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Observation of nonclassical measurement statistics induced by a coherent spin environmentD. D. Bhaktavatsala Rao,^{1,*} Sen Yang,^{2,†} Stefan Jesenski,¹ Emre Tekin,¹ Florian Kaiser,¹ and Jörg Wrachtrup¹¹*Center for Integrated Quantum Science and Technology, MPI for Solid State Research, and Third Institute of Physics, University of Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany*²*Department of Physics, Chinese University of Hong Kong, Shatin, Hong Kong, China*

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We demonstrate the role of measurement back action of a coherent spin environment in the dynamics of a spin (qubit) coupled to it, by inducing nonclassical (quantum random walk–like) statistics on its measurement trajectory. We show how the long lifetime of the spin bath correlates with measurements of the qubit over many repetitions. We use nitrogen vacancy centers in diamond as a model system and projective single-shot readout of the electron spin at low temperatures to simulate these effects. The employed measurement control could be used for purification of spin ensembles to desired quantum states.

DOI: [10.1103/PhysRevA.100.022307](https://doi.org/10.1103/PhysRevA.100.022307)**I. INTRODUCTION**

Quantum mechanics allows us to postselect events that cannot be observed classically [1]. Even though these events are rare we cannot create a classical setup that allows us to observe such an effect. A quantum random walk (QRW) is one such example where the translation operation that can only displace the walker by a single step can, in some (rare) cases, displace it much farther, while keeping the average displacement of the walker identical [2,3]. This comes about from the interference between the possible trajectories of a quantum random walker, which can substantially modify the statistics compared to classical random walks (CRWs). Such quantum walks are shown to be used to implement quantum search algorithms and, also, are considered a universal computational primitive [4,5]. They have applications in the simulation of biological processes [5] and implementation of quantum algorithms [6], and they have been demonstrated on various experimental platforms [7–11].

The discrete-time QRW can be described by the repeated application of a unitary evolution operator U that acts on the combined space of a quantum coin and a quantum system with a larger Hilbert space, a multilevel system (e.g., graph, position, or momentum space). The evolution within the Hilbert space of the walker is conditioned on the state of the quantum coin. By using the superposition states of the coin or by generating them at every evolution step, the walker transitions also get superposed. If the coherence time of the walker is much longer than the total evolution time, the superposition of various evolution paths creates a fundamentally different random walk compared to its classical counterpart [3]. Such conditional evolution is well known in central spin models (Fig. 1), where the central spin (coin) controls the evolution of a quantum bath (walker) conditioned on its spin state [12].

With arbitrary unitary control on the central spin, it would be interesting to know how such systems can mimic a QRW (see Fig. 1). Similarly to the random walk description [3], the bath (walker) state cannot be precisely measured due to the infeasibility of performing large quantum-state tomography on large quantum environments. We show here that by frequently measuring the state of the quantum coin during the evolution, one can imprint features of the bath evolution on to its measurement statistics. Various parameters of the bath state (purity, entanglement, magnetization, etc.) can then be used to characterize the walk (similarly to translations in the real space of the walker). While earlier works demonstrating QRW in spin systems [9] aimed at preparing two-spin (qubit) quantum states whose state probabilities show QRW statistics, we here imprint them on the measurement statistics of the quantum coin, allowing us to achieve a larger walk space, for any general interacting spin system.

Open quantum systems can naturally display non-Markovianity (memory effects) in their dynamics when interacting with environments having long correlation times [13]. Such long correlation times can become useful to keep track of the changes made by the quantum system coupled to it, which is frequently measured. For example, it was shown that repeatedly projecting the quantum system onto a given state can lead to quantum-state engineering of a quantum environment by generating highly entangled or purified states that are useful for quantum information protocols [14–17]. Such projective measurements have also been used for real-time measurements of the nuclear spin environment to which the solid-state qubit is coupled [18]. We here show how the nonclassical statistics of these projective measurements can be used to reveal the conditional quantum evolution of the bath and transduce its physical behavior (such as a QRW). Prior to the description of the physical model and the experimental results, we show how projective measurements of the quantum coin in the well-known one-dimensional (1D) quantum walk can imprint a quantum-walk-like behavior onto its measurement statistics, which we verify experimentally.

