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Authors	Yi, Zelong; Yu, Man; Cheung, Ki Ling
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Impacts of Counterfeiting on a Global Supply Chain

Zelong Yi

Department of Transportation Economics and Logistics Management, College of Economics, Shenzhen University, Nanhai Ave
3688, Shenzhen, Guangdong, China, 518060, yizl@szu.edu.cn

Man Yu

Department of Information Systems, Business Statistics and Operations Management,
The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, manyu@ust.hk

Ki Ling Cheung

Department of Information Systems, Business Statistics and Operations Management,
The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, imcheung@ust.hk

Problem Definition. This paper investigates how a global supply chain is influenced by counterfeits and how the supply chain should effectively take anti-counterfeit actions.

Academic/Practical Relevance. Nowadays impacts of counterfeiting have been increasingly profound on global supply chains. It is critical to understand how counterfeiting impacts supply chains when supply chain members act in their own interests, and how supply chains can effectively combat counterfeiting when all the members can contribute to it. This is the first paper that offers insights into these important questions. In particular, we examine who, among the supply chain members, is in the best position to perform counteracting activities, how these members can cooperate in anti-counterfeiting, and what are the economic implications of the anti-counterfeit actions to the supply chain, individual firms, consumer surplus, and social welfare.

Methodology. We consider a supply chain consisting of a manufacturer and a retailer, and analyze a game-theoretical framework to derive the equilibrium.

Results. The manufacturer prefers to induce the retailer to combat counterfeits rather than to combat himself. Contrary to conventional wisdom, counterfeits can increase the supply chain's profit even in absence of network externality effects. The crux is that the manufacturer lowers wholesale price to incentivize the retailer's counteraction and, consequently, the threat of counterfeits can mitigate double marginalization and benefit the supply chain.

Managerial Implications. Our results demonstrate that a sustainability risk can trigger collaborative endeavors of supply chain members and, thus, be advantageous to the supply chain. The findings also underscore the important role that retailers should play in anti-counterfeiting. Particularly, it can be in the supply chain's interest that the manufacturer does not execute the counteraction, either jointly with the retailer or by himself.

Key words: counterfeit; anti-counterfeit actions; supply chain

1. Introduction

A bright brand image is the most valuable asset for many global firms. It usually serves as a guarantee to consumers about superior product quality and to firms about huge sales and profits. It is estimated that the top 100 brands are collectively worth as much as \$2.6 trillion (Economist.com 2013). Unfortunately, brand-name products are also frequent targets of counterfeits, i.e., similar copies of brand-name goods offered in the marketplace by competing firms, or “counterfeiters”, to take advantage of the authentic brands. The motivation behind commercial counterfeiting is very clear: free-riding on the economic value associated with the brand ownership.

While counterfeited products used to be largely restricted to high-end watches, fashion clothes, designer apparels, accessories, precious jewelries and music recordings, nowadays counterfeiting has become a ubiquitous problem in a wide range of product categories, including pharmaceutical drugs, infant milk powder, automotive parts, computer softwares and aerospace components (Berman 2008). It is reported that illegal imitation of brand-name products costs companies billions of dollars annually and the situation is rapidly aggravating. In 2012, International Anti Counterfeiting Coalition estimates that “[c]ounterfeiting is a \$600 billion a year problem. In fact, it’s a problem that has grown over 10,000% in the past two decades...” (Chaudhry and Zimmerman 2013).

One particular negative impact of counterfeiting is that it may smear image of the genuine brand (Scorpecci 2009), as it ruins uniqueness of the authentic products and thus reduces consumers’ willingness to pay. With such “fake-averse” consumers, market potential for the brand-name owners diminishes due to counterfeits. Another type of consumers consider only the authentic products because the counterfeits are not morally accepted by them. Bloch et al. (1993), Shy and Thisse (1999) and Kwong et al. (2003) show that consumers’ attitudes towards counterfeiting largely influence their willingness to buy counterfeits as well as decisions of authentic-product distributors.

Although counterfeiting takes place in almost all economies, market environment for counterfeiting varies significantly across countries. In particular, counterfeiting technology, public attitudes towards counterfeits, and law enforcements vary from economy to economy. For example, if a long-standing tradition of consuming luxury products exists in a country, it is very likely that counterfeiting technology of luxury goods in this country is more advanced than that in other countries. In light of these market differences, many firms collaborate with local distributors and/or retailers to fight against counterfeit products.

Olson and Granzin (1992) articulate three reasons regarding why anti-counterfeit efforts are more effective if they are through dealer/retailer networks. First, retailers are often in the forefront position to probe fake products and provide information to prosecutors for taking legal actions. Second, they are in a key position to screen out fake products before these products are sold to customers leading to dreadful consequences. Third, they are the ones to interact with and educate customers about

the seriousness and importance of discerning fake products. Because of the strategic position of the retailers, it is imperative for manufacturers to incentivize them to join hands in fighting counterfeits.

The giant on-line retailer, Amazon.com, presents a compelling example. Well aware of the potential ramifications of counterfeiting, it has undertaken a vigorous fight against fake products and “invests heavily - both funds and company energy” to tackle online counterfeiting, despite the fact that Amazon does not produce any of the products. In particular, it has successfully taken legal actions against some counterfeiters (Retail News Insider, 2018). Similarly, the Chinese largest e-retailer, Alibaba invests around 100 million Chinese yuan annually in anti-counterfeiting and assisted in the closure of more than 1,400 counterfeit manufacturing locations in 2016 (China Daily, 2017).

As counterfeiting causes serious threats to brand-owners’ profitability and sustainability in global markets, it is increasingly important to understand how counterfeiting impacts supply chains when supply chain members act in their own interests, and how supply chains can effectively combat counterfeiting when all supply chain members can contribute to it. However, to our best knowledge, no prior study addresses these important questions. In particular, it is unclear who, among the supply chain members, is in the best position to take the responsibility of counteracting counterfeits (from the standpoint of either the brand owner or the entire supply chain), how these members can cooperate in their anti-counterfeit actions, what are the economic implications of the anti-counterfeit actions to the supply chain, individual firms, consumer surplus, and social welfare, and how these results rely on marketing conditions such as customers’ attitudes, forms of combat cost, etc. In this paper, we examine these important issues in a dynamic model.

Specifically, we examine a supply chain where a global manufacturer (i.e., a brand owner) distributes his products through a local retailer who faces the threat of a local counterfeiter in the end market. The presence of both the retailer and the counterfeiter effectively carves the market into two interrelated submarkets: a primary market of genuine products, and a secondary market of counterfeits. Since the counterfeit is an illegal imitation of the authentic products, it can be removed (totally or partially) with the help of regulatory institutions (Financial Times 2011). Typical actions for this removal include patrolling sales spots so that counterfeit dealers cannot easily operate their businesses, taking counterfeiters to court, and enforcing closure of workshops or factories that supply counterfeits. These actions are always associated with some costs, i.e., combat expenses, such as investigations and related research, assistance to governments, legal proceedings against counterfeiters, and so on. In our model the combat expenses are proportional to the firms’ anti-counterfeiting efforts and whoever takes the counteractions bears the combat cost. The manufacturer chooses the wholesale price and also decides whether to take charge of the anti-counterfeiting actions himself or entrust the retailer. The retailer determines the retail price, as well as her combating efforts if she has the right to make the decision. On the other hand, if the manufacturer chooses to take charge himself, he decides his own combating efforts, and the retailer determines only the retail price.

Our model leads to several interesting findings. First, ironically and rather surprisingly, contrary to the conventional wisdom that counterfeiting is detrimental to firms offering authentic products, we show that the threat of counterfeits can increase the supply chain's profit. This is because, due to the presence of counterfeiting, the manufacturer may lower the wholesale price to induce the retailer to take combating actions. As a result, the counterfeit serves as a coordination device to reduce double marginalization in the supply chain. When the reduction in supply chain inefficiency outweighs the combat expense, counterfeiting leads to a higher profit for the supply chain. Likewise, the possibility of having counterfeits can also add positive values to consumer surplus and social welfare. The main mechanisms that drive these results are two-fold: first, a decentralized supply chain in which both parties independently determine prices and optimize their own profits, leading to double marginalization; second, the presence of direct combat as an option for the supply chain to cope with counterfeits. We elaborate on these mechanisms later.

While a majority of the literature regarding benefits of counterfeits focus on positive demand-side externalities, our finding identifies a fundamentally different source of positive values for counterfeits, from the supply side and taking the perspective of a supply chain. To our knowledge, this is the first paper regarding benefits of a sustainability risk (i.e., counterfeiting) to a supply chain.

Second, compared to the manufacturer, the retailer tends to be in a preferable position to combat counterfeits, from the viewpoint of either the manufacturer or the supply chain. This result is valid even if the retailer does not have any advantage over the manufacturer regarding combating costs. This is related to the fact that the combat cost is effectively shared by the two firms when the manufacturer induces the retailer's counteraction (via a reduction in the wholesale price), while it is solely borne by the manufacturer when he takes the combating actions himself.

Third, while the firms' collaboration on anti-counterfeiting is crucial, their simultaneous engagement in combating activities may not be desirable to the brand-name supply chain, as it can dampen the manufacturer's incentive to lower the wholesale price in response to the threat of counterfeiting.

Last but not least, we confirm our main results, mostly by analytical proofs and also with numerical examples, in several variants of the base model, which incorporate three counterfeit-induced consumer behavior (ethical consumers, brand-value loss, and status utility), and extra operational complexities such as a higher production cost for the authentic product than the counterfeit, counterfeiter's endogenous choice of quality, non-linear combat costs, and reactive combating actions. In these model extensions we also derive insights regarding the effects of some key parameters on the supply chain's profits and anti-counterfeiting strategies.

2. Literature Review

Our paper expands the literature on counterfeits. Most of the earlier studies on counterfeits are empirical and descriptive. Based on survey data from fashionable luxury products, Yoo (2005) investigates

the extent to which counterfeit items promote sales of genuine ones. Jacobs et al. (2001) provide a summary of protective measures against counterfeits in practice, including labeling, technological controls, protection by law, and so on. Wilcox et al. (2009) show that social motivations greatly influence consumers' desire for counterfeited luxury brands. Qian (2008) empirically examines how counterfeits impact authentic-product producers in terms of product quality and self-enforcing investment. By analyzing software sales and piracy data from fifty countries between 1996 and 2002, Gu and Mahajan (2005) examine the impact of anti-piracy actions on industry-level sales. Different from these papers, we explore the counterfeiting issue from an analytical perspective.

A stream of analytical research examines how counterfeits influence authentic product manufacturers' profits and how they respond to counterfeiters. Chen and Png (2003) examine how to exert efforts in monitoring, detecting, and prosecuting copyright violation cases for brand-product owners. Sun et al. (2010) examine a global firm's optimal strategy of technology transfer to an emerging market, where manufacturing cost is lower but competition from a local imitator is present. Zhang et al. (2012) show that counterfeits negatively influence the authentic products and to combat counterfeits, a brand-name firm has a stronger incentive to improve her own perceived quality than to affect that of the counterfeit. Lahiri and Dey (2013) prove that, in certain situations, lower piracy enforcement increases the monopolistic firm's incentive to invest in quality, which is contrary to common knowledge that inadequate piracy enforcement hinders product development. Cho et al. (2015) analyze optimal decisions on quality, price and product type for a counterfeiter and provide insights into conditions when anti-counterfeiting strategies, such as quality improvement, price reduction and marketing campaign are effective. Qian et al. (2015) analyze the effects of counterfeiting on goods with both searchable and experiential quality. They characterize how the authentic product provider should differentiate from counterfeiters by the quality levels. Zhang and Zhang (2015) identify the optimal supply chain structure in the presence of deceptive counterfeits. Gao et al. (2017) suggest that copycats with a high physical resemblance and yet a low quality are more likely to defy the deterrence of the incumbent and successfully enter the market. Pun and Deyong (2017) characterize a manufacturer's optimal quality decision when it competes with a copycat in selling to strategic customers. Gao (2018) examines the situation where an authentic drug producer can combat counterfeiters by overt anti-counterfeiting technologies and analyzes the effects of increased technology complexity.

Different from the aforementioned papers, we examine direct combat as an option for a supply chain to cope with counterfeits. That is, a supply chain can legally wipe out, at some cost, an illicit counterfeit from (a portion of) the market. We analyze the supply chain's active decision on direct combat and explore its strategic implications to the entire supply chain. The direct-combat option also differentiates our work from the limit-pricing literature (a detailed review of this literature is provided by Hall 2008), where a lower price is adopted as a strategic deterrence to entry of competitors (see Gao et al. 2017 for an application of limit pricing in anti-counterfeiting).

