Trade and Intellectual Property Rights Protection: 
By Whom and for What?*

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Abstract

In a North-South trade model with innovation and imitation, we investigate the effects of IPR protection and trade protection. Typically, we show that unlike Southern tariff, Northern tariff supplements IPR protection and is not a beggar-thy-neighbor policy. The optimal Northern tariff rate is higher than in conventional models without innovation. The global welfare rises as Northern tariff increases, but declines as Southern tariff increases. This calls for urgent trade liberalization in the South.

Keywords: innovation, imitation, IPRs, tariff, North, South.
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1. Introduction

Although innovation is conducive to economic growth, it has been well understood that markets do not provide appropriate incentives for innovation.\(^1\) Nonetheless, it is not difficult to find solutions to the under-provision of new products and new production processes. In practice, governments have adopted various mechanisms to encourage innovation, including patents, R&D subsidization and patent buy-out.\(^2\) Among those mechanisms, patents are used most widely and, along with copyrights and trademarks, are the major components of IPR (Intellectual Property Rights) protection. However, perhaps partly due to the concern of monopoly price distortion, even in developed countries (referred to here as the North) IPR protection is not strong enough from the social point of view, let alone in less developed countries (referred to here as the South).\(^3\) Thus, the world is crying for stronger IPR protection, and cooperative efforts have been put to the strengthening of IPR protection in many nations.\(^4\) While the world is making progress in this area, it is by no means an easy process. We are aware that some other mechanisms are potential supplements for weak IPR protection although they are not originally designed for such a purpose. Trade policy, such as tariffs, is one of those mechanisms.

Tariffs can either supplement or offset IPR protection. It is therefore important to have a close, careful re-examination of tariffs when IPRs are not fully protected. The present study is motivated by this desire. To achieve this end, we establish a North-South trade model with

\(^1\)Throughout this paper, the words innovation and invention are used interchangeably, and so are the words innovate and invent.

\(^2\)However, each of these mechanisms has its own shortcomings. Patents create monopolies and lead to social deadweight losses. Government subsidizing R&D is much better than the patent policy (Spence, 1984), but it cannot escape from the asymmetric information problem and always invites rent-seeking, leading to inefficient subsidization. Patent buy-out could be potentially superior to the other two, but its drawback has not been fully understood yet due to the lack of practice and theoretical analysis. Kremer (1997) is one of the recent studies on patent buy-out.

\(^3\)Kremer (1997) has a nice summary of the empirical literature on patents. Given the current patent system, social returns to innovation far exceed the private returns, suggesting that innovation is not encouraged sufficiently.

\(^4\)In many recent international agreements, including the Uruguay Round, European Union and the North American Free Trade Agreement, signatories are required to strengthen their national IPR protection over the next decade. See Maskus (1998) for some discussions on this.
innovation and imitation, in which both regions have various degrees of IPR protection, captured by patent length, and trade protection, captured by tariffs. There are also other regional differences such as different inventive abilities. It is fair to assume that innovation originates from the North and imitation takes place in the South. Conventional wisdom tells us that in this type of models, strengthening IPR protection in both regions or raising the Northern tariff will lead to more innovations and less imitations, while increasing the Southern tariff will result in more imitations and less innovations. However, we show that this is not always the case. For example, under certain very plausible circumstances, more imitations are found when the IPR protection becomes stronger or the Northern tariff increases, and less imitations can be associated with a higher Southern tariff.

While the above findings are interesting, our focus is on the welfare effects of tariffs in the presence of innovation and imitation. Basically, we have obtained three results in this regard. First, there is a new rationale for Northern tariff. It has been well understood in the literature of international trade and policy that a country may find it optimal to impose a tariff if the terms-of-trade effect and the rent-shifting effect are sufficiently large.\(^5\) In the present model we find that even if these two positive effects are not large enough to justify the imposition of tariffs, the North may still find it optimal to impose a tariff because of a third positive welfare effect – the Northern tariff provides incentives for innovation and thus benefits consumers. Also because of this effect, the optimal Northern tariff rate is always higher in the present model than in models without innovation. The tariff that is designed to capture this third effect is to protect IPR, not sale. Thus, the Northern tariff supplements IPR protection.

Second, while the Northern tariff is pro-innovation, the Southern tariff, in contrast, is anti-innovation. This differentiates the two tariffs in the following way: the Southern tariff is a beggar-thy-neighbor policy, but the Northern tariff may not be. Third, the global welfare declines as the South raises its tariff rate, but under most circumstances the global welfare rises as the North increases its tariff rate. Hence, if one is allowed to raise tariff, it should be the North, not the South.\(^6\)

\(^5\)There are some other economic justifications for tariffs, such as the infant industry argument and increasing returns to scale. Of course, one can also find the political economy’s arguments for tariffs.

\(^6\)However, if tariffs are designed for the purpose of terms-of-trade improvement or profit-shifting, the Northern tariff and the Southern tariff are not qualitatively different. They are all beggar-thy-neighbor policies and always reduce global welfare.
The above results are striking. Note in the real world, the South maintains much higher trade protection in general and tariffs in particular than the North. This situation is completely upside-down because our results suggest that the opposite would lead to higher global welfare. The strong message that this study is trying to convey is that it is more harmful to keep high tariff in the South than in the North and hence trade liberalization is more urgent and should be carried out at a faster pace in the South than in the North.

Our results are derived from a partial-equilibrium model and so some of them may not hold in a general equilibrium setting. In particular, raising Northern tariff does not always lead to higher global welfare. However, we think the qualitative differences between the Southern and Northern tariffs shown in the present study are intact and so their implications for trade liberalization is more general than the model.

The present paper combines innovation, trade and policies, with an emphasis on welfare analysis. There exists a rich literature of technology and trade, which mainly focuses on the interplay between technology and international trade. More recently, there are also studies on the effects of IPR protection and trade policies. Taylor (1993), Maskus and Penubarti (1995) and Smith (1999) investigate whether strengthening IPR protection induce more trade flows. Horowitz and Lai (1996) and Lai (1998) analyze the effects of IPR protection on the rates of innovation. Grossman and Helpman (1991, ch.6 and ch.10) examine the response of innovation rates to trade and industrial policies. However, it is notable that studies on the welfare effects of trade policies or IPR protection in the content of North-South trade are few. Chin and Grossman (1990), Diwan and Rodrik (1991), Deardorff (1992), Helpman (1993) and Lai and Qiu (1999) all examine how strengthening IPR protection in the South affects welfare in the North, or the South, or both regions. As Grossman and Helpman (1995, p.1327) point out, we still do not have a complete normative analysis of trade policies, especially for a large, open, innovating economy. By addressing some of the issues relevant to this untouched area, the present study

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7In models with IPR and North-South trade, Helpman (1993) is a general-equilibrium one, but many others are partial-equilibrium ones such as Chin and Grossman (1990), Diwan and Rodrik (1991) and Deardorff (1992).

8Raising Northern tariff may not even increase Northern welfare if other benefits associated with free trade, which include more efficient resource allocation, increasing competition and economies of scale, are also considered. We choose not to incorporate these free-trade benefits into the present model so that we can stress the positive pro-innovation effect of Northern tariff.

9Grossman and Helpman (1995) has a comprehensive survey of this literature.
makes a contribution to this growing literature.

The rest of the paper is organized as follows. In Section 2, we construct a North-South trade model with innovation and imitation, and examine the policy effects on equilibrium innovation and imitation; section 3 analyzes Northern tariff; section 4 compares Northern tariff with Southern tariff; and finally, section 5 concludes.

2. The Model

Consider a world comprised of two regions, the North (representing developed countries) and the South (representing less developed countries). Initially the two regions differ only in their abilities to innovate new products and the costs of imitating the newly invented products.\(^\text{10}\) For analytical simplicity, we confine our study to an extreme case, which is not unrealistic, where innovation only takes place in the North and imitation in the South.\(^\text{11}\) There is a set of potential products to be invented, indexed by \(i\) within the range \([0, +\infty)\). Any new product will become obsolete after \(T\) periods since its invention. After that, it is treated as a traditional product. The IPR policy in the North is characterized by patent protection. A patent length \(T_n\), \(T_n \leq T\), means that the Northern government prevents a patented product from being imitated and sold in the North within \(T_n\) periods since its invention. At time zero, let \(M_n\) be the set of new products that have been just invented in the North.

Although the Northern government’s IPR policy cannot be extended to the South, a new product will not be imitated in the South before period \(T_s\), due to a natural delay of imitation or the IPR protection by the Southern government. For whatever reason, it is reasonable to assume \(T_s < T_n\).

The two regions trade with each other. The Northern government imposes a uniform specific

\(^{10}\)In the literature, it is common that one study focuses only on one type of innovation that either generates new products, or improves existing products’ quality, or lowers production costs.

