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ONE-WAY CARSHARING SERVICE DESIGN UNDER DEMAND UNCERTAINTY: A SERVICE RELIABILITY-BASED TWO-STAGE STOCHASTIC PROGRAM APPROACH

--Manuscript Draft--

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**ONE-WAY CARSHARING SERVICE DESIGN UNDER DEMAND UNCERTAINTY: A
SERVICE RELIABILITY-BASED TWO-STAGE STOCHASTIC PROGRAM
APPROACH**

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ABSTRACT

This study proposes a service reliability-based two-stage stochastic program approach for one-way carsharing systems. The objective is to improve the profitability for one-way carsharing operators under demand uncertainty. We first formulate the two-stage stochastic model, where stage 1 determines the strategic and tactic deployments for the regular service, including the number of vehicles, personnel and parking lots. Upon realization of the random demand, stage 2 deploys ad hoc services and optimizes vehicle relocation and personnel assignment. The ad hoc service is integrated into the model, which serves the demand that cannot be satisfied by the regular service. Further, a SR-based gradient descent algorithm is developed to find the optimal SR. Finally, the performance of the proposed model and algorithm is tested with a numerical study, in which sensitivity analysis over the degree of demand variation and ad hoc service cost are conducted. Results show that up to 19% improvement in profit can be achieved by the proposed stochastic approach compared to the solution obtained by the deterministic model.

Keywords: one-way carsharing service design, demand uncertainty, service reliability, two-stage stochastic programming, ad hoc service

1 INTRODUCTION

The convenience and cost-effectiveness of carsharing have made it an attractive alternative to private vehicle and public transport. Empirical studies have demonstrated that carsharing can effectively reduce private vehicle ownership, which further improves the utilization rate of vehicles, saves lands from parking, mitigates traffic congestion and vehicle emissions (1). The advantages of carsharing are further expanded by the synergistic effects of the integration between carsharing and other transportation modes, which can increase the city mobility (2). Carsharing systems can be broadly divided into two categories: round-trip and one-way carsharing systems. Round-trip carsharing systems restrict users to pick up and drop off rental vehicles at the same station, while one-way systems allow users to return rental vehicles to any station. From the perspective of users, one-way carsharing systems are more flexible than round-trip systems. Whilst, though one-way carsharing has the potential to attract more demand, one-way carsharing service has not yet been adopted by many carsharing operators. This is because such a service brings about a series of strategic and managerial complexities (3, 4).

The complexities in one-way carsharing systems can be categorized into strategic, tactic and operational levels (5). Decisions for the locations, amounts and parking capacity of carsharing stations belong to the strategic level. The scale of vehicle fleet and the number of vehicles deployed at each station are at the tactic level, and operational decisions involve vehicle relocation, personnel assignment and price charge for customers. Carsharing operators determine the strategic, tactic and operational plans response to their estimation of carsharing demand. Jorge et al. (6) presented a comprehensive review regarding the demand and supply modeling of carsharing systems, and they highlighted the key challenge for one-way carsharing operation is how to match the uneven distributed spatio-temporally demand. The uneven nature of travelers' demand may lead to carsharing vehicles gradually concentrating in certain regions, limiting the availability of carsharing vehicles in other regions. This brings about not only demand loss which may further result in the reduction in profit, but also the downgrade of quality of service. Therefore, to better match demand, one-way carsharing operators ought to periodically check the distribution of vehicles, and relocate idle vehicles based on their prediction of future demand (4).

Reviewing the current literature in one-way carsharing operation, researchers have made different assumptions when they model the demand. One of the most common methods is to treat demand as a deterministic (7, 8). However, in reality, the demand for carsharing is influenced by a variety of factors, such as time of the day, day of the week, social activities and weather conditions. Strategies designed for deterministic demand might not be able to capture the demand variation and therefore be a less realistic option. Treating carsharing demand as elastic is another research direction. In those studies, carsharing demand is usually assumed to be sensitive to the trip price, the accessibility of carsharing stations, vehicle availability at stations (9–12). The elastic demand setting is able to capture the interactions between carsharing demand and system-related factors such as the supply of vehicles, price, etc. However, those elasticity coefficients can hardly be accurately calibrated due to data limitation.

A more realistic assumption is to treat carsharing demand as stochastic and utilize historical operational data to model the variation in carsharing demand. In the presence of stochastic demand, robust optimization and stochastic optimization are two typical approaches. The first approach focuses on min-max risk control, aiming at producing strategies that are robust to either the worst scenario (on the design margin) or scenarios within safe boundaries. He et al. (13) developed a robust up-to and down-to policy for carsharing reposition problem to account for stochastic

1 demand. In such a two-threshold policy, the repositioning of vehicles is activated only if the
 2 inventory of shared vehicles in any station is out of the prespecified inventory range. Such methods
 3 can lead to conservative deployment plan. Stochastic optimizations, on the other hand,
 4 approximate real-world situations by sampling a set of scenarios and adopt the solution with the
 5 best objective expectation, for instance, the maximum profit or the minimum operational cost. Fan
 6 (14) modeled the stochastic demand using a scenario tree, where each node stands for the states at
 7 a particular time interval and each arc represents the demand realization. Hua et al. (15) considered
 8 the uncertain carsharing demand using a scenario tree-based model as well, which describes a
 9 multistage stochastic process evolving over time. A completed path from the first stage to the end
 10 represents a demand realization scenario, which is sampled with a certain probability. In these
 11 scenario tree-based models, decisions to be made in the downstream nodes depend on the decisions
 12 made in its parent node. Building such a scenario tree requires a large amount of information. The
 13 overall objective is mostly related to the decision made in the very first stage that evaluates its
 14 impacts on the following stages and makes a trade-off between current profit and longer-term
 15 expected profit.