Akin to our paper, several existing studies identify situations where authentic products may become better off by counterfeits. A majority of the literature regarding the benefits of counterfeits center on positive demand-side externalities. For example, Conner (1995) shows that counterfeiting may be valuable to a firm by expanding the overall user base, given network externality effects, i.e., consumers' utility increases in the number of users of either the authentic or the counterfeited product. Xie and Sirbu (1995) show that competitive entry by firms offering compatible products can facilitate product diffusion and thus benefit incumbent monopolies in markets with strong network effects. By examining a diffusion model, Givon et al. (1995) point out that software piracy can be beneficial to the authentic software, since, through word-of-mouth interactions, piracy may significantly accelerate the diffusion of the authentic software and induce more users to adopt the products. Jain (2008) shows that, in a competitive environment, brand-name companies can become better off by piracy, since copying by more price-sensitive consumers can lead to less intensity in price competition.

In contrast to all the aforementioned papers, we identify a different source of positive values for counterfeits, from the viewpoint of a supply chain. We show that a threat of counterfeiting can promote collaborative endeavors of supply chain members and, thus, be advantageous to the supply chain.

3. Model

Consider a brand-name global company selling a product in a foreign country. The manufacturer entrusts a local retailer, who has the know-how of running business in the local market, to distribute the product to end consumers. The brand-name product becomes the target of a local counterfeiter. The counterfeited product is non-deceptive¹ and its attributes are known to all consumers. For example, a pair of "Nike" shoes made of cheap materials and sold for \$10 in a wet market can be easily identified by consumers as a counterfeit (Cho et al. 2015). The brand-name channel members are aware of the counterfeiting issue, and make decisions on the price of the product and on the anti-counterfeit strategy.

The counterfeit differs from the authentic product in quality. Let the quality of the authentic product be $q_a = q$ and that of the counterfeit be $q_f = \beta q$. The coefficient $\beta \in (0, 1)$ represents similarity or substitutability between the authentic product and the counterfeit. It also reflects the technology level of counterfeiting in the local market. The value of β is assumed to be exogenous in the base model. In §6.2.3 we examine the situation where the counterfeiter has some flexibility in choosing the counterfeit quality. We assume that the production costs (fixed or marginal costs) for the authentic product and the counterfeit are equal and both are normalized to zero. This assumption is relaxed in an extension in §6.2.1, where the two goods differ not only in quality, but also in production cost.

¹ Grossman and Shapiro (1988a, 1988b) categorize counterfeits into non-deceptive and deceptive ones, where non-deceptive counterfeits are those for which consumers are aware of the origin and the inferior quality, while deceptive counterfeits are those which consumers buy in faith and honesty without knowing the true quality. In this paper, we follow a majority of analytical papers on counterfeiting to consider non-deceptive counterfeits.

A unique feature of our model is that the counterfeit can be partially or completely removed from the market by the brand-name firms' counteracting efforts. In the base model we assume that the efforts can only be exerted by one firm. This naturally leads to two situations differing in the decision-maker of the anti-counterfeit actions. In the first case which we label as DM, only the manufacturer may combat and he decides the combating efforts to maximize his profits, while in the other case denoted as DR, the retailer decides whether and how much combating efforts to spend. As the brand owner, the manufacturer retains the decision power of which approach, DM or DR, to adopt. In either case, the combat cost is paid by the firm who takes the anti-counterfeit actions. As a benchmark (B), we study in §4 the case of a centralized chain, where a central decision-maker makes all the decisions. In §6.1 we consider an extension where both firms may combat and show that sharing combating efforts may turn out to be detrimental to the channel profits.

Assume that the anti-counterfeit efforts can be monetarily characterized. Specifically, let the expense to completely remove the counterfeit be $c(\beta)$. If $\gamma c(\beta)$ is spent, then in a γ portion of the market, counterfeits are disallowed to disseminate and only authentic products are available for sales. That is, consumers can buy a fake product only in the $(1 - \gamma)$ portion of the market. When γ approaches 1, the anti-counterfeit actions span the entire market and the fake product is completely eliminated; when γ approaches 0, no anti-counterfeit action is taken by either firm and the fake product can be purchased everywhere. We focus on linear combating cost in the base model. In §6.2.2, we consider the case where combating costs are non-linear functions of the combating efforts.

The brand-name supply chain and the counterfeiter independently determine the selling prices for their products. We assume that the supply chain's pricing decision precedes that of the counterfeiter (similar to Cho et al. 2015). This is because the authentic product is usually an incumbent in the market and, as an imitator, the counterfeiter fabricates its product and sets its price after observing the products and prices offered by the authentic-product supply chain. Let the retail prices of the authentic product and of the fake product be p_a and p_f , respectively.

On the demand side, without loss of generality, we normalize the market size to one and assume that each consumer demands at most one unit, either the authentic or the counterfeited product. A consumer has the options of purchasing the authentic product (choice a), purchasing the fake product (choice f), or not buying (choice 0). Given the prices, surpluses from purchasing either product and that from buying nothing are $U_i = \theta q_i - p_i$ for $i = \{a, f\}$ and $U_0 = 0$, respectively, where θ represents the consumers' quality sensitivity. Consumers' quality sensitivity may depend on various factors such as wealth level, social status and personal preference. To capture consumer heterogeneity in quality sensitivity, we assume that θ is uniformly distributed between 0 and 1, i.e., $\theta \sim U(0, 1)$. This consumer-utility model is widely used in marketing literature (see, for example, Zeithammer and Thomadsen 2013 and Shi et al. 2013). In §6.3.1, §6.3.2, and §6.3.3 we further incorporate three

distinctive phenomena of consumer behavior induced by counterfeiting: ethical consumers, brand value loss, and status utility. These phenomena capture heterogeneity in consumers' attitudes towards counterfeits, as well as social status associated with brand-name goods. We prove the robustness of our main results under these phenomena, and derive insights into the effects of each phenomenon.

We consider a dynamic game between the authentic-product supply chain and the counterfeiter. The sequence of events is depicted in Figure 1. The manufacturer, as the brand owner, first chooses whom to combat (i.e., DM versus DR) and then determines a wholesale price (denoted by ω). If DM is chosen, then the manufacturer decides the extent to combat, γ . After observing the manufacturer's combating choice and wholesale price, the retailer decides and announces the retail price p_a , as well as the combat scope γ if DR is chosen. Following that, the counterfeiter (if not completely eliminated from the market) determines her own price p_f . Finally, consumers decide which product (if any) to buy and sales are realized. Note that the firms' anti-counterfeit decision precedes the setting of the retail price. This assumption recognizes that anti-counterfeit actions typically involve long-term strategic moves,² whereas pricing decisions are normally short-term tactical tools that can be adjusted easily. In our model the manufacturer and the retailer independently determine prices to optimize their own profits. This assumption reflects the common practice of decentralized pricing in distribution and retail businesses, which in many countries is enforced by laws (see, for example, Pinsent Masons 2017 and Federal Trade Commission 2020).

Figure 1 Sequence of events: M (Manufacturer) and R (Retailer)

4. Benchmark (B): Centralized Chain

Many global companies like Zara, Armani, and Zegna are vertically integrated and manage their own distribution networks, even across borders (Economist.com 2009). We first study this benchmark with a centralized chain. In this case, the seller first determines the combat level γ and the retail price p_a and subsequently the counterfeiter sets the counterfeit price p_f .

² For example, anti-counterfeit actions typically involve preparation such as coordination with the local government and/or hiring professional agents.

Recall that after the seller spends the combat efforts, only the authentic products are available in a γ portion of the market and both types of products are available in the remaining $1 - \gamma$ portion of the market. Hence, in the γ portion of the market, a consumer buys one authentic product if $U_a \geq 0$; in the remaining $1 - \gamma$ portion, she buys one authentic product if $U_a \geq U_f$ and $U_a \geq 0$ and buys a counterfeit if $U_a < U_f$ and $U_f \geq 0$. Thus, demands for the authentic product and the counterfeit are,

$$\begin{aligned} D_a(\gamma, p_a, p_f) &= \gamma \text{Prob}(U_a \geq 0) + (1 - \gamma) \text{Prob}(U_a \geq U_f, U_a \geq 0) \\ &= \gamma \left(1 - \frac{p_a}{q}\right)^+ + (1 - \gamma) \left(1 - \left(\frac{p_a - p_f}{q(1 - \beta)}\right)^+\right)^+, \end{aligned} \quad (1)$$

and

$$D_f(\gamma, p_a, p_f) = (1 - \gamma) \text{Prob}(U_a < U_f, U_f \geq 0) = (1 - \gamma) \left(\min \left\{ \left(\frac{p_a - p_f}{q(1 - \beta)}\right)^+, 1 \right\} - \min \left\{ 1, \frac{p_f}{\beta q} \right\} \right)^+, \quad (2)$$

respectively, where $x^+ = \max\{x, 0\}$. Clearly, the demand for each type of product decreases (increases) in its own price (quality) and increases (decreases) in the price (quality) of the other product.

We solve the problem by backward induction. First, the counterfeiter determines the counterfeit price $p_f^*(\gamma, p_a)$ by maximizing

$$\pi_f(p_f; \gamma, p_a) = p_f D_f(\gamma, p_a, p_f) = p_f (1 - \gamma) \left(\min \left\{ \left(\frac{p_a - p_f}{q(1 - \beta)}\right)^+, 1 \right\} - \min \left\{ 1, \frac{p_f}{\beta q} \right\} \right)^+. \quad (3)$$

Anticipating the counterfeit price $p_f^*(\gamma, p_a)$, the seller decides γ^* and p_a^* to maximize

$$\pi_a(\gamma, p_a) = p_a D_a(\gamma, p_a, p_f^*(\gamma, p_a)) = p_a \left[\gamma \left(1 - \frac{p_a}{q}\right)^+ + (1 - \gamma) \left(1 - \left(\frac{p_a - p_f^*(\gamma, p_a)}{q(1 - \beta)}\right)^+\right)^+ \right] - \gamma c(\beta). \quad (4)$$

In the following lemma, we characterize the seller's optimal decisions in the benchmark:

LEMMA 1. Consider a centralized chain. Let $\hat{q}^B = \frac{4c(\beta)(2-\beta)}{\beta}$. When $q \geq \hat{q}^B$, $\gamma^* = 1$; otherwise $\gamma^* = 0$. The corresponding optimal price p_i^* , the demand D_i^* and the profit π_i^* are presented in Table 1.

q	γ^*	p_a^*	p_f^*	D_a^*	D_f^*	π_a^*	π_f^*
$q < \hat{q}^B$	0	$\frac{1-\beta}{2-\beta}q$	$\frac{\beta(1-\beta)}{2(2-\beta)}q$	$\frac{1}{2}$	$\frac{1}{2(2-\beta)}$	$\frac{1-\beta}{2(2-\beta)}q$	$\frac{\beta(1-\beta)}{4(2-\beta)^2}q$
$q \geq \hat{q}^B$	1	$\frac{q}{2}$	—	$\frac{1}{2}$	—	$\frac{q}{4} - c(\beta)$	—

Table 1 Optimal results for the centralized chain

By Lemma 1, the seller's optimal strategy is bang-bang, i.e., either eradicating the counterfeits from the entire market, or tolerating its presence everywhere. This is because the seller's marginal benefit from increasing counteracting effort is greater when she starts from a higher combating level, resulting

in a convex revenue function. To see why, note that extra combating effort expands the market segment where the seller enjoys monopolistic power, and allows her to raise the price for the authentic product. Such a price increase, on the other hand, induces some consumers to switch to buy the counterfeits wherever they are available for sales. This competitive effect is weaker when the counterfeits are constrained to a smaller market portion. Hence, when the market is less afflicted with bogus products (i.e., when the starting combating level is higher), the seller is less concerned about competition and, thus, is able to set an even higher mark-up as a reward for her additional combating effort. The resultant convex revenue, together with the linear combating cost, leads to the all-or-nothing strategy. The case with non-linear combating cost is examined in §6.2.2.

Lemma 1 further shows that the firm combats the counterfeit only if her product's quality is high enough (when $q \geq \hat{q}^B$). This is driven by two facts: First, the firm may not be able to generate enough revenues to cover the combat expense when the authentic product's quality is low. Second, when the authentic product's quality is low, for a given β , the threat from counterfeits, whose quality is also low, is insignificant and hence it is not worthwhile to take anti-counterfeit actions.