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FIG. 1. (a) Schematic of the open-system model, where a central spin (NV) is coupled to a bath of nuclear spins. (b) The experimental scheme to test the nonclassicality in measurements induced by a long-lived (coherent) spin environment is shown through a circuit diagram. The NV spin is initialized in the same state, while the bath state is updated from previous (projective) measurements performed on the central spin. The NV spin evolves by interacting with a spin environment for a contact time τ , through U , leading to phase evolution of its spin states.

II. PROBED 1D QUANTUM RANDOM WALK

To understand the origin of nonclassicality and quantum-random-like statistics in the measurements of a qubit coupled to a spin bath, let us consider the well-known QRW problem [3]. In the quantum version of the symmetric classical 1D random walk, an unbiased coin which stochastically determines the choice of the walker to make translations in the physical space (to left or to right) is physically included in the model. Assigning the quantum nature of a two-level (two-independent-choice) system to the coin, the evolution of the walker on a 1D lattice (chain) is simplified to the form

$$U \equiv \exp(ilS^z \otimes \hat{T}) = |\uparrow\rangle\langle\uparrow| \exp(il\hat{T}) + |\downarrow\rangle\langle\downarrow| \exp(-il\hat{T}), \quad (1)$$

where T is the translational operator acting on the walker state, causing spatial transitions of length $\pm l$, and S^z is the two-level operator (Pauli Z) of the coin. Similarly to the CRW, after every step a Hadamard gate performed on the quantum coin determines the walker's further evolution. Together the total evolution is a repeated action of the operator $V = U \cdot (H_C \otimes \mathcal{I})$ on the total Hilbert space, comprised of the coin and the walker subspaces. Starting from the initial state $|\psi\rangle = |s\rangle \otimes |x\rangle = |s, x\rangle$, with $s \in \{|\uparrow\rangle, |\downarrow\rangle\}$, and considering unit translation, $l = 1$, the operator U takes the simple form

$$U = |\uparrow\rangle\langle\uparrow| \otimes \sum_i |i+1\rangle\langle i| + |\downarrow\rangle\langle\downarrow| \otimes \sum_i |i-1\rangle\langle i|. \quad (2)$$

After the first three steps, the initial state $|\psi_0\rangle = |\uparrow, 0\rangle$ would be

$$\begin{aligned} |\psi_3\rangle &\equiv V^3|\psi\rangle = \frac{1}{\sqrt{8}}(|\uparrow, 3\rangle + |\downarrow, 1\rangle + |\uparrow, 1\rangle - |\downarrow, -1\rangle \\ &\quad + |\uparrow, 1\rangle + |\downarrow, -1\rangle - |\uparrow, -1\rangle + |\downarrow, -3\rangle) \\ &= \frac{1}{\sqrt{8}}(|\uparrow, 3\rangle + (|\downarrow\rangle + 2|\uparrow\rangle)|1\rangle - |\uparrow, -1\rangle + |\downarrow, -3\rangle). \end{aligned} \quad (3)$$

As can be seen the phases interfere with and influence the probability distribution. Hence the probability of finding the walker away from the center is highest, $p = (\frac{1}{\sqrt{8}})^2 + (\frac{2}{\sqrt{8}})^2 =$

$\frac{5}{8}$, while in the CRW, the probability remains high in the center for any number of steps.

As we assign a physical nature to the coin, it can be manipulated in a random fashion and measured in any arbitrary space. While unitary manipulations have been studied widely, the role of weak and strong measurements on the quantum coin and its effect on the walker's evolution have received less attention. In this work, we highlight this effect and show that when probing a quantum walker with the quantum coin its measurement statistics too inhibits the QRW-like nonclassical statistics.

To achieve this we repeatedly measure the coin in the basis $|\pm\rangle \equiv \frac{1}{\sqrt{2}}[|\uparrow\rangle \pm |\downarrow\rangle]$, i.e., we evolve the coin for n steps and measure it. The dynamics of such repetitive measurements and evolution in between can be represented as a nonunitary operation acting on the walker's subspace given by

$$\tilde{V}_{\pm} = \frac{1}{2}[\exp(il\hat{T}) \pm \exp(-il\hat{T})], \quad (4)$$

for the respective measurement (projection) of the quantum coin onto the states $|\pm\rangle$.

