Atomic *Zitterbewegung* in Abelian vector gauge potentials

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We propose a scheme to realize *Zitterbewegung* (ZB) with cold atoms in an Abelian vector potential. Two dark states can be created by interacting alkali-metal atoms with three coaxial Gaussian beams. Atoms moving in the subspace spanned by the two dark states feel a vector gauge potential that is nonvanishing only along the laser axis. We show that cold atoms in this Abelian vector potential undergo ZB with a large amplitude detectable in experiment.

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The counterintuitive trembling motion of Dirac electrons, usually called *Zitterbewegung* (ZB), was first discovered by Schrödinger when he examined the behavior in time of the physical observables of free electrons associated with the Dirac equation [1,2]. The oscillatory dynamics of these observables, induced by the interference between the positive- and negative-energy states, is characterized by a frequency of the order of 10^{20} Hz (corresponding to an energy gap of ~ 1 MeV) and an amplitude of oscillation in the electron's position of the order of the Compton wavelength. It is believed that direct experimental observation of these oscillations would be extremely difficult if not impossible [3].

In recent years, there has been a revival in the study of ZB in crystalline solid systems, where the dynamics of particles or quasiparticles is governed by Dirac-like equations and the energy spectrum is similar to the spectrum of the Dirac Hamiltonian. The energy gaps in these systems are usually of the order of meV and the oscillation frequency is typically a few THz. In particular, semiconductor heterostructures [4], graphene [5,6], tunnel-coupled electron-hole bilayers [7], superconductors [8], and photonic crystals [9] have been proposed as candidate systems for the direct observation of ZB.

Contemporaneous with these developments have been an impressive series of experimental and theoretical achievements in ultracold atoms. Sophisticated technologies for creating and manipulating quantum degenerate gases, such as Bose-Einstein condensates [10] and degenerate Fermi gases [11], have led to intriguing new possibilities of simulating in cold atoms numerous remarkable physical phenomena that are experimentally inaccessible in their original counterpart systems. Recently, it was shown that a system governed by a Dirac-like Hamiltonian can be created by interacting cold atoms with laser beams carrying relative phase. Atoms in the system feel a non-Abelian gauge potential [12–14] and undergo ZB as a result [15,16]. The frequency of the ZB is of the order of a few MHz or kHz and the amplitude is comparable to the wavelength of the laser beams. These results show that cold atomic systems are a promising candidate for observing ZB. However, the ZB in these models is thus far restricted to the non-Abelian vector potentials [12,17,18] created in a two-dimensional (2D) laser configuration, and it vanishes when the vector potentials are Abelian.

In this paper, we propose to observe ZB in an Abelian gauge potential created in a one-dimensional (1D) laser configuration. Cold atoms are placed in three coaxial Gaussian beams, in which the vector gauge potential felt by the atoms is nonvanishing only along the laser axis. Therefore, the vector potential is Abelian. However, we show that due to a scalar gauge potential, the velocity operator does not commute with the Hamiltonian; the cold atoms therefore oscillate and a ZB of the center of mass in real space can be detected.

The atomic system under consideration is of the “tripod” type, which can be realized with ^{87}Rb or other alkali-metal atoms. The level scheme and laser configuration are depicted in Fig. 1. Three ground hyperfine states $|1\rangle$, $|2\rangle$, and $|3\rangle$ are coupled to the excited state $|0\rangle$ by three Gaussian beams. The corresponding Rabi frequencies are denoted $\Omega_j(\vec{r}) \equiv \tilde{\Omega}_j(\rho)e^{ik_j z}$, where $\rho = \sqrt{x^2 + y^2}$, z is the coordinate along the laser axis, and $k_j (j=1,2,3)$ is a wave vector. We assume these laser beams are tuned to achieve electromagnetically induced transparency [19].

In the rotating-wave approximation, the interaction Hamiltonian describing the atom-laser interaction is [12] $H_{\text{int}} = -\hbar \sum_{i=1}^3 (\Omega_i |0\rangle \langle i| + \Omega_i^* |i\rangle \langle 0|)$. The eigenstates of H_{int} include two dark states [12] $|a\rangle = (\Omega_2 \Omega_3 |1\rangle - \Omega_1 \Omega_3 |2\rangle) / (\Theta_2 \tilde{\Omega}_3)$ and $|b\rangle = (\Omega_1^* \Omega_3 |1\rangle + \Omega_2^* \Omega_3 |2\rangle - \Theta_2^2 |3\rangle) / (\Theta_2 \Theta_3)$, where $\Theta_j = (\sum_{i=1}^j |\Omega_i|^2)^{1/2}$. The dark states have internal energy $\varepsilon_a(\vec{r}) = \varepsilon_b(\vec{r}) = 0$ and are well separated from the other two “bright” states. We are therefore justified in invoking an adiabatic approximation [12] in which all motion is confined to the dark subspace spanned by $\{|a\rangle, |b\rangle\}$.