It is worth noting that the counterfeit always hurts the firm in the centralized chain regardless of the anti-counterfeiting actions. To see why, note from Table 1 that, if no anti-counterfeit action is taken (i.e., $q < \hat{q}^B$), the counterfeit does not affect the authentic product's demand, which is always $\frac{1}{2}$. However, the infliction of the counterfeit lowers the authentic product's retail price from $\frac{q}{2}$ to $\frac{1-\beta}{2-\beta}q$, which leads to profit loss for the brand-name company.³ When anti-counterfeit actions are taken (i.e., $q \geq \hat{q}^B$), a monopolistic position is retained, which is the same as the no-counterfeit case, yet at a positive anti-counterfeit cost. Hence, a loss in profit is also incurred. In a decentralized system, however, the authentic product channel may get better off by counterfeits, as we show next.

5. Decentralized Chain

In practice, many brand-name manufacturers collaborate with local retailers to distribute products in emerging markets to take advantage of the retailers' hands-on knowledge about the local markets. However, due to double marginalization, the retailer's incentive may not be fully aligned with the one of the supply chain. We examine such a decentralized supply chain in this section.

To facilitate our analysis, consider two simple benchmarks in a decentralized chain. The first benchmark is a standard bilateral monopoly in the absence of counterfeits. Recall the equilibrium outcome for this channel: the wholesale price and retail price are $\frac{q}{2}$ and $\frac{3q}{4}$, respectively, resulting in the manufacturer's profits being $\frac{q}{8}$ and retailer's profits $\frac{q}{16}$. The second benchmark is the case when counterfeits are present and neither the manufacturer nor the retailer combats. It is easy to solve for the equilibrium in this case: the wholesale, retail, and counterfeit prices are $\omega^* = \frac{1-\beta}{2-\beta}q$, $p_a^* = \frac{3(1-\beta)}{2(2-\beta)}q$, and

³ With the counterfeit, the firm's profit is $\frac{1-\beta}{2(2-\beta)}q$. Without the counterfeit, we have shown that the retail price and the demand are $\frac{q}{2}$ and $\frac{1}{2}$ respectively, and the profit is $\frac{q}{4}$. As a result, the profit loss is $\frac{\beta}{4(2-\beta)}q$.

$p_f^* = \frac{3\beta(1-\beta)}{4(2-\beta)}q$, respectively, and the corresponding profits for the manufacturer, the retailer, and the counterfeiter are $\pi_M^* = \frac{1-\beta}{4(2-\beta)}q$, $\pi_R^* = \frac{1-\beta}{8(2-\beta)}q$, and $\pi_f^* = \frac{9\beta(1-\beta)}{16(2-\beta)^2}q$. In the brand-name product channel, given that the other member does not combat, this case specifies the minimum profits that a member can secure without combating. Due to infliction of the counterfeit, the profits of both firms and the channel as a whole are lower in the second benchmark, compared to the counterparts in the first one.

5.1. Manufacturer's Decision on Combating Efforts (DM)

We start with the case where the manufacturer takes charge of the combating actions himself (DM). Under this approach, the manufacturer decides the extent to combat γ , and the wholesale price ω , and then the retailer determines the retail price p_a . Subsequently, the counterfeiter determines the price p_f . The demands for the two types of products are presented in (1) and (2), respectively. Analogous to the benchmark case, at the third stage of the game, $p_f^*(\gamma, p_a)$ is obtained from (3). Then, for given γ and ω , the retailer sets $p_a^*(\gamma, \omega)$ to maximize

$$\pi_R(p_a; \gamma, \omega) = (p_a - \omega) \left[\gamma \left(1 - \frac{p_a}{q} \right)^+ + (1 - \gamma) \left(1 - \left(\frac{p_a - p_f^*(\gamma, p_a)}{q(1-\beta)} \right)^+ \right)^+ \right]. \quad (5)$$

Finally, at the first stage of the game, the manufacturer determines γ^* and ω^* to maximize

$$\pi_M(\gamma, \omega) = \omega \left[\gamma \left(1 - \frac{p_a^*(\gamma, \omega)}{q} \right)^+ + (1 - \gamma) \left(1 - \left(\frac{p_a^*(\gamma, \omega) - p_f^*(\gamma, p_a^*(\gamma, \omega))}{q(1-\beta)} \right)^+ \right)^+ \right] - \gamma c(\beta). \quad (6)$$

The optimal anti-counterfeit strategy under DM is provided in the next proposition:

PROPOSITION 1. *Assume that the manufacturer chooses DM. Let $\hat{q}^M = \frac{8c(\beta)(2-\beta)}{\beta}$. $\gamma^* = 1$ if $q \geq \hat{q}^M$; otherwise $\gamma^* = 0$. The corresponding optimal price p_i^* and the profit π_i^* are presented in Table 2.*

q	γ^*	p_a^*	p_f^*	ω^*	π_R^*	π_M^*	π_f^*
$q < \hat{q}^M$	0	$\frac{3(1-\beta)}{2(2-\beta)}q$	$\frac{3\beta(1-\beta)}{4(2-\beta)}q$	$\frac{1-\beta}{2-\beta}q$	$\frac{1-\beta}{8(2-\beta)}q$	$\frac{1-\beta}{4(2-\beta)}q$	$\frac{9\beta(1-\beta)}{16(2-\beta)^2}q$
$q \geq \hat{q}^M$	1	$\frac{3}{4}q$	—	$\frac{q}{2}$	$\frac{q}{16}$	$\frac{q}{8} - c(\beta)$	—

Table 2 Equilibrium results under DM

Under DM, the threshold above which anti-counterfeit actions are taken, $\hat{q}^M$, is higher than that of the integrated supply chain, $\hat{q}^B$. This means that the brand owner is less stringent on combating counterfeits when he sells products through a local retailer. Due to double marginalization, the manufacturer cannot fully enjoy the gain from anti-counterfeiting in the decentralized case.⁴ Thus, everything else being equal, the manufacturer selling through the decentralized supply chain has a weaker incentive to take anti-counterfeit actions.

⁴ To be more specific, note that, from Table 1 and Table 2, gains from taking anti-counterfeit actions in the centralized chain and in the decentralized DM case are $\frac{\beta}{4(2-\beta)}q - c(\beta)$ and $\frac{\beta}{8(2-\beta)}q - c(\beta)$, respectively. The former is always higher.

We also explore the impact of counterfeiting on firms' profit. When the authentic product's quality is low (i.e., $q < \hat{q}^M$), the manufacturer does not take anti-counterfeit actions. Because of the competition from counterfeit, both firms are worse off.⁵ On the other hand, when the authentic product's quality is high (i.e., $q \geq \hat{q}^M$), the manufacturer takes anti-counterfeit actions and the counterfeit is completely eliminated from the market. In this case, the wholesale price is $\frac{q}{2}$, identical to that without counterfeits. This is because, when the counterfeit is completely eradicated, the manufacturer faces the same market as one without counterfeits and the combat expense is a fixed cost. Thus, under the same wholesale price, the manufacturer cannot recover any of the anti-counterfeit expense from the retailer while the retailer freely enjoys the benefit from anti-counterfeit efforts made by the manufacturer. As a result, the manufacturer incurs a loss in profit ($c(\beta)$) due to counterfeiting while the retailer obtains the same profit as in the case without counterfeits. We will show that the results contrast sharply with those in the case when the retailer is entrusted to make the combating decision (DR).

5.2. Retailer's Decisions on Combating Efforts (DR)

We now proceed to examine the scenario where the retailer decides her efforts to combat the counterfeit. Following backward induction, we first characterize the retailer's optimal decision, given the manufacturer's strategy. Fixing the wholesale price ω , the retailer decides her combat level γ and the retail price p_a , with expectation of the counterfeiter's best reaction $p_f^*(\gamma, p_a)$, where $p_f^*(\gamma, p_a)$ maximizes the counterfeiter's profit:

$$\pi_f(p_f; \gamma, p_a) = p_f(1 - \gamma) \left(\min \left\{ \left(\frac{p_a - p_f}{q(1 - \beta)} \right)^+, 1 \right\} - \min \left\{ 1, \frac{p_f}{\beta q} \right\} \right)^+, \quad (7)$$

and the retailer's profit function is

$$\pi_R(\gamma, p_a; \omega) = (p_a - \omega) \left[\gamma \left(1 - \frac{p_a}{q} \right)^+ + (1 - \gamma) \left(1 - \left(\frac{p_a - p_f^*(\gamma, p_a)}{q(1 - \beta)} \right)^+ \right)^+ \right] - \gamma c(\beta). \quad (8)$$

Denote the retailer's optimal price and combating effort level by $p_a^*(\omega)$ and $\gamma^*(\omega)$, respectively. The lemma below characterizes the condition under which the retailer finds it optimal to combat.

LEMMA 2. *Given ω , there exists a critical function $\bar{q}(\omega)$ such that if $q \geq \bar{q}(\omega)$, then $\gamma^*(\omega) = 1$; otherwise $\gamma^*(\omega) = 0$. Moreover, $\bar{q}(\omega)$ increases in ω .*

Intuitively, under the DR setting, the retailer in a decentralized chain faces exactly the same decision problem as a centralized firm does, except that the per-unit cost is higher at ω . Thus, similar to a centralized firm, the retailer combats only if the brand-name product's quality is sufficiently high, and whenever she does combat, she eradicates the fake products completely from the market.

⁵ From Table 2, the manufacturer's profit and the retailer's profit are $\frac{1-\beta}{8(2-\beta)}q$ and $\frac{1-\beta}{4(2-\beta)}q$, respectively. Recall that without counterfeiting, the manufacturer's profit and the retailer's profit are $\frac{q}{16}$ and $\frac{q}{8}$, respectively.

Note that a lower wholesale price reinforces the retailer's incentive to combat counterfeits. This is evinced by the result that the threshold of q above which counteracting action is taken (i.e., $\bar{q}(\omega)$) increases in ω . The rationale is as follows. A lower wholesale price is equivalent to a lower marginal cost for the retailer. Although a reduction in the wholesale price always improves the retailer's profit, the profit increase is most significant when the retailer is a monopoly in the market. The presence of counterfeiter disallows the retailer to fully take advantage of the lower wholesale price, as any adjustment in her retail price triggers strategic responses of the counterfeiter and of the consumers. Consequently, the retailer is more eager to eliminate competition under a lower wholesale price.

Several special cases are worth noting. First, if the quality q is very low, i.e., $q < \bar{q}(0) = \hat{q}^B$, then the retailer would not combat the counterfeit even with a zero wholesale price. The equilibrium outcome is then the same as the second benchmark mentioned at the beginning of §5. On the other hand, given sufficiently high quality q , the retailer is willing to combat even if the wholesale price is the same as that in the absence of counterfeits, i.e., $\omega = \frac{q}{2}$. This occurs if and only if $q \geq \bar{q}(\frac{q}{2})$, or equivalently, $q \geq \bar{\bar{q}}$, where $\bar{\bar{q}}$ is a threshold depending only on β and $c(\beta)$. In this case, the equilibrium outcome coincides with the first benchmark of standard bilateral monopoly. For intermediate values of product quality, $\hat{q}^B \leq q < \bar{\bar{q}}$, define $\bar{\omega}(q)$ as the highest wholesale price that induces the retailer to combat. That is, $\bar{\omega}(q) := \max\{\omega : q \geq \bar{q}(\omega), \hat{q}^B \leq q < \bar{\bar{q}}\}$. By Lemma 2, $\bar{\omega}(q)$ increases in q , i.e., when the quality of the authentic product is lower, a deeper discount in the wholesale price is needed to incentivize the retailer to combat.

We now proceed to examine the manufacturer's optimal pricing decision. Anticipating the retailer's best response, the manufacturer chooses wholesale price ω to maximize his profit:

$$\pi_M(\omega) = \omega \left[\gamma^*(\omega) \left(1 - \frac{p_a^*(\omega)}{q} \right)^+ + (1 - \gamma^*(\omega)) \left(1 - \left(\frac{p_a^*(\omega) - p_f^*(\gamma^*(\omega), p_a^*(\omega))}{q(1 - \beta)} \right)^+ \right)^+ \right]. \quad (9)$$

Recall from Lemma 2 that the manufacturer can either slash the price to motivate the retailer to combat, or maintain a high price and tolerate the counterfeit. Thus, the optimal wholesale price reflects the manufacturer's tradeoff between reduction of his margin and presence of the counterfeit. Particularly, for $q \in (\hat{q}^B, \bar{\bar{q}})$, the manufacturer compares his profit when the retailer combats under the wholesale price $\omega = \bar{\omega}(q)$, and his reservation profit (i.e., when neither firm combats and $\omega = \frac{1-\beta}{2-\beta}q$). Proposition 2 shows that the former profit dominates if q is higher than a threshold, $\hat{q}^R$.