As the coin is collapsed to one of the measurement basis states, the walker state too collapses in some random basis and will be the starting point for further evolution. The coin is reset to its initial state and the walk continues with this new state of the walker. The measurements of the coin influence the walker state by enhancing and biasing the measurements on one of the eigenstates of the measurement operator M . For example, in the above example a measurement on the coin after three steps of the walk onto the state $M_+ \equiv |+\rangle\langle+|$ will collapse the walker to the unnormalized state $\frac{1}{\sqrt{4}}[(|+3\rangle + |-3\rangle) + \frac{3}{2}|+1\rangle - \frac{1}{2}|-1\rangle]$. The normalization constant will determine the success probability of the measurement M_+ , which in the current case is 0.889. Starting from this new walker state the QRW will continue until the next measurement, where the walker state is again reset to a new one.

By performing four measurements ($N = 4$) on the coin, with 16 possible outcomes, i.e., $++++, +++-, \dots, ----$, we obtain the success probability of obtaining the 16 measurement strings. In Fig. 2 we show how measurement statistics inhibit classical behavior when the walker is classical [Figs. 2(a) and 2(b)] and nonclassical behavior when the walker is quantum [Figs. 2(c) and 2(d)]. In the case of the QRW, the probability is maximal at the extreme points corresponding to identical measurement results in every measurement, and this feature becomes sharper with an increasing number of measurements. For the CRW, all the paths are equally probable and yield a constant value which we show in Fig. 2(b).

Thus the measurement statistics can provide indirect evidence of the physical nature of the walk. Though a QRW is conventionally verified by measuring the probability distribution of the walker, achieving the same experimentally is a tedious task, as it involves full quantum-state tomography [19], which is not straightforward to achieve for the solid-state spin systems that we consider in this work. With a well-controllable quantum coin and its projective measurement, one can indirectly confirm the quantum nature of the walker and its evolution, which we demonstrate in the present work.

FIG. 2. The probability of finding the random walker at a position x is shown for both (a) classical (CRW) and (c) quantum (QRW) walks. The corresponding measurement statistics for (b) CRWs and (d) QRWs displaying the probability of obtaining a measurement sequence M_n for $N = 4$ measurements performed after 20 steps of the walk.

III. PHYSICAL SYSTEM

We model our physical system through a central spin model consisting of a single central electron spin, the nitrogen vacancy (NV) center in diamond, coupled to an internal nuclear spin bath [20]. The NV center has an electronic spin ($S = 1$) ground state, while the surrounding nuclear spin bath consists of a single ^{14}N nuclear spin ($I = 1$) and a bath of ^{13}C nuclear spins, in addition to a few other paramagnetic defect centers in diamond [21]. The nuclear spins that constitute the spin bath are assumed to be weakly coupled and hence noninteracting in the present study. The electron-nuclear spin interactions in diamond material have been extensively studied over the past decade [21–23]. While the evolution of the central electron spin coupled to the spin bath has mostly been studied by classical readout, i.e., by normal measurements averaged over many repetitions, the possibility of using measurement results as feedback to modify the bath state has been explored using quantum dots in [24]. We here study the use of projective quantum measurements. This results in nontrivial bath dynamics and in turn the central spin dynamics which is reflected in the nonclassical measurement statistics that we show in this work.

The dipolar coupling between the spin bath and the NV center is determined by the quantization axis of the NV center, rendering all components of the dipolar coupling nonzero. In the reference frame of the central spin (NV) one can think of the spins of the bath as being randomly distributed in a three-dimensional plane comprising the S -spin polarization and the plane perpendicular to it. With the magnetic field aligned along the central spin axis (say the NV z axis) the Hamiltonian that determines the dynamics is given by [16,20]

$$H = S^z \otimes \sum_k \vec{g}_k(r) \cdot \vec{I}_k + \omega \sum_k I \otimes I_z^{(k)}, \quad (5)$$

where g_k is the strength of the dipolar coupling between the NV (central) spin and the k th nuclear spin, dependent on the spatial separation between these spins. The random spatial location of the spins with respect to the NV center leads to inhomogeneous couplings such that the bath has neither conserved quantities nor preferred symmetries. The external field ω (the nuclear Zeeman term) under which the nuclear spins precess is assumed to be uniform over the entire sample and is along the z direction. The dynamics generated by the above Hamiltonian can be exactly solved [12]. In the S^z basis of the central spin ($|1\rangle, |-1\rangle$), the above Hamiltonian can be rewritten as [16]