\(^{11}\)The qualitative aspects of our results so derived remain robust even if we allow both regions to innovate and imitate. We have included a discussion on this in the concluding section. In fact, this is a common assumption in the literature, for example, Krugman (1979), Grossman and Helpman (1991, ch.11), and Helpman (1993). Grossman and Helpman (1995, p.1327) also provide several reasons for this, “First, firms in the South have shown only limited ability to develop innovative products of their own. Second, several of the governments of less developed nations have been somewhat lax in their enforcement of foreign intellectual property rights. Finally, the low wage rates of the South make it an especially attractive place for copying some kinds of products, because successful imitators can expect to earn substantial profits in their competition against innovators who bear higher labor costs”.

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tariff $\tau_n$ on all products imported from the South, and the Southern government imposes a uniform specific tariff $\tau_s$ on all products imported from the North.\footnote{A country’s welfare will be higher if the government levies different tariff rates on different products. However, this requires the government to possess sufficient information about firms’ characteristics, which is rare. Thus, we implicitly assume that information asymmetry rules out the case for differential tariff rates.}

Consumers in the two regions have identical utility functions. In every period $(t)$, consumers in each region $j$ (with $j = n$ for the North and $s$ for the South) derive utility from consuming the new products and a composite of traditional products:\footnote{Specific utility functions are used in many other studies including Krugman (1979), Grossman and Helpman (1991, ch.11) and Helpman (1993). The main results obtained in the present study should be still valid using a general utility function.}

$$u_j(t) = \int_{M_n} x_j(i)\alpha di + X_j, \quad 0 < \alpha < 1, \quad j = n, s,$$

where $x_j(i)$ is the quantity of product $i$, $i \in M_n$, and $X_j$ is the quantity of traditional goods consumed by region $j$. To simplify the notation, we define $\epsilon = 1/(1 - \alpha)$, $A = (1 - \alpha)\alpha^{(1+\alpha)\epsilon}$ and $B = \alpha^2\epsilon(1 + \alpha)^{\alpha\epsilon}$.

While the price of the traditional goods is normalized to one in both regions, the price of new product $i$ in region $j$ is denoted by $p_j(i)$. Since there is no lending and borrowing, in each period, consumers in region $j$ maximize the period’s utility under the budget constraint: $E_j \geq \int_{M_n} p_j(i)x_j(i)di + X_j$. Hence, the derived demands for new products are

$$p_j(i) = \alpha x_j(i)^{\alpha - 1}, \quad i \in M_n, \quad j = n, s,$$

which have constant elasticity equal to $\epsilon$. Note that the specific form of the utility function considered above implies that we confine our analysis to independent products. Allowing substitution among the new products will greatly complicate the analysis without altering the results qualitatively.

2.1. The Market

Let us consider the Northern market first. Suppose product $i$ has been invented by a Northern firm, called Northern firm $i$. The firm will be guaranteed a monopoly position in the Northern market for $T_n$ periods and in the Southern market for $T_s$ periods. To sharpen our focus on innovation, we assume that the innovation costs of different products vary (see next subsection),
but their production costs are the same. Specifically, and for simplicity, assume identical and constant marginal cost of production, which is equal to \(c_n\), for all firms in the North. Then, under the IPR protection in the North, i.e., for \(t \leq T_n\), Northern firm \(i\)'s operational profit (i.e., profit not taking into account the innovation costs) in the Northern market is \(\pi_{nn}(i) = [p_n(i) - c_n]x_n(i)\), where subscript \(nn\) stands for a Northern firm in the Northern market. In equilibrium,

\[
p_n(i) = \alpha^{-1}c_n, \quad x_n(i) = \alpha^2c_n^{-\epsilon}, \quad \text{and} \quad \pi_{nn}(i) = Ac_n^{-\alpha\epsilon} \quad \text{for} \quad t \leq T_n.
\]

After \(T_n\), all patents expire. If product \(i\) has not been imitated by the South, Northern firm \(i\) remains a monopolist in the market. In this case the market equilibrium is just the same as (1). However, the product may have already been imitated by the South.\(^{14}\) In that case, the Northern firm in the Northern market will face import competition by the Southern imitator in all periods after \(T_n\). Given that there are many new products available for imitation, we assume that if a product is imitated, it will be imitated by only one Southern firm.\(^{15}\) Corresponding to \(c_n\) in the North, the identical and constant marginal cost of production in the South is denoted by \(c_s\). Then, Southern firm \(i\), which imitates product \(i\), has the following export profit:

\[
\pi_{sn}(i) = [p_n(i) - c_s - \tau_n]x_{sn}(i), \quad \text{where} \quad x_{sn}(i) \quad \text{is the firm’s export volume (subscript \(sn\) stands for a Southern firm in the Northern market).}
\]

Northern firm \(i\) produces \(x_{nn}(i)\) to this market and has the operational profit equal to \(\pi_{nn}(i) = [p_n(i) - c_n]x_{nn}(i)\). Hence, the total supply of product \(i\) in the Northern market is \(x_n(i) = x_{nn}(i) + x_{sn}(i)\). Suppose firms in the same market compete in quantities à la Cournot. To ensure that we have the most interesting case where both the Northern and Southern firms in the same market produce positive amounts, we make the following assumption, which is necessary and sufficient,

**Assumption 1:** \(\alpha(c_s + \tau_n) < c_n < \alpha^{-1}(c_s + \tau_n)\).

The resulting equilibrium in the Northern market for product \(i\) is, for \(t > T_n\),

\[
p_n(i) = \frac{c_n + c_s + \tau_n}{1 + \alpha}, \quad x_n(i) = \left[\frac{\alpha(1 + \alpha)}{c_n + c_s + \tau_n}\right]^\epsilon, \quad \pi_{nn}(i) = B\left[\frac{(c_n - \alpha(c_s + \tau_n))^2}{(c_n + c_s + \tau_n)^{1+\epsilon}}\right], \quad \pi_{sn}(i) = B\left[\frac{(c_n - \alpha(c_s + \tau_n))^2}{(c_n + c_s + \tau_n)^{1+\epsilon}}\right].
\]

\(^{14}\)It will be clear from the next subsection that if a product is ever imitated by the South, it will be done right after \(T_n\).

\(^{15}\)This will be the case if imitation cost is not low and competition reduces market profit drastically. Moreover, in this model, if we explicitly allow imitators to choose products for imitation, they will choose to imitate different products. Limited resources also disallow a single imitator to imitate many products. All these tend to support the above assumption.
Clearly, the Northern tariff raises $\pi_{nn}(i)$ and reduces $\pi_{sn}(i)$, shifting profit from the Southern firm to the Northern firm.

We now examine the Southern markets. For $t \leq T_s$, Northern firm $i$, as a monopolist, exports its product to the South. Similar to (1), we have the following equilibrium (subscript $ns$ stands for a Northern firm in the Southern market),

$$
p_s(i) = \alpha^{-1}(c_n + \tau_s), \quad x_s(i) = \alpha^2(c_n + \tau_s)^{-\epsilon} \quad \text{and} \quad \pi_{ns}(i) = A(c_n + \tau_s)^{-\alpha\epsilon} \quad \text{for} \quad t \leq T_s. \quad (3)
$$

For $t > T_s$, if its product has not been imitated, Northern firm $i$ continues to supply the Southern market as a monopolist and (3) describes the equilibrium outcomes. If, however, its product has been imitated, it competes against the local imitating firm. Under Assumption 2 below, which is necessary and sufficient for positive supply by both the Northern and Southern firms, we obtain the equilibrium in (4).

**Assumption 2:** $\alpha(c_n + \tau_s) < c_s < \alpha^{-1}(c_n + \tau_s)$.

For $t > T_s$, let $\pi_{ns}(i)$ (respectively $\pi_{ss}(i)$) denote Northern (respectively Southern) firm $i$’s operational profit derived from the Southern market,

$$
p_s(i) = \frac{c_n + c_s + \tau_s}{1 + \alpha}, \quad x_s(i) = \left[\frac{\alpha(1 + \alpha)}{c_n + c_s + \tau_s}\right]^\epsilon, \quad \pi_{ns}(i) = B \left[\frac{c_s - \alpha(c_n + \tau_s)}{(c_n + c_s + \tau_s)^{1+\epsilon}}\right]^2, \quad \pi_{ss}(i) = B \left[\frac{(c_n + \tau_s - \alpha c_s)^2}{(c_n + c_s + \tau_s)^{1+\epsilon}}\right]. \quad (4)
$$

The Southern tariff raises $\pi_{ss}(i)$ and reduces $\pi_{ns}(i)$, shifting profit from the Northern firm to the Southern firm.

In summary, we have derived the product market equilibria in the North for two distinct periods (i.e., before $T_n$ and after $T_n$), respectively, and those in the South before and after $T_s$, respectively. In particular, we have calculated every firm’s operational profit derived from each market in each time period.