PROPOSITION 2. *Assume that the manufacturer chooses DR. There exists a critical value $\hat{q}^R \in (\hat{q}^B, \bar{\bar{q}})$ such that $\gamma^* = 1$ if $q \geq \hat{q}^R$, and $\gamma^* = 0$ otherwise. The equilibrium strategies and profits are in Table 3.*

Since $\hat{q}^R > \hat{q}^B$, similar to the manufacturer in the DM model, the retailer in the DR model is less eager to combat the counterfeiter than an integrated firm. The reason is also similar: compared to

Figure 2 The manufacturer's equilibrium wholesale price ω^* under DR, as a function of q : $c(\beta) = 0.2\beta^2$ and $\beta = 0.2$. The solid line is for the case with counterfeit, and the dashed line is for the case without counterfeit.

q	γ^*	p_a^*	p_f^*	ω^*	π_R^*	π_M^*	π_f^*
$q < \hat{q}^R$	0	$\frac{3(1-\beta)}{2(2-\beta)}q$	$\frac{3\beta(1-\beta)}{4(2-\beta)}q$	$\frac{1-\beta}{2-\beta}q$	$\frac{1-\beta}{8(2-\beta)}q$	$\frac{1-\beta}{4(2-\beta)}q$	$\frac{9\beta(1-\beta)}{16(2-\beta)^2}q$
$\hat{q}^R \leq q \leq \bar{q}$	1	$\frac{q+\bar{\omega}(q)}{2}$	—	$\bar{\omega}(q)$	$\frac{(q-\bar{\omega}(q))^2}{4q} - c(\beta)$	$\frac{\bar{\omega}(q)(q-\bar{\omega}(q))}{2q}$	—
$q > \bar{q}$	1	$\frac{3}{4}q$	—	$\frac{q}{2}$	$\frac{q}{16} - c(\beta)$	$\frac{q}{8}$	—

Table 3 Equilibrium results under DR

an integrated firm, the retailer cannot fully enjoy the benefit of anti-counterfeiting due to double marginalization, and thus is less willing to spend the efforts. Hence, our finding reveals another reason why noncooperative decision making in a distribution channel may be harmful: it induces the firms to tolerate higher levels of counterfeiting.

In Figure 2 we illustrate the equilibrium wholesale price ω^* as a function of q . For $q < \hat{q}^R$, ω^* increases in q . However, a downward adjustment is made at $q = \hat{q}^R$ to motivate the retailer to take anti-counterfeit actions. For all $q \geq \hat{q}^R$, the wholesale price is always lower than or equal to that in the case without counterfeits. In particular, it is strictly lower than the no-counterfeit wholesale price when q is between $\hat{q}^R$ and $\bar{q}$. Thus, double marginalization is dampened therein compared with the no-counterfeit case, suggesting that the supply chain's performance can be improved due to the threat of counterfeits. We will elaborate on this point in next subsection.

5.3. DM versus DR

With the equilibrium outcomes under the DM and DR cases characterized in §5.1 and §5.2, a natural question is which combat approach should be adopted by the manufacturer. In preparation, we first examine which firm, the manufacturer or the retailer, is more keen to take anti-counterfeiting actions. The lemma below gives the answer:

LEMMA 3. $\hat{q}^R < \hat{q}^M$. That is, the retailer in the DR model is more keen to combat the counterfeit than the manufacturer in the DM model.

Lemma 3 indicates that the retailer in the DR model is more aggressive than the manufacturer in the DM model to eliminate the counterfeits. To see why, recall that under DM, the combat expense is solely borne by the manufacturer and the retailer free rides on the manufacturer's efforts. Nevertheless, under DR, the combat cost is shared between the two firms whenever the wholesale price is lower than the case without counterfeits. In such cases, the manufacturer's profit in the DR case is always lower than the case without counterfeits, even when the counterfeits are removed from the market by the retailer. Hence, the manufacturer in fact pays some cost to retain the monopolistic position, by lowering the wholesale price. On the other hand, for the retailer, she effectively pays less than the full combat cost, $c(\beta)$, to take anti-counterfeit actions due to assistance from the manufacturer. The sharing of anti-counterfeiting costs incentivizes the retailer to be more proactive in combating counterfeits than the manufacturer. This logic also leads to the preferences between DR and DM, as shown in Proposition 3.

PROPOSITION 3. *Both the manufacturer and the supply chain prefer DR to DM.*

Proposition 3 implies that, in equilibrium, the global manufacturer always prefers to authorize the local retailer to combat the counterfeit. As discussed earlier, compared with DM, the DR approach effectively transfers part of the combat expense from the upstream firm to the downstream firm, and thus is preferable to the manufacturer. Furthermore, the supply chain also favors the retailer taking charge of the anti-counterfeiting actions, because (i) the counterfeit is more likely to be removed from the market under DR (Lemma 3), and (ii) the wholesale price is lower under DR when the counterfeit is eliminated in both DM and DR models (Tables 2 and 3), implying mitigated double marginalization. We will elaborate on the latter effect next.

5.4. Impact of Counterfeiting

By comparing the equilibrium outcome with or without counterfeiting, we characterize the impact of counterfeiting on the supply chain's profit:

THEOREM 1. (i) *There exists a critical value $\hat{\beta}$ such that the counterfeit benefits the supply chain if and only if $\beta < \hat{\beta}$ and $\hat{q}^R \leq q < \frac{16(2-\beta)(2-3\beta)}{\beta(6-7\beta)}c(\beta)$.*

(ii) *The manufacturer is worse off by counterfeits while the retailer is better off if and only if $\beta \leq \frac{2}{3}$ and $\hat{q}^R \leq q < \tilde{q}$, where $\tilde{q}$ is a critical value greater than $\hat{q}^R$.*

When firms are confronted with counterfeits that seduce consumers with low price, one may expect that the supply chain is always worse off. However, Theorem 1 (i) suggests the opposite: the threat of counterfeits may benefit the supply chain. This counterintuitive finding reflects a positive effect of counterfeiting: it induces the manufacturer to lower the wholesale price and thus reduces the double marginalization in a decentralized chain. If the positive effect dominates the combat expense, an improvement in the total profit of the supply chain is achieved. Theorem 1 (i) shows that such

improvement is materialized for intermediate values of q . If the authentic product is of low quality, firms do not have enough incentive to take anti-counterfeit actions and the allure of fake products on the market leads to a profit loss to the legitimate supply chain. On the other hand, if the quality of the authentic product is high, combat is desirable but double marginalization is not significantly reduced. This is because, in such a case, the retailer has a strong incentive to combat by herself and the manufacturer does not need to cut the wholesale price substantially to motivate her (refer to Figure 2: the difference between the wholesale price with counterfeit and that without counterfeit diminishes in q for $q > \hat{q}^R$). Consequently, counterfeiting is beneficial to the supply chain only if q is intermediate, as illustrated in the left panel of Figure 3.

Figure 3 Impact of counterfeiting on supply chain profits (left) and on social welfare (right): $\beta = 0.15$ and $c(\beta) = a\beta^2 + 0.04$. For the right panel, the dashed line represents the threshold $\hat{q}^R$ as a function of a .

From a broader perspective, a necessary condition for counterfeits boosting supply chain profitability is the supply-chain partners' *asymmetric* motivation towards combating the counterfeiter. This is evinced by Table 4, which summarizes the impact of counterfeits on supply chain profits under various combinations of firms' incentives. Counterfeiting benefits the supply chain only if the manufacturer would invest to combat counterfeiter while the retailer would not do so without any external incentive (that is, the upper right corner of the table). On the other hand, when the firms' incentives for combating are symmetric (i.e., the upper left or lower right corner of the table), the threat of counterfeit always hurts the supply chain, due to either the combating cost (if the counterfeit is eradicated), or brand piracy (if it remains on the market).

The first part of Theorem 1 focuses on supply chain profit. Part (ii) of the theorem specifies the impact of counterfeits on individual firms in the authentic-product supply chain. The manufacturer is worse off, irrespective of the combating outcome in equilibrium: If the retailer does not combat, both the wholesale price and market sales are lower in presence of counterfeiting. Hence, the manufacturer's

	Retailer would invest to combat counterfeiter without incentive from Mfgr (i.e., $q > \bar{q}$)	Retailer would not invest to combat counterfeiter without incentive from Mfgr (i.e., $q \leq \bar{q}$)
Mfgr would invest to combat counterfeiter (i.e., $q \geq \hat{q}^R$)	Counterfeiting hurts supply chain due to combating cost	Counterfeiting benefits supply chain if and only if q is intermediate and β is low
Mfgr would not invest to combat counterfeiter (i.e., $q < \hat{q}^R$)	Never occurs in equilibrium	Counterfeiting hurts supply chain due to name-brand bandit

Table 4 Impact of counterfeiting on supply chain

profit is reduced by the counterfeits. If the retailer combats, the authentic product monopolizes the market and the manufacturer's profit as a function of wholesale price ω is the same as that without counterfeit. Nevertheless, different from the counterfeit-free case, the manufacturer's wholesale price is sometimes subject to an upper bound, $\bar{\omega}(q)$, so as to provide combating incentive for the retailer, which renders a lower profit for the manufacturer due to counterfeiting. On the other hand, the retailer may enjoy a reduction in the wholesale price, and thus become better off by the threat of counterfeits.

Next we examine the impact of counterfeits on consumer surplus based on the equilibrium outcome. We define consumer surplus as surplus for buyers of both the authentic product and the counterfeit (Gao et al., 2017; Qian et al., 2015; Chen and Png, 2003; Cho et al., 2015). We prove in Proposition 4 that consumer surplus is improved due to counterfeits. To see why, consider two cases: if $q < \hat{q}^R$, the counterfeit serves as a cheaper alternative for consumers and the authentic product's price drops due to infliction of counterfeit, both benefiting the consumers. If $q \geq \hat{q}^R$, both the wholesale price and the retail price are lower due to threat of counterfeit, also improving consumer surplus. We caution that this result holds only to the extents covered by our model. Under a status-utility model, Gao et al. (2017) presents an example where the presence of counterfeit hurts overall consumer surplus. The phenomenon occurs when, with a very high probability (95%), consumers are unable to identify the counterfeit as fake, i.e., the counterfeit is close to perfectly deceptive. In the example, the loss of status utility for the authentic product buyers overshadows other surplus gains from the presence of counterfeit, and leads to an overall reduction of consumer surplus. This is, to our best knowledge, the first and only paper in the counterfeiting literature exemplifying negative impact of counterfeiting on overall consumer surplus. In this paper we consider only non-deceptive counterfeit. In §6.3.3 we show that our result (counterfeiting improves overall consumer surplus) is preserved when we extend the base model to incorporate status utility (see Proposition S.8 in the supplementary document). Thus, our study complements Gao et al. (2017) and suggests that the negative impact of counterfeiting on overall consumer surplus is most pronounced when it is highly deceptive.

We define social welfare as consumer surplus plus profits for the supply chain and the counterfeiter (Gao et al., 2017; Qian et al., 2015; Qian, 2014). Interestingly, counterfeiting can also increase social welfare, as shown in Proposition 4.

PROPOSITION 4. *Counterfeits improve consumer surplus. Furthermore, there exists a unique $\tilde{q}'$ such that $\tilde{q}' \geq \hat{q}^R$ and counterfeits improve social welfare if $q < \tilde{q}'$ and reduce social welfare otherwise.*

The shaded area in right panel of Figure 3 illustrates the cases where the counterfeit improves social welfare. Interestingly, when the counterfeit is removed from the market at a positive cost (i.e., $q \geq \hat{q}^R$), the social welfare may be improved. This is because the reduced double marginalization and the increased supply chain profit can outweigh the expense in taking anti-counterfeit actions.

We note that the main mechanisms driving these results are two-fold. First, a decentralized supply chain in which both parties independently determine prices and optimize their own profits, leading to double marginalization. Second, the presence of direct combat as an option for the supply chain to cope with counterfeits. In particular, the supply chain's ability to legally wipe out the illicit counterfeiter from (a portion of) the market triggers wholesale price reduction, which further leads to mitigation of double marginalization. To elaborate, consider the extreme cases where the combat cost approaches either zero or infinity. In these cases, the problem effectively boils down to either a pure combat problem or a pure duopoly pricing problem. In either case, it can be easily verified that the supply chain's reduction of double marginalization leading to a higher profit will never happen. It is only when combat cost falls into a finite range of values such that an interplay between combat and pricing decisions is viable and drives our surprising results.

