Publication information

Title	A quantum interpretation of the physical basis of mass-energy equivalence
Author(s)	Chang, Donald Choy
Source	Nil
Version	Preprint version
DOI	Nil
Publisher	Nil

Copyright information

© The Author(s)

Notice

This version is available at HKUST Institutional Repository via

<http://hdl.handle.net/1783.1/109043>

If it is the author's pre-published version, changes introduced as a result of publishing processes such as copy-editing and formatting may not be reflected in this document.

For a definitive version of this work, please refer to the published version.

<http://repository.ust.hk/ir/>

A quantum interpretation of the physical basis of mass-energy equivalence

Donald C. Chang

Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

Email: bochang@ust.hk

Abstract

We know energy and mass of a particle can be connected by $E = mc^2$. What is the physical basis of this relation? Historically, it was thought to be based on the principle of relativity (PR). A careful examination of the literature, however, indicated that this understanding is not true. Einstein did not derive this relation from PR. Instead, his argument was mainly based on *thought experiments* which focused on the similarity between radiation and matter. Following this hint, we suspect that the mass-energy equivalence could be based on the quantum property of wave-particle duality. We know photon and electron can behave as a particle as well as a wave. Such a wave property could make the particle behave differently from Newtonian mechanics. Indeed, using a wave model which treats particles as excitations of the vacuum, we show that the mass-energy equivalence relation can be directly derived based on the quantum relations of Planck and de Broglie. This wave hypothesis has several advantages; not only can it explain naturally why particles can be created in the vacuum; it also predicts that a particle cannot travel faster than the speed of light. This hypothesis can also be tested in experiment.

Keywords: Mass-energy equivalence; quantum mechanics; relativity; vacuum; wave-particle duality

1. Introduction

It is well known that the foundation of modern physics is built on two pillars, i.e., quantum mechanics (QM) and the special theory of relativity (STR). They are generally thought to be independent from each other. However, very little is known about the connection between these two theories. For example, could some of the so-called “relativistic effects” have an origin in the QM? Furthermore, many people are not aware that there are important discrepancies between the physical assumptions of QM and STR. For example, the view of vacuum is very different between these two theories. As a classical theory, STR assumes that the vacuum is an empty space between matters; otherwise there will be a resting frame in the universe. Quantum mechanics, on the other hand, assumes that the vacuum is not an empty space; it is just the ground state of the quantum system. Otherwise, it will not be possible to explain the creation of particles upon excitation.

In this review, we would like to summarize recent attempts to sort out the similarity and difference between the implications of STR and QM. More specifically, we will focus on examining the physical basis of the well-known mass-energy equivalence relation $E = mc^2$, which plays a tremendous role in modern physics. What is the physical basis behind this relation? In the current literature, it is often stated that the energy-mass relation is a consequence of STR. A closer examination of the published record, however, indicated that this is not the case (for details, see below). Recently, we found that the energy-mass relation is more likely to be based on wave-particle duality in quantum mechanics. This paper is a brief review of recent works leading to this new understanding.

2. Was the energy-mass relation derived from the principle of relativity?

The mass-energy equivalency relation $E = mc^2$ is one of most well-known formula in modern physics. Historically, this relation was thought to be arisen from the principle of relativity (PR).¹⁻⁵ However, a more careful examination of the literature indicates that such thinking is not really true.⁶⁻¹² First, similar concepts of mass-energy equivalency had been suggested by various scientists before the publication of STR.¹³⁻¹⁶ At the end of the 19th century, several European physicists had proposed that mass can be related to the energy-content of an object. For example, in 1881, J. J. Thomson showed that the magnetic field generated by a moving charged sphere could induce an effective mass on the sphere.¹³ Oliver Heaviside in 1889 further suggested that the effective mass m should be proportional to E / c^2 .¹⁴ Wien and Max Abraham called this the “electromagnetic mass”.^{17,18} The energy-mass relation $E = mc^2$ was first mentioned by Poincare in a paper published in 1900.¹⁵ In 1904, Fritz Hasenöhl also proposed the concept relating energy with mass in a series of papers entitled “*On the theory of radiation in moving bodies*”.^{19,20} According to W. Fadner, there were many discussions on the topic of mass-energy equivalence before 1905.⁸

Furthermore, although Einstein had actively promoted the concept of mass-energy equivalence, his arguments were not based on first principles; instead, they were mostly based on demonstrations in special hypothetical situations (called “*thought experiments*”).^{3,8-12} For example, the first publication of Einstein on this topic was a very short paper published in 1905, in which he considered an object sending out two identical pulses of light at opposite directions.²¹ The energy of each of these light pulses is designated $\frac{1}{2}L$ as measured in the stationary frame S . After the emission of the light pulses, the total energy of the object was measured in the stationary frame S and moving frame S' . He argued that, the difference between the energy measured in the S and S' frames is equal to the kinetic energy K of the object. Using the Taylor expansion and ignoring higher order terms, he showed that, *if a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2* .²¹

His derivation, however, had several problems: (1) It was not a general derivation, instead, it was a result obtained from a special hypothetical situation. (2) The obtained result $E = mc^2$ was

an approximation; it only applied when the moving speed is much smaller than c . (3) His assumption that the total energy is a linear sum of the resting energy and the kinetic energy is incorrect for an object moving at high speed. (4) His hypothetical “*thought experiment*” was unrealistic; it cannot be performed in practice.^{9,12} In addition, this paper was criticized earlier by H. E. Ives for using a circular logic.²²

Einstein was aware of the shortcomings of his derivation and thus wrote many more papers on this topic in the following years.²³⁻²⁸ However, most of Einstein’s *thought experiments* had nothing to do with the principle of relativity; instead, they were mainly based on his intuitive thinking that radiation and matter could behave similarly.^{3,12} For example, Einstein proposed another *thought experiment* in 1906 based on a center-of-gravity argument.²³ The following is a simplified version of his argument as summarized by A. P. French.³

Fig. 1. Einstein’s thought experiment. When a burst of radiation energy was emitted from the left end of the box and travelled to the right end, the box would recoil from its initial position (a) to a final position (b) due to its momentum changes. Einstein then estimated the mass of the radiation based on the momentum involved. (Reproduced based on Ref. 3).

