SUSTAINABLE ROUTE PLANNING AND GUIDANCE SERVICES

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ABSTRACT

This study developed a mixed-equilibrium model and tradeoff analysis to consider the implementation of route planning and guidance (RPG) services. Three perspectives were considered: the users, the service providers, and the traffic management agency. Each of these parties has distinctive objectives that are often in conflict with each other. By modeling the market penetration of RPG services in an elastic manner, analogous to a supply-demand equilibrium, and overlaying these objectives on the same decision plane, the technique allows one to balance these objectives and come up with sustainable RPG services. The model could also be used to investigate the sensitivity of different government policies toward these RPG services. Numerical results of a small network are provided to discuss the tradeoff analysis in detail.

1. INTRODUCTION

Route planning and guidance (RPG) are part of the user services of the Intelligent Transportation Systems (ITS) in the United States (ITS America, 1995). The rationale is that by disseminating traffic information, RPG would allow equipped vehicles to spread from the congested to the less congested areas. As a result, the travel time of both the equipped vehicles and the overall system would be reduced. Most of the present understanding of RPG's system impacts has come from traffic models.

The crux of the modeling efforts focuses on ways to model the route choice behaviors of equipped versus unequipped vehicles. The focus has been mostly on travel time reductions due to RPG under various exogenous parameters such as market penetration of this service, the level and extent of the information provided, and the level of congestion. Other efforts consider that due to a lack of reliable information on traffic conditions or route alternatives, many vehicles could not travel on the shortest routes. It is believed that RPG could remove this perception error and hence improve the route choice quality of its users. Based on this hypothesis, Koutsopoulos and Lotan (1990) considered RPG as a mixed-equilibrium problem in which the equipped vehicles follow the UE routes and the unequipped vehicles follow the SUE routes. The UE (SUE) routes are derived by the assumption of having perfect (imperfect) information. This approach is now well accepted and coded in some traffic models. In reality, people pay for RPG because it is beneficial, perhaps due to its convenience or its potential travel time saving. Thus, the demand for this service is elastic and the market penetrations of this service would be non-uniform even in the same network. Emmerink et al (1994) provided a framework to analyze the market potential of RPG from an economic point of view. Recently, Yang (1998) developed a mixed-equilibrium model that captures this demand relationship endogenously. The model determined the market penetration by a continuous increasing function of the information benefit.
This study models the market penetration of RPG services in an elastic manner and models the equipped and unequipped travelers as following two SUE traffic patterns, with the equipped travelers having lower variations of the network travel times. This is in view of the fact that having and providing perfect traffic information is an idealized situation, which is unlikely to occur in the near future.

To fully analyze the effect of RPG services, one must consider the perspectives of three parties, simplified in this analysis as:
1. Service provider, whose objective is profit-driven
2. Traffic management agency, whose objective is to reduce congestion
3. Users, whose objective is to save travel time at an affordable service charge
In general, these three objectives are often in conflict, deeming tradeoff analysis necessary. This study also provides a methodology to analyze the intriguing relationship between these three objectives and illustrates the methodology and concepts through a simple example. It is difficult to generalize the results based on limited computational experiences but the formulation developed herein can be applied to a general network. The outline of this paper is as follows. Section 2 describes the formulation. Section 3 presents an approach to solve the proposed formulation. Section 4 depicts the numerical experiments. Finally, Section 5 provides some concluding remarks.

2. **MIXED-EQUILIBRIUM MODEL FORMULATION**

We model the mixed-equilibrium problem with elastic market penetrations as a nonlinear complementarity problem (NCP). Consider a strongly connected network \([N, A]\) where \(N\) is the set of nodes and \(A\) the set of directed links and use the following variable notations:

\[
f_p^{rs}, \hat{f}_p^{rs} \quad \text{route flow of equipped (unequipped) travelers on path } p \text{ between OD pair } rs
\]