6. Extensions and Discussions

In this section, we explore several variants of the base model: shared combating efforts, wholesale price postponement, ethical customers, brand value loss, and non-linear combat cost. Detailed analysis for these extensions is provided in a supplementary document, where the numbering of sections, propositions, theorems, and figures starts with an "S" for easy reference (for example, §SA and Proposition S.1). Below we describe each extended model and summarize the findings.

6.1. Shared Combating Efforts

In the base model the manufacturer's choice between DM and DR arrangements precludes the possibility of both firms spending some anti-counterfeiting efforts simultaneously. This may reflect very well many practical situations where jointly taking combating actions in foreign markets requires much involved collaboration among firms and is thus formidable. In this extension we study shared combating efforts and provide another reason why the firms should not share combating efforts: it may result in lower supply chain profit.

We assume that both the manufacturer and the retailer can contribute to anti-counterfeiting and independently choose their efforts to maximize their own profits. Let γ_1 and γ_2 be the manufacturer's and the retailer's combating efforts, respectively, where $\gamma_1, \gamma_2 \geq 0$ and $\gamma := \gamma_1 + \gamma_2 \in [0, 1]$. The sequence of events is depicted in Figure 4. Within this extension, we assume $\beta < \frac{2}{5}$.

Figure 4 Sequence of events when both firms may combat: M (Manufacturer) and R (Retailer)

Below we highlight a few key results to illustrate the effects of sharing efforts. In a subgame where the wholesale price is ω and the manufacturer's combating efforts is γ_1 , denote the retailer's optimal price and combating effort level by $p_a^*(\omega, \gamma_1)$ and $\gamma_2^*(\omega, \gamma_1) \in [0, 1 - \gamma_1]$, respectively. The lemma below characterizes the condition under which the retailer finds it optimal to combat.

LEMMA 4. *Given ω and γ_1 , if $q \geq \hat{q}(\omega, \gamma_1)$, then $\gamma_2^*(\omega, \gamma_1) = 1 - \gamma_1$; otherwise $\gamma_2^*(\omega, \gamma_1) = 0$. Moreover, $\hat{q}(\omega, \gamma_1)$ increases in ω and decreases in γ_1 .*

As shown in Lemma 4, for a given wholesale price ω , the retailer is more likely to take combating actions when the manufacturer's effort level γ_1 increases. This is consistent with our earlier discussion in §4 that the marginal benefit of counteraction increases as the market inflicted by counterfeits shrinks, since the retailer's sales reduction from raising retail price is lower.

Same as in the base model, for a given γ_1 , the wholesale price ω determines whether the retailer would combat. By Lemma 4, the manufacturer can either slash the price to motivate the retailer to combat, or maintain a high price and fight counterfeits alone. Denote the optimal wholesale price by $\omega^*(\gamma_1)$. Figure 5 depicts the manufacturer's profit as a function of his combating level γ_1 , given that the optimal wholesale price $\omega^*(\gamma_1)$ is deployed for each γ_1 . It shows that the manufacturer may find it optimal to spend some combating efforts himself (e.g., $\gamma_1^* > 0$ on the right panel). Lemma 5 proves that sharing efforts is optimal only if the brand-name product quality q is intermediate.

LEMMA 5. *For $q < \frac{8c(\beta)(1-\beta)}{\beta}$, $\gamma_1^* = \gamma_2^* = 0$. For $q > \frac{4c(\beta)(2-\beta)^2}{\beta^2}$, $\gamma_1^* = 0$ and $\gamma_2^* = 1$. For $\frac{8c(\beta)(1-\beta)}{\beta} \leq q \leq \frac{4c(\beta)(2-\beta)^2}{\beta^2}$, γ_1^* may be positive, and whenever it is positive, the counterfeit is completely removed from the market, i.e., $\gamma_1^* + \gamma_2^* = 1$.*

Intuitively, sharing combating efforts ($\gamma_1, \gamma_2 > 0$) may be preferable to the manufacturer when it is less costly than to promote the retailer's combat through wholesale price reduction. Essentially, the manufacturer does not have to lower the wholesale price as much when he directly participates in the counteraction. However, it can be detrimental to the supply chain as a whole, as proved below.

THEOREM 2. *There exists a critical value $\hat{q}^U$ such that $\hat{q}^U > \hat{q}^R$, and if $\hat{q}^R \leq q < \hat{q}^U$ and $0 < \gamma_1^* < 1$, the supply chain profit under shared combating efforts is lower than that in the base model.*

When $\hat{q}^R \leq q < \hat{q}^U$ and $0 < \gamma_1^* < 1$, by Proposition 3 and Lemma 5, the counterfeit is completely eliminated regardless of whether combating efforts are shared. Nevertheless, Theorem 2 shows that in such cases, the supply chain is worse off by effort sharing. This is because the supply chain regains its monopolistic market power with or without effort sharing and yet, the wholesale price is higher with shared efforts, implying a higher level of double marginalization and a lower supply chain profitability. Hence, even in the absence of additional coordination cost, combat effort sharing may not be desirable from the perspective of the supply chain, as it dampens the manufacturer's incentive to lower the wholesale price in response to counterfeiting.

Figure 5 The manufacturer's profit $\pi_M(\omega^*(\gamma_1), \gamma_1)$ as a function of γ_1 : $\beta = 0.35$, $c(\beta) = 0.3\beta^2$, $q = 0.7$ (left) and $q = 0.76$ (right).

6.2. Additional Operational Complexities

6.2.1. Different Production Costs To highlight the effects of quality difference between the authentic product and the counterfeit, we have thus far normalized the production costs for both goods to be zero. In practice the counterfeiter may incur a lower production cost due to its inferior quality. In this subsection (with details in §SB) we extend the base model by considering different costs for the authentic product and the counterfeit. To explicitly recognize the dependence of production cost on product quality, we assume that the costs for the authentic product and the counterfeit are $c_a q$ and $c_f q$, respectively, and without loss of generality, set $c_f = 0$. We further assume $0 < c_a < \frac{2(1-\beta)}{2-\beta}$ to ensure the profitability of the authentic product even if neither the manufacturer nor the retailer combats the counterfeit.

Theorem S.1 confirms that the key results continue to hold when the authentic and counterfeit goods differ in production costs. Furthermore, as the cost difference increases (i.e., a higher c_a for a given q), the counterfeit is more likely to survive in the market, due to a greater cost disadvantage of the authentic supply chain (see the left panel of Figure 6). For the same reason, the manufacturer

tends to be more reluctant to lower its wholesale price for the retailer's counteraction. Thus, the region where the supply chain benefits from the threat of counterfeiting eventually disappears as c_a becomes very high (refer to the right panel of Figure 6). It is interesting, however, to note that the benefit region initially expands in c_a when c_a is low. To see the rationale behind this result, note that, for a given wholesale price, the retailer's profit, regardless of whether she combats, is independent of the production cost, i.e., independent of c_a for a given q . Thus, the highest wholesale price to induce the retailer's counteraction is also independent of c_a . When c_a is low, the manufacturer is willing to incentivize retailer to combat, and in such a case, the gap between its wholesale price (independent of c_a) and the optimal wholesale price in the absence of counterfeiting (increasing in c_a) widens as c_a increases. Therefore, the effect of mitigating double-marginalization is strengthened as c_a increases from zero, which explains the initial expansion of the better-off region. We examine an alternative setting in which the production costs are quadratic functions of quality and observe similar results numerically, as detailed in §SB.2.

Figure 6 Impact of production cost on the retailer's combat action (left) and supply chain profit (right): $\beta = 0.15$ and $c(\beta) = 0.5\beta^2 + 0.04$.

6.2.2. Non-linear Combat Cost So far we assume that the combat cost, $\gamma c(\beta)$, is linear in the combating effort γ . In this extension we demonstrate the robustness of our results under various non-linear combat cost functions including fixed cost, concave cost, and convex cost. These combat cost functions correspond to a wide range of circumstances in practice. For example, a fixed cost applies in a one-time legal lawsuit situation and the combat result will be all or nothing depending on the verdict of the court trial. A fixed cost is also appropriate when a one-time administrative effort is conducted to provide customs agents of the government for border inspection the recognizable differences between counterfeit and authentic goods, and with samples of the latter. On the other hand, the linear cost in the base model may capture staff inspection in the market dispatched by

brand owner in which the inspection cost is linearly increasing in the size of the market. Finally, concave (convex) cost assumptions correspond to the decreasing (increasing) marginal cost scenarios. In particular, concave cost is perceivable when a one-time administrative effort is coupled with market inspection, both discussed above, constituting both a sizable fixed cost and a variable cost to the total combat costs. In §SC we confirm with analytical proofs that our main results are robust under any fixed or concave combat cost, as well as under a particular form of convex combat cost.

We also note that, while in most cases considered in the paper the optimal combat strategy is bang-bang, partial combat is sometimes optimal when the combat cost is convex in the combating effort. Through a numerical study in §SC.3.2, we find that, as the combat cost becomes more convex, partial combat is more likely to be optimal. In such cases, the manufacturer may need to offer a deeper price discount to induce retailer's counteraction against counterfeit, and, consequently, the supply chain may be more often better off by counterfeiting.

6.2.3. Counterfeit's Endogenous Quality In the base model it is assumed that the counterfeit's quality is exogenously given. Qian (2008) articulates a few reasons why counterfeiters has very limited (if any) flexibility in choosing its quality, including the facts that counterfeiters have limited technology available relative to the brand, and also in some countries, counterfeiters, not legitimately registered and have no permits for imports, cannot import fancy materials and equipment. Nevertheless, when the counterfeiter can indeed choose its quality, how does the endogenous quality decision affect the authentic supply chain's combatting action? We explore this question in this extension, and present in §SD a model of quality improvement by the counterfeiter.

Since the counterfeiter's quality decision, as a long-term strategic choice, is usually made at the outset and remains fixed during the selling horizon, all the results in the base model, which are derived under a given counterfeit quality, are preserved under endogenous counterfeit quality. Furthermore, we find that the counterfeiter is better off by the flexibility in choosing quality level as it allows her to preempt counteraction from the authentic supply chain. Specifically, through an improvement of quality, the counterfeiter can hinder the combatting actions by raising the combat cost. In such a case, the supply chain is unlikely to benefit from counterfeiting since counteraction does not occur. Hence, as illustrated by Figure S.12, when the quality-improvement cost decreases, it is more likely for the counterfeit to improve quality and yet less likely for it to benefit the supply chain.

6.2.4. Reactive Combating In the base model the authentic supply chain is able to take preemptive action against the counterfeit and eliminate it from the market before its sales commences. In §SE we prove that the supply chain prefers such a preemptive arrangement, to the situation where the counterfeit is noticed and combatted only ex post (see Theorem S.3). The conclusion is drawn through a comparison of our base model with a two-period dynamic model of reactive combating, where the combatting decisions are made in the second period.

6.3. Counterfeit-Induced Consumer Behavior

6.3.1. Ethical Consumers Our model can be extended to incorporate a group of ethical consumers who are prevalent in practice (Chen and Png 2003; Jain 2008; Lahiri and Dey 2013). These consumers consider counterfeiting to be morally unacceptable, and therefore do not regard counterfeits as substitutes for authentic products. To capture such behavior, we assume that there are two types of consumers in the market: non-ethical and ethical. Specifically, the fraction of ethical consumers is $\alpha \in (0, 1)$, and the remaining fraction, $1 - \alpha$, is unethical in the sense that they embrace counterfeits as an option. Alternatively, the parameter α can also be interpreted as the portion of consumers who are unaware of the counterfeit or who are loyal to authentic products.