Suppose a rigid box with mass M and length L is floating in space (see Fig. 1.). It is stationary initially. At $t = 0$, a burst of radiation wave (with energy E) is emitted from the left end of the box. The radiation carries momentum E/c . Because the total momentum of the system is conserved, the box should acquire a momentum equal to $-E/c$. Then, the box would recoil with a speed v ,

$$v = -\frac{E}{Mc} . \quad (1)$$

The radiation wave would reach the right end of the box at a later time, $t = L/c$ (assuming $v \ll c$). Suppose the radiation energy is totally absorbed by the box, the box would obtain a momentum equal and opposite to the initial burst of momentum and bring the box to rest again. Then, the net result of this process is to displace the box for a distance Δx ,

$$\Delta x = v\Delta t = -\frac{EL}{Mc^2} . \quad (2)$$

Einstein argued that the center of gravity of the box should remain unchanged before and after the radiation process. The only way to counter act the movement of the rigid box is to assume that *the radiation should carry with it the equivalent of a mass m* , such that

$$mL + M\Delta x = 0. \quad (3)$$

Combining Eqs. (2) and (3), we have

$$m = \frac{E}{c^2} \quad \text{or} \quad E = mc^2. \quad (4)$$

This suggests that *the radiation wave can have an effective mass equal to its energy divided by c^2* .

Thus, one can see that Einstein's derivation of the mass-energy relation in this *thought experiment* was not based on the principle of relativity. Instead, it was based on his intuitive thinking that *radiation wave can behave like a material object*. This *thought experiment*, however, actually violated the basic principle of STR, since it assumed the entire box move simultaneously when a burst of radiation wave was emitted from its left end at $t = 0$. That would imply that information can be transmitted faster than the speed of light. This problem was recognized by Einstein himself in his later paper.²⁴

In recent years, many historical reviews on the derivation of the mass-energy relation had been published.⁸⁻¹¹ In one recent review,¹¹ it was concluded that: "*Einstein was aware of the shortcomings of his derivation (in 1905 and 1906) and wrote a half dozen more papers over the next 40 years trying to patch things up but arguably never succeeded.*" A number of physicists had also expressed criticisms on Einstein derivation of the mass-energy relation.^{10,22,29,30} An elaborated review of Einstein's attempt to derive the mass-energy relation was recently given by E. Hecht in a series of papers published in the *American Journal of Physics*.^{9,31} His conclusion was that: "*Einstein produced about 18 virtuoso derivations and demonstrations all aimed at establishing the mass-energy principle. ... although each of them gave evidence for the applicability of $E_0=mc^2$ to a particular set of circumstances, no one derivation, or collection of them taken together, succeeded in providing a definitive proof of its complete generality.*"⁹

3. Could the mass-energy equivalence be based on quantum concepts?

The above reviews raised an important question: If the mass-energy equivalence relation was not derived from PR, what can be its physical basis? We suspect that the mass-energy equivalence is not a classical phenomenon; instead, it could be a quantum effect. Indeed, one may recall that in Einstein's *thought experiments*, his argument was mainly based on his intuitive thinking that the radiation wave behaves like a material object. This inspires us to think: Could the mass-energy equivalence be based on the quantum property of wave-particle duality?

We know photon and electron can behave as a particle as well as a wave. Such a wave property can make the particle behave differently from a point mass. This means that, a quantum object can have properties that are not found in classical mechanics. As shown in the following, if one treats the radiation wave as a photon, one can easily derive the mass-energy equivalence based on well-known quantum relations.

Let us first recall what the definition of “mass” is. From Newtonian mechanics, we know the “mass” (m) of an object is related to its momentum (p),

$$p = mv. \quad (5)$$

In the case of a photon, its speed is c . From the de Broglie relation, we know the momentum of a photon is $p = \hbar k$. (Here $\hbar$ is Planck’s constant divided by 2π). So, the “mass” of a photon can be defined as

$$m = p / c = \hbar k / c. \quad (6)$$

Since $k = 2\pi / \lambda = \omega / c$, using Planck’s relation, the above equation becomes

$$m = \hbar k / c = \hbar \omega / c^2 = E / c^2. \quad (7)$$

Hence, if one regards the quantized light wave as a particle, one can easily derive the mass-energy relation of a photon, i.e.,

$$E = mc^2. \quad (8)$$

In the case of a photon, its mass is the “*effective mass*”, not “*rest mass*”. Since this effective mass is derived from the particle’s momentum, we may call it the “*inertial mass*”. Recently, we demonstrated that this *inertial mass* is also equal to the “*gravitational mass*” of the photon.³²

Thus, at least in the case of a photon, one can show that the mass-energy equivalence is a consequence of its quantum property. For particles with nonzero rest mass, it is more complicated to derive the mass-energy equivalence relation. Nevertheless, as shown in the following, such a relation can indeed be derived based on the wave properties of the particle.

4. Physical nature of a particle from a quantum view

A fundamental question in quantum mechanics is that: *What is the physical nature of a particle?* Is the particle a point object or a wave packet? It was observed in experiments that a particle can behave both like a wave or a corpuscle. This is called “wave-particle duality”. In most textbooks, the particle itself is often regarded as a point object, but its distribution can be described by a wave. This is called the “Copenhagen interpretation”.³³ Such a view, however, is not universally accepted. Many prominent physicists, including Schrödinger, de Broglie and Einstein, did not agree with such interpretation.³³

Following the spirit of de Broglie and Einstein, we think the concepts of matter and radiation can be unified. The simplest hypothesis is to regard free particles (with or without rest mass) as excitation waves of the vacuum. From the Planck's relation³⁴ and the photoelectric effect³⁵, we know a photon can behave like a particle. But from Maxwell's theory, we also know that photon is a type of electro-magnetic wave. Thus, one may see that, in the microscopic view, a particle is an excitation wave; but in the macroscopic view, it can behave like a corpuscular object. (See Fig. 2.)

Fig. 2. Two views of a particle. The classical view vs the quantum view.

We know the massive particle (such as electron) propagates like a wave (called “matter wave”) in the vacuum; its motion is described by a quantum wave equation. Here, we propose that, like the photon, the matter wave of a particle is also an excitation wave of the vacuum. This hypothesis can be justified by a number of observations. First, it had been shown that electrons can be diffracted from a crystal following the Bragg's diffraction law, just like light waves.^{36,37} Second, the wave nature of an electron is also demonstrated in the double-slit experiment or in the operation of a transmission electron microscope.^{38,39} Furthermore, we know particles can be created or annihilated in the vacuum. If the particle is a point mass like a bullet, how can it be created from nowhere or disappear suddenly? The most plausible explanation is that the particle is an excitation wave of the vacuum medium, so that it can be excited by an energetic stimulation.