\([f, \hat{f}] \text{ the set of } \{f_p^{rs}\} \cup \{\hat{f}_p^{rs}\} \text{ with dimension } n_i = \sum_{rs} |P_p^{rs}| \text{ equals the total number of routes in the network; } P_p^{rs} \text{ is the set of routes connecting } rs
\]

\[
\eta_p^{rs} \quad \text{route time on } p \text{ between OD pair } rs, \text{ which is a function of } f, \hat{f}
\]

\([n] \text{ the set of } \{\eta_p^{rs}\} \text{ with dimension } n_i = \sum_{rs} |P_p^{rs}| \text{ equals the total number of routes in the network}
\]

\[q^{rs}, \hat{q}^{rs} \quad \text{demand of equipped (unequipped) travelers between OD pair } rs.
\]

\([q] \text{ the set of } \{q^{rs}, \hat{q}^{rs}\} \text{ with dimension } 2|R_S|
\]

\([\theta, \hat{\theta}] \quad \text{travel time variation of equipped (unequipped) travelers, which could be interpreted}
\]

\([\text{as the information qualities available to these different travelers}
\]

\[
w_p^{rs}, \hat{w}_p^{rs} \quad \text{proportion of the equipped (unequipped) travelers taking route } p
\]

The equipped travelers follow the SUE conditions with the route flows split by the logit model, expressed as (Sheffi, 1985):

\[
w_p^{rs} = \frac{\exp(-\theta \cdot \eta_p^{rs})}{\sum_{k \in P_p^{rs}} \exp(-\theta \cdot \eta_k^{rs})}, \quad p \in P_p^{rs}, rs \in RS
\]
A higher $\theta$ means smaller travel time variations and, therefore, RPG services with better information quality. The logit-based SUE assignment results in the following route flows:

$$f_p^n = w_p^n \cdot q_p^n, \, rs \in RS, p \in P^n$$

(2)

One may state (2) in an equivalent complementary form:

$$f_p^n \left( f_p^n - w_p^n \cdot q_p^n \right) = 0, \forall rs \in RS, \forall p \in P^n$$

(3)

That is, if $f_p^n > 0$ then (2) must be satisfied or $f_p^n$ is apportioned according to (1). Theoretically, the nature of the SUE assignment would assign positive flows to each route, rendering the case of $f_p^n = 0$ nonexistent.

The treatment of the unequipped travelers is identical. We write:

$$\hat{w}_p^n = \frac{\exp(-\hat{\theta} \cdot \eta_p^n)}{\sum_{k \in P^n} \exp(-\hat{\theta} \cdot \eta_k^n)}, \, p \in P^n, rs \in RS$$

(4)

$$\hat{f}_p^n = \hat{w}_p^n \cdot \hat{q}_p^n, \, rs \in RS, p \in P^n$$

(5)

$$\hat{f}_p^n \left( \hat{f}_p^n - \hat{w}_p^n \cdot \hat{q}_p^n \right) = 0, \forall rs \in RS, \forall p \in P^n$$

(6)

The variable $\hat{\theta}$ can be interpreted as the familiarity of the network conditions of these travelers without RPG services. This is a characteristic of the network.

We model the market penetration of RPG services in an elastic manner, which affects the distribution of $q^n$ and $\hat{q}^n$. Note that this elastic market penetration, analogous to a supply-demand equilibrium, is modeled per origin-destination (OD) pair but is not uniform across all OD pairs. The total travel demand between OD pair $rs$, $\bar{q}^n$, is assumed to be fixed. We model the elastic market penetration as:

$$q^n = \frac{\bar{q}^n}{1 + \exp(C_Y - B \cdot \phi^n)}$$

(7)

$C_Y$ is the net or out-of-pocket service charge. The parameter $B$ is the value of time of the travelers. This is a characteristic of the network. The travel time saving of the equipped travelers on OD pair $rs$ is defined as:

$$\phi^n = \left( \sum_p \hat{w}_p^n \cdot \eta_p^n \right) - \left( \sum_p w_p^n \cdot \eta_p^n \right), \, \forall rs \in RS$$

(8)

The first term on the right hand side represents the average travel time of the unequipped vehicles on OD pair $rs$. The second term is the average travel time of the equipped vehicles for the same OD pair. Besides being a function of $B$ and $C_Y$, the demand for the service is a function of the route flows $f_p^n, \hat{f}_p^n$ that eventually determine the travel time saving $\phi^n$. Since both types of travelers are modeled as following the SUE conditions, there is no guarantee that the travel times of all equipped travelers will be lower than those of their unequipped counterparts. However, we anticipate that the average travel time of the equipped travelers will be lower if the market penetration is at a significant level.