16 In existing carsharing stochastic optimization studies, quality of service (QoS), often
 17 defined to be the ratio of realized demand to the total demand, is an important output to evaluate
 18 the performance of the model. Lu et al. (4) proposed a two-stage stochastic integer programming
 19 approach to maximize carsharing operators' profitability and QoS under demand uncertainty.
 20 However, in their model, the relocation effects are not taken into consideration and QoS is an
 21 output of their model. In our study, we propose a stochastic modeling approach which treats service
 22 reliability (SR) as a decision variable. SR is a common concept in service design problems (16,
 23 17). In the carsharing operation context, SR refers to the proportion of demand that can be satisfied
 24 by the available carsharing operator-owned vehicles. Then, those extra demand (due to demand
 25 stochasticity) that cannot be served by existing carsharing vehicles will be served by outsourced
 26 mobility services (e.g. ridehailing, taxi, etc.) at a higher cost. Setting a high SR indicates allocating
 27 more budget to fixed resources and fewer budget to outsourced ad hoc service, which may lead to
 28 the waste of fixed resources when the real demand is low. While setting a low SR can save money
 29 from purchasing fixed resources but may require lots of outsourced ad hoc services when the real
 30 demand is high. Then the key point is to determine the proper SR at each station at each time
 31 interval to guarantee the profitability of carsharing systems. To this end, we propose a SR-based
 32 two-stage stochastic model to maximize the profitability of one-way carsharing systems under
 33 demand uncertainty.

34 We summarize three major advantages of applying the SR-based stochastic optimization
 35 to handle demand uncertainty in one-way carsharing systems. First, the introduction of SR
 36 decouples the complexities between long-term fixed strategic and tactic deployments and short-
 37 term operational schemes into two stages: the first stage is to determine the SR level at each station
 38 at each time interval and the second stage is to evaluate the profitability of such a SR level design
 39 under a series of scenarios.

40 Second, SR enables us to incorporate ad hoc service with regular service. Various types of
 41 innovative transport service can serve as ad hoc service such as ride-hailing and micro-transport.
 42 Once SR is determined, the maximum capability of the regular service is also determined. Any
 43 demand that cannot be satisfied by the regular service can be met by the ad hoc service, which is
 44 generally at a higher cost than the former. As SR is solved in the first stage, carsharing operators
 45 can decide when and where to active the ad hoc service. This shows the potential of applying our
 46 model to integrate carsharing with other transport modes.

Third, the SR-based model is flexible and applicable to different transport networks. Spatio-temporally uncertain demands are modeled by a set of distributions. Carsharing operators can change these distributions if they observe an update in customer demand patterns and adapt their market strategies. Our model also allows the distributions of demand at different stations at different times of the day to be different. This makes the representation of demand uncertainty to be more flexible and practical.

The contributions of this study are further summarized as follows:

- From the modeling perspective, we propose a service reliability-based two-stage programming model to account for the uncertain demand for one-way carsharing systems. A SR-based gradient solution algorithm is applied to solve the proposed stochastic optimization model.
- From the practical perspective, we propose a comprehensive one-way carsharing planning and operation strategy that 1) from the strategic planning level determines the optimal fleet and personnel deployment, and 2) from the operational level integrates the regular operator-based relocation approach with the ad hoc service.

The rest of paper is structured as follows. Section 2 presents assumptions and model formulations. Section 3 elaborates the solution algorithm for finding the best service reliability at each station. Section 4 presents the results of our model with a case study. Section 5 concludes our work.

2 MODEL FORMULATION

2.1 Assumptions

Without loss of generality, we make the following assumptions in this study:

- Historical carsharing demand patterns are known. In the deterministic problem, we assume the travel demand is known beforehand. In the stochastic demand model, the demand uncertainty is modeled to follow a series of distributions. We assume demand distributions can be obtained from carsharing historical operation data.
- All customer trip requests shall be satisfied. If the trip request cannot be served by the regular service, ad hoc service should be hired to satisfy the excessive demand.
- Carsharing operators charge customers by unit time interval of vehicle use.
- Travel time is discretized into time intervals where each interval represents 0.5 hour in this study.

The notations are summarized as follows.

Sets and parameters

N	Set of stations
t, t_{ij}^0	Time interval index
T	Set of time intervals $T = \{1, 2, \dots, T_{max}\}$
Δ	Duration of one time interval
p	Charge for one-way carsharing users per time interval
c_f	Fuel cost per time interval
c_r	Relocation cost per time interval
c_{mc}	Movement cost of personnel per time interval
c_s	Rental cost of a parking lot per day

c_v	Amortized cost of a vehicle per day
c_p	Personnel payment per time interval
AC	Cost of ad hoc service per time interval
$\tilde{d}_{ij}^t$	Random demand originated from station i to j at time interval t
τ_{ij}	Travel time from station i to j
N_v	Maximum number of vehicles
N_p	Maximum number of personnel

Decision variables

x_{ij}^t	Number of personnel relocating cars from station i to j at the beginning of time interval t
y_{ij}^t	Number of personnel moving from station i to j by other modes at the beginning of time interval t
z_{ij}^t	Number of hiring ad hoc service from station i to j at the beginning of time interval t
$\mathbf{s}$	The vector of the initial number of parking lots at each station. $\mathbf{s} = [s_1, \dots, s_N]$
$\mathbf{n}^0$	The vector of the initial number of vehicles at each station. $\mathbf{n}^0 = [n_1^0, \dots, n_N^0]$
$\mathbf{m}^0$	The vector of the initial number of personnel at each station. $\mathbf{m}^0 = [m_1^0, \dots, m_N^0]$

Auxiliary variables

n_i^t	Number of vehicles at station i at the end of time interval t
m_i^t	Number of personnel at station i at the end of time interval t

2.2 Dterministic Problem

We start by formulating the deterministic problem where the customer demand is fixed. Figure 1 (a) shows the vehicle flows of station 1 in a network with N stations over T time intervals. There are three types of arcs representing three states of vehicles: idle arc, relocation arc and service arc, where the arc flow represents the number of vehicles. The travel time between each OD pair can be read from the horizontal time interval axis. In this study, the travel time between origin station and destination station is assumed to be symmetric.