Hence, it is reasonable to assume that both radiation waves and matter waves are excitation waves of the same vacuum medium.⁴⁰ In fact, this point is supported by the observation that no object can travel faster than the speed of light. We know the speed of an excitation wave is determined solely by the physical properties of the transmitting medium. Since the limiting speeds of all particles (with or without rest mass) are found to be the same, matter wave and radiation wave must be carried by the same vacuum medium. In other words, all particles are

excitation waves of the vacuum; their wave functions then represent different solutions of the same wave equation.

In summary, this wave model proposes that: (1) *Both photons and massive particles are quantized excitation waves of the vacuum.* (2) *Different types of particles are represented by different excitation modes of the same vacuum.* Based on this hypothesis, one will see in the following that the mass-energy equivalence relation can emerge naturally from the wave properties of the particle.

5. Mechanism of wave propagation in the vacuum

5.1. *It was implied from the Maxwell theory that the vacuum could behave like a dielectric medium*

In order to derive the wave equation describing the excitation waves of the vacuum, we need to find out the physical nature of the vacuum medium first. This is a major challenge. At present, our knowledge about the vacuum is rather limited; many hypothetical models had been proposed before and their views are highly diverse.^{7,41-52} In order to find a credible hypothesis to start our work, we have actively searched the literature. In this process, we discovered a very useful suggestion given by Maxwell in his work on electrodynamics. His theory indirectly implied that the vacuum must behave like a dielectric medium.⁵² The following is a summary of his argument.

An important discovery in the 19th century was Ampere's Law, which gives the relation between the electric and magnetic fields. This law was originally known as⁵³

$$\nabla \times \mathbf{H} = \mathbf{J} \quad . \quad (9)$$

This description of Ampere's Law was included in the Maxwell equations proposed in 1862. But later, Maxwell was not satisfied with it, because this equation would violate the condition of conservation of charge, i.e.,

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_e}{\partial t} \quad . \quad (10)$$

In order to fix this problem, Maxwell proposed to add a new term $\partial \mathbf{D} / \partial t$ into the right hand-side of Eq. (9).⁵³ ($\mathbf{D}$ is called the “electric displacement” or “charge displacement”.) The justification for this was that, if the medium is a dielectric material, there will be both positive and negative charges embedded in it. Then, when the dielectric medium is exposed to an electric field, it will induce a displacement of the dielectric charges. A time variation of the displaced charge should produce a displacement current ($\mathbf{J}_d$). Therefore, the current given in Eq. (9) should include both the externally applied current $\mathbf{J}$ and the internally induced displacement current $\mathbf{J}_d$, i.e.,

$$\mathbf{J}_{\text{total}} = \mathbf{J} + \mathbf{J}_d = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad . \quad (11)$$

Based on this consideration, Maxwell proposed to revise the Ampere's law to a new form,

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} . \quad (12)$$

This equation was included in the final form of the Maxwell's equations.⁵³ Thus, the key to justify the revision of Ampere's law was really dependent on the assumption of vacuum being a dielectric medium.

5.2. Wave propagation in the vacuum

Using the above equation, Maxwell was able to construct his theory of light propagation. When he investigated the propagation of electro-magnetic waves in the vacuum, Maxwell argued that only the external current $\mathbf{J}$ is zero, but the displacement current is not zero. This is equivalent to say, *he made a hidden assumption that the vacuum is a dielectric medium rather than an empty space*. With this assumption, the Ampere's law in the vacuum now became

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} . \quad (13)$$

This can be combined with Faraday's law,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (14)$$

to set up a wave equation for the propagation of $\mathbf{E}$ or $\mathbf{H}$. We know the electric and magnetic inductions are proportional to the electric and magnetic fields, such that

$$\mathbf{D} = \varepsilon \mathbf{E} , \quad (15)$$

$$\mathbf{B} = \mu \mathbf{H} , \quad (16)$$

where ε and μ are the dielectric permittivity and magnetic permeability. (In the vacuum, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$.) Combining the above four equations, one can easily derive the wave equation

$$\nabla^2 \mathbf{D} - \frac{1}{c^2} \frac{\partial^2 \mathbf{D}}{\partial t^2} = 0 \quad (17)$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is equal to the speed of light. One can derive similar wave equations by replacing the physical variable $\mathbf{D}$ with either $\mathbf{E}$ or $\mathbf{B}$. One may recognize that the above equation is identical to the wave propagation equation of light. Thus, the key assumption in Maxwell's theory of light propagation was to regard the vacuum as a dielectric medium. If one treats the vacuum as an empty space, the $\partial \mathbf{D} / \partial t$ term in Eq. (12) should vanish. Then, it will be impossible to derive the wave equation of light.

Mathematically speaking, one can replace $\mathbf{D}$ in Eq. (17) with any linear functions of $\mathbf{E}$ or $\mathbf{B}$, including the magnetic vector potential $A^\mu = (\phi, \mathbf{A})$. However, although all these wave equations

are mathematically equivalent, their physical meanings are not. In a classical mechanical system, such as an elastic solid, we know the physical parameter responsible for wave propagation is usually associated with a local displacement of the wave medium. Since Maxwell's theory implied that the vacuum is a dielectric medium, it will be natural to regard the propagating wave as a movement of the charge displacement $\mathbf{D}$. Nevertheless, in order to provide a general wave equation for describing the excitation waves in the vacuum, we may use a generic wave function ψ to represent all possible physical variation within the vacuum medium. That is, Eq. (17) can be generalized to be written as

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0. \quad (18)$$

We propose that this is the basic wave equation (BWE) for all types of excitation waves of the vacuum medium. Now, if one accepts our assumption that matter waves and radiation waves are both excitation waves of the same vacuum medium, one will expect that the wave function of the matter wave should also obey the same wave equation as shown in Eq. (18). Here ψ can have different solutions, representing different excitation modes. These modes may represent different types of free particle. At this point, we already know this hypothesis is correct for at least one particle, i.e., the photon. In the following, we will show that this hypothesis is also valid for matter wave representing a massive particle.