In the dark subspace, we can write the Hamiltonian describing the center-of-mass motion as $H = \sum_{\eta'} \Sigma_{\eta} |\eta'\rangle H_{\eta'\eta} \langle \eta|$ ($\eta, \eta' = a, b$), where the operator

FIG. 1. (Color online) “Tripod” scheme using ^{87}Rb to generate an Abelian vector gauge potential.

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$$H_{\eta'\eta} = \frac{1}{2M}(\delta_{\eta'\eta}p^2 + \vec{p} \cdot \vec{A}_{\eta'\eta} + \vec{A}_{\eta'\eta} \cdot \vec{p}) + U_{\eta'\eta} \quad (1)$$

acts on the atomic center of mass. Here,

$$\vec{A}_{\eta'\eta}(\vec{r}) \equiv -i\hbar\langle\eta'|\vec{\nabla}|\eta\rangle \quad (2)$$

is an effective vector potential, while

$$U_{\eta'\eta}(\vec{r}) = \frac{\hbar^2}{2M}\langle\vec{\nabla}\eta'|\vec{\nabla}\eta\rangle + \langle\eta'|V|\eta\rangle \quad (3)$$

is an effective scalar potential [20]. The trapping potential $V(\vec{r})$ is taken to be diagonal in the $\{|1\rangle, |2\rangle, |3\rangle\}$ basis, with components $V_1(\vec{r})$, $V_2(\vec{r})$, and $V_3(\vec{r})$.

We assume the three coaxial Gaussian beams have the same waist (which is much larger than the wavelength) and introduce parameters $\tan\phi = \tilde{\Omega}_1(\rho)/\tilde{\Omega}_2(\rho)$ and $\sin\theta = \Theta_2/\Theta_3$ for simplicity. It is easy to verify that both ϕ and θ are constants determined by the intensity ratios of the laser beams and the dark states $|a\rangle$ and $|b\rangle$ depend only on z , but not on ρ or the azimuthal angle $\varphi = \arctan(y/x)$. The ρ and φ components of the vector potential (2) are consequently zero; only the z component is nonvanishing. Therefore, the vector potential is Abelian. We can write $\vec{A}_{\eta'\eta} = A_{\eta'\eta}\hat{z}$, with

$$A_{aa} = \hbar[(k_3 + k_2)\cos^2\phi + (k_3 + k_1)\sin^2\phi], \quad (4a)$$

$$A_{bb} = \hbar\cos^2\theta[(k_3 - k_2)\cos^2\phi + (k_3 - k_1)\sin^2\phi], \quad (4b)$$

$$A_{ab} = \frac{1}{2}\hbar(k_2 - k_1)\cos\theta\sin 2\phi\exp[-i(k_1 + k_2)z]. \quad (4c)$$

The vector potential is thus a constant since $k_2 = -k_1$. The scalar potential U has a similar structure

$$U_{aa} = [V_1 + \zeta(k_3 + k_2)^2]\cos^2\phi + [V_2 + \zeta(k_3 + k_1)^2]\sin^2\phi, \quad (5a)$$

$$U_{bb} = V_3\sin^2\theta + \{[V_2 + \zeta(k_3 - k_2)^2]\cos^2\phi + [V_1 + \zeta(k_3 - k_1)^2]\sin^2\phi\}\cos^2\theta, \quad (5b)$$

$$U_{ab} = [(V_1 - V_2)/2 + \zeta k_3(k_2 - k_1)]\cos\theta\sin 2\phi \times \exp[-i(k_1 + k_2)z], \quad (5c)$$

in which $\zeta = \hbar^2/2M$. It is convenient to write the matrices A and U as $A = a_0 + \vec{\sigma} \cdot \vec{a}$ and $U = u_0 + \vec{\sigma} \cdot \vec{u}$, where $\vec{\sigma}$ is a vector of Pauli matrices, $a_0 = \frac{1}{2}(A_{aa} + A_{bb})$, and $\vec{a} = (a_x, a_y, a_z) = (A_{ab}, 0, \frac{1}{2}(A_{aa} - A_{bb}))$, with similar expressions for u_0 and $\vec{u}$.