(a) Vehicle movements

FIGURE 1 Illustration of one-way carsharing vehicle movements

Each shared vehicle is either driven by customers, relocated by personnel or idle at the current station. The number of available vehicles at station i at the end of time interval t is denoted by n_i^t . In particular, n_i^0 , i.e., $t = 0$, represents the initial fleet deployment at station i . Similarly, m_i^0 denotes the number of initial personnel at station i . Denote d_{ij}^t as the number of vehicles driven by customers, i.e., customer demand from station i to j at time interval t and x_{ij}^t as the number of vehicles relocated by personnel from station i to j ($j \neq i$) at the beginning of time interval t . The dynamics of vehicles at station i are illustrated by Figure 1 (b). The total outflow of vehicles at station i within time interval t is $\sum_{j \in N \setminus \{i\}} (d_{ij}^t + x_{ij}^t)$ and the total inflow is $\sum_{j \in N \setminus \{i\}} \sigma_{ji} (d_{ji}^{t^0} + x_{ji}^{t^0})$, where $d_{ji}^{t^0}$ and $x_{ji}^{t^0}$ denote the number of vehicles driven by customers and the number of vehicles under relocation, departing from station j at time interval t_{ji}^0 and will arrive at station i at current time interval t . Specifically, $t_{ji}^0 = t - \tau_{ji}$ and σ_{ji} is the binary indication that $\sigma_{ji} = 0$ if $t_{ji}^0 \leq 0$; otherwise $\sigma_{ji} = 1$. This appears in the first several time intervals when no vehicle arrives at station i . Therefore, the flow constraints of vehicles are given by Equation 2. Equation 3 constricts that the maximum outflow of vehicles at station i at time interval t should not exceed its available vehicles at the end of the last time interval $t - 1$. Similarly, the flow constraints of personnel are given by Equation 4 and Equation 5.

$$\sigma_{ji} = \begin{cases} 0, & \text{if } t_{ji}^0 \leq 0 \\ 1, & \text{if } t_{ji}^0 > 0 \end{cases} \quad (1)$$

$$n_i^t = n_i^{t-1} - \sum_{j \in N \setminus \{i\}} (d_{ij}^t + x_{ij}^t) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} (d_{ji}^{t^0} + x_{ji}^{t^0}) \quad \forall i \in N, t \in T \quad (2)$$

$$\sum_{j \in N \setminus \{i\}} (d_{ij}^t + x_{ij}^t) \leq n_i^{t-1} \quad \forall i \in N, t \in T \quad (3)$$

$$m_i^t = m_i^{t-1} - \sum_{j \in N \setminus \{i\}} (x_{ij}^t + y_{ij}^t) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} (x_{ji}^{t^0} + y_{ji}^{t^0}) \quad \forall i \in N, t \in T \quad (4)$$

$$\sum_{j \in N \setminus \{i\}} (x_{ij}^t + y_{ij}^t) \leq m_i^{t-1} \quad \forall i \in N, t \in T \quad (5)$$

Consider a problem where a carsharing operator needs to initially deploy shared vehicles (n_i^0) in a set of stations, allocate relocators (m_i^0) to rebalance these vehicles over T time intervals and the purchase a number of parking lots (s_i) at each station. As such, carsharing operators need to tailor a set of decision vector variables ($\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}$), the relocation scheme of vehicles ($\mathbf{x}$) and the movement assignment of personnel by other transport modes ($\mathbf{y}$) at each time interval. The deterministic problem with fixed demand $\mathbf{d} = \{d_{ij}^t\}, \forall i \neq j \in N, t \in T$ can be formulated as:

$$\begin{aligned} \max_{\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}, \mathbf{x}, \mathbf{y}} \pi = & (p - c_f) \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} d_{ij}^t - c_r \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} x_{ij}^t \\ & - c_{mc} \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} y_{ij}^t - c_v \sum_{i \in N} n_i^0 - c_p \sum_{i \in N} m_i^0 - c_s \sum_{i \in N} s_i \end{aligned} \quad (6)$$

subject to
Vehicle-related constraints (1), (2), (3) and

$$n_i^t \leq s_i \quad \forall i \in N, t \in \{0, T\} \quad (7)$$

$$\sum_{i \in N} n_i^0 \leq N_v \quad (8)$$

Personnel-related constraints (4), (5) and

$$\sum_{i \in N} m_i^0 \leq N_p \quad (9)$$

$$x_{ij}^t, y_{ij}^t, n_i^t, m_i^t, s_i \in \mathbb{N}^0 \quad \forall i \in N, j \in N \setminus \{i\}, t \in T \quad (10)$$

The objective function Equation (6) intends to maximize the profit for carsharing operators, i.e., the different between revenue, and operating and fixed cost: (i) the first term represents the total revenue over a given operating T interval where $(p - c_f)$ is the net revenue from one unit of vehicle-time interval and the rest summation is the total service time; (ii) the second term is the total relocation cost that personnel relocate the idle vehicles from station i to j over T intervals; (iii) the third term is the total travel cost of idle personnel moving from station i to j by other transportation modes at the travel cost of c_{mc} . For simplicity, we assume the time that personnel travel from station i to j is equal to τ_{ij} ; (iv) the last three terms are fixed costs: total parking cost, total vehicle amortized cost and the total salary for personnel, respectively. As for the constraints, Equation 1 and Equation 2 are vehicle flow constraints. Equation 3 represents the available number of vehicles constraints at the beginning of each time interval. Equation 7 defines the parking capacity limitation at each station. Equation 8 states the total number of vehicles should not exceed the maximum vehicle budget N_v . Similar to vehicle constraints, Equation 4 represents staff flow constraints, Equation 5 represents the available number of staff constraints at the beginning of each time interval and Equation 9 represents the maximum staff budget constraint. Equation 10 requires that all variables should be integer. In sum, the deterministic problem constitutes a MILP

In the deterministic problem, the objective is to determine the optimal initial deployment: fleet size n_i^0 , staff size m_i^0 and the number of parking lots s_i at each station, and tailor an optimal operational scheme x and y under the given travel demand.