6. The wave equation and wave function of a free particle in the vacuum

Following the spirit of de Broglie and Schrödinger, we proposed that both radiation wave and matter wave are excitation waves of the vacuum. Then, one may expect the wave functions of photon and massive particles to be associated with different solutions of the basic wave equation (BWE) of the vacuum. The simplest solution of BWE is a plane wave $\psi_k \sim e^{i(k \cdot x - \omega t)}$, where k and ω are the wave vector and frequency, respectively. By substituting this wave function into Eq. (18), one obtains its dispersion relation

$$\omega = ck. \quad (19)$$

Using the Planck's relation and the de Broglie relation, the quantized wave should obey the following energy-momentum relation,

$$E = cp, \quad (20)$$

which is known to be the energy-momentum relation of a photon. Thus, we can identify this plane wave solution of BWE as the wave function of the light wave (photon).

To find the wave function of the matter wave, the solution is more complicated. Since the matter wave represents a particle with non-zero rest mass, the cross section of the wave function should be more localized in the transverse plane. In another word, since a massive particle would

behave like a mass point in the classical limit, the oscillating amplitude of its wave function should be concentrated at its trajectory. This suggests that the wave function of a massive particle (moving in the direction $\hat{\mathbf{k}}$) not only depends on the spatial coordinate along its trajectory (i.e., $\hat{\mathbf{k}} \cdot \mathbf{x}$), it also varies in the transverse plane ($\hat{\mathbf{k}} \times \mathbf{x}$). Thus, one may assume that the general wave function representing a free particle has the form

$$\psi_{\hat{\mathbf{k}}}(\mathbf{x}, t) = \psi_T(\hat{\mathbf{k}} \times \mathbf{x}) \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t), \quad (21)$$

where ψ_L is the longitudinal component of the wave function which describes the travelling wave along the particle's trajectory, and ψ_T is the transverse component of the wave function. Substituting Eq. (21) into Eq. (18), one can use the technique of separation of variables to obtain two coupled equations:

$$\left\{ \begin{array}{l} \left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) = \ell^2 \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) \\ \nabla^2 \psi_T(\hat{\mathbf{k}} \times \mathbf{x}) = -\ell^2 \psi_T(\hat{\mathbf{k}} \times \mathbf{x}), \end{array} \right. \quad (22)$$

$$(23)$$

where ℓ is a fitting constant to be determined later. By solving Eqs. (22) and (23) separately, one can show that the general solution of ψ is⁵⁴

$$\psi_{\hat{\mathbf{k}}}(\mathbf{x}, t) = a J_n(\ell r) e^{\pm i n \theta} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}, \quad (24)$$

where J_n is Bessel function of the first kind; n is an integer or half integer; r and θ represent the radial distance and the azimuthal angle of the space vector in the transverse plane. (a is a normalizing constant.) The dispersion relation of this wave function is

$$\omega^2 = (k^2 + \ell^2) c^2. \quad (25)$$

As expected, this wave function behaves like a travelling wave moving along its trajectory $\hat{\mathbf{k}}$. Due to the presence of the Bessel function, $\psi_{\hat{\mathbf{k}}}$ varies in a diminishing oscillating manner in the transverse plane (i.e., in directions perpendicular to the particle's trajectory). (See **Fig. 3**). The wave length of this transverse oscillation is $\lambda_T = 2\pi / \ell$. As it turns out, the transverse wave length λ_T is found to be identical to the ‘‘Compton wave length’’ of a particle.⁵⁵

Fig. 3. The general solution of the wave equation of the vacuum. The wave function is assumed to be a product of a longitudinal wave function and a transverse wave function. The particle is travelling along the direction $\mathbf{k}$. In this plot, the parameter n is equal to 0.

In the following section, we will show that Eq. (25) is equivalent to the energy-momentum relation of a massive particle. This suggests that the solution of BWE as presented in Eq. (24) should represent the wave function of a massive particle.

7. Correspondence between wave and particle properties in the quantum view

The wave function shown in Eq. (24) contains several wave parameters, ω , $\mathbf{k}$, and ℓ . What are their corresponding meanings in a particle view? If one accepts the concept that a particle is simply a quantized wave packet, one can easily identify ω and $\mathbf{k}$ as the energy (E) and momentum ($\mathbf{p}$) of the particle based on the Planck's relation $E = \hbar \omega$ and the de Broglie relation $\mathbf{p} = \hbar \mathbf{k}$. Then, what is the physical meaning of ℓ ?

If one multiplies Eq. (25) with $\hbar^2$ and using the Planck's relation and the de Broglie relation, one can obtain

$$E^2 = c^2 p^2 + c^2 \hbar^2 \ell^2. \quad (26)$$

Recall that the particle velocity (v) is equal to the group velocity of the wave packet,⁵⁶

$v = \frac{\partial \omega}{\partial k} = \frac{\partial E}{\partial p}$, and the particle mass m is defined by $p = mv$, one can obtain from the above equation that

$$m = \frac{\hbar \ell / c}{(1 - v^2 / c^2)^{1/2}}. \quad (27)$$

When v approaches zero, m equals the rest mass, m_0 . The above equation becomes

$$m_0 = \frac{\hbar \ell}{c} . \quad (28)$$

This indicates that the wave parameter ℓ is associated with the rest mass of the particle. In another word, the transverse oscillation of the matter wave is characterized by the Compton wave length $\lambda_T = 2\pi / \ell = h / m_0 c$. Combining the above two relations, one has

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}} . \quad (29)$$

Substituting Equation (28) into Equation (26), the dispersion relation becomes

$$E^2 = p^2 c^2 + m_0^2 c^4 , \quad (30)$$

which is exactly the energy-momentum relation of a massive particle. Furthermore, by combining the above two equations with $p = mv$, one can directly show that the total energy of a particle is proportional to its moving mass,

$$E = mc^2 . \quad (8)$$

Thus, this demonstrated that the mass-energy equivalence relation is really originated from the wave properties of the particle. In addition, from the energy-momentum relation as expressed in Eq. (30), it is obvious that, when the particle is at rest ($p = 0$), the rest-energy of a particle is proportional to its rest mass, i.e.,

$$E_0 = m_0 c^2 . \quad (8a)$$

Hence, the mass-energy equivalence relation is not only applicable for a photon, as suggested by Einstein's earlier works,^{23,57} it also holds for a particle with non-zero rest mass.

