An Analysis of Singularity in Single Fibre Pull-Out Test Using BEM

Y. Dai^{1,2}, J.K. Kim¹, X. Ji² and S.T. Xue²

¹ Department of Mechanical Engineering, Hong Kong University of Science and Technology,
Clear Water Bay, Hong Kong

² Department of Engineering Mechanics and Technology, Tongji University,
Shanghai, 200092, China

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Abstract

The single fibre pull-out test is one of the major mesomechanical methods which are designed to measure the interface properties between the fiber and matrix in composites. An axisymmetrical boundary element method is used for the single fibre composite model to obtain the stress distributions at the fibre-matrix interface. The numerical results show that the interface radial and shear stress components, σ_r and τ_{rz} , are singular at the fiber entry and the embedded fiber end, and vary linearly with the axial distance, z , in a log-log plot. It is proposed that the stress components, σ_r and τ_{rz} , have the form $z^\alpha F_{ij}(\theta)$, where α is the singularity exponent and $F_{ij}(\theta)$ is the angular distribution function. It is shown that the singularity exponent depends on the loading geometry of the model and the properties of the composite constituents, especially the Young's modulus ratio of fiber to matrix.

1. Introduction

Rigorous micromechanics analyses have been performed [1,2] to obtain the stress distributions in the composite constituents in the single fibre pull-out test. In the theoretical analysis, the condition of interface debonding has been defined by two distinct approaches: the shear strength criterion and the fracture mechanics approach, assuming that the interface failure occurs purely in the mode II shear. In addition to the analytical micromechanics models, numerical analyses based on finite element models (FEMs) have also been performed, which may provide more accurate description of the stress profiles. The FEM analyses [3,4] identified high concentrations of the radial and shear stress components at the fibre-matrix interface near free edges, which may imply the possibility of interface debonding in combined mode I and II failure, as opposed to the conventional assumption of in mode II shear. It follows that an appropriate debond criterion should be established to account for the singular behaviour of the multi-axial interface stresses.

A boundary element method (BEM) is used in the present analysis to evaluate the stress profiles corresponding to processing and materials variables. Special emphasis is placed on study of the singular nature of the interface radial and shear stresses near the fibre ends and of important parameters which may affect the stress singularity.

2. Boundary Element Model

A boundary element analysis is carried out on a composite model which consists of a fibre embedded in a cylindrical matrix. The axi-symmetric loading geometry of the practical test allows a

Figure 1 Boundary element model for single fibre pull-out test.

two-dimensional analysis. The loading geometry and the boundary conditions are chosen to represent those of the actual experimental technique, as shown in Figure 1. The fibre and matrix are assumed to be isotropic and linear elastic, and perfectly bonded at the interface. The residual thermal stresses are not specifically considered in the present analysis which may arise from the differential thermal contraction upon cooling from the processing temperature. A constant stress is uniformly applied to the fibre at its free end, $z=-l$, while the bottom surface of the matrix is fixed at its end, $z=L_m$. For an efficient boundary element analysis of a bi-material problem, the composite model is divided into two regions with their boundary being the fibre-matrix interface.

3. Results and Discussion

The material properties, $E_f/E_m = 100$, $\nu_f = 0.3$ and $\nu_m = 0.3$, and the geometric factors $a/b = 0.05$, $L/b = 2$, $l/a = 2$ and $L_m/b = 3$ are used as a standard condition. The essential trends are calculated for a fully bonded interface. To study the effects of effective fibre volume fraction and Young's modulus, the parameters a/b and E_f/E_m are also varied. The interface radial and

Figure 2 (a) Interface radial and (b) shear stress profiles along the fiber length.

stress profiles, normalised with the external stress, are plotted as a function of axial distance in Figure 2. It is found that both the interface radial and shear stresses exhibit two peaks, each at the fibre entry ($z = 0$) and at the embedded fibre end ($z = L$), where the stress fields become singular. The absolute values obtained at the former end ($z = 0$) are found to be slightly greater than those at the latter end ($z = L$) for the material properties studied, suggesting the interface debond initiation at the fibre entry in preference to the embedded fibre end, if a simple debond criterion based on the maximum stress is used and the stress singularity is not considered.

Figure 3 Log-log plots of interface radial and shear stress profiles along the fiber length for varying E_f/E_m : (a) at fibre entry; (b) at embedded fibre end.

Figure 4 Log-log plots of interface radial and shear stress profiles along the fiber length for varying a/b ; (a) at fiber entry; (b) at embedded fiber end.

The log-log plots of the interface stress profiles along the fibre length are presented in Figures 3 and 4 for varying Young's modulus ratio, E_f/E_m , and fibre to matrix ratio, a/b , respectively. It is worth noting that the log stress curves become linear with respect to z -axis both near the fibre entry and the embedded fibre end, with their slopes being characteristic finite values. The slopes vary in a regular manner with the Young's modulus ratio, E_f/E_m , while it is almost constant regardless of the radius ratio, a/b , at a given end of fibre. We propose that the stress distributions near the fibre entry and the embedded fibre end have the form:

$$\sigma_y = z^\alpha F_y(\theta) \quad (1)$$

where α is the singularity exponent, and $F_{ij}(\theta)$ is the angular function for the interface stress components, σ_{ij} . α and $F_{ij}(\theta)$ can be determined from the least-squares fitted log-log plots of the interface stresses:

$$\log|\sigma_{ij}| = \alpha \log z + \log|F_{ij}(\theta)| \quad (2)$$

Figure 5 Singularity exponents, α , of the interface radial and shear stresses at fiber entry and embedded fiber end for varying (a) E_f/E_m and (b) a/b .

The absolute values are used for the stress terms, σ_{ij} and $F_{ij}(\theta)$, as the interface stresses are often negative with respect to z-axis, depending on the loading geometry and the boundary conditions. The singularity exponent, α , thus obtained from Eq. (2) is presented in Figure 5. It is found that the singularity exponent varies rapidly with the Young's modulus ratio, E_f/E_m , for small values of E_f/E_m , and approaches a plateau value when E_f/E_m is greater than about 100. The relationship between the singularity exponent and Young's modulus ratio is opposite at the fibre entry and the embedded fibre end (Figure 5(a)). In contrast to the effect of E_f/E_m , the singularity exponents, α , for both the interface radial and shear stresses (Figure 5(b)) are insensitive to radius ratio of fibre to matrix, a/b . It is very interesting to note that the singularity exponents, α , obtained for the interface radial and shear stresses at a given fibre end are almost identical. This result indicates that the singularity exponent is characteristic to material properties and loading conditions. In other words, the singularity exponent is a material constant characteristic to a fibre-matrix interface given the loading condition, regardless of the stress component.

4. Summary

A boundary element analysis is performed to evaluate the interface stresses in the single fibre pull-out test. The following results can be summarized.

- (1) Both the interface radial and shear stresses exhibit two peaks, each at the fibre entry and the embedded fibre end.
- (2) A singularity exponent is proposed, which is characteristic to the fibre and matrix properties.
- (3) The singularity exponents of the interface radial and shear stresses should be the same at a given fiber end.
- (4) The singularity exponents of the interface radial and shear stresses vary with the Young's modulus ratio, E_f/E_m , but are independent of the radius ratio, a/b .

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