### 2.2. Innovation and Imitation

We now turn to examining innovation and imitation. To avoid complication, we assume away two important features often associated with innovation and perhaps imitation as well, i.e., uncertainty and knowledge spillover, which are not critical for our results. Northern firm $i$
invests in R&D to invent product \( i \) if and only if the stream of operational profits over the entire \( T \) periods is sufficiently large to cover the innovation costs, and so does Southern firm \( i \) for its imitation decision. For convenience, we order the products in a way such that a product with a higher index \( i \) has a higher innovation cost. In this way, we can view that \( i \) represents the degree of product sophistication. Accordingly, we adopt the simplest possible innovation cost structure for product \( i \): \( b_0 + bi \) where \( b_0 \geq 0 \) and \( b > 0 \). It is also assumed that imitation costs are higher for more sophisticated products (i.e., higher \( i \)) and for simplicity they are defined as \( e_0 + ei \) where \( e_0 \geq 0 \) and \( e > 0 \). Now it is obvious that if Northern firm \( i \) ever invests in innovation, it does so at \( t \) equal to zero and if Southern firm \( i \) ever imitates, it does so right after \( T_s \). Assume zero discount on future profits for convenience.

It proves advantageous to consider imitation first. Suppose all products \([0, +\infty)\) are available for imitation. If Southern firm \( i \) imitates product \( i \), it sells the product to the Southern market after \( T_s \), earning operational profit \( \pi_{ss}(i) \) in each period as given in (4). It also sells to the Northern market after \( T_n \), earning operational profit \( \pi_{sn}(i) \) in each period as given in (2). Thus, Southern firm \( i \)'s net profit (taking into account imitation costs) is

\[
\Pi_s(i) = \int_0^T \pi_{ss}(i) dt + \int_0^T \pi_{sn}(i) dt - e_0 - ei = \pi_s - e_0 - ei,
\]

where

\[
\pi_s = B(T - T_n) \frac{(c_n + \tau_n - \alpha c_s)^2}{(c_n + c_s + \tau_n)^{1+\epsilon}} + B(T - T_n) \frac{[c_n - \alpha(c_s + \tau_n)]^2}{(c_n + c_s + \tau_n)^{1+\epsilon}}.
\]

(5)

Since the operational profits are the same for all imitators, but the imitation costs increase in \( i \), it is manifest that there exists a unique \( m_s \) such that all products with \( i \leq m_s \) are imitated and none of the products with \( i > m_s \) will be imitated. The cut-off firm \( m_s \) is the one having zero net profit, \( \Pi_s(m_s) = 0 \), or

\[
m_s = \frac{\pi_s - e_0}{e}. \tag{6}
\]

We now consider innovation. Recall that under IPR protection, Northern firm \( i \), if it invests in innovation, earns monopoly profit \( \pi_{nn}(i) \) as given by (1) from its home market in every period of \( t \leq T_n \), and monopoly profit \( \pi_{ns}(i) \) as given by (3) from export to the South in every period of \( t \leq T_s \). It receives the same amount of operational profit in each market in every period after the IPR protection expires if its product has not been imitated. If, however, its product is imitated by Southern firm \( i \), its operational profit in the Northern market in each period is
$\pi_{nn}(i)$ as given by (2) and that in the Southern market is $\pi_{ns}(i)$ as given by (4). By definition, Northern firm $i$'s net profit (taking into account innovation costs) is

$$\Pi_n(i) = \int_0^T [\pi_{nn}(i) + \pi_{ns}(i)] dt - b_0 - bi = \pi_n - b_0 - bi,$$

where the sum of operational profits $\pi_n$ is equal to

$$\pi_1 = AT[c_n^{-\alpha} + (c_n + \tau_n)^{-\alpha}] \text{ for } i > m_s; \quad (7)$$

otherwise (i.e., $i \leq m_s$), $\pi_n$ is equal to

$$\pi_0 = AT_n c_n^{-\alpha} + AT_s (c_n + \tau_n)^{-\alpha} + B(T - T_n) \left[\frac{(c_n + \tau_n - \alpha c_n)^2}{(c_n + c_s + \tau_n)^{1+\varepsilon}} + B(T - T_s) \frac{(c_n - \alpha(c_n + \tau_n))^2}{(c_n + c_s + \tau_s)^{1+\varepsilon}}\right].$$

Corresponding to $m_s$ for imitation, let us define $m_n$ and $m_o$ that satisfy $\pi_1 - b_0 - bm_n = 0$ and $\pi_0 - b_0 - bm_o = 0$, respectively. Then,

$$m_n = \frac{\pi_1 - b_0}{b} \quad \text{and} \quad m_o = \frac{\pi_0 - b_0}{b}. \quad (8)$$

Obviously, $m_n > m_o$.

We are now ready to construct the innovation set. Referring to Figures 1, 2, or 3, let us depict the three profit lines, $\pi_s$, $\pi_0$ and $\pi_1$, along with the imitation cost line $IM = e_0 + ei$ and the innovation cost line $IN = b_0 + bi$ in a diagram with the horizontal axis indexed by $i$. Regarding the positions of the three profit lines, we know for sure that $\pi_1$ is above $\pi_0$, but $\pi_s$ could be anywhere. However, we only depict the case where $\pi_s$ is below $\pi_0$ as we can easily check that the relative position of $\pi_s$ to the others does not matter for our analysis. Recalling that $M_n$ stands for the innovation set, we let $M_s$ be the imitation set. In general, there are three possible combinations of the innovation and imitation sets, depending on the relative magnitudes of the three cut-off points, $m_s$, $m_n$ and $m_o$.

First, $m_s \leq m_o < m_n$. Referring to Figure 1, all Northern firms with $i \leq m_n$ have non-negative net profits ($\Pi_n(i)$) and so the innovation set $M_n$ is equal to $[0, m_n]$, while the imitation set $M_s$ is equal to $[0, m_s]$. We call this Case 1, or the partial imitation case.

Second, $m_o < m_s < m_n$. It is interesting to note that, with reference to Figure 2, all Northern firms $i \leq m_n$ except $i \in (m_o, m_s]$ have non-negative net profits ($\Pi_n(i)$) and so make their innovation investments, but Northern firms in the range of $(m_o, m_s]$ find their operational profits $\pi_0$ not large enough to cover their innovation costs. As a result, $M_n = [0, m_o] \cup (m_s, m_n]$
and \( M_s = [0, m_o] \). We refer to this as Case 2, or the *missing innovation case*. This is the most interesting case and is the focus of our subsequent study.

Lastly, \( m_n \leq m_s \). As in Figure 3, it is clear that \( M_n = M_s = [0, m_o] \). We call this Case 3, or the *full imitation case*.

Let us briefly examine how innovation and imitation costs jointly determine which of the above three configurations as the equilibrium outcome. Given that innovation costs are not too low, suppose the imitation costs are very small, e.g., both \( e_0 \) and \( e \) are close to zero. Then, \( m_s \) is likely to be greater than \( m_n \) and we will have Case 3. As \( e \) increases, \( m_s \) decreases. We will firstly observe Case 2 and later observe Case 1.

\(<\text{Figures 1, 2 and 3 about here}>\)

### 2.3. Policy Effects on Innovation and Imitation

Given policy parameters, the preceding subsection has characterized the possible equilibrium outcomes about innovation and imitation. In this subsection, we explore how various policies affect the innovation and imitation sets.

First, we examine the effects of IPR protection and trade policy on profits. Direct differentiation gives:

\[
\begin{align*}
\frac{\partial \pi_1}{\partial \tau_s} &< 0, \quad \frac{\partial \pi_1}{\partial T_n} = \frac{\partial \pi_1}{\partial T_s} = \frac{\partial \pi_1}{\partial \tau_n} = 0, \\
\frac{\partial \pi_0}{\partial \tau_s} &< 0, \quad \frac{\partial \pi_0}{\partial T_n} > 0, \quad \frac{\partial \pi_0}{\partial T_s} > 0, \quad \frac{\partial \pi_0}{\partial \tau_n} > 0, \\
\frac{\partial \pi_s}{\partial \tau_s} &> 0, \quad \frac{\partial \pi_s}{\partial T_n} < 0, \quad \frac{\partial \pi_s}{\partial T_s} < 0, \quad \frac{\partial \pi_s}{\partial \tau_n} < 0. 
\end{align*}
\]

(9)

While most of the above inequalities are easily established, we derive the least obvious one below.

\[
\frac{\partial \pi_o}{\partial \tau_n} = B(T - T_n)[(1 + \alpha + \epsilon)c_n - \alpha e(c_s + \tau_n)] \frac{c_s + \tau_n - \alpha c_n}{(c_n + c_s + \tau_n)^{2+\epsilon}}.
\]