For definiteness, we now assume that the trapping potential $V(\vec{r}) = V(\rho)$ is generated by a fourth Gaussian beam (not shown in Fig. 1) tuned far below resonance. The main effect of this beam is to produce a tight confinement in the x - y plane. However, its polarization can also be varied to produce a fictitious magnetic field [21,22] that separates V_2 from $V_1 = V_3$ (for the configuration shown in Fig. 1). The splitting is maximum for circular polarization and vanishes for linear polarization; its magnitude is also proportional to the fine-structure splitting of the $^2P_{1/2}$ and $^2P_{3/2}$ excited states.

Since the atoms are “frozen” in the x - y plane, we only need to consider the center-of-mass motion along the z axis. The Hamiltonian of this quasi-1D system is

$$H = h_0 + \frac{\hbar}{2}\vec{\sigma} \cdot \vec{\omega}, \quad (6)$$

where $h_0 = p_z^2/2M + a_0p_z/M + u_0$ and $\vec{\omega} = (2/\hbar)\vec{u} + (2p_z/\hbar M)\vec{a}$, in which the potentials $V_i(\rho)$ are replaced by their mean values $\bar{V}_i = \langle V_i(\rho) \rangle$. Since p_z commutes with H , the eigenvalues and eigenvectors of H are

$$E_{\pm}(k) = E_0(k) \pm \frac{1}{2}\hbar\omega(k), \quad (7a)$$

$$|+\rangle = |k\rangle\left(\cos\frac{\alpha}{2}|a\rangle + \sin\frac{\alpha}{2}|b\rangle\right), \quad (7b)$$

$$|-\rangle = |k\rangle\left(-\sin\frac{\alpha}{2}|a\rangle + \cos\frac{\alpha}{2}|b\rangle\right), \quad (7c)$$

where $|k\rangle$ is an eigenket of p_z with eigenvalue $\hbar k$, $E_0(k) = \hbar^2 k^2/2M + a_0\hbar k/M + u_0$, $\omega(k) = |\vec{\omega}(k)|$, $\vec{\omega}(k) = (2/\hbar)\vec{u} + (2k/M)\vec{a}$, and $\tan\alpha(k) = \omega_x(k)/\omega_z(k)$.

The energy spectrum consists of an upper band $E_+(k)$ and a lower band $E_-(k)$, separated by a gap of $\hbar\omega(k)$. To express $\omega(k)$ in a simple form, it is convenient to define a wave number $k = k_0$ such that $\vec{\omega}_0 \equiv \vec{\omega}(k_0)$ is perpendicular to $\vec{a}$. This gives

$$\omega(k) = \omega_0\sqrt{1 + \chi_0^2(k - k_0)^2}, \quad (8)$$

in which $\omega_0 = (2/\hbar)|\vec{u} \times \vec{a}|$ and $k_0 = -M\vec{u} \cdot \vec{a}/(\hbar a^2)$. The constant $\chi_0 = 2a/M\omega_0$ is a characteristic length analogous to the reduced Compton wavelength $\chi_C = \hbar/mc$. Note that the unit vectors $\hat{a}$, $\hat{y}$, and $\hat{\omega}_0$ form an orthogonal (although not necessarily right-handed) basis.

From the definition of ω_0 , we see that the energy gap does not vanish unless $\vec{u} \parallel \vec{a}$ (i.e., unless $[U, A] = 0$). Explicit values for the parameters ω_0 and χ_0 in Eq. (8) may be obtained from the expressions

$$\vec{u} \times \vec{a} = \hat{y}(\hbar/2)\cos\theta\sin^2\theta\sin 2\phi\{k_2(\bar{V}_2 - \bar{V}_3) + (k_2 - k_3)[(\bar{V}_1 - \bar{V}_2)/2 + \zeta k_2(k_2 + k_3)]\}, \quad (9)$$

$$a^2 = \hbar^2\{k_2^2\cos^2\theta\sin^2 2\phi + [k_3\sin^2\theta + k_2(1 + \cos^2\theta)\cos 2\phi]^2/4\}. \quad (10)$$