From (7), one can see that even without travel time saving, there would still be a certain market penetration if the system cost is not high. This models the intrinsic value of having the service, including such benefits as "sense of security", convenience, etc. On the other hand, if
both the value of time and travel time saving are high, the term \( \exp(C_N - B \cdot \phi^r) \) approaches zero, which makes the market penetration close to 100%. However, higher travel time saving would attract more subscribers to the service, which would then result in a lower travel time saving. With the total travel demand \( \hat{q}^r \) modeled as a constant in this formulation, one can define the amount of unequipped travelers

\[
\hat{q}^r = \hat{q}^r - q^r
\]

We formulate this mixed-equilibrium problem as a nonlinear complementarity problem (NCP) in the form of:

\[
x \cdot F^T(x) = 0, x \geq 0
\]

by letting:

\[
x = (f, \hat{f}) \in R^{n+n},
\]

and

\[
F^T(x) = \begin{bmatrix}
(f^r - w^r \cdot \hat{q}^r, \forall rs \in RS, \forall p \in P^r)^T \\
(\hat{f}^r - \hat{w}^r \cdot \hat{q}^r, \forall rs \in RS, \forall p \in P^r)^T
\end{bmatrix} \in R^{n+n}.
\]

The terms on the right hand side of (12) is determined through the substitutions of (1), (4), (7), (8), and (9). Actually, equations (10)-(12) are merely rewrites of the SUE conditions defined in (1)-(3) and (4)-(6). The solution to the NCP (10)-(12) satisfies the SUE conditions of both the equipped and unequipped travelers simultaneously.

3. FORMULATION OF DIFFERENT OBJECTIVES

3.1 Users: maximize user benefits (i.e., \( \max UB \))

We define the user benefit on OD pair \( rs \) (\( ub^r \)) and total user benefit (\( UB \)) to be:

\[
ub^r = B \cdot \phi^r - C_N
\]

\[
UB = \sum_n ub^r
\]

The first term on the right hand side (RHS) of \( ub^r \) is the value of the travel time saving. \( C_N \) is the net service charge, which could consist of two terms:

\[
C_N = C_S - S
\]

where \( C_S \) is the actual charge of RPG services; while \( S \) is introduced to model possible subsidies provided by the government to lower the net service charge. In circumstances without government subsidy, \( C_N, C_S \) are equal.

3.2 Service providers: maximize profit (i.e., \( \max P \))

We define the profit function \( P \) to be:

\[
P = Q \cdot C_S - \left[ \beta \cdot \theta + \int_0^\theta (\lambda + e^{-\mu}) \, dx \right]
\]

The first term on the right hand side is the revenue. \( Q = \sum_n q^r \) is the total number of users, while \( C_S \) is the service charge per user. The bracket represents the cost of providing the
services. \( \beta \) is the cost of collecting and processing data per information quality, which is fixed regardless of the number of users. This term models the infrastructure and operating costs to provide information quality at \( \theta \). The second term in the bracket is the cost of serving \( Q \) users, which is variable. As is customary, we further assume that this variable cost per user is decreasing with the number of users. The parameter \( \lambda \) is the ultimate cost per user with a large user base and \( \mu \) can be interpreted as capturing the economy of scale of providing the service. They should correspond to the network characteristics.

3.3 Traffic management agency: Maximize the reduction in total system travel time (i.e., \( \max RT \))

The total system travel time is: 
\[
TSTT = \sum_{r} \sum_{p} (f_{p}^{r} + f_{p}^{''r}) \cdot \eta_{p}^{r},
\]
which is the sum of the travel times of the equipped and unequipped travelers on each route. We define a related measure, called the relative reduction in \( TSTT \) (\( RT \)), to capture the change in \( TSTT \) before and after the implementation of RPG services. That is,
\[
RT = \frac{TSTT^{a} - TSTT^{b}}{TSTT^{b}} \times 100\%.
\]
(16)
The superscripts \( b \) and \( a \) refer to the cases of "before" and "after" the service implementation respectively.