(10)

Using \( c_n > \alpha(c_s + \tau_n) \) from Assumption 1 to replace \( c_n \) within the bracket \([\ ]\) in the above equation, we have

\[
\frac{\partial \pi_o}{\partial \tau_n} > B(T - T_n)\alpha(1 + \alpha)(c_s + \tau_n) \frac{c_s + \tau_n - \alpha c_n}{(c_n + c_s + \tau_n)^{2+\epsilon}} > 0.
\]

Figure 4 visually indicates the above policy effects. Based on Figure 4, Lemma 1 below is straightforward.
Lemma 1: The IPR protection has the following effects

\[
\frac{\partial m_n}{\partial T_n} = 0, \quad \frac{\partial m_o}{\partial T_n} > 0, \quad \frac{\partial m_s}{\partial T_n} < 0, \quad \text{and} \quad \frac{\partial m_n}{\partial T_s} = 0, \quad \frac{\partial m_o}{\partial T_s} > 0, \quad \frac{\partial m_s}{\partial T_s} < 0.
\]

The trade policy effects are

\[
\frac{\partial m_n}{\partial \tau_n} = 0, \quad \frac{\partial m_o}{\partial \tau_n} > 0, \quad \frac{\partial m_s}{\partial \tau_n} < 0, \quad \text{and} \quad \frac{\partial m_n}{\partial \tau_s} < 0, \quad \frac{\partial m_o}{\partial \tau_s} < 0, \quad \frac{\partial m_s}{\partial \tau_s} > 0.
\]

At the end of the previous subsection, we have outlined the dependence of each equilibrium case on various combinations of innovation and imitation costs. With Lemma 1, we can discuss the equilibrium outcome based on various combinations of policies. Let us begin with an equilibrium in Case 3. Now hold \( \tau_s \) fixed but let any or all of \( T_n, T_s, \) and \( \tau_n \) increase. Consequently, \( m_o \) increases and \( m_s \) decreases. Firstly, we will see that as \( m_s \) continues to decrease, it becomes smaller than \( m_n \). Case 2 is in place. As \( T_n, T_s \) and \( \tau_n \) continue to increase, \( m_s \) drops further and \( m_o \) increases further. Eventually, \( m_s \) becomes smaller than \( m_o \) and we will have Case 1 in equilibrium.

While Lemma 1 gives the policy implications for the cut-off points, it does not provide direct results of policy effects on the innovation set and imitation set. We report these results in Propositions 1 and Corollary 1 below. It is worth emphasizing that strengthening IPR protections does not always enlarge the innovation set and reduce the imitation set. Moreover, the trade policy effects on innovation and imitation sometimes may also go against what one should normally expect.

Proposition 1: (i) If \( T_n, T_s, \) or \( \tau_n \) increases, then in Case 1, \( M_s \) shrinks but \( M_n \) remains unchanged. In Case 2 and Case 3, both \( M_n \) and \( M_s \) expand.

(ii) If \( \tau_s \) increases, then in all three cases, \( M_s \) expands in Case 1, but shrinks in Case 2 and Case 3.

Proof: Let us only consider small policy changes so that the equilibrium outcome will not switch from one case to another. In addition, let us not consider \( m_o = m_s \), a very special
situation in Case 1. With this qualification, the proof is straightforward based on Lemma 1 and Figures 1 to 4. \(\square\)

While the policy effects on \(M_n\) are clear, those on \(M_s\) are not straightforward. Proposition 1 warns that careful scrutiny is needed before making precise claims about the impacts of these policies on the innovation and imitation sets. Generally speaking, stronger IPR protection in both regions (i.e., higher \(T_n\) and \(T_s\)) and stronger trade protection by the North would lead to higher profits for the Northern firms, but lower profits for the Southern firms, and hence encourage innovation and discourage imitation. Stronger trade protection by the South has just the opposite effects. According to Proposition 1, the effects on \(M_n\) are consistent with the general belief, except with a minor qualification that in Case 1, \(M_n\) does not respond to changes in \(T_n, T_s\) and \(\tau_n\). The key to the understanding of this non-responsiveness case is that there is no imitation threat to the existing Northern firms at or close to the cut-off point \(m_n\) and so they enjoy monopoly positions in all markets in all periods. As a result, changes in \(T_n, T_s\) and \(\tau_n\) do no harm or benefit them (note from Lemma 1 that \(m_n\) is only affected by \(\tau_s\)).

On the contrary, the imitation set may be reduced or enlarged by the policy changes, depending upon which equilibrium case we have. Let us focus our discussion on the seemingly counter intuitive scenarios. First, stronger IPR protection and/or Northern trade protection results in more imitations in Case 2 and Case 3 (Proposition 1(i)). In Case 3, the innovation set is so small (due to high innovation costs) that all products are imitated by the South (due to low imitation costs). Moreover, the imitation costs are so low that many Southern firms with \(i\) greater than \(m_o\) are eager to imitate but the corresponding products simply have not yet been invented by the North. As the policies change to stimulate more innovations in the North, more products are available for imitation. Although the same policy changes lower Southern firms profits, some still find positive profits from imitating the newly available products. Consequently, the imitation set is expanded. In Case 2, the pro-innovation policy changes make some products in the otherwise missing-innovation range \((m_o, m_s]\) (see Figure 2) available in the market. As in Case 3, this creates more opportunities for imitation and we will see the Southern firms with \(i\) slightly greater than \(m_o\) start to imitate, leading to a larger imitation set.

Second, stronger Southern trade protection reduces imitations in Case 2 and Case 3. The intuition is similar to that given above. A higher tariff imposed by the Southern government
lowers innovation incentives. Indirectly, imitation opportunities are reduced, which in turn make
some Southern firms find their corresponding products not available for imitation.

If we are interested in knowing the policy effects on the percentage of products being imitated,
we can derive a corollary based on Proposition 1. The results are unambiguous except for Case
2.

Corollary 1: (i). In Case 1, an increase in $T_n$, $T_s$, or $\tau_n$, or a decrease in $\tau_s$ would lower
imitation ratio $M_s/M_n$.

(ii). In Case 2, the policy effects on imitation ratio $M_s/M_n$ are ambiguous.

(iii). In Case 3, imitation ratio $M_s/M_n$ is not affected by any policy change.

3. Northern Protection: For Sale or Innovation?

From now on we switch our focus and turn to welfare analysis. In this section, we characterize
the optimal Northern tariff and its relation to the degree of IPR protection. In particular, we
argue that Northern tariff protects not just sale, but also innovation. In the next section, we
will compare and contrast Northern tariff and Southern tariff. In both sections, Our primary
purpose is to derive results that otherwise are not obtained in conventional trade models, i.e.,
models without product innovation and imitation. To achieve this end as clearly as possible, we
focus on Case 2 and Case 3 since the policy effects on innovation and imitation are more drastic
in these two cases than in Case 1. In what follows, we provide detailed analyses for Case 2 but
only brief discussions for Case 3.

Suppose we are in Case 2 and to avoid unnecessary complication, we make the assumption
that we remain in Case 2 for all $\tau_n \geq 0$, which satisfies Assumption 1.

Based on the previous analysis, we have, for $t \leq T_n$,

$$u_n(t) = \int_0^{m_o} x_n(i)^\alpha di + \int_{m_s}^{m_o} x_n(i)^\alpha di + X_n = (m_n - m_s + m_o)(1 - \alpha) \left( \frac{\alpha^2}{c_n} \right)^{\alpha e} + E_n,$$

and for $t > T_n$,

$$u_n(t) = m_o(1 - \alpha) \left[ \frac{\alpha(1 + \alpha)}{c_n + c_s + \tau_n} \right]^{\alpha e} + (m_n - m_s)(1 - \alpha) \left( \frac{\alpha^2}{c_n} \right)^{\alpha e} + E_n.$$  

We use $V_n$ to denote the total import to the North, which is

$$V_n = \int_{T_n}^{T} \int_0^{m_o} x_{sn}(i) di \ dt = (T - T_n)Bm_o(1 + \alpha) \frac{c_n - \alpha(c_s + \tau_n)}{(c_n + c_s + \tau_n)^{1+e}}.$$
To reduce notation, let \( \tau = (\tau_n, \tau_s, T_n, T_s) \) be the set of policy instruments and \( m = (m_o, m_n, m_s) \) the collection of the cut-off points. Northern welfare, which is denoted by \( W_n(\tau) \) or \( w_n(\tau, m) \), is defined as the weighted sum of consumer utility, producer profits and tariff revenue. Let \( \mu_0 \in [0, 1] \) be the weight assigned to profits and \( \mu_1 \in [0, 1] \) to tariff revenue. Then,

\[
W_n(\tau) = w_n(\tau, m) = \int_0^{T_n} u_n(t)dt + \int_{T_n}^T u_n(t)dt + \mu_0 \int_0^{m_o} \Pi_o(i)di + \mu_0 \int_{m_o}^{m_s} \Pi_o(i)di + \mu_1 \tau_n V_n,
\]

\[
= [T(m_n - m_s) + T_n m_o](1 - \alpha) \left( \frac{\alpha^2}{c_n} \right)^{\alpha} + (T - T_n) m_o (1 - \alpha) \left( \frac{\alpha (1 + \alpha)}{c_n + c_s + \tau_n} \right)^{\alpha}\n
+ T E_n + \mu_0 \left[ m_o (\pi_0 - b_0) - \frac{b}{2} m_o^2 + (m_n - m_s)(\pi_1 - b_0) - \frac{b}{2} (m_n^2 - m_s^2) \right] + \mu_1 \tau_n V_n. \quad (13)
\]