2.3 The Two-stage Stochastic Formulation

FIGURE 2 Illustration of spatio-temporal demand uncertainty. LB, UB and μ denote the lower bound, upper bound and the average of demand, respectively. Each element represents the demand distribution at a station at one time interval

The nature of demand uncertainty introduces dynamic complexities to planning and operation. As shown in Figure 2, carsharing demand at each station at each time interval is characterized by a specific type of distribution or a set of distributions. If adopting the strategies and operational plans solved in deterministic cases, the overall profitability of carsharing operators becomes questionable.

To account for demand uncertainty, we formulate the stochastic problem as a two-stage stochastic program. Two types of services are integrated into the model: regular carsharing service and ad hoc service. Regular service is operated to maintain the required SR whereas ad hoc service provides a supplementary approach to satisfy those excessive carsharing demand. More specifically, when demand does not exceed the number of available vehicles at a carsharing station, such demand will be served by the regular carsharing service. Due to the demand uncertainty, it is possible that at certain times of the day and certain stations, the actual demand exceeds carsharing service capability. Those excessive demand will be served by the ad hoc service at a higher cost. The stochastic problem can be divided into two stages. The first stage is to determine the regular deployment, i.e., fleet size n_i^0 , staff size m_i^0 and the number of parking lots s_i at each station. The second stage optimizes the operational scheme under stochastic demand and stage 1 deployment constraints. It should be noted that these two stages are coupled together. The initial decisions in stage 1 determine the available regular resources which are then implemented in stage 2 under stochastic demand scenarios. In real-world cases, it is feasible to accommodate all carsharing travel demand by offering sufficient resources, such as vehicles, personnel and parking lots, even though sometimes these resources are not fully utilized. This, however, may introduce unnecessary fixed

cost. Hence, to determine the initial deployments in stage 1, the risks and opportunities of profit in stage 2 must be simultaneously considered.

The goal of two-stage stochastic program is to maximize the expected profit through reasonably deploying fleet size, personnel size and parking capacity at each station, simultaneously considering the cost of hiring ad hoc service for unserved demand under demand uncertainty. In this study, we resort to scenario simulation to approximate real-world stochastic demand. Let $H = \{1, 2, \dots, \theta\}$ be the scenario set, which consists θ scenarios in total. Assume each scenario $h \in H$ has equal probability of $p_h = 1/\theta$ with $\sum_{h \in H} p_h = 1$. As for each scenario, we assume the cost of ad hoc service for each unserved demand $z_{ij}^{t,h}$ from station i to j at time interval t is AC . AC can also be interpreted as the ‘regret’ cost or penalty due to the unreasonable initial deployment decided in stage 1. Let $\tilde{d}_{ij}^{t,h}$ be the stochastic demand from station i to j at time interval t in the scenario h . Then demand that can be served by the regular service deployment is given by Equation 11.

$$d_{ij}^{t,h} = \tilde{d}_{ij}^{t,h} - z_{ij}^{t,h} \quad \forall i \neq j \in N, t \in T, h \in H \quad (11)$$

If the actual random demand $\tilde{d}_{ij}^{t,h}$ is under the capacity of regular carsharing service, there will be no need to hire ad hoc service; otherwise, each excessive unserved demand will be satisfied by ad hoc service at a higher operating cost. The corresponding original two-stage stochastic problem is formulated as:

P0: the original stochastic problem

$$\max_{\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}} \Phi(\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}) = \bar{Q}(\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}) - c_v \sum_{i \in N} n_i^0 - c_p \sum_{i \in N} m_i^0 - c_s \sum_{i \in N} s_i \quad (12)$$

subject to:

(8), (9), and

$$n_i^t, m_i^t, s_i \in \mathbb{N}^0 \quad \forall i \in N, t \in T \quad (13)$$

where

$$\begin{aligned} \bar{Q}(\mathbf{n}^0, \mathbf{m}^0, \mathbf{s}) = & \max_{\mathbf{x}^h, \mathbf{y}^h, \mathbf{z}^h} \sum_{h \in H} p_h ((p - c_f) \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} d_{ij}^{t,h} \\ & - AC \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} z_{ij}^{t,h} - c_r \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} x_{ij}^{t,h} \\ & - c_{mc} \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} y_{ij}^{t,h}) \quad h \in H \end{aligned} \quad (14)$$

$$n_i^{t,h} = n_i^{t-1,h} - \sum_{j \in N \setminus \{i\}} (d_{ij}^{t,h} + x_{ij}^{t,h}) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} (d_{ji}^{t,h} + x_{ji}^{t,h}) \quad \forall i \in N, t \in T, h \in H \quad (15)$$

$$\sum_{j \in N \setminus \{i\}} (d_{ij}^{t,h} + x_{ij}^{t,h}) \leq n_i^{t-1} \quad \forall i \in N, t \in T, h \in H \quad (16)$$

(7), and

$$m_i^{t,h} = m_i^{t-1,h} - \sum_{j \in N \setminus \{i\}} (x_{ij}^{t,h} + y_{ij}^{t,h}) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} (d_{ji}^{t^0,h} + x_{ji}^{t^0,h}) \quad (17)$$

$\forall i \in N, t \in T, h \in H$

$$\sum_{j \in N \setminus \{i\}} (x_{ij}^{t,h} + y_{ij}^{t,h}) \leq m_i^{t-1} \quad \forall i \in N, t \in T, h \in H \quad (18)$$

$$x_{ij}^{t,h}, y_{ij}^{t,h}, z_{ij}^{t,h} \in \mathbb{N}^0 \quad \forall i \in N, j \in N \setminus \{i\}, t \in T \quad (19)$$

P0 constitutes two dependent stages: the initial deployment cost and the expected revenue. The objective function of stage 2 involves the expected revenue which is dependent on the deployments determined in stage 1 and the demand realization under stochastic scenarios. Stage 2 could not be solved without knowing the initial plans of fleet size n_i^0 , personnel size m_i^0 and number of parking lots s_i at each station. In other words, n_i^0, m_i^0 and s_i are solved from the stage-1 problem and used as initial constraints in stage-2 problem. On the other hand, initial deployments must take the random demand into consideration, which influences the achievable revenue from stage 2. The coupled interactions between stage-1 problem and stage-2 problem make this problem hard to tackle.