With consumer heterogeneity in ethics, demands for the authentic product and the counterfeit in (1) and (2) become

$$\begin{aligned} D_a(\gamma, p_a, p_f) &= (1 - \gamma) [\alpha \text{Prob}(U_a \geq 0) + (1 - \alpha) \text{Prob}(U_a \geq U_f, U_a \geq 0)] + \gamma \text{Prob}(U_a \geq 0) \\ &= (1 - \gamma) \left[\alpha \left(1 - \frac{p_a}{q}\right)^+ + (1 - \alpha) \left(1 - \max \left\{ \frac{p_a - p_f}{q(1 - \beta)}, 0 \right\}\right)^+ \right] + \gamma \left(1 - \frac{p_a}{q}\right)^+, \end{aligned}$$

and

$$\begin{aligned} D_f(\gamma, p_a, p_f) &= (1 - \gamma)(1 - \alpha) \text{Prob}(U_a < U_f, U_f \geq 0) \\ &= (1 - \gamma)(1 - \alpha) \left(\min \left\{ \max \left\{ \frac{p_a - p_f}{q(1 - \beta)}, 0 \right\}, 1 \right\} - \min \left\{ 1, \frac{p_f}{\beta q} \right\} \right)^+, \end{aligned}$$

respectively. As expected, the demand for the authentic (counterfeit) product increases (decreases) in the portion of ethical consumers. Essentially, with a fraction of ethical consumers, the counterfeit is less influential and firms have less incentive to take anti-counterfeit actions. Our main results remain unchanged, as proved in Proposition S.1.

While the retailer is less willing to combat the counterfeit with more ethical consumers, the supply chain is more likely to benefit from counterfeits. As shown in left panel of Figure 7, the shaded area, where the supply chain benefits from counterfeits, expands in the fraction of ethical consumers α . This is because, as α increases, the manufacturer needs to slash the wholesale price more aggressively to give the retailer extra incentive to combat and consequently double marginalization is further reduced.

6.3.2. Brand Value Loss Brand value loss reflects consumers' attitudes towards the counterfeit-influenced market. As discussed in the introduction, with the presence of counterfeit, some consumers will leave the market and do not consider buying the product because uniqueness of the authentic brand is affected or ruined. We refer to them as fake-averse consumers. Assume that a portion of consumers, $\kappa \in (0, 1)$, are fake-averse. That is, when counterfeiting is not completely eliminated from the market ($\gamma < 1$), the total market size shrinks to $K(\gamma) = 1 - (1 - \gamma)\kappa$. Thus the market size increases in the combat scope. If the supply chain does not take any anti-counterfeit action ($\gamma = 0$), the total market size shrinks to $1 - \kappa$ due to consumers' fake-averse behavior.

Figure 7 Impact of counterfeiting on supply chain profits with ethical consumers: $\beta = 0.15$ and $c(\beta) = 0.8\beta^2$ (left) and with brand value loss: $\beta = 0.05$ and $c(\beta) = 8\beta^2$ (right).

Under the brand-value-loss effect, we fully characterize the equilibrium outcomes under the DR and DM models, and exemplify with a numerical example that supply chain profitability is sometimes boosted by threat of counterfeiting (see Proposition S.2). Interestingly, with a stronger effect of brand value loss, the supply chain is less likely to benefit from counterfeits. As illustrated in the right panel of Figure 7, the area where supply chain benefits from counterfeits shrinks as λ increases. The underlying reason is as follows: as more consumers become fake-averse, the loss in brand value due to counterfeiting is aggravated, implying a greater gain from anti-counterfeiting and, thus, the retailer is increasingly self-motivated to combat. Hence, the manufacturer does not need to distort the wholesale price as much to provide incentive, and, consequently, the mitigation of supply-chain inefficiency by counterfeiting is less pronounced.

6.3.3. Status Utility As aforementioned, luxury and high-end products are among the frequent and common targets for counterfeiting. Consumers may purchase these goods not only for their use values, but also due to the social status associated with the brands. In this extension (with details in §SH) we incorporate a status utility into the consumer-choice model and evaluate its effects on anti-counterfeiting strategies in a supply chain. We adopt the status-utility model developed by Rao and Schaefer (2013) and Gao et al. (2017), which captures the impact of counterfeit sales on the utility of authentic-product buyers.

The equilibrium outcome under the status-utility model confirms that our main results continue to hold (specifically, in Proposition S.6). In addition, we find that the status utility widens the utility gap between authentic product buyers and nonbuyers (including counterfeit buyers and consumers who make no purchase), and thus increases consumers' willingness to pay for the branded items. This effect renders the sales of the authentic product more profitable and in turn supports its supply chain to fight more vigorously with the counterfeit (refer to Proposition S.7). As the manufacturer does

not need to provide as much incentive for the retailer's counteraction, it is less likely for a threat of counterfeiting to benefit the authentic supply chain (see Figure S.16 for an illustration).

7. Conclusion

The past two decades have witnessed an explosion of counterfeiting problems around the globe. In this paper, we examine influence of counterfeits on a global supply chain comprised of a manufacturer and a local retailer. We analyze the global supply chain's optimal strategy in taking anti-counterfeit actions and how counterfeiting impacts firms' pricing and profitability.

While it is widely believed that counterfeiting is detrimental to brand owners, we find that the threat of a counterfeiter may in fact boost supply chain profits. This is because the imminent attack by counterfeiting triggers collaborative endeavors of the supply chain members: the manufacturer lowers the wholesale price to compensate the retailer for her combating expenses, which mitigates double marginalization and improves overall supply chain efficiency. We demonstrate that the collaborative endeavors are driven by an interesting asymmetry in the manufacturer's and the retailer's incentives to combat counterfeits. We also investigate the impacts of counterfeiting on consumer surplus and social welfare and find that both can be improved.

In terms of who should stand on the combat front-line, we show that the manufacturer generally prefers to incentivize the retailer to tackle the counterfeit, instead of doing so himself. This is because the combat cost is (indirectly) shared by the two firms in the former case, while it is solely borne by the manufacturer in the latter. Further, it can be of the supply chain's best interest that the manufacturer does not execute the combating activities, either jointly with the retailer or by himself.

We show, with either analytical proofs or numerical examples, that the benefits from counterfeiting are preserved under a few variants of the base model, including three counterfeit-induced consumer behavior (ethical consumers, brand value loss, and status utility), and additional operational complexities such as a higher production cost incurred by the authentic product than the counterfeit, non-linear combat cost, endogenized counterfeit quality, and reactive combating. We also find that the presence of ethical consumers can reinforce the benefit of supply chain due to counterfeiting, while both brand value loss and status utility have opposite effects. In addition, the authentic supply chain's disadvantage in production cost and inability to forestall counterfeiting, as well as the counterfeiter's flexibility to choose quality, may all reduce the possibility for the authentic supply chain to gain from the threat of counterfeiting.

The positive value of counterfeits explored in prior research lies in the network externality effects: the overall market size increases due to counterfeits, since consumers' utility increases in the number of users of either the authentic products or the counterfeits (Conner and Rumelt 1991; Conner 1995). Nevertheless, we identify an alternative way that counterfeits may positively contribute to authentic product: alleviation of double marginalization. This effect may outweigh the combat cost such that the

overall profitability of the supply chain is enhanced. In this regard, this paper gains a new perspective, from the supply side, on how counterfeits can benefit authentic products, even in mature markets where the impact of network externality effects is weak or absent.

Note that our analysis does not capture all the negative externalities caused by counterfeits, hence we caution that counterfeits are beneficial to supply chains and consumers only to the extents covered by our model. For instance, it is often said that, because of counterfeits, authentic brand owners are disincentivized to innovate with new products, which is not studied here. In fact, there have been debates in the economics literature on the relationship between competitive threat and innovation. To this end, we refer to Qian (2008, 2014) who presents both theoretical and empirical results to argue that under certain circumstances authentic product owners are driven to innovate with higher quality products to cope with the market entry of counterfeit products. Qian (2014) states that, “the brand seeks to innovate only when the counterfeit quality is within a certain range. If the counterfeit quality is too low, it does not post sufficient threat to induce innovation. If the quality is too high, innovation may not alleviate competition.” Interestingly, this coincides with our finding that counterfeits benefit a supply chain only when the authentic product quality is neither too high nor too low. Even though product innovation is beyond the scope of this paper, we believe that a properly aligned supply chain also helps the chain partners to fight counterfeits effectively. It is through product design and supply chain alignment, among other initiatives, that companies may reap in the near- or even long-term under the threats of counterfeits.

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Appendix

This appendix contains proofs for a few key results in the main model. Other proofs are provided in an online appendix and a supplementary document.

Proof of Lemma 1. Without loss of optimality, we shall focus on $p_a \leq q$. Maximizing the counterfeiter's profit function in (3), we obtain

$$p_f^*(\gamma, p_a) = \begin{cases} \frac{\beta}{2}p_a, & \text{if } p_a < \frac{2q(1-\beta)}{2-\beta}, \\ p_a - (1-\beta)q, & \text{if } \frac{2q(1-\beta)}{2-\beta} \leq p_a \leq \frac{2-\beta}{2}q, \\ \frac{\beta q}{2}, & \text{otherwise,} \end{cases} \quad (10)$$

which is lower than p_a . Substituting it into (1), we have

$$D_a(\gamma, p_a) = \begin{cases} \gamma \left(1 - \frac{p_a}{q}\right) + (1 - \gamma) \left(1 - \frac{2-\beta}{2q(1-\beta)} p_a\right), & \text{if } p_a < \frac{2q(1-\beta)}{2-\beta}, \\ \gamma \left(1 - \frac{p_a}{q}\right), & \text{if } p_a \geq \frac{2q(1-\beta)}{2-\beta}. \end{cases}$$

Then,

$$\pi_a(\gamma, p_a) := \begin{cases} \pi_a^I(\gamma, p_a) = p_a \left(\gamma \left(1 - \frac{p_a}{q}\right) + (1 - \gamma) \left(1 - \frac{2-\beta}{2q(1-\beta)} p_a\right) \right) - \gamma c(\beta), & \text{if } p_a < \frac{2q(1-\beta)}{2-\beta}, \\ \pi_a^{II}(\gamma, p_a) = p_a \gamma \left(1 - \frac{p_a}{q}\right) - \gamma c(\beta), & \text{if } p_a \geq \frac{2q(1-\beta)}{2-\beta}. \end{cases} \quad (11)$$

Let $p_a^*(\gamma) := \arg \max_{p_a} \pi_a(\gamma, p_a)$ and $\Pi_a(\gamma) := \pi_a(\gamma, p_a^*(\gamma))$. Then, for $\beta \leq \frac{2}{3}$, $p_a^*(\gamma) = \frac{1-\beta}{2-\beta-\gamma\beta}q$, and $\Pi_a(\gamma) = \frac{1-\beta}{2(2-\beta-\gamma\beta)}q - \gamma c(\beta)$ and $\frac{d\Pi_a(\gamma)}{d\gamma} = \frac{1-\beta}{2(2-\beta-\gamma\beta)^2}\beta q - c(\beta)$, where $\frac{d\Pi_a(\gamma)}{d\gamma}$ increases in γ . For $\beta > \frac{2}{3}$, if $\gamma \leq \frac{2(1-\beta)}{\beta}$, $p_a^*(\gamma) = \frac{1-\beta}{2-\beta-\gamma\beta}q$, $\Pi_a(\gamma) = \frac{1-\beta}{2(2-\beta-\gamma\beta)}q - \gamma c(\beta)$ and $\frac{d\Pi_a(\gamma)}{d\gamma} = \frac{1-\beta}{2(2-\beta-\gamma\beta)^2}\beta q - c(\beta)$, where $\frac{d\Pi_a(\gamma)}{d\gamma}$ increases in γ ; otherwise, if $\gamma > \frac{2(1-\beta)}{\beta}$, $p_a^*(\gamma) = \frac{q}{2}$, $\Pi_a(\gamma) = \gamma \frac{q}{4} - \gamma c(\beta)$ and $\frac{d\Pi_a(\gamma)}{d\gamma} = \frac{q}{4} - c(\beta)$, where $\frac{d\Pi_a(\gamma)}{d\gamma}$ is independent of γ . Note that $\left. \frac{d\Pi_a(\gamma)}{d\gamma} \right|_{\gamma=\left(\frac{2(1-\beta)}{\beta}\right)^-} = \frac{1-\beta}{2\beta}q - c(\beta) < \left. \frac{d\Pi_a(\gamma)}{d\gamma} \right|_{\gamma=\left(\frac{2(1-\beta)}{\beta}\right)^+} = \frac{q}{4} - c(\beta)$. Then, $\frac{d\Pi_a(\gamma)}{d\gamma}$ always increases in γ . Hence, for any β , $\Pi_a^*(\gamma)$ is convex in γ . Then, the optimal combat level can be determined by comparing the two extreme points $\gamma = 0$ and $\gamma = 1$. Note that $p_a^*(0) = \frac{1-\beta}{2-\beta}q$ and $p_a^*(1) = \frac{q}{2}$. So we have $\Pi_a(0) = \pi_a^I(0, p_a^*(0)) = \frac{1-\beta}{2(2-\beta)}q$ and $\Pi_a(1) = \pi_a^{II}(1, p_a^*(1)) = \frac{q}{4} - c(\beta)$. Let $\hat{q}^B = \frac{4c(\beta)(2-\beta)}{\beta}$. We then have $\gamma^* = 1$ when $q \geq \hat{q}^B$ and $\gamma^* = 0$ otherwise. For $q \geq \hat{q}^B$, $p_a^* = \frac{q}{2}$ and by (1) and (4), we have $D_a^* = \frac{1}{2}$ and $\pi_a^* = \frac{q}{4} - c(\beta)$. For $q < \hat{q}^B$, we have $p_a^* = \frac{1-\beta}{2-\beta}q$ and $p_f^* = \frac{\beta(1-\beta)}{2(2-\beta)}q$. Then, by (1) and (4), $D_a^* = \frac{1}{2}$ and $\pi_a^* = \frac{1-\beta}{2(2-\beta)}q$. By (2) and (3), $D_f^* = \frac{1}{2(2-\beta)}$ and $\pi_f^* = \frac{\beta(1-\beta)}{4(2-\beta)^2}q$. $\square$