The Northern government’s objective is to maximize Northern welfare by choosing a non-negative \( \tau_n \) subject to Assumption 1. Recall from Lemma 1, \( \partial m_n / \partial \tau_n = 0 \). Thus, assuming that the optimal tariff rate is an interior solution, it must satisfy the following first-order condition:

\[
\frac{\partial W_n}{\partial \tau_n} = \frac{\partial w_n}{\partial \tau_n} + \frac{\partial w_n}{\partial m_o} \frac{\partial m_o}{\partial \tau_n} + \frac{\partial w_n}{\partial m_s} \frac{\partial m_s}{\partial \tau_n} = 0. \quad (14)
\]

We examine each welfare term of (14) in turn. First,

\[
\frac{\partial w_n}{\partial \tau_n} = - \frac{m_o B(T - T_n)(1 - \alpha)}{(c_n + c_s + \tau_n)^\alpha} + \mu_0 m_o \frac{\partial \pi_o}{\partial \tau_n} + \mu_1 \left( V_n + \frac{\tau_n}{\partial \tau_n} \right). \quad (15)
\]

Equation (15) is the welfare effect of tariff that we usually see in models with imperfect competition but without innovation and imitation. Northern tariff reduces consumer surplus, increases firms’ profits, and generates government revenue. If import subsidy is not allowed, the optimal tariff could be zero or positive, depending upon whether the terms of trade are improved and how much profit is shifted from the foreign exporters to the local firms.\(^{16}\) For ease of discussion, let \( \tau_n \) be the optimal tariff rate when changes in innovation and imitation are ignored. Then, \( \tau_n = 0 \) if \( \partial w_n / \partial \tau_n \leq 0 \) and \( \tau_n > 0 \) otherwise.

We now turn to the second term of (14), i.e., the welfare effect of the resulted changes in \( m_o \). Using the zero profit condition at \( i = m_o \) (i.e., \( \Pi_o(m_o) = 0 \)), we obtain

\[
\frac{\partial w_n}{\partial m_o} \frac{\partial m_o}{\partial \tau_n} = \alpha^\alpha (1 - \alpha) \left[ T_n \left( \frac{\alpha}{c_n} \right)^\alpha + (T - T_n) \left( \frac{1 + \alpha}{c_n + c_s + \tau_n} \right)^\alpha \right] \frac{\partial m_o}{\partial \tau_n} + \mu_1 \tau_n \frac{V_n}{m_o} \frac{\partial m_o}{\partial \tau_n} > 0. \quad (16)
\]

\(^{16}\)Helpman and Krugman (1989, Chapter 6) have a good analysis on this issue.
Note, $\partial m_o/\partial \tau_n > 0$ and as a result, the innovation set expands (Proposition 1). Thus, the first term of (16) is positive because consumers benefit from the increase of product varieties. Moreover, Northern tariff also expands the imitation set (Proposition 1 again), and hence the North imports more varieties of imitated products. As a result, tariff revenue increases, or mathematically the second term of (16) is positive.

Finally, the welfare effect of changing $m_s$, the last term of (14), is:

$$\frac{\partial w_n}{\partial m_s} \frac{\partial m_s}{\partial \tau_n} = \left[ T(1 - \alpha) \left( \frac{\alpha^2}{c_n} \right)^{\alpha} + \mu_0(\pi_1 - b_0 - bm_s) \right] \frac{\partial m_s}{\partial \tau_n} > 0. \quad (17)$$

Raising tariff $\tau_n$ lowers $m_s$ ($\partial m_s/\partial \tau_n < 0$) and so expands the innovation set (Proposition 1). Consequently, consumers benefit from more product varieties, and there are more Northern firms earning positive profits. These together raise Northern welfare.

We now combine all the effects discussed above for the first-order condition (14). Let $\tau_n^*$ be the optimal tariff rate that satisfies (14). Then, $\tau_n^* = 0$ if $\partial W_n/\partial \tau_n \leq 0$ and $\tau_n^* > 0$ otherwise. From the above analysis, we know $\partial W_n/\partial \tau_n > \partial w_n/\partial \tau_n$, and hence, $\tau_n^* \geq \tau_n$. More specifically, whenever $\tau_n > 0$, we must have $\tau_n^* > 0$ and $\tau_n^* > \tau_n$; and in some cases, $\tau_n = 0$, but $\tau_n^* > 0$.

The above analysis and result for Case 2 also apply to Case 3. As the analysis for Case 3 is so similar to that for Case 2, we just bring out a few points that need attention. To get the utility and welfare for Case 3, we set $m_s = m_n$ in (11), (12), and (13). For the first-order condition, set $\partial w_n/\partial m_s = 0$ in (14) and so the last term vanishes. The individual welfare effects (15) and (16) remain the same for Case 3.

We summarize the above results in Proposition 2 below.

**Proposition 2**: There is a new rationale for Northern tariff. Suppose the equilibrium is in Case 2 or Case 3. Then, it is more likely that a Northern tariff will increase the Northern welfare over free trade in the present model than in other trade models without innovation and imitation. Whenever it is optimal to impose tariff, the optimal tariff rate in the presence of innovation and imitation is strictly higher than that in the absence of innovation and imitation.

What exactly is the new rationale for Northern tariff? Note, when $T_n = T$, $\partial m_o/\partial \tau_n = \partial m_s/\partial \tau_n = 0$. In this case, there is no import from the South and the tariff plays no role.
Moreover, this is the only case that Proposition 2 does not apply to. It suggests that whenever
the IPR protection is not perfect (i.e., $T_n < T$), there is room for trade policy to supplement
the IPR protection. We demonstrate this point below. To isolate the new motive, we assume
$\mu_0 = \mu_1 = 0$ in the welfare function. This enables us to focus on the case where tariff is not for
profit shifting or terms-of-trade improving. We investigate how the optimal tariff rate depends
on the degree of IPR protection. Assuming the second-order condition holds, i.e., $\partial^2 W_n/\partial T_n^2 < 0$.
By totally differentiating the first-order condition (14), we obtain
\[
\frac{\partial \tau_n}{\partial T_n} = - \frac{\partial}{\partial T_n} \left( \frac{\partial W_n}{\partial \tau_n} \right) / \left( \frac{\partial^2 W_n}{\partial \tau_n^2} \right),
\]
which implies $\text{sgn} \left( \frac{\partial \tau_n}{\partial T_n} \right) = \text{sgn} \left[ \frac{\partial}{\partial T_n} \left( \frac{\partial W_n}{\partial \tau_n} \right) \right]$.
This sign is negative and the similar relationship holds for $\tau_n$ and $T_s$ (see Appendix). Therefore,
we have the following proposition:

**Proposition 3**: Suppose the equilibrium is in Case 2 or Case 3. The Northern tariff, which
maximizes Northern welfare with $\mu_0 = \mu_1 = 0$, is higher if the IPR protection in the North or
the IPR protection in the South is weaker. Mathematically, $\partial \tau_n/\partial T_n < 0$ and $\partial \tau_n/\partial T_s < 0$.

**PROOF**: See Appendix. □

To understand that the Northern tariff is used to supplement weak IPR protection, let us
first compare the similar welfare effect resulted from an increase in Northern IPR protection and
that from a higher Northern tariff. Note, with $\mu_0 = \mu_1 = 0$, the welfare is simply the consumer
surplus, which decreases if the prices are higher but increases when there are more product
varieties (i.e., greater $m_o$). Since we cannot find any (costless) policy that will lower the prices
and at the same time stimulate innovation, policies that maximize welfare should be combined
to balance the following two sides: prices are not too high and innovations are not too few. First,
based on (13), we easily observe the two conflicting effects of increasing $T_n$. On the one hand,
consumer surplus is reduced because all goods are charged at their monopoly prices for a longer
period of time. On the other hand, $m_o$ is larger and hence consumer surplus increases due to
more product varieties. Second, those two conflicting welfare effects are also present when $\tau_n$
increases: A rise in the tariff results in higher prices paid by consumers for all products during
the import periods and thus lowers consumer surplus; however, greater profits for the Northern
innovators are assured by a higher tariff and therefore there are more innovations (i.e., $m_o$ is
larger), giving rise to greater consumer surplus. Clearly, the Northern tariff plays a similar role as the Northern IPR protection and so the former can be used to supplement the latter when the Northern government has more flexibilities to adjust its tariff rate than the patent length. For example, if the IPR protection is too weak ($T_n$ too small), meaning that $m_o$ is not big enough from the social point of view, we should raise the tariff to stimulate innovation. If, however, the IPR protection is already very strong ($T_n$ too big), welfare can be increased by depressing the prices, through lowering the tariff rate.