2.4 Reliability-based Two-stage Stochastic Formulation

The intricacy of **P0** lies on the uncertainties in demand, which couples two stage decisions. To tackle this problem, we propose a service reliability (SR) concept that the demand can be served by the initial deployment. Carsharing operators take both fixed cost and perceived profit into consideration and thus decide which level of SR to operate. In this study, SR is treated as an immediate variable that decouples the original two-stage problem. Let ρ_{ij}^t denote the SR provided for regular service demand from station i to j at time interval t . Let $\boldsymbol{\rho}$ denote the set of all ρ_{ij}^t . Assume the random demand $\tilde{d}_{ij}^t$ follows a specific distribution and its relative accumulative distribution function is $\varphi_{\tilde{d}_{ij}^t}^{-1}(\cdot)$. Denote $\bar{d}_{ij}^t$ as the demand that can be realized at the SR level of ρ_{ij}^t . Therefore, $\bar{d}_{ij}^t$ can be obtained by Equation 20 and Equation 21.

$$\Pr\{\tilde{d}_{ij}^t \leq \bar{d}_{ij}^t\} = \rho_{ij}^t \quad (20)$$

$$\bar{d}_{ij}^t = \varphi_{\tilde{d}_{ij}^t}^{-1}(\rho_{ij}^t) \quad (21)$$

With a given $\boldsymbol{\rho}$, demand to be served by regular service is thus fixed. Further, the initial deployments of fleet size, personnel size and the number of parking lots can be obtained in the stage-1 problem. The expected revenue can also be calculated by running all stochastic demand scenarios in stage-2 problem. The SR-based two-stage stochastic formulation is presented as follows:

P1: Stage-1 problem

$$\begin{aligned} \max_{n^0, m^0, s} \pi = & (p - c_f) \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} \bar{d}_{ij}^t - c_r \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} x_{ij}^t \\ & - c_{mc} \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} y_{ij}^t - c_v \sum_{i \in N} n_i^0 - c_p \sum_{i \in N} m_i^0 - c_s \sum_{i \in N} s_i \end{aligned} \quad (22)$$

1 subject to

$$n_i^t = n_i^{t-1} - \sum_{j \in N \setminus \{i\}} (\bar{d}_{ij}^t + x_{ij}^t) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} (\bar{d}_{ji}^t + x_{ji}^t) \quad \forall i \in N, t \in T \quad (23)$$

$$\sum_{j \in N \setminus \{i\}} (\bar{d}_{ij}^t + x_{ij}^t) \leq n_i^{t-1} \quad \forall i \in N, t \in T \quad (24)$$

2 and (1), (4), (5), (7), (8), (9), (10)

3

4 Based on the stage-1 decisions (n^0, m^0, s) , the following stage-2 problem is solved for
5 each demand scenario:

6

7 P2: Stage-2 problem

8

$$\begin{aligned} \max_{x^h, y^h, z^h} Q_h = & (p - c_f) \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} (\bar{d}_{ij}^{t,h} - z_{ij}^{t,h}) - AC \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} z_{ij}^{t,h} \\ & - c_r \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} x_{ij}^{t,h} - c_{mc} \sum_{t \in T} \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \tau_{ij} y_{ij}^{t,h} \quad h \in H \end{aligned} \quad (25)$$

9 subject to

$$n_i^{t,h} = n_i^{t-1,h} - \sum_{j \in N \setminus \{i\}} ((\bar{d}_{ij}^{t,h} - z_{ij}^{t,h}) + x_{ij}^{t,h}) + \sum_{j \in N \setminus \{i\}} \sigma_{ji} ((\bar{d}_{ji}^{t,h} - z_{ji}^{t,h}) + x_{ji}^{t,h}) \quad \forall i \in N, t \in T, h \in H \quad (26)$$

$$\sum_{j \in N \setminus \{i\}} ((\bar{d}_{ij}^{t,h} - z_{ij}^{t,h}) + x_{ij}^{t,h}) \leq n_i^{t-1,h} \quad \forall i \in N, t \in T, h \in H \quad (27)$$

$$n_i^{0,h} = n_i^0 \quad \forall i \in N, h \in H \quad (28)$$

$$m_i^{0,h} = m_i^0 \quad \forall i \in N, h \in H \quad (29)$$

10 and (7), (16), (17), (18)

11

12 **P2** is solved for single h scenario, and is repeated H times. The expected revenue
13 considering stochastic demand and ad hoc service is determined by the following equation:

14

$$\bar{Q}(\rho) = \sum_{h \in H} p_h Q_h \quad (30)$$

15

16 Combining two stages, the objective of reliability-based two-stage problem is:

17

$$\max_{\rho, n^0, m^0, s} \pi(\rho) = \bar{Q}(\rho) - c_v \sum_{i \in N} n_i^0 - c_p \sum_{i \in N} m_i^0 - c_s \sum_{i \in N} s_i \quad (31)$$

With the fixed SR ρ , the last three terms in the objective function Equation 31 represent the fixed cost including total cost of parking lots, total fixed cost of vehicles and salary of staff. These decision variables are solved in **P1**. The first term is the expected revenue, which involves stochastic demand scenarios and is solved in **P2**. The objective of the SR-based two-stage stochastic model is to determine the optimal ρ that maximizes the objective function. Here, we develop a SR-based gradient descent method over ρ to solve **P1** and then **P2** repeatedly, until we obtain the optimal solution of ρ . The procedure is presented in the Section 3.