Proof of Proposition 1. Similar to (11), we obtain

$$\pi_R(p_a; \gamma, \omega) = (p_a - \omega) \begin{cases} \gamma \left(1 - \frac{p_a}{q}\right) + (1 - \gamma) \left(1 - \frac{2-\beta}{2q(1-\beta)} p_a\right), & \text{if } p_a < \frac{2q(1-\beta)}{2-\beta}, \\ \gamma \left(1 - \frac{p_a}{q}\right), & \text{if } p_a \geq \frac{2q(1-\beta)}{2-\beta}. \end{cases}$$

First we note that, if $\omega > q$, then the retailer cannot make any positive profit regardless of combating or not. In this case, trivially, she will not sell in the market. Next, we focus on the case $\omega \leq q$. We also note that, if $p_a < \frac{2q(1-\beta)}{2-\beta}$, the local maximum is attained at $p_a = \min \left\{ \max \left\{ \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}, \omega \right\}, \frac{2q(1-\beta)}{2-\beta} \right\}$; if $p_a \geq \frac{2q(1-\beta)}{2-\beta}$, the local maximum is attained at $p_a = \max \left\{ \frac{q+\omega}{2}, \frac{2q(1-\beta)}{2-\beta}, \omega \right\} = \max \left\{ \frac{q+\omega}{2}, \frac{2q(1-\beta)}{2-\beta} \right\}$. It is further noted that $\frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2} < \frac{q+\omega}{2}$.

Note that $\frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ increases in γ . Then, if $\omega > \frac{2q(1-\beta)}{2-\beta}$, we have $\frac{2q(1-\beta)}{2-\beta} < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ for any γ ; if $\omega < \frac{2-3\beta}{2-\beta}q$, we have $\frac{2q(1-\beta)}{2-\beta} > \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ for any γ ; otherwise, $\frac{2q(1-\beta)}{2-\beta}$ can be greater or less than $\frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$, depending on the value of γ . We then have the following cases:

(i) $\omega > \frac{2q(1-\beta)}{2-\beta}$: In this case, we have $\frac{2q(1-\beta)}{2-\beta} < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2} < \frac{q+\omega}{2}$. Hence, if $p_a < \frac{2q(1-\beta)}{2-\beta}$, $\pi_R(p_a; \gamma, \omega)$ increases in p_a ; if $p_a \geq \frac{2q(1-\beta)}{2-\beta}$, $\pi_R(p_a; \gamma, \omega)$ increases in p_a when $p_a < \frac{q+\omega}{2}$ but decreases in p_a when $p_a \geq \frac{q+\omega}{2}$. Then, we have $p_a^*(\gamma, \omega) = \frac{q+\omega}{2}$ and $\Pi_R(\gamma, \omega) := \pi_R(p_a^*(\gamma, \omega); \gamma, \omega) = \left(\frac{q+\omega}{2} - \omega\right)\gamma\left(1 - \frac{q+\omega}{2q}\right) = \gamma \frac{(q-\omega)^2}{4q}$, and $\pi_M(\gamma, \omega) = \omega\gamma\left(1 - \frac{q+\omega}{2q}\right) - \gamma c(\beta) = \gamma \frac{\omega(q-\omega)}{2q} - \gamma c(\beta)$, which is linear in γ . As such, given ω , the optimal combat can only be either 0 or 1.

(ii) $\frac{2-3\beta}{2-\beta}q < \omega < \frac{2q(1-\beta)}{2-\beta}$: In this case, we have $\omega < \frac{2q(1-\beta)}{2-\beta}q$ for any $\gamma \in [0, 1]$, or equivalently $\omega < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$. Consider two sub-cases. If $\gamma > \frac{2-\beta}{\beta} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)}$, we have $\omega < \frac{2q(1-\beta)}{2-\beta} < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2} < \frac{q+\omega}{2}$ and hence the optimal price is $p_a^*(\gamma, \omega) = \frac{q+\omega}{2}$. We have $\Pi_R(\gamma, \omega) := \pi_R(p_a^*(\gamma, \omega); \gamma, \omega) = (\frac{q+\omega}{2} - \omega)\gamma(1 - \frac{q+\omega}{2q}) = \gamma \frac{(q-\omega)^2}{4q}$, and $\pi_M(\gamma, \omega) = \pi_M^I(\gamma, \omega) := \omega\gamma(1 - \frac{q+\omega}{2q}) - \gamma c(\beta) = \gamma \frac{\omega(q-\omega)}{2q} - \gamma c(\beta)$, which is a linear function of γ with a slope of $s^I = \frac{\omega(q-\omega)}{2q} - c(\beta)$.

To proceed, if $\gamma < \frac{2-\beta}{\beta} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)}$, we have $\omega < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2} < \frac{2q(1-\beta)}{2-\beta} < \frac{q+\omega}{2}$ and hence the optimal retail price is either $p_a = \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ or $p_a = \frac{q+\omega}{2}$; that is, $\Pi_R(\gamma, \omega) := \pi_R(p_a^*(\gamma, \omega); \gamma, \omega) = \max \left\{ \left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right), \gamma \frac{(q-\omega)^2}{4q} \right\}$. More specifically, for $\gamma < \hat{\gamma} = \frac{(2q(1-\beta)-\omega(2-\beta))^2}{\beta(\omega^2(2-\beta)-2q(2\omega-q)(1-\beta))}$, we have $\left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right) > \gamma \frac{(q-\omega)^2}{4q}$, $p_a^*(\gamma, \omega) = \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ and

$$\begin{aligned} \pi_M(\gamma, \omega) &= \pi_M^{II}(\gamma, \omega) := \omega \left[\gamma \left(1 - \frac{p_a^*(\gamma, \omega)}{q} \right) + (1-\gamma) \left(1 - \frac{2-\beta}{2q(1-\beta)} p_a^*(\gamma, \omega) \right) \right] - \gamma c(\beta) \\ &= \omega \left[\gamma \left(1 - \frac{1-\beta}{2-\beta-\gamma\beta} - \frac{\omega}{2q} \right) + (1-\gamma) \left(1 - \frac{2-\beta}{2(2-\beta-\gamma\beta)} - \frac{2-\beta}{2(1-\beta)} \frac{\omega}{2q} \right) \right] - \gamma c(\beta) \\ &= \omega \frac{2(1-\beta)q - (2-\beta-\gamma\beta)\omega}{4(1-\beta)q} - \gamma c(\beta), \end{aligned}$$

which is a linear function of γ with a slope of $s^{II} = \frac{\beta\omega^2}{4(1-\beta)q} - c(\beta)$; For $\hat{\gamma} < \gamma < \frac{2-\beta}{\beta} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)}$, we have $\left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right) < \gamma \frac{(q-\omega)^2}{4q}$, $p_a^*(\gamma, \omega) = \frac{q+\omega}{2}$, $\pi_M(\gamma, \omega) = \pi_M^I(\gamma, \omega) := \omega\gamma(1 - \frac{q+\omega}{2q}) - \gamma c(\beta) = \gamma \frac{\omega(q-\omega)}{2q} - \gamma c(\beta)$, which is also a linear function of γ with a slope of $s^I = \frac{\omega(q-\omega)}{2q} - c(\beta)$. Note that, at $\gamma = \frac{2-\beta}{\beta} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)}$, we have $\left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right) - \gamma \frac{(q-\omega)^2}{4q} = -\frac{((2-\beta)\omega - (2-3\beta)q)^2}{4q\beta(2-\beta)} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)} < 0$, i.e., $\left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right) < \gamma \frac{(q-\omega)^2}{4q}$. Hence, $\hat{\gamma} < \frac{2-\beta}{\beta} \frac{2q(1-\beta)-\omega(2-\beta)}{4q(1-\beta)-\omega(2-\beta)}$.

To sum up, the manufacturer's profit function is $\pi_M^I(\gamma, \omega)$ for $\gamma > \hat{\gamma}$ and $\pi_M^{II}(\gamma, \omega)$ for $\gamma < \hat{\gamma}$. We note that, for any ω , $\pi_M^{II}(\gamma, \omega) - \pi_M^I(\gamma, \omega) = \omega \frac{(1-\gamma)(2-\beta)}{4(1-\beta)} \left(\frac{2q(1-\beta)}{2-\beta} - \omega \right) > 0$. That is, the profit function is discontinuous at $\gamma = \hat{\gamma}$ and decreases at this discontinuous point. Also note that $\pi_M(0, \omega) = \pi_M^{II}(0, \omega)$, and $\pi_M(1, \omega) = \pi_M^I(1, \omega) = \pi_M^{II}(1, \omega)$. Besides, we have $s^I - s^{II} = \frac{\omega(q-\omega)}{2q} - \frac{\beta\omega^2}{4(1-\beta)q} = \frac{\omega(2q(1-\beta) - (2-\beta)\omega)}{4(1-\beta)q} > 0$. Hence, while the potential maximizer may be $\gamma = 0$, $\hat{\gamma}$ or 1, note that $\gamma = \hat{\gamma}$ is always dominated by $\gamma = 0$ or 1. As such, given ω , the optimal combat level can only be either 0 or 1.

(iii) $\omega < \frac{2-3\beta}{2-\beta}q$: In this case, we have $\omega < \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2} < \frac{q+\omega}{2} < \frac{2q(1-\beta)}{2-\beta}$. Then, by FOC, $p_a^*(\gamma, \omega) = \frac{1-\beta}{2-\beta-\gamma\beta}q + \frac{\omega}{2}$ and $\Pi_R(\gamma, \omega) := \pi_R(p_a^*(\gamma, \omega); \gamma, \omega) = \left(\frac{1-\beta}{2-\beta-\gamma\beta}q - \frac{1}{2}\omega \right) \left(\frac{1}{2} - \frac{2-\beta-\gamma\beta}{4(1-\beta)} \frac{\omega}{q} \right)$. Substituting this result into (6), we have

$$\begin{aligned} \pi_M(\gamma, \omega) &= \omega \left[\gamma \left(1 - \frac{p_a^*(\gamma, \omega)}{q} \right) + (1-\gamma) \left(1 - \frac{2-\beta}{2q(1-\beta)} p_a^*(\gamma, \omega) \right) \right] - \gamma c(\beta) \\ &= \omega \left[\gamma \left(1 - \frac{1-\beta}{2-\beta-\gamma\beta} - \frac{\omega}{2q} \right) + (1-\gamma) \left(1 - \frac{2-\beta}{2(2-\beta-\gamma\beta)} - \frac{2-\beta}{2(1-\beta)} \frac{\omega}{2q} \right) \right] - \gamma c(\beta) \\ &= \omega \frac{2(1-\beta)q - (2-\beta-\gamma\beta)\omega}{4(1-\beta)q} - \gamma c(\beta), \end{aligned}$$

which is also linear in γ . As such, given ω , the optimal combat can only be either 0 or 1.