We now turn to the substitutability of $\tau_n$ for $T_s$. Unlike $T_n$, $T_s$ has a single effect on the Northern welfare through its influence on product variety. $T_s$ raises $m_o$. Thus, as the Southern IPR protection becomes weaker, the North can raise its tariff rate to at least partially compensate the Northern innovators so that their innovations will not decrease too much. On the other hand, when the Southern IPR protection becomes stronger, the North worries less about innovation incentives but more about high consumer prices and thus tariff rate should be lowered to dampen the prices.

Here is a summary of this section. We have demonstrated (in Proposition 2) that there is a pro-innovation element in the optimal Northern tariff. This is a new justification (motive or rationale) for the Northern tariff. This tariff is to protect innovation, not sale. Moreover, in Proposition 3, we have shown that this tariff is higher (lower) if the IPR protection, in the North or South, becomes weaker (stronger). Hence, we have answered the question of “protection for what”. The issue of “protection by whom” will be dealt with in the next section.

4. Tariff Protection: By the North or the South?

The pro-innovation feature of the Northern tariff seems to imply that maybe the Northern tariff is desirable not only for the North but also for the world. What about the Southern tariff? If we allow tariff protection, which region should be allowed to use it, the North or the South? To answer these questions, we contrast the different welfare effects of the Northern tariff and the Southern tariff.\textsuperscript{17} As the discussion for Case 3 is analogous to that for Case 2, we confine our analysis to Case 2 below to avoid repetition.

\textsuperscript{17}Recall that we have known their different impacts on innovation and imitation that are summarized in Proposition 1.
First, the Southern tariff’s impact on Northern welfare. Taking derivative gives:

\[
\frac{\partial W_n}{\partial \tau_s} = \frac{\partial w_n}{\partial \tau_s} + \frac{\partial w_n}{\partial m_o} \frac{\partial m_o}{\partial \tau_s} + \frac{\partial w_n}{\partial m_s} \frac{\partial m_s}{\partial \tau_s} + \frac{\partial w_n}{\partial m_n} \frac{\partial m_n}{\partial \tau_s}.
\]

(18)

Note, we can sign each term in (18) based on Lemma 1, (16), (17),

\[
\frac{\partial w_n}{\partial \tau_s} = \mu_0 m_o \frac{\partial \tau_0}{\partial \tau_s} + \mu_0 (m_n - m_s) \frac{\partial \tau_1}{\partial \tau_s} < 0, \quad \text{and} \quad \frac{\partial w_n}{\partial m_n} = T (1 - \alpha) \left( \frac{\alpha^2}{c_n} \right)^{\alpha} > 0.
\]

The first term on the right-hand side of (18) is negative. This is the familiar profit-shifting result in the strategic trade literature and it has captured the total effect in the conventional trade model of imperfect competition without innovation and imitation. Thus, foreign tariff is detrimental to home welfare by reducing the home producers’ profits.

In the present model with innovation and imitation, the Southern tariff is more harmful to the North than in the conventional model. To see this, recall from Proposition 1 that an increase in \( \tau_s \) shrinks both the innovation set and imitation set because the Southern tariff reduces Northern firms’ profits. As a result, in the North, consumers have fewer product varieties, firms earn less profit and the government collects less revenues. The Northern welfare unambiguously decreases. Mathematically, the second, third and fourth terms on the right-hand side of (18) are all negative. Based on this analysis we immediately establish the following proposition:

**Proposition 4**: The Southern tariff is a beggar-thy-neighbor policy. Its adverse effect on the Northern welfare is more serious in the present model with innovation and imitation than in the conventional model without innovation and imitation.

We now turn to considering the impact of the Northern tariff on Southern welfare. We should derive the Southern welfare function first. Similar to (11) and (12), we obtain

\[
u_s(t) = (m_n - m_s + m_o)(1 - \alpha) \left( \frac{\alpha^2}{c_n + \tau_s} \right)^{\alpha} + E_s, \quad \text{for} \ t \leq T_s,
\]

and

\[
u_s(t) = m_o(1 - \alpha) \left[ \frac{\alpha(1 + \alpha)}{c_n + c_s + \tau_s} \right]^{\alpha} + (m_n - m_s)(1 - \alpha) \left( \frac{\alpha^2}{c_n + \tau_s} \right)^{\alpha} + E_s, \quad \text{for} \ t > T_s.
\]

The total import by the South is

\[
V_s = (T - T_s) B m_o (1 + \alpha) \frac{c_s - \alpha(c_n + \tau_s)}{(c_n + c_s + \tau_s)^{1+\epsilon}} + [T(m_n - m_s) + T \alpha m_o] \frac{\alpha^{2\epsilon}}{(c_n + \tau_s)^{\epsilon}}.
\]

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The Southern welfare function, which can be denoted by $W_s(\tau)$ or $w_s(\tau, m)$, is defined as the weighted sum of consumer surplus, producer profit and tariff revenue. As we are not trying to derive the optimal Southern tariff rate, we assume $\mu_0 = \mu_1 = 1$ for simple exposition. Hence,

$$W_s(\tau) = w_s(\tau, m)$$

$$= [T(m_n - m_s) + T_s m_o](1 - \alpha) \left( \frac{\alpha^2}{c_n + \tau_s} \right)^{\alpha} + (T - T_s)m_o(1 - \alpha) \left[ \frac{\alpha(1 + \alpha)}{c_n + c_s + \tau_s} \right]^\alpha$$

$$+ TE_s + m_o(\pi_s - e_0) - \frac{\epsilon}{2} m_o^2 + \tau_s V_s.$$  

Differentiating with respect to $\tau_n$, we obtain

$$\frac{\partial W_s}{\partial \tau_n} = \frac{\partial w_s}{\partial \tau_n} + \frac{\partial w_s}{\partial m_o} \frac{\partial m_o}{\partial \tau_n} + \frac{\partial w_s}{\partial m_s} \frac{\partial m_s}{\partial \tau_n}. \quad (19)$$

The first term on the right-hand side of (19), which is equal to $m_o \frac{\partial \pi_s}{\partial \tau_n}$, is negative, the result of profit shifting. Because of this, the Southern welfare is reduced. However, the second and third terms are both positive, which tend to raise the Southern welfare. To see this, recall $\frac{\partial m_o}{\partial \tau_n} > 0$ and $\frac{\partial m_s}{\partial \tau_n} < 0$ from Lemma 1, and note

$$\frac{\partial w_s}{\partial m_o} = \alpha^{\alpha}(1 - \alpha) \left[ T_s \left( \frac{\alpha}{c_n + \tau_s} \right)^{\alpha} + (T - T_s) \left( \frac{1 + \alpha}{c_n + c_s + \tau_s} \right)^{\alpha} \right]$$

$$+ \tau_s(T - T_s)B(1 + \alpha) \frac{c_s - \alpha(c_n + \tau_s)}{(c_n + c_s + \tau_s)^{1+\epsilon}} > 0, \quad (20)$$

$$\frac{\partial w_s}{\partial m_s} = -T(1 - \alpha) \left( \frac{\alpha^2}{c_n + \tau_s} \right)^{\alpha} - \tau_s(T - T_s) \left( \frac{\alpha^2}{c_n + \tau_s} \right)^{\epsilon} < 0.$$  

According to Proposition 1, the Northern tariff expands both the innovation set and imitation set. As a result, in the South, consumers enjoy more product varieties, more imitators enter and earn positive profits, and the government collects more tariff revenues. All these lead to greater welfare for the South.

It is manifest from the above analysis that because the Northern tariff is pro-innovation, its adverse effect on the Southern welfare, if any, is smaller in the present model with innovation and imitation than in the conventional model without innovation and imitation. Depending on the relative degrees of the positive and negative effects, the Northern tariff may even raise the Southern welfare, in which case it is no longer a beggar-thy-neighbor policy.
It is not only a possibility that the South can benefit from Northern tariff. We can actually show that it is really true for some values of $\tau_n$. As $W_s$ is a multivariable function including $\tau_n$, whether $\partial W_s / \partial \tau_n > 0$ or not depends on various combinations of many parameters and thus finding the necessary and sufficient conditions is intractable. Nevertheless, our purpose is to show that under some meaningful conditions Northern tariff is beneficial to the South. Proposition 5 below provides some results on this.

