation, different levels of random noise were added to these synthetic images. The clinical experiments were performed on a set of MRA images from TubeTK [2].

Fig. 2. First row: helix-tubes. Second row: synthetic trees. $2^{nd}-6^{th}$ columns: 2D view with noise levels $\mathcal{N} \in \{0, 0.04, 0.1, 0.3, 0.5\}$ respectively.

The proposed method (Ψ_f) was compared with classical random walks (*RW*), *OOF* and the Hessian [6] with direction coherence enforcement framework in [17] (OOF_{Coh} , HES_{Coh}) and the ranking orientation responses of path operators (*RORPO*) [11]. We also compared the gradient symmetry quantification ($\mathcal{M}(\mathbf{x})$) to see the effects of SH transform. For the proposed method, we set the order of the SH series to 4 and the number of sampled directions was equal to 64. The radii for Ψ_f , OOF_{Coh} and HES_{Coh} were in the range of 1 to 10 with steps equal to 0.5 in the synthetic experiments, while there were in the range of 1 to 20 with steps equal to 0.5 in the clinical tests. For *RORPO*, the length of the minimum path, the factor and the number of scales were set to 2, 1.5 and 10, respectively for synthetic data and for the clinical parts, they were set to 2, 1.5 and 20, respectively. We optimized the thresholding values in the scope of 0.02 to 0.8 with steps equal to 0.02 for binarization. Dice score was employed for statistic comparison, which is given by $\frac{2TP}{2TP+FP+FN}$.

Table 1. Dice on Helix-tubes and synthetic trees

Synthetic data	Methods	Noise level								
		0	0.02	0.04	0.06	0.1	0.2	0.3	0.4	0.5
Helix-tubes	$\mathcal{M}(\mathbf{x})$	99.79%	97.15%	96.54%	95.71%	94.74%	90.93%	86.52%	82.30%	77.09%
	Ψ_f	99.20%	96.82%	96.33%	95.88%	94.79%	92.75%	89.81%	86.79%	83.84%
	<i>RORPO</i>	100.00%	97.80%	88.17%	76.75%	57.44%	0.33%	0.00%	0.00%	0.00%
	<i>OOFCoh</i>	98.78%	92.23%	90.08%	90.50%	91.47%	89.68%	86.09%	80.79%	75.10%
	<i>HESCoh</i>	98.84%	95.94%	95.26%	94.35%	92.13%	82.92%	73.49%	62.79%	53.29%
	<i>RW</i>	100.00%	51.70%	51.10%	36.72%	48.26%	49.07%	30.23%	53.92%	36.44%
Synthetic trees	$\mathcal{M}(\mathbf{x})$	99.70%	93.33%	92.66%	91.10%	90.90%	89.60%	87.08%	84.23%	80.69%
	Ψ_f	99.62%	92.27%	91.55%	91.41%	90.99%	89.77%	88.17%	86.48%	84.79%
	<i>RORPO</i>	99.97%	99.83%	95.80%	86.18%	67.16%	0.74%	0.00%	0.00%	0.00%
	<i>OOFCoh</i>	99.77%	94.79%	93.74%	92.76%	90.42%	89.55%	86.54%	85.18%	81.27%
	<i>HESCoh</i>	98.73%	94.51%	93.39%	92.22%	90.45%	85.14%	75.09%	59.89%	47.01%
	<i>RW</i>	100.00%	77.80%	46.93%	56.01%	26.12%	58.11%	41.76%	40.00%	54.50%

The numerical results are listed in Table 1 and Table 2. It is clear that both $\mathcal{M}(\mathbf{x})$ and Ψ_f exhibit an accurate and stable performance on both types of synthetic data with different noise levels. When the noise level is low, $\mathcal{M}(\mathbf{x})$ performs slightly better than Ψ_f due to the antipodally-symmetry enforced by the even order tensor. However, when the noise level is high, the difference between $\mathcal{M}(\mathbf{x})$ and Ψ_f becomes distinct and Ψ_f presents a better ability on curvilinear structure analysis thanks to the SH deconvolution in the computation of fODF. Despite the high performance when noise level is 0, *RW* suffers from the inhomogeneous intensity caused by noise. Similar to *RW*, *RORPO* achieves high scores with no noise and low noise levels but it fails in the cases of high noise levels. In the clinical experiments, we will see that noise of the clinical data will still be retained by the feature map of *RORPO*.

Table 2. Dice on Group 1 of Vascusynth

Noise level	0			0.04			0.1		
Bif num.	16	36	56	16	36	56	16	36	56
Ψ_{fODF}	93.37%	95.02%	92.73%	83.82%	88.42%	84.09%	79.27%	85.58%	80.64%
<i>RORPO</i>	98.70%	98.19%	95.04%	73.92%	71.45%	65.14%	45.40%	40.12%	36.29%
<i>OOFCoh</i>	97.84%	98.22%	97.61%	77.92%	81.67%	78.37%	75.01%	78.93%	76.05%
<i>HESCoh</i>	94.13%	91.45%	94.30%	84.17%	84.28%	84.73%	77.12%	80.48%	78.85%

For the clinical experiments, Ψ_f was compared with *OOFCoh* and *RORPO* qualitatively. Segmentation results of Ψ_f are shown in Fig. 3 with different thresholding values (0.04 and 0.14). The comparison of vascular maps generated from the above-mentioned methods is presented in Fig. 4. It is noted that promising segmentation can be achieved by Ψ_f despite high noise levels and intensity

inhomogeneity, while vessels of small scales can be obtained under low thresholding value. Although both Ψ_f and OOF_{Coh} use the similar graphical framework, OOF_{Coh} fails outside the region of the Circle of Willis. $RORPO$ does generate relative good vascular responses but as shown in the zoom region of the green rectangle, the noise of background and some unwanted segments are retained.

Fig. 3. From left to right: original image and two segmentation results with distinct thresholding values (0.04 and 0.14).

Fig. 4. From left to right: the vascular maps of Ψ_f , OOF_{Coh} and $RORPO$.

5 Conclusion

We have introduced a spherical harmonics framework for curvilinear structure analysis, which can be regarded as the higher-order tensor analysis in the frequency domain. Comparing to the higher-order tensor, the spherical harmonics, combining with the orientation distribution function, is more efficient and convenient to use. We have also presented a novel gradient symmetry quantification and fit it into the spherical harmonics framework. Three groups of experiments have been carried out and the results demonstrate that our method is robust to noise.

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