According to Eq. (9), the value of ω_0 is determined primarily by the splitting $\bar{V}_2 - \bar{V}_3$ (a fine-structure effect), since $k_2 - k_3$ is proportional to the hyperfine splitting of the ground state. If we neglect the small difference between k_2 and k_3 , then $a = \hbar k_2(1 - \sin^2\theta\sin^2\phi)$ and

$$\omega_0 = \left| \frac{\bar{V}_2 - \bar{V}_3}{\hbar} \frac{\sin^2\theta\cos\theta\sin 2\phi}{1 - \sin^2\theta\sin^2\phi} \right|, \quad (11)$$

$$\chi_0 = \left| \frac{4\zeta k_2}{\bar{V}_2 - \bar{V}_3} \frac{(1 - \sin^2 \theta \sin^2 \phi)^2}{\sin^2 \theta \cos \theta \sin 2\phi} \right|. \quad (12)$$

The key parameters ω_0 and χ_0 are therefore controlled by the values of $\bar{V}_2 - \bar{V}_3$, θ , and ϕ .

Let us now consider the dynamics of the center of mass. In the Heisenberg picture, the velocity operator $v(t) = dz/dt$ is given by $v = (i/\hbar)[H, z] = (p_z + A)/M$. Its explicit time dependence can be calculated by applying the Baker-Hausdorff lemma [23] to the definition $v(t) = \exp(iHt/\hbar)v(0)\exp(-iHt/\hbar)$, which yields

$$Mv(t) = Mv(0) - \vec{\sigma} \cdot (\hat{\omega} \times \vec{a}) \sin \omega t + \vec{\sigma} \cdot [\hat{\omega} \times (\hat{\omega} \times \vec{a})](1 - \cos \omega t). \quad (13)$$

Here, $\hat{\omega}$ and ω are functions of k , while $\vec{\sigma}$ is independent of time. Upon integrating the velocity with respect to time, we obtain the position z of the center of mass

$$z(t) = z(0) + v(0)t + \vec{\sigma} \cdot (\hat{\omega} \times \vec{a})(\cos \omega t - 1)/(M\omega) + \vec{\sigma} \cdot [\hat{\omega} \times (\hat{\omega} \times \vec{a})](\omega t - \sin \omega t)/(M\omega). \quad (14)$$

In addition to the classical linear motion of a free particle, Eqs. (13) and (14) exhibit the sinusoidal oscillations characteristic of ZB. Evidently, such terms can only arise if v does not commute with H [24], since otherwise $v(t) = v(0)$. In our system, the commutator $[v, H] \neq 0$ because the vector potential A in $v = (p_z + A)/M$ does not commute with the scalar potential U in H . As shown above, the condition $[U, A] \neq 0$ is necessary and sufficient for the existence of a nonvanishing energy gap $\hbar\omega_0$.

The amplitude of the ZB in Eq. (14) is proportional to $\chi(k) \equiv 2a/M\omega(k) = \chi_0\omega_0/\omega(k)$. Remarkably, this amplitude diverges in the limit $\omega \rightarrow 0$. This is similar to the case of semiconductors with Rashba spin-orbit coupling, where the ZB diverges in the neighborhood of a point of contact between two spin-split bands [24]. Note, however, that for very small ω , the ZB oscillations may become so slow that they are difficult to observe.

To illustrate the significance of the above results, we shall calculate the expectation value of $z(t)$ using experimentally reasonable values for the various physical parameters. We assume that the atoms are initially prepared in a wave packet $|\Psi\rangle = \int dk \varphi(k)|k\rangle|\hat{n}\rangle$, where $\varphi(k)$ is the wave function in momentum space and $|\hat{n}\rangle$ is an eigenket of $\hat{n} \cdot \vec{\sigma}$ with eigenvalue 1. Since $\langle \hat{n} | \vec{\sigma} | \hat{n} \rangle = \hat{n}$, the expectation value of $z(t)$ for the state $|\Psi\rangle$ is given by

$$\langle z(t) \rangle = \langle z(0) \rangle + \langle v(0) \rangle t + \langle \hat{n} \cdot (\hat{\omega} \times \vec{a})(\cos \omega t - 1)/(M\omega) \rangle + \langle \hat{n} \cdot [\hat{\omega} \times (\hat{\omega} \times \vec{a})](\omega t - \sin \omega t)/(M\omega) \rangle, \quad (15)$$

in which $\langle v(0) \rangle = (\langle p_z \rangle + a_0 + \hat{n} \cdot \vec{a})/M$. We can maximize the ZB with certain choices of $\hat{n}$. For example, if $\hat{n} = \hat{\omega}_0 \times \hat{a} = \pm \hat{y}$, then

$$\langle z(t) \rangle = \langle z(0) \rangle + \langle v(0) \rangle t + \frac{1}{2} \chi_0 \langle (\cos \omega t - 1) \omega_0^2 / \omega^2 \rangle, \quad (16)$$

while if $\hat{n} = \hat{a}$, then

FIG. 2. (Color online) *Zitterbewegung* of a wave packet of ^{87}Rb atoms in an Abelian vector potential.