3 SERVICE RELIABILITY BASED SOLUTION ALGORITHM

In the above, we decompose the original stochastic problem **P0** into a SR-based two-stage stochastic problem **P1-P2** by introducing an interval variable ρ . Yet the key focus is to find the optimal ρ^* . The procedure developed in the following, namely SR-based gradient descent algorithm, aims at finding a (local) optimal solution (along the opposite descent direction to maximize profit) until reaching the convergence criterion. Unlike other heuristic algorithms that cannot ensure the solution quality, the proposed algorithm is capable to find the local optimal solution. Further, instead of cutting away the feasible region to find the optimal solution as most of traditional Multi-cut or L-shaped methods do, SR-based gradient descent algorithm searches for the ascent direction through ρ . Last, the advantage of applying this algorithm is further enhanced by the concept of SR vector ρ . Recall that ρ represents the probability, or the service reliability, that the demand can be covered by the regular service. Thus, one can decide which level of service to offer in the specific regions during specific time interval by adding pre-specified service reliability ρ_{ij}^t into algorithm constraints. This enables carsharing operators flexibly adjust their market strategies to maximize their profits while their budgets are limited.

SR-based gradient descent algorithm is briefly described as follows: initialize a feasible solution vector $\rho, \forall \rho_{ij}^t \in [0, 1), t \in T, i \neq j \in N$ in step 1. Based on the initial vector ρ , step 2 and step 3 solve stage-1 problem and stage-2 problem sequentially to obtain the total profit in Equation 31. If the objective function dose not reach the stopping criterion (step 4), then, calculate the descent gradient of ρ in step 5. Due to the discrete nature of decision variables (i.e., (n_k^0, m_k^0, s_k)), it is impossible to obtain the descent gradient of ρ at a time. As such, we calculate the partial derivative with respect to each element in vector ρ through perturbation analysis. Step 2-5 will run repeatedly until the stopping criterion is satisfied. The algorithm is shown in Figure 3.

Optimization Algorithm for ρ in Pseudocode

Start

Load inputs: demand data; travel time data

Step 1: Initialization

- (a) Determine the feasible region of ρ ;
- (b) Set $k = 1$; historical optimal scheme $\Omega = \phi$; initialize $\rho_k = (\rho_{ij}^t)_k, \forall \rho_{ij}^t \in [0, 1), t \in T, i \neq j \in N$;
- (c) Initialize Adam parameters: step size $\lambda = 0.01$; small amount variable $\partial = 10^{-8}$ (used for numerical stabilization); the first and second moment variables $u = 0, r = 0$; decay rates for moment estimations $\omega_1 = 0.9, \omega_2 = 0.999$ (suggested default);
- (d) Initialize the cumulative indication $\alpha = 0$ and maximum cumulative indication number $\beta = 3$.

Step 2: Solve stage-1 problem

Solve stage-1 problem (**P1**) based on the fixed ρ_k , and then attain optimal scheme (n_k^0, m_k^0, s_k) ;
If $(n_k^0, m_k^0, s_k) \in \Omega$, take the corresponding objective function as $\pi(\rho_k)$; else, go to **Step 3**.

Step 3: Solve stage-2 problem**3.1:** Calculate the average optimal objective value $\bar{Q}(\rho_k)$

Solve **Equation 25** (n_k^0, m_k^0, s_k) from **Step 2** and attain the Q_h for each scenario $h \in H$;
Attain the average profit $\bar{Q}(\rho_k)$ by calling **Equation 30**;

3.2: Update historical optimal scheme set Ω

Attain the optimal objective function $\pi(\rho_k)$ of stage-2 problem using **Equation 31**;
Add current result $((n_k^0, m_k^0, s_k), \pi(\rho_k))$ to Ω ;

Step 4: Stopping criterion

If $|\pi(\rho_k) - \pi(\rho_{k-1})| / \pi(\rho_{k-1}) < \epsilon, \alpha = \alpha + 1$; else reset $\alpha = 0$. If $\alpha > \beta$ stop; else proceed to **Step 5**;

Step 5: Determine the optimal ρ **5.1:** Calculate the gradient $\nabla\pi(\rho_k)$ over ρ^d , where d denotes one element in ρ_k .

Given $\rho_k = (\rho^1, \rho^2, \dots, \rho^G)$ and its corresponding solution $((n_k^0, m_k^0, s_k), \pi(\rho_k))$, where G is the number of elements in ρ_k .

5.1.1: Do perturbation analysis of ρ^d

- (a) Set $\rho_k^d = (\rho^1, \rho^2, \dots, \rho^d + \delta^d, \dots, \rho^G)$, where δ^d is a small positive number (let $\delta^d = 0.05$).
- (b) If $\rho^d + \delta^d$ is beyond the feasible region of ρ^d , go to (d); else, proceed to **5.1.1(c)**.
- (c) Solve **P1** for $(n_k^{0,d}, m_k^{0,d}, s_k^d)$ with new ρ_k^d . If $(n_k^{0,d}, m_k^{0,d}, s_k^d) \neq (n_k^0, m_k^0, s_k)$, go to **Step 5.2**; else, set $\delta^d = \delta^d + 0.05$ and go back to **5.1.1(a)**.
- (d) Set $\rho_k^d = (\rho^1, \rho^2, \dots, \rho^d - \delta^d, \dots, \rho^G)$. Solve **P1** for $(n_k^{0,d}, m_k^{0,d}, s_k^d)$ with new ρ_k^d . If $(n_k^{0,d}, m_k^{0,d}, s_k^d) \neq (n_k^0, m_k^0, s_k)$, go to **Step 5.1.2**; else, set $\delta^d = \delta^d - 0.05$.
- (e) If $\rho^d - \delta^d \geq 0$, go to **5.1.1(d)**; else, the sensitivity of ρ^d is zero.

5.1.2: Calculate the gradient of ρ^d

If $(n_k^{0,d}, m_k^{0,d}, s_k^d) \notin \Omega$, solve **P2** to obtain $\pi(\rho_k^d)$ with ρ_k^d ;

The gradient of ρ^d : $\frac{\Delta\pi(\rho_k)}{\Delta\rho^d} = \frac{\pi(\rho_k^d) - \pi(\rho_k)}{\rho^{d'} - \rho^d}$, where $\rho^{d'} = \rho^d + \delta^d$, or $\rho^d - \delta^d$

5.1.3: Calculate the whole gradient vector

If $d \neq G$, set $d = d + 1$, go to **Step 5.1.1**. Otherwise, we have obtained the whole set gradients of $\pi(\rho_k)$ over all ρ^d . Denote this vector as $\nabla\pi(\rho_k)$.