$$H = H^+ |1\rangle\langle 1| + H^- |-1\rangle\langle -1|, \quad (6)$$

$$H^\pm = \sum_k \omega_k I_z^\pm \pm \sum_k \vec{g}_k(r) \cdot \vec{I}_k,$$

where $H^\pm$ are the nuclear spin-bath operators. The simple form of $H^\pm$ makes it easy to diagonalize, so that we obtain a closed-form equation for the time-evolution operator of the total system,

$$U(t) = U_I^+(t) |1\rangle\langle 1| + U_I^-(t) |-1\rangle\langle -1|, \quad (7)$$

where $U_I^\pm(t)$ govern the dynamics of the spin environment conditioned on the state of the NV spin [16].

In the absence of an external field the Hamiltonian describing the system-bath interaction takes a simpler form, $H = S^z \otimes \hat{B}$, that could be mimicked by a classical noise model, where the bath operator is $\hat{B} = \sum_k \vec{g}_k \cdot \vec{I}_k$. By considering the initial bath state to be completely unpolarized the bath operator can be replaced by a random magnetic field to obtain similar dynamics in the two-level system (TLS). Starting from an initial superposition state $|\psi\rangle = \frac{1}{\sqrt{2}}[|1\rangle + |-1\rangle]$, the coherence of the TLS at any later time t is given by

$$C(t) \equiv \langle S^x \rangle = \int_{-b_m}^{b_m} d\omega G(\omega) \cos(\omega t), \quad (8)$$

where b_m represent all possible energy eigenstates of the bath and G is the bath distribution function [12,13]. Due to the dynamics governed by Eq. (6), the coherence is lost at a rate γ determined by the width of the distribution $G(\omega)$. This sets the T_2^* time of the spin. On the other hand, the dephasing time T_2 could be made longer by employing dynamical decoupling sequences [25,26]. The simpler form of the interaction given above does not lead to any bath dynamics, i.e., $\rho_B(t) = \rho_B(0)$, when the initial bath state is completely mixed. For this reason, the bath acts like background noise on the TLS and almost resembles a classical noise source for the TLS. The changes in the bath state could result from the measurement back action's inducing a nonunitary action on its state through the operators

$$V_{\pm}(t) = (U_I^{\pm}(t) \pm U_I^{-}(t))/2, \quad (9)$$

depending on the measurement result 0 or 1. One could compare this to the nonunitary operator given in Eq. (4). Now upon repeated application of this operator the measurement statistics of the central spin should follow a similar nonclassical behavior, shown in Fig. 2. For example, a measurement outcome that finds the central spin in state 0 all n times will give rise to a measurement string $M_1 = \{0, 0, \dots, 0\}$, and the bath state corresponding to such a measurement trajectory is given by [15]

$$\rho_B^{(1)}(t) = \frac{V_+ \rho_B(0) V_+^{\dagger}}{\text{Tr}[V_+ \rho_B(0) V_+^{\dagger}]}, \quad (10)$$

where $V = \cos^n(\hat{B}t)$. Clearly $\rho_B^{(1)}(t) \neq \rho_B(0)$, indicating the modified bath state for a given trajectory M_1 . Similarly, all bath states corresponding to the 2^n measurements can be found. Interestingly, one can verify that the total averaged state equals the actual state at time t , i.e., $\sum_M \rho_B^{(M)}(t) = \rho_B(t)$, confirming the *ergodicity* of system when averaged over all possible measurements. The hierarchy of changes in the system caused by the bath and then in the bath caused by the system will continue until a steady state is reached that has a comparatively higher purity than its initial state. This polarization gain also adds on to the measurement statistics, explaining the preferred choice of measurement statistics as we discuss below.

Here one can make the analogy to the probed quantum random walk problem discussed earlier. As the evolution in the walker's space is conditioned on the state of the quantum coin through Eq. (3), we have shown in Fig. 2(d) that by frequently measuring the state of the quantum coin, and using the measurement back action on the walker dynamics, one can obtain measurement statistics that exactly mimic the QRW behavior [see Fig. 2(c)]. Similarly for the spin-bath case, the initial thermal state of the bath, which is equally probable to be found in all possible states, tends to get polarized by measurements, and the amount of polarization follows the probability distribution of the measurement statistics as shown in Fig. 3. From our simulation we clearly observe the signature of bath polarization directly reflected onto the measurement statistics of the central spin.