Furthermore, from the above analysis, one can see that there is a *one-to-one correspondence* between particle properties and wave properties. Our results suggest that *energy*, *momentum* and *mass* can all be treated on the same footing in a wave perspective. (See **Fig. 4**). We may add that, although the dispersion relation we obtained in the above was based on a continuous wave solution, the particle is really represented by a wave packet, in which there is a narrow distribution in ω and k . In fact, our earlier work has shown that, the quantum conditions of a particle (including the Planck's relation, the de Broglie relation and the Heisenberg uncertainty principle) are all associated with the properties of a wave packet, not the continuous wave.⁵⁸

Wave-particle duality		
Particle View	Connecting rule	Wave View
Energy: E	$E = \hbar \omega$	Frequency $\omega = \frac{2\pi}{T}$
Momentum: p	$p = \hbar k$	Wave vector $k = \frac{2\pi}{\lambda_L}$
Rest mass: m_0	$m_0 = \hbar \ell / c$	Transverse wave vector $\ell = \frac{2\pi}{\lambda_T}$
Velocity: v	$v = \frac{\partial \omega}{\partial k}$	Group velocity $v = \frac{\partial \omega}{\partial k}$
E - p relation: $E^2 = c^2 p^2 + m_0^2 c^4$	all of above	Dispersion relation: $\omega^2 = c^2 (k^2 + \ell^2)$

Fig. 4. Wave-particle duality. The *one-to-one correspondence* between particle properties and wave properties in the quantum view.

8. This wave model is compatible with present theories of quantum mechanics

To further justify our hypothesis that particles are quantized excitation waves of the vacuum, we will show that the quantum mechanical wave equations for different particles can be derived based on this wave hypothesis. In our earlier discussion, we have already shown that the wave model is fully capable to describe the wave movement of a photon, since our basic wave equation is similar to the known wave equation of light. The remaining question is whether this wave model can also describe the wave movement of particles with nonzero rest mass (i.e., matter waves). In the current literature, the quantum theories used to describe the motion of massive particles are the Klein-Gordon equation^{59,60} (for scalar particles) or the Dirac equation (for leptons). Can this model be compatible with those quantum mechanical equations?

In the following, we will show that this is indeed the case. As shown in Eq. (21), the wave function representing an excitation wave in the vacuum medium has both a longitudinal component (ψ_L) and a transverse component (ψ_T). Using the technique of separation of variables, we have shown that the longitudinal wave (ψ_L) is described by Eq. (22),

$$\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t) = \ell^2 \psi_L(\hat{\mathbf{k}} \cdot \mathbf{x}, t). \quad (22)$$

From discussion in the last section, we know that ℓ is connected with the rest mass m_0 through Eq. (28). If we treat ψ_L as the wave function ϕ of the scalar particle, the above equation becomes

$$\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \phi = \left(\frac{m_o c}{\hbar} \right)^2 \phi . \quad (31)$$

This is identical to the *Klein-Gordon equation*. This analysis suggests that the wave function described by the Klein-Gordon equation mainly represents the longitudinal component of the travelling wave of the particle (with rest mass m_o). In another word, the Klein-Gordon equation describes only the movement of the excitation wave along the particle's trajectory; it does not concern with the oscillation of the vacuum medium in the transverse plane.

The above analysis demonstrated that the Klein-Gordon equation is fully compatible with our wave model. The wave function of a massive particle is just one special solution of the basic wave equation of the vacuum.

Now, one may ask what about the situation of a lepton. Can one demonstrate that the wave function of an electron is also a special solution of the basic wave equation of the vacuum?

This is indeed possible. Once the Klein-Gordon equation is derived, one can further derive the Dirac equation by linearizing the Klein-Gordon equation. Eq. (31) can be rearranged as

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi_e - \left(\frac{m_o c}{\hbar} \right)^2 \psi_e = 0 , \quad (32)$$

where ψ_e is the wave function associated with an electron (which is not required to be a scalar function). As suggested by Dirac,⁶¹ the left-hand side of the above equation can be decomposed into the product of two operations, i.e.,

$$\left(i \frac{1}{c} \frac{\partial}{\partial t} - i \alpha \cdot \nabla + \beta \frac{m_o c}{\hbar} \right) \left(i \frac{1}{c} \frac{\partial}{\partial t} + i \alpha \cdot \nabla - \beta \frac{m_o c}{\hbar} \right) \psi_e = 0 . \quad (33)$$

Then, the above equation can be decoupled into separated linearized equations, such that

$$\left(i \frac{1}{c} \frac{\partial}{\partial t} + i \alpha \cdot \nabla - \beta \frac{m_o c}{\hbar} \right) \psi_e = 0 . \quad (34)$$

However, in order to make Eq. (33) to satisfy the Klein-Gordon equation, both the parameters α and β must be treated as a 4 x 4 matrix,

$$\alpha_i = \begin{pmatrix} -\sigma^i & \mathbf{0} \\ \mathbf{0} & \sigma^i \end{pmatrix}, \quad \beta = \begin{pmatrix} \mathbf{0} & \sigma^o \\ \sigma^o & \mathbf{0} \end{pmatrix},$$

where the various σ^i are 2×2 Pauli spin matrices. Furthermore, the wave function ψ_e is now becoming a four-component column matrix (called the “Dirac spinor”).⁶¹ The linearized equation arisen from Eq. (33) is called the “*Dirac equation*”.

Thus, one can see that the *Dirac equation* was derived from the *Klein-Gordon equation*, which in turns was derived from the *basic wave equation of the vacuum*. Therefore, the solution

of the Dirac equation must be a special solution of the wave equation of the vacuum. This suggests that the Dirac theory of electron is fully compatible with the wave model discussed in this paper.

Furthermore, it has been shown that the *Schrödinger equation* can be obtained from the Dirac equation under the condition when the Coulomb potential energy and kinetic energy are small in comparing to the particle mass.⁶² Such condition is usually satisfied in atomic physics or condensed matter physics. Since this derivation procedure was well known in the literature,^{62,63} we will not repeat it here. Nevertheless, it is clear that the Schrödinger wave equation is also compatible with our wave model.

From the above discussions, one can see that the wave functions of different particles are indeed special solutions of the basic wave equation of the vacuum. This result suggests that the hypothesis proposed in this wave model is well justified.

9. Why must a particle not travel faster than the speed of light?

A major postulation of STR is that no object can travel faster than the speed of light. But there was no direct justification of this postulation. This speed limit, however, can be easily justified in the quantum wave model discussed in this paper. The fact that a particle cannot travel faster than the speed of light is a natural consequence of the wave property of the particle.