5.2: Update ρ_k with Adam algorithm

- (a) Update biased moments: $u = \omega_1 u + (1 - \omega_1) \nabla\pi(\rho_k)$ and $r = \omega_2 r + (1 - \omega_2) \nabla\pi(\rho_k) \otimes \nabla\pi(\rho_k)$
- (b) Correct bias in moments: $\hat{u} = u / (1 - \omega_1^k)$ and $\hat{r} = r / (1 - \omega_2^k)$
- (c) Compute the update amount: $\Delta = -\lambda \hat{u} / (\sqrt{\hat{r}} + \partial)$
- (d) Set $\tilde{\rho}_{k+1} = \rho_k + \Delta$. Project $\tilde{\rho}_{k+1}$ onto the feasible region derived in **Step 1**. Denote this new feasible reliability as ρ_{k+1} . Set $k = k + 1$. Go to **Step 2**.

End

FIGURE 3 SR-based gradient descent algorithm

4 CASE STUDY

In this section, the proposed SR-based two-stage stochastic programming model is tested on a three-node network over six time intervals where the duration of each time interval is 0.5 hour. The algorithm is coded in Matlab 2017b using CPLEX as the solver and is ran on an eight-core 3.4 GHz personal computer with 16 GB RAM. Each case is solved with 30 randomly generated scenarios. Each scenario takes around 0.8 second, up to 4 seconds.

4.1 Network Illustration and Parameters Setup

The illustrative network is depicted in Figure 4, where the specific travel times between OD pairs are counted by time interval, and are shown on the arcs. The average demand between each OD pair is shown in Table 1. The total number of trip requests within three hours is 496.

FIGURE 4 Illustrative network and OD travel time (in time intervals)

TABLE 1 OD Demand Data (Mean Value)

Time interval	1-2	1-3	2-1	2-3	3-1	3-2	Total
1	12	12	13	2	11	1	51
2	21	21	21	4	19	2	88
3	33	32	33	6	30	3	137
4	27	26	27	5	25	3	113
5	16	15	16	3	14	1	65
6	10	10	10	2	9	1	42
Total	119	116	120	22	108	11	496

The parameters of the benchmark case are determined according to the realistic operational data the one-way carsharing service in Australia. The hourly charge p for users is \$50, i.e., \$25 for each time interval. The fixed amortized cost of each shared vehicle c_v is calculated by assuming the depreciation rate of 2% per month (for 50 months) and a new vehicle valued around \$18,000, which leads to $c_v = 12$ \$/day. The basic hourly payment for personnel c_p is \$10. An additional subsidy $c_{mc} = 0.5$ \$/h is given to personnel when they move to targeted stations for relocation task by other transport modes (e.g. e-scooter). The price of the ad hoc service AC is set to be 80 \$/h, which is set according to the average hourly ride-hailing/taxi service rate. This AC is higher than the price of regular service offered by one-way carsharing operators. The average parking cost c_s for each parking lot is 5 \$/h. The hourly fuel cost for a customer trip c_f and relocation trip c_r is assumed to be 2.5 \$/h. Besides, the maximum parking capacity at each station is 75 and the maximum total fleet size is set to be the sum of the maximum parking capacity of three stations, i.e., 225 vehicles. The maximum number of personnel to hire is 100 persons. For simplicity, the

stochastic demand at each station at each time interval is assumed to follow a normal distribution with the average demand shown in Table 1 as its mean and its variance is proportional to its mean, which can be calculated by the coefficient of variation (COV). A large value of COV indicates higher demand variability. For comparison purpose, the COV of demand scenario in the benchmark case is set to be 0.2. Each case is tested with 30 random scenarios.

4.2 Performance of the SR-based Two-stage Stochastic Approach

As the algorithm to solve the SR-based two-stage programming model is inherently non-convex (lying in the demand inverse function in stage 2), the choice of different starting points might affect the final solution and the converge process.

FIGURE 5 The converge process of different starting point

Figure 5 shows converge process of the objective function value with the starting points varying from 0.1 to 0.7. When the starting point is over 0.7, the vehicle fleet size needed to cover the regular demand exceeds the maximum fleet budget. Although starting from different initial points, the final objective function values converge to be very close. Because the maximum optimal profit is obtained at the starting point of 0.6 and this point requires less iterations to converge, the rest analysis will choose the starting point as 0.6.

4.3 Sensitivity Analysis on Demand Variation Degree

For comparison and evaluation, the number of deployed vehicles (No. Vehs), the number of personnel (No. Pers), the average optimal profit (Ave profit), the average regular service cost (Ave RS cost), the average number of demand satisfied by regular service (Ave RS demand), the average relocation time (Ave Rel time), the average personnel movement time (Ave Move time), the average ad hoc service time (Ave AD time), the average vehicle idle time (Ave Veh idle time) and the average personnel idle time (Ave Per idle time) are reported. Note that all averages are obtained from 30 scenarios. Besides, to further inspect states of each vehicle and each personnel, the expected vehicle idle time (Exp Veh idle time) and the expected personnel idle time (Exp Per idle time) are calculated, which take the expectation of average vehicle idle time and average personnel idle time on the number of vehicles and personnel, respectively.