FIG. 3. The simulated probability of obtaining a measurement string M_n for four measurements (top) is compared with the achievable relative purity ($P_R = \text{Tr}[(\rho_B^{(M)})^2]/\text{Tr}[(\rho_B(0))^2]$) of the bath state (bottom) for the corresponding measurement string M_n . We have considered a spin bath with $N = 7$ spins and couplings as given in the Appendix.

IV. EXPERIMENTS

To demonstrate these effects we implement this protocol in a low-strain (≈ 1.2 -GHz) NV center to suppress strain-induced effects, e.g., lowering the symmetry of the NV and altering the configuration of the excited state [20]. For experiments an NV center along the [111] orientation is chosen [27]. In the experiment a small magnetic field is applied along the NV axis, to cancel the earth's magnetic field, thereby making the net B field at the location of the NV center 0. Due to this the $|\pm 1\rangle_e$ are degenerate and hence only the bright superposition, i.e., $|b\rangle \equiv [|\pm 1\rangle + |0\rangle]/\sqrt{2}$, couples to $|0\rangle_e$ by a microwave transition, while the other superposition, $|d\rangle \equiv [|\pm 1\rangle - |0\rangle]/\sqrt{2}$, remains dark and does not couple to $|0\rangle_e$. These two states ($|b\rangle, |d\rangle$) form the TLS, where the population in one state can be projectively read out by mapping it to state $|0\rangle_e$. To ensure projective readout of the NV spin state $|0\rangle_e$, the experiments are performed at low temperatures (4 K), at which the optical selection rules allow one to distinguish the electronic spin states $|\pm 1\rangle_e$ and $|0\rangle_e$ with a high fidelity through single-shot readout [28]. The upper limit of the nuclear spin coherence time, given by the electron spin's time T_1 , reaches minutes at low temperatures [27]. Various steps in the protocol are schematically shown in Fig. 1, where we initialize the electron spin in state $|0\rangle$ by resonantly exciting it in the A_1 transition [Fig. 4(a)], followed by a microwave transition, $|0\rangle_e \rightarrow |b\rangle$, and further its phase evolution through interaction with the spin bath. Finally, another microwave transition maps the population $|b\rangle \rightarrow |0\rangle_e$, and then, by resonantly exciting the center in the E_x transition, we projectively read out the NV center, i.e., we observe fluorescence if it is only in state $|0\rangle_e$ and dark otherwise.

FIG. 4. (a) Schematics of the level diagram and the pulse sequence used for experiments. (b) The free induction decay of the coherence of the NV spin $C(t)$ is plotted as a function of the time t . (c) The coincidence of finding a given measurement index i , corresponding to a measurement outcome which is the binary representation of i , is shown for four consecutive measurements. Measurements were performed at intervals of τ . The theoretical simulations are detailed in the Appendix. (d) Left: Raw data for a measurement sequence (string) consisting of 246 measurement results (with evolution time $\tau = 1.2 \mu s$) repeated 600 times. Violet color corresponds to the detection of at least one photon ($|b\rangle$), and yellow color symbolizes the measurement of no photon ($|d\rangle$), where $|b(d)\rangle = \frac{1}{\sqrt{2}}[|+1\rangle \pm |-1\rangle]$. Right: Two measurement trajectories. (e) From the raw data shown in (d) we have extracted the probability of various measurement trajectories for the case of 40 consecutive measurements (dashed blue line). In the same plot we also show the expected Gaussian statistics (solid red line) when all the measurements are equally probable, i.e., in the absence of back action.