In classical mechanics, the vacuum is regarded as an empty space. Then, since there is no friction or resistance in this empty space, a particle should be able to travel as fast as it can. In another word, there is no reason to limit the travelling speed of the particle. However, in the quantum wave model presented here, the vacuum is not empty. As we mentioned earlier, the vacuum is thought to behave like a dielectric medium. With this understanding, one can easily see why a particle cannot travel faster than the speed of light. Because the particle is a quantized excitation wave of the vacuum, its traveling speed is determined by the group velocity of its wave packet, which always travels at a speed less than the phase velocity of the wave. This phase velocity, on the other hand, is solely determined by the physical property of the transmitting medium. From Eq. (18), we know the phase velocity of the excitation wave in the vacuum is the speed of light, i.e., $c = 1 / \sqrt{\mu_0 \epsilon_0}$. So naturally, the particle cannot travel faster than c .

The fact that the maximum speeds for all particles (with or without rest mass) are the same (i.e., the speed of light c) has a very important implication; it suggests that massive particles and photons are excitation waves of the same vacuum medium (which has the same μ_0 and ϵ_0). Otherwise their limiting velocities should be different.

Based on the quantum wave model presented in the above, one can easily show that a particle should not travel faster than the speed of light. Recall that the dispersion relation of the excitation wave of the vacuum is given in Eq. (25),

$$\omega^2 = (k^2 + \ell^2) c^2. \quad (25)$$

Since the traveling speed of the particle is equal to the group velocity of the wave packet, i.e., $v = \frac{\partial \omega}{\partial k}$. From Eq. (28), we know the parameter ℓ is proportional to the rest mass m_o . In the case of a photon, which has no rest mass, $\ell = 0$; its dispersion relation becomes $\omega = ck$. Thus, its group velocity is equal to its phase velocity ($\omega / k = c$), that is,

$$v = \frac{\partial \omega}{\partial k} = c. \quad (35)$$

This explains *why a photon always travels at the speed c*. For a particle with nonzero rest mass, $\ell \neq 0$. From Eq. (25), its group velocity is

$$v = \frac{\partial \omega}{\partial k} = \frac{ck}{\sqrt{k^2 + \ell^2}} < c. \quad (36)$$

This predicts that *a massive particle can travel at any speed* depending on its momentum $\hbar k$, and *its maximum speed cannot exceed the speed of light*. So, this so called “relativistic assumption” is actually a reflection of the fact that the particle is a quantum wave packet.

10. Why does the mass of a particle increase with its speed?

In the quantum wave model presented in this paper, it was predicted from Eq. (29) that the *mass* of a particle is dependent on its *travelling speed* v , such that

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}. \quad (29)$$

The concept of mass being speed-dependent is not new. At the end of the 19th century, Lorentz developed a modified aether theory to explain the experimental results of Michelson and Morley.⁶⁴ Lorentz showed that the mass of an electron parallel to the direction of motion is $m_L = \gamma^3 m_0$ and the mass perpendicular to the direction of motion is $m_T = \gamma m_0$, where $\gamma = 1 / \sqrt{1 - v^2 / c^2}$ is the “Lorentz factor”.¹⁶ In 1905, Einstein published his famous paper on relativity.⁶⁵ He also realized that the mass of an object could be speed-dependent. More specifically, he showed that the mass of an “electron” can change with its traveling speed, such that

$$\left\{ \begin{array}{l} \text{Longitudinal mass} = \frac{m_0}{(1 - v^2 / c^2)^{3/2}} \end{array} \right. \quad (37)$$

$$\left\{ \begin{array}{l} \text{Transverse mass} = \frac{m_0}{1 - v^2 / c^2} \end{array} \right. \quad (38)$$

Several experiments had been conducted at the beginning of the 20th century to measure particle mass at different speeds.^{18,66-68} Their results generally supported the prediction of Eq. (29), but not the results of Einstein's 1905 paper. Since then, similar findings had been reported in many subsequent experiments.⁶⁹⁻⁷¹

Why does the particle's mass change with its speed? This is a mystery in classical physics. However, if one accepts the quantum wave model, it can be explained easily.

In classical physics, the vacuum is an empty space, and the particle is regarded as a point mass (like a tiny bullet). When an object moves in the vacuum, it can be accelerated continuously. Its increase of momentum is reflected in gaining speed. But in the wave model, the particle is a quantized excitation wave of the vacuum. No matter how much energy is given to the particle, its group velocity can only approach the phase velocity of the vacuum (the speed of light), but cannot exceed it.

Once there is a speed limit for the particle, it can be predicted that the mass must change with speed. Why? We know that the momentum of a particle is the product of mass and velocity,

$$p = mv. \quad (5)$$

Suppose a particle is accelerated in an accelerator and one can apply an unlimited amount of energy to accelerate the particle, the energy and momentum of the particle can increase continuously. Since the particle's speed v has an upper limit c , the particle cannot increase its speed further when its speed approaches the speed of light. Its momentum p can only be increased through the increase of mass. Hence, the energy absorbed by the particle is not used to increase its speed v , but is mainly used to increase its mass m . That is to say, the speed limit of the particles must be compensated by the increase in mass; otherwise the energy and momentum of the particle cannot continue to increase.

In other words, the faster a particle travels, the harder it is to be accelerated. This implies that, the larger the velocity of the particle, the greater its inertial mass. Thus, one can predict that the inertial mass of this particle will increase with its speed. And all this is because the particle is a quantized excitation wave; its travelling speed cannot exceed the phase velocity of the vacuum medium, which is the speed of light.

11. Can this quantum interpretation be tested in experiment?

Finally, no matter how logical a physical theory is, its ultimate judgement is dependent on whether its prediction can be supported in experiment. Then, can one propose an experiment to test the two different models of mass-energy equivalence discussed in this paper? Namely, is the $E = mc^2$ relation due to the principle of relativity or due to the quantum wave nature of the particle?

We have recently proposed such an experiment, which is based on a key difference between quantum mechanics and STR, that is, they made different assumptions about the vacuum.⁷² In STR, the vacuum is regarded as an empty space; there should be no fixed resting frame in the universe. In our quantum wave model, particles are regarded as excitation waves of the vacuum; such a non-empty vacuum will naturally form a resting frame. So, one can differentiate these two models by testing whether there is a fixed resting frame in our universe or not.

This can be done by measuring precisely the moving mass of two identical particles traveling in opposite directions.⁷² From Eq. (29), we know the moving mass of a particle is speed-dependent. If the STR is correct, all inertial frames will be equivalent. Then, the laboratory frame can be chosen as the stationary frame. The moving mass should be the same regardless of the particle's moving direction in the laboratory frame. But if the vacuum is a dielectric medium as implied from the Maxwell theory, there is a resting frame in our universe. One will observe a difference in the moving mass when the particle changes its traveling direction. This is because the speed of the particle is a vector sum of the particle's velocity relative to the laboratory frame and the velocity of the laboratory frame relative to the vacuum. (See **Fig. 5**).