**Proposition 5 :** Suppose the equilibrium is in Case 2 or Case 3.

(i). Given all other parameter values, $\partial W_s / \partial \tau_n > 0$ if $b_0$ is sufficiently large (i.e., there exists $b_0$ such that the inequality holds for all $b_0 > b_0$).

(ii). Given $(\alpha, c_s, T_s, T)$, $\partial W_s / \partial \tau_n > 0$ at $\tau_n = 0$ if $c_n$ is small relative to $c_s$, or if $T_n$ is not big relative to $T$ (i.e., there exists $c$ such that the inequality holds for all $c_n/c_s < c$, or there exits $R$ such that the inequality holds for all $T_n/T < R$).

PROOF: See Appendix. □

It is clear why we need those conditions. When $b_0$ is large or $T_n$ is small, there are not enough innovations in the North. Under these circumstances, the South hopes that the North innovate more, in whatever way, even at the South’s expenses. Raising the Northern tariff is one of the policies, although by no means the best one, which will encourage innovation. In some cases, the expense of increasing the Northern tariff on the South could be too large. But this will not occur if $c_n$ is very small relative to $c_s$, because the profits shifted by the Northern tariff from the Southern firms to the Northern firms are quite limited since the Northern firms are very efficient relative to the Southern firms.

Tariffs imposed by the North and the South have very different consequences on the other regions. How are they different in effecting the global welfare? A related question is that if we allow just one region to impose tariff, be it the North or the South. The above analysis seems to indicate the superiority of the Northern tariff over the Southern tariff. In the rest of this section, we seek a direct answer to these questions.

Global welfare is the sum of the Northern welfare and the Southern welfare and so it is defined as

$$W(\tau) = W_n(\tau) + W_s(\tau), \quad \text{or} \quad w(\tau, m) = w_n(\tau, m) + w_s(\tau, m).$$
Differentiating with respect to the Southern tariff rate, we obtain
\[
\frac{\partial W(\tau)}{\partial \tau_s} = \frac{\partial w}{\partial \tau_s} + \frac{\partial w}{\partial m_o} \frac{\partial m_o}{\partial \tau_s} + \frac{\partial w}{\partial m_s} \frac{\partial m_s}{\partial \tau_s} + \frac{\partial w}{\partial m_n} \frac{\partial m_n}{\partial \tau_s} < 0.
\]

The first term on the right-hand side is negative. This is the result from conventional trade models without innovation and imitation that any region’s tariff lowers global welfare even if it may raise the policy-adopting region’s welfare. All other terms are also negative because the Southern tariff reduces the Northern firms’ innovation incentives and so are detrimental to both regions. Hence, the Southern tariff unambiguously reduces global welfare, and it reduces the global welfare more in the presence of innovation and imitation than in the absence of innovation and imitation.

Differentiating the global welfare with respect to the Northern tariff rate, we obtain
\[
\frac{\partial W(\tau)}{\partial \tau_n} = \frac{\partial w}{\partial \tau_n} + \frac{\partial w}{\partial m_o} \frac{\partial m_o}{\partial \tau_n} + \frac{\partial w}{\partial m_s} \frac{\partial m_s}{\partial \tau_n}.
\]

The first term on the right-hand side is negative. However, both the second and third terms are positive, because Northern protection increases product varieties and thus both regions benefit. Therefore, innovation reduces the detrimental effect of the Northern tariff on global welfare. It is even possible that the global welfare is improved from a rise in the Northern tariff if the positive innovation effect is sufficiently strong.

Although the above analysis seems to suggest the dominance of the Northern tariff over Southern tariff in the sense that \(\partial W(\tau)/\partial \tau_n > \partial W(\tau)/\partial \tau_s\) for all \(\tau_n = \tau_s\), we are unable to prove this analytically. While this strong result cannot be generally proved in the present study, it is easy to find examples in which \(\partial W(\tau)/\partial \tau_n > 0\) for all \(\tau_n\) that does not violate Assumption 1. In these cases, the differences between the Northern and Southern tariffs in their global welfare effects are put in sharp contrast. Figure 5 is one of these examples where the global welfare rises continuously as the Northern tariff increases, but declines as the Southern tariff increases.

Figure 5 depicts the global welfare as a function of the Northern tariff when the Southern tariff is set at zero, and the global welfare as a function of the Southern tariff when the Northern tariff is set at zero, respectively. The two curves are drawn based on the following specific parameter values: \(\alpha = 0.5, T = 30, T_n = 20, T_s = 5, b_0 = c_0 = 0, b = 2, e = 1, c_n = c_s = 1\). Moreover, without loss of generality, we set \(E_n = E_s = 0\) for simplicity. Given these values, Assumptions 1 and 2 limit the tariffs, \(\tau_n\) and \(\tau_s\), within the range of \([0, 1]\).
While $W(\tau)$ may not always monotonically increase in $\tau_n$ under some other parameter specifications, we cannot find any case whereby the inequality, $\partial W(\tau)/\partial \tau_n > \partial W(\tau)/\partial \tau_s$ for all $\tau_n = \tau_s$, is violated. Therefore, we conclude that from the world’s point of view, we should allow tariff protection by the North, not the South, if we have to choose between them.

5. Concluding Remarks

In a North-South trade model with innovation and imitation, we have shown how trade policies and IPR policies in the two regions affect innovation and imitation. Since innovation originates from the North and imitation occurs in the South, we normally expect that strengthening IPR protection in both regions and raising the Northern tariff will result in more innovation and less imitations, while raising the Southern tariff will lead to more imitations and less innovations. But that is not always the case. For example, it is possible that more imitations are associated with stronger IPR protection and higher Northern tariffs, and less imitations result from an increase of the Southern tariff.

The central message from this study comes from its answers to the two questions: what are the Northern tariff protecting and should the North or the South be allowed to raise tariffs? We have argued that unlike in conventional trade models without innovation and imitation, the Northern tariff protects not only sale but also innovation and thus supplements weak IPR protection. Because of the consideration of innovation and imitation, the Northern tariff and the Southern tariff are qualitatively different. The former may not be a beggar-thy-neighbor policy but the latter is. The global welfare declines as the South raises its tariff, but under most circumstances it rises as the North raises its tariff. Hence, we should allow the North, not the South, raise tariff.

Robustness of the results. As any other economic model, this model is built upon a number of assumptions. Among these, those deserving attention are the specific utility function, the independent products, and the places for innovation and imitation. As it is not difficult to see that the qualitative results of the paper will not be altered if we use a general utility function and assume differentiated products, let us focus our discussion on the implications of allowing imitation to take place in the North and innovation in the South. Obviously, if the two regions
are completely identical, the Northern tariff and the Southern tariff will have the same effects on innovation, imitation and global welfare. The minimum assumptions that we need to maintain for a North-South model are: (a) IPR protection is much stronger in the North than in the South, (b) the ability to innovate is much higher in the North than in the South, and (c) (maybe because of cheaper labors) the cost of imitation is lower in the South than in the North. Consider Southern innovation first. The IPR difference (a) alone would have resulted in much more innovations in the North than in the South even if both regions have equal abilities to innovate. Their difference in innovative abilities further widens the gap. Therefore, we would have the situation that the North does most of the innovation and the South does only a little bit. Then, the Northern tariff and the Southern tariff would still be very different, albeit in a lesser degree than in the present model, in their effects on innovation and global welfare.

Now coming to Northern imitation. First, although the Northern tariff and the Southern tariff affect imitation differently in the present model, that difference is not the major factor that distinguishes them in terms of global welfare effects. What really matters here is their different impacts on innovation. Hence, the assumption that only the South has imitation is not crucial. Second, even though we allow imitation to take place in the North, Northern imitations would be much less than Southern imitations for two reasons. On the one hand, IPR protection is much stronger in the North than in the South as stated in (a) above, and so it is less profitable for a Northern imitator to copy a product than for a Southern imitator to do so even if the cost of imitation is the same for both. On the other hand, the lower imitation cost in the South, as stated in (c) above, guarantees that more imitations come from this region. Therefore, because most imitations concentrate in the South, the policy effects obtained in the present model will not be altered in any significant way.

We should reiterate once more that in this paper we are not trying to argue for raising trade protection anywhere. It is quite the opposite. The central message we want to convey is that trade protection by the South is more detrimental than by the North.
Appendix

A. Proof of Proposition 3.

We prove the result for Case 2 first.