Let $\omega^*(\gamma) := \arg \max_{\omega} \pi_M(\gamma, \omega)$ and $\Pi_M(\gamma) := \pi_M(\gamma, \omega^*(\gamma))$. It suffices to compare $\Pi_M(0)$ and $\Pi_M(1)$ to obtain the optimal combat level. Note that $\pi_M(0, \omega) = \omega \left(\frac{2(1-\beta)q - (2-\beta)\omega}{4(1-\beta)q} \right)^+$ and $\pi_M(1, \omega) = \omega \frac{q-\omega}{2q} -$

$c(\beta)$. Then, $\omega^*(0) = \frac{1-\beta}{2-\beta}q$ and $\omega^*(1) = \frac{q}{2}$; so we have $\Pi_M(0) = \frac{1-\beta}{4(2-\beta)}q$ and $\Pi_M(1) = \frac{q}{8} - c(\beta)$. Let $\hat{q}^M = \frac{8c(\beta)(2-\beta)}{\beta}$. Then, $\gamma^* = 1$ when $q \geq \hat{q}^M$ and $\gamma^* = 0$ otherwise. For $q \geq \hat{q}^M$, $\omega^* = \frac{q}{2}$, $p_a^* = \frac{3}{4}q$, $D_a^* = \frac{1}{4}$, $\pi_R^* = \frac{q}{16}$ and $\pi_M^* = \frac{q}{8} - c(\beta)$. For $q < \hat{q}^M$, $\omega^* = \frac{1-\beta}{2-\beta}q$, $p_a^* = \frac{3(1-\beta)}{2(2-\beta)}q$, $p_f^* = \frac{3\beta(1-\beta)}{4(2-\beta)}q$, $D_a^* = \frac{1}{4}$, $D_f^* = \frac{3}{4(2-\beta)}$, $\pi_R^* = \frac{1-\beta}{8(2-\beta)}q$, $\pi_M^* = \frac{1-\beta}{4(2-\beta)}q$ and $\pi_f^* = \frac{9\beta(1-\beta)}{16(2-\beta)^2}q$. $\square$

Proof of Proposition 2. As in Lemma 2, we have, given ω , $\gamma^*(\omega) = 1$ if $q > \hat{q}^B = \frac{4c(\beta)(2-\beta)}{\beta}$ and $\omega \leq \bar{\omega}(q)$; $\gamma^*(\omega) = 0$ otherwise. Specifically,

$$\bar{\omega}(q) = \begin{cases} \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}}, & \text{if } q < \frac{4c(\beta)(2-\beta)^2}{\beta^2}, \\ q - 2\sqrt{qc(\beta)}, & \text{if } q \geq \frac{4c(\beta)(2-\beta)^2}{\beta^2}. \end{cases} \quad (12)$$

Next, we characterize the manufacturer's optimal strategy based on the value of q . Specifically, by $\bar{\omega}(\bar{q}) = \frac{\bar{q}}{2}$, we have $\bar{q} = 16c(\beta)$ for $\beta > \frac{2}{3}$ and $\bar{q} = \frac{32c(\beta)(1-\beta)(2-\beta)}{\beta(6-7\beta)}$ for $\beta \leq \frac{2}{3}$. Note that the case of $q < \hat{q}^B$ means the retailer will not combat even if the wholesale price is zero and $q > \bar{q}$ corresponds to the situation where the retailer will combat even without wholesale price reduction. Also note that these two cases coincide with the two benchmarks. Thus, we will focus on $q \in (\hat{q}^B, \bar{q})$ in this proof.

Then, by (9), (10) and (S.2), we have

$$\gamma^*(\omega) = \begin{cases} 1, & \text{if } \omega \leq \bar{\omega}(q), \\ 0, & \text{if } \omega > \bar{\omega}(q), \end{cases} \quad \text{and} \quad \pi_M(\omega) = \begin{cases} \omega \frac{q-\omega}{2q}, & \text{if } \omega \leq \bar{\omega}(q), \\ \omega \left(\frac{2(1-\beta)q - (2-\beta)\omega}{4(1-\beta)q} \right)^+, & \text{if } \omega > \bar{\omega}(q). \end{cases}$$

Note that $\bar{\omega}(q) \frac{q-\bar{\omega}(q)}{2q} > \bar{\omega}(q) \frac{2(1-\beta)q - (2-\beta)\bar{\omega}(q)}{4(1-\beta)q}$ for any $\bar{\omega}(q)$ and $\pi_M(\omega)$ decreases at this discontinuous point. It then indicates that $\pi_M(\omega)$ can be potentially maximized at $\omega = \bar{\omega}(q)$, $\frac{1-\beta}{2-\beta}q$ and $\frac{q}{2}$, which correspond to the discontinuous point, the FOC points of $\omega \frac{2(1-\beta)q - (2-\beta)\omega}{4(1-\beta)q}$ and $\omega \frac{q-\omega}{2q}$, respectively. To find the optimal solution, we need to compare $\bar{\omega}(q)$, $\frac{1-\beta}{2-\beta}q$ and $\frac{q}{2}$. Note that $\frac{1-\beta}{2-\beta}q < \frac{q}{2}$ always holds, so we have the following three cases: (i) $\bar{\omega}(q) < \frac{1-\beta}{2-\beta}q < \frac{q}{2}$; (ii) $\frac{1-\beta}{2-\beta}q < \bar{\omega}(q) < \frac{q}{2}$; (iii) $\frac{1-\beta}{2-\beta}q < \frac{q}{2} < \bar{\omega}(q)$. Specifically, (i) happens if $\hat{q}^B < q < \frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)}$; (ii) happens if $\frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)} < q < \bar{q}$; (iii) happens if $q > \bar{q}$. These cases are graphically depicted in Figure A1.

Figure A1 An illustration for $\pi_M(\omega)$: $c(\beta) = \frac{1}{30}$ and $\beta = 0.3$. Left panel for case (i) with $q = 0.85$; Middle panel for case (ii) with $q = 1$; Right panel for case (iii) with $q = 1.5$.

(i) $\hat{q}^B < q < \frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)}$: By noting that $\frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)} - \frac{4c(\beta)(2-\beta)^2}{\beta^2} = -\frac{12c(\beta)(2-\beta)^2(1-\beta)}{\beta^2(3-\beta)} < 0$, i.e., $\frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)} < \frac{4c(\beta)(2-\beta)^2}{\beta^2}$, we have $\bar{\omega}(q) = \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}}$. In this case, ω^* can be either $\bar{\omega}(q)$ or $\frac{1-\beta}{2-\beta}q$ and $\pi_M^* = \max \left\{ \bar{\omega}(q) \frac{q-\bar{\omega}(q)}{2q}, \frac{1-\beta}{4(2-\beta)}q \right\}$. The indifference condition for the manufacturer between charging $\bar{\omega}(q) = \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}}$ while inducing the retailer to combat and charging $\frac{1-\beta}{2-\beta}q$ while not inducing the retailer to combat is then given by $\bar{\omega}(q) \frac{q-\bar{\omega}(q)}{2q} = \frac{1-\beta}{4(2-\beta)}q$. Then, there exists a threshold $\hat{q}^R = \frac{-3+4\beta+\sqrt{\beta(2-\beta)}}{17\beta-9} \frac{16c(\beta)(2-\beta)}{\beta} \in (\hat{q}^B, \frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)})$, such that the manufacturer induces the retailer to combat if and only if $q \geq \hat{q}^R$.

(ii) $\frac{8c(\beta)(2-\beta)^2}{\beta(3-\beta)} < q < \bar{q}$: In this case, the profit function increases when $\omega \leq \bar{\omega}(q)$ but decreases when $\omega > \bar{\omega}(q)$. Hence, there must be $\omega^* = \bar{\omega}(q)$ and the retailer always combats.

(iii) $q > \bar{q}$: In this case, the profit function increases when $\omega \leq \frac{q}{2}$ but decreases when $\omega > \frac{q}{2}$. Hence, there must be $\omega^* = \frac{q}{2}$ and the retailer always combats.

Therefore, combining (i), (ii) and (iii), the retailer combats if $q \geq \hat{q}^R$ and does not combat if $q < \hat{q}^R$. The corresponding optimal wholesale price is as follows:

$$\omega^* = \begin{cases} \frac{1-\beta}{2-\beta}q, & \text{if } q < \hat{q}^R, \\ \bar{\omega}(q), & \text{if } \hat{q}^R \leq q < \bar{q}, \\ \frac{q}{2}, & \text{otherwise.} \end{cases}$$

By the expressions of $\bar{q}$ and $\bar{\omega}(q)$, for $\beta > \frac{2}{3}$, we have

$$\omega^* = \begin{cases} \frac{1-\beta}{2-\beta}q, & \text{if } q < \hat{q}^R, \\ \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}}, & \text{if } \hat{q}^R \leq q < \frac{4c(\beta)(2-\beta)^2}{\beta^2}, \\ q - 2\sqrt{qc(\beta)}, & \text{if } \frac{4c(\beta)(2-\beta)^2}{\beta^2} \leq q \leq 16c(\beta), \\ \frac{q}{2}, & \text{otherwise,} \end{cases}$$

and for $\beta \leq \frac{2}{3}$, by $\frac{32c(\beta)(1-\beta)(2-\beta)}{\beta(6-7\beta)} < \frac{4c(\beta)(2-\beta)^2}{\beta^2}$, we have

$$\omega^* = \begin{cases} \frac{1-\beta}{2-\beta}q, & \text{if } q < \hat{q}^R, \\ \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}}, & \text{if } \hat{q}^R \leq q < \frac{32c(\beta)(1-\beta)(2-\beta)}{\beta(6-7\beta)}, \\ \frac{q}{2}, & \text{otherwise.} \end{cases}$$

Moreover, D_a^* and p_a^* can be obtained by (1) and $p_a^* = \frac{1-\beta}{2-\beta-\gamma^*\beta}q + \frac{\omega^*}{2}$; D_f^* and p_f^* can be obtained by (2) and $p_f^* = \frac{\beta}{2}p_a^*$; π_f^* , π_R^* and π_M^* can be obtained by (3), (8) and (9). The results are as in Table A1.

$\beta > \frac{2}{3}$	p_a^*	p_f^*	ω^*	π_R^*	π_M^*	π_f^*
$q < \hat{q}^R$	$\frac{3(1-\beta)}{2(2-\beta)}q$	$\frac{3\beta(1-\beta)}{4(2-\beta)}q$	$\frac{1-\beta}{2-\beta}q$	$\frac{1-\beta}{8(2-\beta)}q$	$\frac{1-\beta}{4(2-\beta)}q$	$\frac{9\beta(1-\beta)}{16(2-\beta)^2}q$
$\hat{q}^R \leq q \leq \frac{4c(\beta)(2-\beta)^2}{\beta^2}$	$\frac{q+\omega_1^R}{2}$	—	ω_1^R	$\frac{(q-\omega_1^R)^2}{4q} - c(\beta)$	$\frac{\omega_1^R(q-\omega_1^R)}{2q}$	—
$\frac{4c(\beta)(2-\beta)^2}{\beta^2} < q < 16c(\beta)$	$\frac{q+\omega_2^R}{2}$	—	ω_2^R	0	$\frac{\omega_2^R(q-\omega_2^R)}{2q}$	—
$q \geq 16c(\beta)$	$\frac{3}{4}q$	—	$\frac{q}{2}$	$\frac{q}{16} - c(\beta)$	$\frac{q}{8}$	—
$\beta \leq \frac{2}{3}$	p_a^*	p_f^*	ω^*	π_R^*	π_M^*	π_f^*
$q < \hat{q}^R$	$\frac{3(1-\beta)}{2(2-\beta)}q$	$\frac{3\beta(1-\beta)}{4(2-\beta)}q$	$\frac{1-\beta}{2-\beta}q$	$\frac{1-\beta}{8(2-\beta)}q$	$\frac{1-\beta}{4(2-\beta)}q$	$\frac{9\beta(1-\beta)}{16(2-\beta)^2}q$
$\hat{q}^R \leq q \leq \frac{32c(\beta)(1-\beta)(2-\beta)}{\beta(6-7\beta)}$	$\frac{q+\omega_1^R}{2}$	—	ω_1^R	$\frac{(q-\omega_1^R)^2}{4q} - c(\beta)$	$\frac{\omega_1^R(q-\omega_1^R)}{2q}$	—
$q > \frac{32c(\beta)(1-\beta)(2-\beta)}{\beta(6-7\beta)}$	$\frac{3}{4}q$	—	$\frac{q}{2}$	$\frac{q}{16} - c(\beta)$	$\frac{q}{8}$	—

Table A1 Equilibrium results under DR for $\beta > \frac{2}{3}$ (upper) and $\beta \leq \frac{2}{3}$ (bottom). Specifically,

$$\omega_1^R = \sqrt{\frac{2(1-\beta)q(\beta q - 4c(\beta)(2-\beta))}{\beta(2-\beta)}} \text{ and } \omega_2^R = q - 2\sqrt{qc(\beta)}.$$