We first measure the decay of NV spin coherence [free induction decay (FID)] due to bath interaction [see Fig. 4(b)], i.e., the time taken, $|b\rangle \rightarrow \frac{1}{2}(|b\rangle\langle b| + |d\rangle\langle d|)$. We experimentally find that the NV spin initialized in state $|b\rangle$ is completely decohered as described above over a time scale of $T_2^* \sim 1.2 \mu s$ [see Fig. 4(b)]. We then perform four measurements (and repeat them 75 312 times for statistics), which will result in $2^4 = 16$ possible measurement strings (represented as basis states of a four-qubit system in the 0, 1 basis), and their occurrence probabilities will determine the nonclassicality in their statistics and the coherent nature of the spin bath. The choice of interaction τ is set from the FID behavior shown in Fig. 4(b). To mimic the classical coin, we choose the time $\tau = T_2^* = 1.2 \mu s$, such that the NV spin state has become completely depolarized, and the probability of finding it in either of its orthogonal states is equal to 0.5, representing a classical coin. If the initial conditions are similar in every experimental run, all 16 possibilities should occur with equal probability. On the contrary, for the statistics shown in Fig. 4(c), there is a high probability of measurement results that are identical, i.e., $M_0 = 0$ and $M_{15} = 1111$, which decreases with increasing heterogeneity. This is due to the fact that though we reset the NV spin after each measurement, given the long spin lifetime (T_{2n}^* , T_{1n}), the nuclear spins do

not reset to their initial fully mixed state (as $\tau \ll T_{1n}$) but, rather, get projected to a different state that has a modified bath distribution $G(\omega)$ [see Eq. (8)] after each measurement, affecting later measurements. Thus a different bath state at each measurement modifies all future measurements. While the total evolution time for the four measurements including the spin readout and initialization time is well within $< 25 \mu s$, the coherence time of the bath spins exceeds 4 s [27].

We further observe symmetries in the statistics that are similar to a probed 1D QRW described earlier. For example, measurements with similar total bit values, i.e., strings 0001, 0100, 1000, and 0100, occur with equal probability. Thus one can bin the measurement results with the total bit value M_n . For n measurements the total number of measurement strings with value M_n follows the binomial distribution $C_{|n-M_n|}^n$ ($0 \leq M_n \leq n$). When all the measurements are equally probable $M_n = 0$ and $M_n = n$ should occur with an exponentially lower probability. In contrast, we find them here to occur with the highest probability.

In analogy to statistical physics and thermodynamics, if we consider the observed FID measurement result at a given time τ to be a macroscopic variable, then the total number of measurements (strings) and their distribution will determine the microscopic states of the system. While the average of the

statistics agrees with the observed FID results at these respective times, the probability with which different measurements are chosen adds on to them nonclassical statistics induced by measurement back action. For example, if the measurement results are all equally probable, then such a distribution (when written in the computational basis of a four-qubit system) has a maximum entropy of 0.0625 [29], and the FID result corresponding to such statistics is a fully depolarized NV spin as shown in Fig. 4(b) at time $\tau = 1.2 \mu\text{s}$. On the other hand, the distribution shown in Fig. 4(c) gives a similar FID result but the measurement purity of the observed measurements is lower, 0.1594, indicating a preferential ordering or polarization in measurement statistics. To understand the origin of this purity, we made numerical simulations using a finite bath with four to seven spins and obtained a measurement purity of 0.1437. The errors responsible for the observed differences between theory and experiment [Fig. 4(c)] are detailed in the Appendix. The measurement back-to-back action on the system-bath dynamics thus reveals the physical nature of the bath that cannot be mimicked by classical noise models [13].

To further confirm the correlations in subsequent measurements shown in Fig. 4(c) for higher measurement numbers, we performed an experiment with $n = 246$ measurements that was repeated 600 times [Fig. 4(d)]. From this we obtain the statistics shown in Fig. 4(e). One can clearly observe the deviation from the expected Gaussian distribution (red squares) when all the measurement strings are equally probable. Though the averaging for these measurements is not as high as for Fig. 4(c) we clearly observe the predicted non-Gaussian (nonclassical) measurement statistics. From these data we also analyzed the possibility of the bath's approaching a steady state. We show this in Fig. 5. Since each measurement modi-

FIG. 5. (a) Raw data for a measurement sequence (string) consisting of 246 measurement results (with evolution time $\tau = 1.2 \mu\text{s}$) repeated 600 times. Violet color corresponds to the detection of at least one photon ($|b\rangle$), and yellow color symbolizes the measurement of no photons ($|d\rangle$). (b) From the raw data we evaluate the probability of finding a similar measurement result (corresponding to spin state $|b\rangle$) after n identical measurement results in any given measurement string described above.

fies the bath state, finding an identical result in m consecutive measurements will influence the $(m + 1)$ th measurement. The monotonically increasing probability and its saturation could hint at a steady state for the bath. Such steady states could have nonzero polarization, which we have shown earlier.