1 **TABLE 2 The Number of Vehicles and Personnel under Different COVs**

COV	No. Vehs	No. Pers	Ave profit (\$)	Ave RS cost (\$)	Ave RS demand	Ave Rel time (h)	Ave Move time (h)	Ave AD time (h)	Ave idle time (h)	Veh time	Ave Per idle time (h)	Exp idle time (h)	Veh time	Exp idle time (h)	Per time
0.1	193	14	11,935.86	6,595.98	492.20	8.90	1.45	2.5	211.10		31.65	1.09		2.26	
0.2	199	15	11,476.41	6,761.43	488.73	12.70	3.77	5.4	235.78		28.53	1.18		1.90	
0.3	199	24	10,513.63	7,067.17	477.00	17.92	6.00	8.7	244.36		48.08	1.23		2.00	
0.4	209	32	9,743.45	7,593.55	472.87	25.35	9.85	10.4	283.45		60.80	1.36		1.90	
0.5	204	32	8,698.61	7,443.14	459.43	27.92	11.87	19.5	280.15		56.22	1.37		1.76	

2

4 **FIGURE 6 Average profit, relocation time, movement time and ad hoc service time versus**
5 **COV**

7
8 Table 2 shows the impacts of degree of demand variation in the combined service one-way
9 carsharing system. The parameters are set as the benchmark ones other than COV. It can be seen
10 that when COV increases from 0.1 to 0.5, while the total number of vehicles are close among these
11 five cases, the total number of personnel increases, rising from 14 persons to 32 persons (more
12 than twice as the former). Besides, when the demand variation becomes larger, the average profit
13 decreases gradually from \$11,935.86 to \$8,698.61 with more than 27% reduction compared to the
14 former. In the meanwhile, the regular service cost also increases, resulting from the increases in
15 the number of vehicles and personnel, average relocation time and average movement time, which
16 lead to a higher expenditure of fixed cost (vehicles, personnel and parking lots) and operational
17 cost (relocation and personnel movement). The increase of average ad hoc service time means that
18 ad hoc service is activated more often. In addition, when increasing COV, the average idle times
19 of both vehicles and personnel rise with minor fluctuations, as the consequence of the increase in
20 the number of vehicles and personnel. However, while the expected vehicle idle time increases
21 gradually, the expected personnel idle time shows a decreasing trend, although the number of

personnel gets larger. These results indicate that when demand fluctuates widely, a more economical way to reduce the adverse effects of demand variation is to deploy more personnel, which is with more flexibility, rather than deploy a large fleet size. Besides, as depicted in Figure 6, when increasing COV, the ranges of profit, movement time, relocation time and ad hoc service time become wider. This means more variations arise in these operating aspects with the increase in demand variation.

Solve the deterministic model for the optimal profit and test the performance of the solution with 30 scenarios under different COV cases. The profits are \$11,880.37, \$11,269.55, \$9,882.34, \$8,570.74 and \$7,252.37 corresponding the five COV cases above. Compared to profits obtained by the deterministic model, the SR-based two-stage model achieves 0.46%, 1.84%, 6.39%, 13.68% and greatly 19.94% improvement in the profit when COV ranges from 0.1 to 0.5, respectively. These significant improvements on profit demonstrate satisfactory performance of the proposed model, which improves the profitability for one-way carsharing systems under demand variations.

4.4 Sensitivity Analysis on Ad hoc Service Cost

The ad hoc service cost are important factors to the performance on this combined one-way carsharing system.

TABLE 3 Impact Analysis of Ad Hoc Service Cost

Ad hoc service cost (\$/h)	No. Vehs	No. Pers	Ave profit (\$)	Ave RS cost (\$)	Ave RS demand	Ave Rel time (h)	Ave Move time (h)	Ave AD time (h)	Ave Veh idle time (h)	Ave Per idle time (h)	Exp Veh idle time (h)	Exp Per idle time (h)
60	198	14	11,581.05	6,686.45	487.90	12.12	3.57	5.92	232.77	26.32	1.18	1.88
70	200	15	11,514.26	6,789.08	489.03	12.82	3.98	5.25	238.80	28.20	1.19	1.88
80	200	15	11,478.44	6,804.83	489.17	12.97	4.08	5.12	238.82	27.95	1.19	1.86
90	201	16	11,430.97	6,879.03	489.97	13.40	4.07	4.67	241.80	30.53	1.20	1.91
100	205	17	11,362.61	6,915.73	491.40	14.03	4.28	3.87	253.63	32.68	1.24	1.92

Table 3 shows that the increase in the cost of ad hoc service leads to increasing number of vehicles and personnel. There is an approximately 2% reduction in the average profit, when the ad hoc service cost increases from 60 to 100 \$/h. The average regular service cost increases, which results from the increasing number of vehicles and personnel, the increasing average relocation time and the average movement time, suggesting that vehicles and personnel are actively utilized to reduce the use of ad hoc service. This can also be seen in a reduction of ad hoc service time, which suggests less ad hoc service is activated. These results reflect that when the cost of ad hoc service increases, carsharing operators tend to enlarge the numbers of fleet and personnel to avoid using the expensive ad hoc service. However, this leads to the increase in the average idle time of both vehicles and personnel.

5 CONCLUSION

This study proposes a two-stage stochastic model to improve the profitability for one-way carsharing system under demand uncertainty. An important concept of service reliability is introduced, which decouples the complexities between long-term strategic and tactic deployments, and short-term operational schemes. The model is divided into two stages: the first stage determines the regular service including the number of vehicles, personnel and parking lots under a certain SR level, whereas the second stage evaluates the profitability of the regular service and determines the optimal ad hoc service in response to demand variation. Ad hoc service is flexibly

1 integrated into the one-way carsharing system, which serves the demand that cannot be satisfied
2 by regular service at a higher cost than regular service. A SR-based gradient descent algorithm is
3 developed to solve for the best SR at each station at each time interval. Performance of the
4 proposed model and the algorithm is tested with an illustrated network. The impacts of the degree
5 of demand variation and the cost ad hoc service on the combined service one-way carsharing
6 system are evaluated. The proposed model can be used as a decision support system, which is easy
7 to be transferred to other networks.

8 The proposed stochastic approach can be further extended in the following aspects. First,
9 the algorithm efficiency is mostly affected by calculating the discrete gradient of profit over each
10 OD pair at each time interval, which is obtained by conducting the perturbation analysis. Thus,
11 further improvements can be developed for the current algorithm, for instance, developing a
12 heuristic algorithm for efficient gradient calculation. Second, the proposed model shows the
13 potential coordination between carsharing operator and outsourced mobility service providers.
14 However, the specific operational mechanism is not discussed in detail. Future studies can focus
15 on modelling the sharing mechanism between these types of services so that the two operators can
16 reach the win-win situation.
17

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