4. SOLVING THE MIXED-EQUILIBRIUM FORMULATION

We reformulate the NCP (10)-(12) to an unconstrained optimization. This reformulation would make available a large number of solution techniques already developed for unconstrained optimizations. This reformulation primarily draws upon the use of a gap function.

**Definition 1:** Let \( \Omega \) be the set of solutions to the NCP formulation (10)-(12), and \( \Psi = \{x \geq 0, F(x) \geq 0\} \). A function \( G: R^{2n} \to R \) is a gap function for the NCP formulation if the following three conditions are satisfied:

1. \( G(x) = 0 \)
2. \( G(x) = 0 \iff x \in \Omega \)
3. \( \min_{x \in \Psi} G(x) = 0 \) is a global minimum

In essence, the gap function provides a convenient measure of convergence for developing solution algorithms. By minimizing \( G \) over \( x \), a point in \( \Omega \) is obtained. That is:
\[
\min_{x \in \Psi} G(x) \quad \text{s.t.} \quad x \in \Psi \quad \text{where} \quad \Psi = \{x \geq 0, F(x) \geq 0\}.
\]
(17)

Previous applications of gap functions for the traffic equilibrium problem are reviewed in Lo and Chen (1999a). These applications are either nonconvex and/or nonsmooth and may pose limitations in computational efficiency if these problems are not carefully addressed. Recently, Fischer (1992) noted that the following simple, two-variable, convex function:
\[
\phi(a,b) = \sqrt{a^2 + b^2} - (a + b),
\]
has this property:
\[
\phi(a,b) = 0 \iff a \geq 0, b \geq 0, ab = 0.
\]
(19)
A gap function is proposed based on squaring \( \phi(a,b) \) (Facchinei and Soares, 1995):
\[ \varphi(a,b) = \frac{1}{2} \varphi^2(a,b) \]. In Lo and Chen (1999b), the following propositions were proved.

1. \( \varphi(a,b) = 0 \) if and only if \( a \geq 0, b \geq 0, ab = 0 \)
2. \( \varphi(a,b) \geq 0 \) for all \( (a,b) \in \mathbb{R}^2 \)
3. \( \varphi \) is continuously differentiable on \( \mathbb{R}^2 \), in particular, \( \nabla \varphi(0,0) = (0,0) \)

This gap function is also applicable to this mixed-equilibrium problem. Specifically, we define the gap function as: \( G(x) = \sum_{i=1}^{n} \varphi(x_i, F_i(x)) \) where \( i \) represents the \( i \)th component in (11) and (12). One advantage of this gap function is that the nonnegativity constraints in (10) are automatically satisfied according to the property of (19) when the gap function \( G(x) \) and hence each individual term \( \varphi(x_i, F_i(x)) \) in \( G(x) \) becomes zero. Solving the NCP is thus equivalent to finding the unconstrained global solutions of the problem \( \{ \min G(x) \} \).

5. **NUMERICAL STUDY**

To discuss the tradeoff analysis in a more detailed fashion, we choose a simple network for ease of results exposition. The example network used in this study is shown in Figure 1. The network consists of 3 nodes, 3 links and 2 OD pairs, with the link performance functions shown in Figure 1. The two OD pairs are from node 1 to node 3 and from node 2 to node 3. OD pair (1,3) has two routes while OD pair (2,3) has only one route. For OD (2,3), there is no travel time saving between the equipped and unequipped drivers since there is only one route between the OD pair. The market penetration is solely determined by the cost of RPG services regardless of traffic conditions. As for OD pair (1,3), the equipped drivers can gain travel time saving by suitably choosing the route with shorter travel time.