Detailed design of this proposed experiment had been given in our earlier paper.⁷² In short, we will measure the moving masses of two identical particles travelling in opposite directions with the same speed (u); one particle travels to the left, while the other travels to the right, as shown in **Fig. 5**. If the STR is correct, the laboratory frame can be regarded as the stationary frame; there is no speed difference between the two particles. The measured moving mass of the two particles will be the same. On the other hand, if the vacuum is the resting frame, the particle travelling to the right should have a combined speed faster than that travelling to the left. The measured moving mass of the two particles will be different.

Fig. 5. The basic idea of a proposed experiment designed to test whether there is a resting frame in our universe due to the presence of a non-empty vacuum.

We proposed to conduct this experiment in a way similar to the Michelson-Morley experiment. Since the experimental apparatus is fixed at the surface of the Earth, its orientation depends on the Earth's rotation. Thus, the direction of the particle movement will change at different times of the day or in different seasons of the year. (See **Fig. 6**). The speed of the Earth can be estimated from the Earth's orbital speed, the speed of the Sun in our galaxy, and the speed of the Milky Way *vs* the rest of the universe. We estimated that the mass variation due to the change of apparatus orientation can be in the order of $\frac{\Delta m}{m_0} = \frac{2v_0 u}{c^2} \approx 2 \times 10^{-4}$.⁷² Such a change of particle mass should be measurable using existing technology. For simplicity, we suggested to use electron as the testing particle in this experiment.

Fig. 6. A simplified diagram showing the basic idea of the experimental design. If there is a resting frame in our universe, it is expected that, for two electrons traveling in opposite directions (right and left), there will be a difference in their moving mass. This mass difference will be seasonal dependent and change with the time of the day. (a) A top view of the movement of the Earth around the Sun. (b) The R and L arms of the apparatus are pointing to the East-West direction. Because of the movement of the Earth, the orientation of the apparatus is different in different times of the day. Thus, the electrons moving toward Right and Left will have different velocities relative to the resting frame of our universe. This means that the difference in their moving mass will also change with the hours in a day.

12. Conclusion

- This work proposed a novel explanation for the physical basis of mass-energy equivalence. Unlike the conventional thinking, we found that the relation $E = mc^2$ was not derived from the principle of relativity. Instead, it is due to the wave-particle duality of a particle. It is the quantum nature of the particle that accounts for the equivalence between mass and energy.
- Using a wave model which treats particles as excitation waves of the vacuum, we showed that both the mass-energy equivalence and the speed-dependence of mass are direct results of the wave behavior of a particle. The so-called “*relativistic energy-momentum relation*” is actually originated from the dispersion relation of matter wave.

- There is a *one-to-one correspondence* between particle properties and wave properties. We showed that *energy, momentum and mass* can all be treated on the same footing in a wave perspective.
- This wave model has several advantages. Besides providing an explanation for mass-energy equivalence, it can also explain why particles can be created in the vacuum and why a particle cannot travel faster than the speed of light.
- Finally, this wave interpretation can be tested in a proposed experiment.

Acknowledgements

I thank Prof. K. K. Phua for inviting me to write this review. I thank Ms. Lan FU for her assistance. This work was partially supported by the Macro-Science Program, Hong Kong University of Science and Technology (DCC18SC01) and the Hong Kong RGC grant (RMGS20SC01).

References

1. G. D. Coughlan and J. E. Dodd, *The Ideas of Particle Physics: An Introduction for Scientists* (Cambridge University Press, Cambridge England; New York, 1991).
2. M. S. Longair, *Theoretical Concepts in Physics: An Alternative View of Theoretical Reasoning in Physics* (Cambridge University Press, UK, 1984), pp. 259-273.
3. A. P. French, *Special Relativity* (W. W. Norton & Company, 1968), pp. 1-29.
4. L. B. Okun, *Phys. Today* **42** (1989) 31.
5. J. B. Hartle, *Gravity: An Introduction to Einstein's General Relativity* (Addison-Wesley, San Francisco, 2003).
6. H. E. Ives, *Science* **91** (1940) 79.
7. E. Whittaker, *A History of the Theories of Aether and Electricity* (Thomas Nelson and Sons Ltd, London, 1951).
8. W. L. Fadner, *Am. J. Phys.* **56** (1988) 114.
9. E. Hecht, *Am. J. Phys.* **79** (2011) 591.
10. M. Jammer, *Concepts of Mass in Contemporary Physics and Philosophy* (Princeton University Press, Princeton, N.J, 2000), pp. 62-89.
11. T. Rothman, Was Einstein the first to invent $E = mc^2$? *Sci. Am. e-publication* (2015) <https://www.scientificamerican.com/article/was-einstein-the-first-to-invent-e-mc2/>.
12. D. C. Chang, *J. Mod. Phys.* **9** (2018) 215.
13. J. J. Thomson, *Philos. Mag.* **11** (1881) 229.
14. O. Heaviside, *Philos. Mag.* **27** (1889) 324.
15. H. Poincaré, *Arch. Néerl. Sci. Exactes Nat.* **2** (1900) 252.
16. H. A. Lorentz, *Proc. R. Neth. Acad. Arts Sci.* **6** (1904) 809.
17. W. Wien, *Ann. Phys.* **310** (1901) 501.
18. M. Abraham, *Phys. Z.* **4** (1902) 57.
19. F. Hasenöhr, *Ann. Phys.* **15** (1904) 344.
20. F. Hasenöhr, *Ann. Phys.* **16** (1905) 589.
21. A. Einstein, *Ann. Phys.* **17** (1905) 639. English translation: A. Einstein, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* (Dover Publications, New York, 1952), pp. 69-71.
22. H. E. Ives, *J. Opt. Soc. Am.* **42** (1952) 540.
23. A. Einstein, *Ann. Phys.* **20** (1906) 627. English translation: "The principle of conservation of motion of the center of gravity and the inertia of energy" in *The Collected Papers of Albert Einstein, The Swiss Years:*