Define \( \tilde{m}_o = m_o / B(T - T_n) \) and \( \tilde{m}_s = m_s / B(T - T_n) \). Then, from (5), (6), (10) and (8),

\[
\frac{\partial \tilde{m}_o}{\partial T_n} = \frac{1}{B(T - T_n)} \frac{\partial m_o}{\partial T_n} = \frac{1}{b} \frac{\partial}{\partial T_n} \left[ \frac{(c_s + \tau_n - \alpha c_n)^2}{(c_n + c_s + \tau_n)^{1+\varepsilon}} \right],
\]

\[
\frac{\partial \tilde{m}_s}{\partial T_n} = \frac{1}{B(T - T_n)} \frac{\partial m_s}{\partial T_n} = \frac{1}{e} \frac{\partial}{\partial T_n} \left[ \frac{(c_n - \alpha (c_s + \tau_n))^2}{(c_n + c_s + \tau_n)^{1+\varepsilon}} \right].
\]

We divide both sides of the first order condition (14) (noting that \( \mu_0 = \mu_1 = 0 \)) by the common factor \( B(T - T_n) \), and define \( \Delta \) as

\[
\Delta = \frac{1}{B(T - T_n)} \frac{\partial W_n}{\partial T_n} = -\frac{(1 - \alpha)m_o}{(c_n + c_s + \tau_n)^{1+\varepsilon}} + \frac{\partial w_n}{\partial m_o} \frac{\partial \tilde{m}_o}{\partial T_n} + \frac{\partial w_n}{\partial m_s} \frac{\partial \tilde{m}_s}{\partial T_n}.
\]

Then,

\[
\frac{\partial}{\partial T_n} \left( \frac{\partial W_n}{\partial T_n} \right) = -B \Delta + B(T - T_n) \frac{\partial \Delta}{\partial T_n}.
\]

First, at the optimal tariff rate, \( \Delta = 0 \) by the first order condition (14). Second, because neither \( \frac{\partial \tilde{m}_o}{\partial T_n} \) nor \( \frac{\partial \tilde{m}_s}{\partial T_n} \) is a function of \( T_n \),

\[
\frac{\partial \Delta}{\partial T_n} = -\frac{1 - \alpha}{(c_n + c_s + \tau_n)^{1+\varepsilon}} \frac{\partial m_o}{\partial T_n} + \frac{\partial^2 w_n}{\partial T_n \partial m_o} \frac{\partial \tilde{m}_o}{\partial T_n} + \frac{\partial^2 w_n}{\partial T_n \partial m_s} \frac{\partial \tilde{m}_s}{\partial T_n} = 0.
\]

Following (8), (9), (16) and (17) (noting \( \mu_0 = \mu_1 = 0 \)), we have

\[
\frac{\partial m_o}{\partial T_n} > 0, \quad \frac{\partial^2 w_n}{\partial T_n \partial m_o} = \alpha \alpha (1 - \alpha) \left[ \left( \frac{\alpha}{c_n} \right)^{\alpha} - \left( \frac{1 + \alpha}{c_n + c_s + \tau_n} \right)^{\alpha} \right] < 0, \quad \text{and} \quad \frac{\partial^2 w_n}{\partial T_n \partial m_s} = 0.
\]

Therefore, \( \partial \Delta / \partial T_n < 0 \). The inequality \( \partial \tau_n / \partial T_n < 0 \) follows immediately.

We now turn to prove \( \partial \tau_n / \partial T_s < 0 \). First, the following relationship holds,

\[
\text{sgn} \left( \frac{\partial \tau_n}{\partial T_s} \right) = \text{sgn} \left[ \frac{\partial}{\partial T_s} \left( \frac{\partial W_n}{\partial T_n} \right) \right].
\]

Second, based on (5), (6), (10), (8), (16) and (17), we know that none of the following terms is a function of \( T_s \),

\[
\frac{\partial w_n}{\partial m_o}, \quad \frac{\partial w_n}{\partial m_s}, \quad \frac{\partial m_o}{\partial \tau_n}, \quad \frac{\partial m_s}{\partial \tau_n}.
\]

Thus, from (14) and (15), we have

\[
\frac{\partial}{\partial T_s} \left( \frac{\partial W_n}{\partial T_n} \right) = -B(T - T_n)(1 - \alpha) \frac{\partial m_o}{(c_n + c_s + \tau_n)^{1+\varepsilon}} < 0,
\]

because \( \partial m_o / \partial T_s > 0 \) according to (8) and (9). The result follows.
Finally, for Case 3, we simply need to repeat the above proof by dropping the terms with $m_s$. The result then follows easily. ∎

**B. Proof of Proposition 5.**

Since the last term of (19) is positive in Case 2 and zero in Case 3, a sufficient condition for $\partial W_s/\partial \tau_n > 0$ in Case 3 will be also a sufficient condition in Case 2. Thus, we focus on Case 3.

First, from (5), we have

$$\frac{\partial \tau_n}{\partial \tau_n} = B(T - T_n)[c_n - \alpha(c_s + \tau_n)](1 + 2\alpha + \varepsilon)c_n - \alpha^2\varepsilon(c_s + \tau_n)}{(c_n + c_s + \tau_n)^{2+\varepsilon}}.$$

From (10) and (8), we have $\partial m_o/\partial \tau_n$. Substituting these into (19) and then multiplying both sides by $b(c_n + c_s + \tau_n)^{2+\varepsilon}/B(T - T_n)$, we have

$$\frac{b(c_n + c_s + \tau_n)^{2+\varepsilon} \partial W_s}{B(T - T_n) \partial \tau_n} > \Phi(\tau_n), \quad \text{and so } \frac{\partial W_s}{\partial \tau_n} > 0 \text{ if } \Phi(\tau_n) > 0,$$

where

$$\Phi(\tau_n) = \frac{\partial W_s}{\partial m_o}(c_s + \tau_n - \alpha c_n)[(1 + \alpha + \varepsilon)c_n - \alpha\varepsilon(c_s + \tau_n)]$$

$$- (\pi_0 - b_0)[c_n - \alpha(c_s + \tau_n)][(1 + 2\alpha + \varepsilon)c_n - \alpha^2\varepsilon(c_s + \tau_n)],$$

where $\partial w_s/\partial m_o$ is given in (20). Clearly, given all other parameters satisfying Assumptions 1 and 2, if $b_0$ is sufficient large (close to $\pi_0$), then $\Phi(\tau_n)$ will be positive.

We now find other sufficient conditions when $b_0$ is small. Define $\Phi_1(\tau_n)$ as equal to $\Phi(\tau_n)$ except $b_0 = 0$ and dropping the last term of $\partial w_s/\partial m_o$, which is positive. Then we have $\Phi(\tau_n) > \Phi_1(\tau_n)$ for all $\tau_n$.

Using Assumption 2, we have $\alpha c_s - c_n < \tau_s < c_s/\alpha - c_n$. Then,

$$\frac{\alpha}{c_n + \tau_s} > \frac{\alpha^2}{c_s}, \quad \frac{1 + \alpha}{c_n + c_s + \tau_s} > \frac{\alpha}{c_s},$$

and

$$\frac{AT_s}{(c_n + \tau_s)^{\alpha^2}} < \frac{AT_s}{(ac_s)^{\alpha^2}} \frac{(c_s - \alpha c_n - \alpha \tau_n)^2}{(c_n + c_s + \tau_s)^{1+\varepsilon}} < (1 - \alpha)^2(1 + \alpha)^{-\alpha\varepsilon}c_s^{-\alpha\varepsilon}.$$  

Furthermore, define parameter $k$ as $k = c_n/c_s$. Then the constraint on $k$ implied by Assumptions 1 and 2 is $\alpha < k < 1/\alpha$.

Substituting all the above into $\Phi_1(0)$, we have

$$\frac{1}{c_s^{\alpha\varepsilon}}\Phi_1(0) > \Phi_2(k, T_n),$$

where

$$\Phi_2(k, T_n) \equiv \alpha^{2\alpha}(1 - \alpha)(1 - \alpha k)[T - (1 - \alpha^\alpha)T_s][(1 + \alpha + \varepsilon)k - \alpha]\frac{AT_s}{k^{\alpha^2}} + \frac{AT_s}{\alpha^{\alpha^2}} + B(T - T_n)\frac{(1 - \alpha k)^2}{(1 + k)^{1+\varepsilon}} + B(T - T_s)(1 - \alpha)^2(1 + \alpha)^{-\alpha\varepsilon}.$$  

Under the constraint for $k$, $\Phi_2(k, T_n)$ decreases in both $k$ and $T_n$. Obviously, for small $k$ that is close to $\alpha$, $\Phi_2(k, T_n) > 0$. There also exits $T_n$ such that $\Phi_2(k, T_n) > 0$. For example, if $(\alpha, T, T_s, k) = (0.5, 30, 5, 0.75)$, then $\Phi_2(k, T_n) = 3.49 - 0.63(5.25 + 0.03T_n) > 0$, if and only if $T_n < 9.5$. ∎
References