**Figure 1**  The example network

[Diagram of the network with relevant equations]

Specifically, the values of the parameters used in this study are: \( \lambda = 0.5, \mu = 10, B = 15, \beta = 70, \tilde{q}^{23} = 3, \tilde{\omega} = 0.01 \), which are assumed to be fixed. Such values are chosen for illustration purposes. By fixing the above parameters while varying the decision variables \( C_N, \theta \) and solving the resultant mixed-equilibrium, one obtains the performance of the three objectives corresponding to the \( C_N, \theta \) values. We then plot the contours of three objective values on the same \( C_N \times \theta \) plane, as shown in Figures 2-3.
Figure 2 shows the case of a relatively low travel demand (4 units) from node 1 to node 3. Three types of contours are shown as depicted in the legend. The arrows correspond to the direction whereby each of the objective values can be further increased. In the case of profit, for example, the service provider can increase its profit by reducing the information quality of the RPG service while maintaining the same service charge. The objectives $UB$ and $RT$, to different extents, can be increased by providing better information qualities at lower service charges. In the extreme cases, in the ranges of $C_N, \theta$ shown in the figure, the optima for $UB, RT$ coincide at the highest $\theta$ value and the lowest $C_N$ value. Physically, it implies giving the best traffic information to the public freely. While this may be good for the travelers and the traffic management agency, it is not financially viable due to the high information acquisition costs. This will only happen if the government themselves act as a service provider without considering information acquisition costs or profit.

The darkened lines in Figure 2 show the bottom lines of each of the objectives. For the RPG service to be acceptable to each of the three parties, all of these conditions must hold:

1. the user benefits must be nonnegative ($UB \geq 0$)
2. the profit must be nonnegative ($P \geq 0$)
3. the reduction in total travel time must be nonnegative ($RT \geq 0$)

Violating the first condition implies that there is no market for the service. Violating the second condition implies that there is no private provider of the service. Violating the third condition would unlikely be approved by the traffic management agency. Therefore, the intersection between these three regions defines the feasible region for implementing RPG services. According to Figure 2, at a relatively low demand, there is no feasible region.

At a higher demand level (6 units), Figure 3 shows the relationships between the three objective values. The optimum reduction in total travel time, $RT_{\text{max}}$, still occurs with providing the best information at no charge to the users. The maximum user benefit, $UB_{\text{max}}$, however, shifts leftward to the case with lower information quality while maintaining the service charge at zero. One way to explain is the following. At the service charge of zero, the user benefit comes solely from the value of the travel time saving. If all travelers become users, there is no or a small travel time saving. The demand level also influences the consideration. In low congestion, the travel time saving would likely be small regardless of the information quality. The same would occur with very heavy congestion. It would seem that at the demand level of six, travel time saving is sensitive to the information quality. This formulation involves solving several intertwined simultaneous equations. This result simply shows the difficulty of anticipating the outcomes if one was not equipped with an appropriate model.

This scenario of higher demand does have a feasible region, as shown shaded in Figure 3. The private sector can create a viable market situation to provide the service. If left alone, however, the private sector would probably plan the service charge and information quality to rest at their optimum, $P_{\text{max}}$, which occurs at a service charge of 3.4 and information quality of $\theta = 0.09$. Even at this solution, both the system and the users gain compared with the case without the service. Total system travel time is reduced by 4% and the user benefits are positive. The objective values of each of the parties are shown in the scenario of "Base case" in Table 1.

The government, however, can do better by introducing policies to shift the equilibrium to two other points. The first one is referred to as $RT_{2^{nd} \text{Best}}$ in Figure 3, which is the second best
solution for the traffic management agency. It lies on the intersection of the zero-profit line and the highest $RT$ contour on this line. It defines the situation wherein companies will start to have the incentive to provide RPG services in a competitive market. The government may then rely on the RPG services provided by the private sector to drive down the total system travel time. From Figure 3, the government can achieve this second best solution by regulating the service charge to 3.1 and improving the information quality for the service providers. As shown in the second scenario in Table 1, the final outcome is that $RT$ increases from the base case of 4% to 5.4% and the user benefits also increase from 0.4 to 1.1.