In conclusion, we have shown that when measuring a quantum system coupled to a coherent spin environment, the measurement back action modifies the environment, which in turn influences the subsequent measurement, thus generating correlations among them. The microscopic detail on the occurrence of various measurement strings helps us to understand the nonclassical features in quantum measurements and system-bath interactions, allowing for the purification (cooling) of a quantum environment into desired target states [15,16]. By finding the measurement distributions that result in observable average behavior of the system, we will be able to understand the statistical nature of measurements and, further, their analogy with thermodynamics [30,31].

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APPENDIX

1. Simulations and experiments

For the interaction between the NVC and the spin bath [given in Eq. (5) of the text], the main consideration is choosing the density and spatial coordinates of the bath spins. As the bath spins are localized and are dipolar coupled to the central spin (NV center), the bath coupling spectrum is greatly influenced by the distance (r) between the central spin and the bath spin due to the $1/r^3$ dependence of the coupling. As the magnetic field is 0 and the initial bath state is thermal (fully mixed state), the Hamiltonian is commuting; only the root-mean-square field caused by each bath spin in the NV center is important, and not their actual distribution. We consider a diamond lattice with a typical concentration of 1.1% ^{13}C and an NV center at the depth of $6 \mu\text{m}$ from the surface. With the dipolar coupling between the electron and the nuclear spins decaying as $1/r^3$, the typical volume around the NV center in which the bath spins interact with strengths higher than 10 Hz would be 10 nm^3 .

From the experimental observations on diamond samples with similar ^{13}C concentrations, only a few (6–10) spins with couplings larger than 100 Hz are relevant for the dynamics (see Table I). One can model the observed statistics from a purely classical model or from a quantum spin-bath model.

a. Classical model

One can model the observed non-Gaussian statistics purely from classical conditional statistics. As each measurement

TABLE I. Dipolar coupling strengths of some ^{13}C nuclear spins to the central NV electron spin.

g_x (kHz)	4.5838	-6.4172	-10.0842	-3.6670	6.4172	-1.8335	4.5838
g_z (kHz)	-0.9167	-8.2508	6.4172	-7.3340	-8.2508	-12.8345	-11.9178
g_y (kHz)	-8.2508	-0.9167	2.7502	-7.3340	-6.4172	0.00	-4.5838

modifies the bath, and by taking into account the modified FID behavior, the coherence of the qubit at τ changes with every measurement. As each measurement increases the probability of obtaining the same measurement, we model a stochastic process where a random number chosen determines the result based on the threshold, and this changes after every measurement. For example, in Fig. 4(c) we average the four measurement experiments over a million trials and obtain the average statistics for the 16 possible measurement results. The agreement is quite good, confirming the spin-bath averaging. In this simulation we assumed a 10% increase in the probability of obtaining a similar result after a given measurement.

b. Quantum model

For the quantum model we choose a certain bath distribution, which matches the observed free induction decay. For example, a typical choice of bath couplings that we use in our simulations is as follows: We would like to note that the results are generic enough to observe for any arbitrary choice of bath-spin distribution. For a more exact treatment of the problem, one needs to perform spin spectroscopy (detection) measurements using pulse control ($XY8$) schemes to scan the ^{13}C spin bath.

We start from an initial thermal state of the bath (fully mixed) and an NV spin in state $|0\rangle$. By applying a π pulse at a frequency of 2.8 GHz, we generate state $|b\rangle$. Using the above couplings and solving the dynamics, we obtain the FID of the central spin, which is initialized in the superposition state $|b\rangle$; i.e., the decay of population from state $|b\rangle$ is plotted as a function of time [see Fig. 4(b)]. From this we find time T_2^* to be around $1.2\ \mu\text{s}$, consistent with our experimental observations, and choose it as the measurement time for simulations. We would like to note that similar FID behavior can be obtained with various spin-bath coupling spectra and hence the above choice is not unique.