- Writings, 1900-1909, Vol. 2, Translated by Anna Beck*, eds. J. Stachel, D. C. Cassidy, J. Renn, and R. Schulmann (Princeton Univ. Press, 1990), pp. 200-206.
24. A. Einstein, *Ann. Phys.* **23** (1907) 371. English translation: "On the inertia of energy required by the relativity" in *The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1900-1909, Vol. 2, Translated by Anna Beck*, eds. J. Stachel, D. C. Cassidy, J. Renn, and R. Schulmann (Princeton Univ. Press, 1990), pp. 238-250.
 25. A. Einstein, *Phys. Z.* **10** (1909) 817.
 26. A. Einstein, *Ann. Phys.* **35** (1911) 898. English translation: "On the influence of gravitation on the propagation of light" in *The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1909-1911, Vol. 3, Translated by Anna Beck*, eds. M. J. Klein, A. J. Kox, J. Renn, and R. Schulmann (Princeton Univ. Press, 1990), pp. 379-387.
 27. A. Einstein, *Bull. Am. Math. Soc.* **41** (1935) 223.
 28. A. Einstein, *Technion Yearbook* **5** (1946) 16.
 29. R. Hazelett and D. Turner, *The Einstein Myth and the Ives Papers* (Devin-Adair Co, Old Greenwich, Conn., 1979).
 30. H. Arzelies, *Rayonnement et Dynamique du Corpuscule Charge Fortement Accelere* (Gauthier-Villars, Paris, 1966), pp. 74-79.
 31. E. Hecht, *Am. J. Phys.* **77** (2009) 799.
 32. D. C. Chang, *Optik* **174** (2018) 636.
 33. A. Messiah, *Quantum Mechanics, Vol. 1* (John Wiley & Sons, New York, 1965), pp. 47-48.
 34. M. Planck, *Verhandl. Dtsch. phys. Ges.* **2** (1900) 237.
 35. A. Einstein, *Ann. Phys.* **17** (1905) 132. English translation: A. Einstein, *Am. J. Phys.* **33** (1965) 367.
 36. C. J. Davisson and L. H. Germer, *Nature* **119** (1927) 558.
 37. G. P. Thomson and A. Reid, *Nature* **119** (1927) 890.
 38. R. Bach, D. Pope, S. Liou, and H. Batelaan, *New J. Phys.* **15** (2013) 033018.
 39. R. F. Egerton, *Physical Principles of Electron Microscopy* (Springer, 2016).
 40. D. C. Chang, arXiv preprint physics/0404044v2 (2017).
 41. A. Einstein, "Ether and the theory of relativity (speech at Univ. of Leiden, May 5, 1920)," in *The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1918-1921, Vol. 7, Translated by Alfred Engel*, eds. M. Janssen, R. Schulmann, J. Illy, C. Lehner, and D. K. Buchwald, (Princeton Univ. Press, 2002), pp. 160-182.
 42. P. A. M. Dirac, *Math. Proc. Camb. Philos. Soc.* **30** (1934) 150.
 43. P. W. Higgs, *Phys. Rev. Lett.* **13** (1964) 508.
 44. J. D. Barrow, *The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe* (Pantheon Books, New York, 2000).
 45. G. Altarelli, "Chapter 2. Gauge Theories and the Standard Model" in *Landolt-Börnstein - Group I Elementary Particles, Nuclei and Atoms, Volume 21A "Theory and Experiments"*, ed. H. Schopper (Springer-Verlag Berlin Heidelberg, 2008).
 46. E. Zeidler, *Quantum Field Theory III: Gauge Theory: A Bridge between Mathematicians and Physicists* (Springer Berlin Heidelberg, 2011).
 47. S. M. Carroll, *Something Deeply Hidden: Quantum Worlds and the Emergence of Spacetime* (Dutton, Boston, Mass, 2019).
 48. A. H. Guth and D. I. Kaiser, *Science* **307** (2005) 884.
 49. X. Wen, *Phys. Rev. D* **68** (2003) 065003.
 50. L. Boi, *The Quantum Vacuum: A Scientific and Philosophical Concept, from Electrodynamics to String Theory and the Geometry of The Microscopic World* (Johns Hopkins University Press, Baltimore, 2011).
 51. K. Huang, *A Superfluid Universe* (World Scientific, New Jersey, 2017).
 52. D. C. Chang and Y. Lee, *J. Mod. Phys.* **6** (2015) 1058.
 53. M. S. Longair, *Theoretical Concepts in Physics: An Alternative View of Theoretical Reasoning in Physics* (Cambridge University Press, Cambridge; New York, 1984), pp. 37-59.
 54. D. C. Chang, arXiv preprint physics/0505010v2 (2017).
 55. D. C. Chang, *J. Mod. Phys.* **4** (2013) 21.
 56. A. Messiah, *Quantum Mechanics* (John Wiley & Sons, New York, 1965), pp. 45-59.
 57. A. Einstein, *Jahrbuch der Radioaktivität und Elektronik* **4** (1907) 411. English translation: "On the relativity principle and the conclusions drawn from it" in *The Collected Papers of Albert Einstein, The Swiss Years: Writings, 1900-1909, Vol. 2, Translated by Anna Beck*, eds. J. Stachel, D. C. Cassidy, J. Renn, and R. Schulmann (Princeton Univ. Press, 1990), pp. 252-311.
 58. D. C. Chang, *Chin. Phys. B* **26** (2017) 040301.

59. O. Klein, *Z. Phys.* **37** (1926) 895.
60. W. Gordon, *Z. Phys.* **40** (1926) 117.
61. P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1981), pp. 253-276.
62. W. N. Cottingham and D. A. Greenwood, *An Introduction to the Standard Model of Particle Physics* (Cambridge University Press, Cambridge, 1998), pp. 71-72.
63. J. J. Sakurai, *Advanced Quantum Mechanics* (Addison-Wesley, Reading, 1973), pp. 75-89.
64. A. A. Michelson and E. W. Morley, *Am. J. Sci.* **34** (1887) 333.
65. A. Einstein, *Ann. Phys.* **17** (1905) 891. English translation: A. Einstein, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* (Dover Publications, New York, 1952), pp. 37-65.
66. W. Kaufmann, *Göttinger Nachrichten* **2** (1901) 143.
67. W. Kaufmann, *Phys. Z.* **4** (1902) 54.
68. A. H. Bucherer, *Ann. Phys.* **333** (1909) 513.
69. M. M. Rogers, A. W. McReynolds, and F. T. Rogers, *Phys. Rev.* **57** (1940) 379.
70. P. S. Faragó and L. Jánossy, *Nuovo Cim* **5** (1957) 1411.
71. W. Bertozzi, *Am. J. Phys.* **32** (1964) 551.
72. D. C. Chang, *Eur. Phys. J. Plus* **132** (2017) 140.