Another possible solution is the users' second best solution, denoted as $UB_{2^{nd} \text{Best}}$, in Figure 3. Similarly, it lies on the intersection between the zero-profit line and the highest $UB$ contour on the line. Such a solution can be reached by regulating the service charge to 1.5 while roughly keeping the same information quality as what would maximize $UB$. The results of this policy are summarized in the third scenario in Table 1. This policy regulation increases $RT$ slightly by 0.4%. The big winners are the users, with $UB$ increases from the base case of 0.4 to 2.1.

This modeling framework allows sensitivity analyses of pricing instruments in addition to the regulatory ones discussed above. Specifically, the government may choose to subsidize the data acquisition costs for the service providers or subsidize the service charge for the users. In each of these cases, assume that the government imposes no regulations on the service charge or information quality and the private sector is allowed to come up with its profit maximization solution. The results of applying these subsides are summarized in the last two scenarios in Table 1. The fourth scenario represents the situation where the government subsidizes the information acquisition cost ($\beta$ decreases to 60). In the last scenario, the government subsidizes 2 monetary units to each user. Both types of subsidies benefit all parties as compared with the base scenario. All objectives are driven upward for each case, though the extra reduction in total travel time is modest in both cases. What is not anticipated is that subsidizing the service charge actually doubles the profit of the service provider. This is due to increases in the market penetration of OD pair (1,3) from 60% to 77%, with the service provider collecting 5.2 units for each user.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Profit</th>
<th>Travel time reduction (%)</th>
<th>User benefits</th>
<th>Solution $(C_x, \theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>7.2</td>
<td>4</td>
<td>0.4</td>
<td>(3.3, 0.09)</td>
</tr>
<tr>
<td>restricting to RT 2$^{nd}$ Best</td>
<td>0</td>
<td>5.4</td>
<td>1.1</td>
<td>(3.1, 0.22)</td>
</tr>
<tr>
<td>restricting to UB 2$^{nd}$ Best</td>
<td>0</td>
<td>4.4</td>
<td>2.1</td>
<td>(1.5, 0.12)</td>
</tr>
<tr>
<td>Information subsidy ($\beta = 60$)</td>
<td>7.8</td>
<td>4.5</td>
<td>0.7</td>
<td>(3.7, 0.14)</td>
</tr>
<tr>
<td>Service charge subsidy ($S = 2$)</td>
<td>14.5</td>
<td>4.6</td>
<td>1.2</td>
<td>(3.2, 0.14)</td>
</tr>
</tbody>
</table>

The focus here is not on what are government policies. The tradeoff is likely case specific and network dependent. We demonstrated that the outcome is not easily anticipated, even with the small network example used in this study. Even though the results generated in this study are not directly transferable, the model and the tradeoff analysis developed in this study can be applied and extended to other general networks. Results similar to what is expressed in Figures 2-3 and Table 1 would provide important insights for considering government policies.
Figure 2  Tradeoff of the three objectives with four units of demand

Figure 3  Tradeoff of the three objectives with six units of demand
6. CONCLUDING REMARKS

This study developed a mixed-equilibrium model and tradeoff analysis to consider the implementation of RPG services. Three perspectives were considered: the users, service providers, and traffic management agency. Each of these parties has distinctive objectives that are often in conflict with each other. By modeling the market penetration of RPG services in an elastic manner, analogous to a supply-demand equilibrium, and overlaying these objectives on the same decision plane, the technique allows one to trade these objectives and plan sustainable RPG services. The model could also be used to investigate the sensitivity of different government policies toward these RPG services.

The numerical results, albeit based on a small network, are quite revealing. In general, they show that it may not be possible to determine a unique set of decisions to optimize all objectives. Moreover, the outcomes are sensitive to the total demand, the network, and various cost parameters, rendering the outcomes difficult to be anticipated by intuitions. This points to the need of developing and calibrating a model for its analysis. We hope that the model discussed in this paper will be useful in this regard. Directly using an unconstrained optimization approach to solve this mixed-equilibrium problem may not be the most efficient, albeit it is simple. We are developing other approaches to solve this type of problem.

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