Starting again from the initial conditions described above, we let the NV spin (central spin) evolve for a time $\tau = 1.2\ \mu\text{s}$, and by mapping back the $|b\rangle \rightarrow |0\rangle$, we projectively read out its population. Immediately after the measurement, which takes about $3\ \mu\text{s}$, we reinitialize the NV spin to its initial state and continue the evolution and subsequent measurements. With an initialization time $\sim 2\ \mu\text{s}$ and microwave pulses of width $0.25\ \mu\text{s}$, the total time of the experiment for N measurements adds up to $T \approx 6N\ \mu\text{s}$. Even for $N \sim 100$, we are still within the lifetime of the nuclear spin, and hence no thermal resetting of the nuclear spin takes place. Due to this condition, we assume that the spin bath remains coherent during our entire experiment, leading to the observed nonclassical measurement statistics. After N repetitive measurements, we wait for about 10 s, so that the nuclear spin bath relaxes to its thermal state, and a new experimental run can start.

The main sources of error in our experiment come about from the imperfect single-shot measurement (90%) and other

additional error processes that occur during optical excitation. For example, there is a finite probability for the spin to jump from the excited level $E_{x,y}$ to $A_{1,2}$ [27], making a bright event dark, and hence 0 appears as 1. Equally, a dark event could appear bright with a finite probability. Due to these errors the relative probabilities among various measurement strings do not coincide exactly with the theory model for smaller numbers of measurements. For example, in Fig. 2(c), taking into consideration the 10% error in each measurement and averaging over 75 312 experiments, each data point has an average error of $< 1\%$.

2. Purity of bath and measurement statistics

With every projection of the NV spin, there is a slight modification of the spin-bath state, thus directly affecting its purity. For example, in the two-spin case discussed above, if we find the projection of the NV spin onto state $|0\rangle$ for some τ , then there is a high probability of finding the bath spins in the state $\rho_B^{(1)} = |\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|$. Thus projecting the NV spin has increased the purity of the bath spins from 0.25 to 0.5. Now if the measurement is repeated with the bath being in the new state, $\rho_B^{(1)}$, there is an even higher probability of finding the NV spin in state $|0\rangle$. Thus the purifying bath also alters the probability of the measurement outcomes, thus leading to an effective purification of the measurement statistics.

To quantify the purity of measurement statistics, we map the outcome measurement to a single two-level system (qubit). Then N measurements would correspond to N qubits, whose basis states are nothing but the possible outcomes of N measurements. The fictitious N -qubit state ρ_M is always diagonal, and its elements give the probabilities of occurrence of 2^N possible measurement outcomes. The purity of measurement statistics is then defined as $\text{Tr}\rho_M^2$. For example, in the simulations involving a bath spin with four spins, this mapping would lead to a measurement purity of $P_M = 0.1594$, while a back-action-free measurement would give an equally probable measurement statistic, which has the minimum purity, $P_M = 1/16$.

In addition to the measurement purity, each measurement string alters the purity of the spin bath due to the nonunitary evolution operators V . Hence, there is a varying purity for the initially unpolarized spin-bath state for each individual measurement string. We show these different states in Fig. 3, and the average purity of the bath caused by these measurements is then found to be $P_B = 0.112$.

In our experiments, as we do not perform quantum tomography of the bath, we are limited to the measurement statistics, and for the experimentally observed statistics, we find $P_M = 0.1437$. Clearly the simulations show a higher purity than the experiment. The inconsistency with the theoretical model arises from the optical excitation and readout errors detailed earlier. Due to these errors the relative probabilities among

various measurement strings do not coincide exactly with the theory model for smaller numbers of measurements.

3. Effects of excited-state hyperfine coupling

The NV is polarized to the $m_s = 0$ state by resonant excitation in the A_1 transition, and is read out resonantly by excitation in the $E_{x/y}$ transition. Only during this time is the nuclear spin bath prone to excited-state hyperfine coupling. We have shown in [27] that there is almost zero (negligible) hyperfine coupling between the electron and the nuclear spins

in the A_1 level, while the nuclear spin suffers dephasing when excited to the energy states $E_{x/y}$. As we are always diagonal in the nuclear spin basis during the entire protocol, laser illumination of the NV center has no effect on the nuclear spin dynamics. The situation is dramatically different in the case where there is a strong external magnetic field, which sets the quantization direction of the nuclear spins. Measurements in this case are shown to stabilize the bath in coherent N -partite entangled states [16], which are then highly sensitive to the readout laser pulses.

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