$$\langle z(t) \rangle = \langle z(0) \rangle + \langle v(0) \rangle t - \frac{1}{2} \chi_0 \langle (\omega t - \sin \omega t) \omega_0^3 / \omega^3 \rangle. \quad (17)$$

These calculations are easiest to perform as integrals in k space.

Experimentally, a 1D wave packet of alkali-metal atoms can be prepared as a quasimonomode guided atom laser [25–30] or generated directly by cooling atoms via velocity-selective coherent population trapping (VSCPT) [31–33]. Here we take a wave packet in the dark state $|a\rangle$ from VSCPT as an example. In this case, $\hat{n} = \hat{z}$ and

$$\langle z(t) \rangle = \langle z(0) \rangle + \langle v(0) \rangle t + \frac{1}{2} s \chi_0 \langle (\omega t - \sin \omega t) \omega_0^2 \omega_x / \omega^3 \rangle, \quad (18)$$

in which $s \equiv \hat{y} \cdot (\hat{\omega}_0 \times \hat{a}) = \pm 1$. The momentum distribution $|\varphi(k)|^2$ of the wave packet, as verified in the experiment [33], can be approximated by a Lorentzian function

$$|\varphi(k)|^2 = \frac{\gamma/\pi}{(k - k_0)^2 + \gamma^2}, \quad (19)$$

where γ is the half width at half maximum (HWHM). It can be taken as $k_L/25$ [33], where $k_L \equiv |k_1| = |k_2|$ is the wave number of the first two laser beams in Fig. 1, which can act as cooling beams in VSCPT. Using this initial wave packet, we have numerically integrated Eq. (18); the oscillating part of the center-of-mass motion is shown in Fig. 2.

The frequency of the oscillation is around 700 Hz, as compared to 10^{20} Hz for Dirac electrons and 10^{12} Hz for electrons in semiconductor systems. The amplitude of the ZB in the center-of-mass position is around $0.5 \mu\text{m}$, comparable to the wavelength of the laser beams. These results show the possibility of imaging the ZB oscillation using optical means. Moreover, numerical simulations indicate that the amplitude of the ZB can be increased by reducing the HWHM γ of the initial wave packet. Experimentally, a narrower wave packet in k space (hence a broader wave in real space) can be achieved by keeping the cooling laser beams on longer in VSCPT [33,34]. In the limit $\gamma \rightarrow 0$, the ZB component (i.e., the sinusoidal term) in Eq. (18) is $\langle z(t) \rangle_{\text{ZB}}$

$= -\frac{1}{2}s\chi_0 \sin \alpha(k_0) \sin \omega_0 t$, the amplitude of which is of the order of $2 \mu\text{m}$ —much larger than the resolution limit of a charge coupled device (CCD) camera.

Finally, we give a comment on the necessity of a non-Abelian vector gauge potential in atomic ZB. As mentioned above, previous papers on atomic ZB [15,16] used 2D laser configurations and non-Abelian vector potentials. In both of these models, the ZB vanishes in the limit of an Abelian vector potential [35]. Here, however, we show that in a purely 1D configuration, one can still achieve a nonconserved velocity operator if the scalar potential does not commute with the vector potential, thus producing ZB in an Abelian vector potential. One characteristic of our scheme is that the three Gaussian beams in the 1D configuration are coaxial and have the same waist radius; the amplitude ratios between these laser beams and the vector potential felt by atoms are hence constant. This makes our model suitable for an ex-

tremely broad initial wave packet (in real space), which is believed to be favorable for the detection of ZB [16].

In conclusion, we have presented a scheme to realize ZB in cold atoms with an Abelian vector potential. This vector potential can be produced in a 1D laser configuration using simple Gaussian beams. For a real wave packet of cold atoms generated in a VSCPT experiment, we estimate the amplitude of ZB in the position that the center of mass is comparable to the wavelength of the laser beams. Experimental detection of such ZB should be feasible using current technologies.

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