Persistent currents in cold atoms

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We propose a scheme to realize persistent mass and spin currents in cold atoms using optically generated gauge potentials. Two degenerate dark states can be created by coupling “tripod” atoms to three Laguerre-Gaussian laser beams carrying relative orbital angular momentum. Atoms in these dark states feel opposite effective magnetic fields of the flux-tube type and circulate in opposite directions around the axis of the tube, generating persistent mass currents in the system. Persistent spin currents can also be generated, with or without spin polarization.

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Persistent charge currents in mesoscopic rings or cylinders threaded by a magnetic flux have attracted appreciable interest for nearly half a century. In these mesoscopic systems, if the phase coherence length of electrons is larger than the size of the system, an equilibrium current, which is a periodic function of the threaded magnetic flux, can flow without dissipation as long as the flux is kept on [1]. Early work in the 1960s, dealing with flux quantization in superconductors, suggested such currents can exist in flux quantized states of a superconducting ring or cylinder [2,3]. Later, based on the fact that elastic scattering (due to some static potential) does not destroy the coherence of the wave function even over the whole circumference of the ring, such currents were proposed and detected in normal metal rings [4–9]. In recent years, such currents were also observed in semiconductor systems, such as GaAs/GaAlAs self-assembled rings [10–12].

Spin-orbit coupling also produces interesting effects in these mesoscopic systems. Numerous studies show that persistent spin currents, accompanied by a net charge current or not, can be associated with a Berry phase [13–15]. Recently, in a ferromagnetic Heisenberg ring, a persistent spin-polarized charge current carried by bosonic excitations was predicted as a direct analog of the persistent charge current in normal metal rings [14]. Very recently, it was shown that Rashba spin-orbit coupling in a mesoscopic hybrid ring can induce a pure persistent spin current without net charge current [15].

The past decade has also witnessed impressive experimental and theoretical achievements in ultracold atoms [16,17]. One driving force behind these studies is the possibility to adopt quantum degenerate gases as playgrounds to simulate different physical systems [18]. Recently, theoretical studies [19–22] showed that effective magnetic fields (EMFs) acting on the center of mass of neutral atoms—analogueous to physical magnetic fields acting on charge carriers in solid-state systems—can be generated by coupling the internal states of cold atoms to laser beams carrying relative orbital angular momentum (OAM). On the experimental side, a vector gauge potential was successfully created for ⁸⁷Rb atoms [23]. This opens a way to study in cold atoms

numerous remarkable quantum phenomena associated with magnetic fields in solid-state physics, such as the Aharonov-Bohm effect, geometric phases, and persistent current. Very recently, Ryu *et al.* found that a persistent mass current, analogous to the persistent flows in superfluid helium and superconducting rings, can be induced to flow in a toroidal trap by directly transferring OAM from optical modes to cold atoms [24]. The realization of persistent flow facilitates our understanding of some longstanding important issues in several fields of physics, such as the critical velocity in liquid helium and the relationship between superfluidity and Bose-Einstein condensation [18].

In this paper, we propose a scheme to create persistent mass and spin currents in cold atoms using optically generated EMFs. We show that an analog of a mesoscopic ring system [1] can be realized in ultracold atoms. In this system, the atom-laser interaction creates two dark states in which atoms feel opposite EMFs of the “flux-tube” type. Atoms in these states are therefore driven to circulate in opposite directions about the laser axis, giving rise to persistent mass currents. Since the internal atomic states carry spin, persistent spin currents are also generated. By varying the laser intensity, we can produce spin currents with or without mass current and spin polarization, directly analogueous to the spin-polarized or nonpolarized persistent charge currents in semiconductor systems.

To establish that this mesoscopic analogy is meaningful, we note that the phase coherence length of an interacting Bose gas in two dimensions (2D) is $L_\phi \sim r_m \exp(T_{2D}/2T)$ [[16], p. 451], where r_m is a cutoff length for classical sound waves, T is the temperature, and $T_{2D} = 2\pi\hbar^2 n_{2D}/Mk_B$, in which $n_{2D} = N/A$ is the number of particles per unit area and M is the particle mass. Thus, in a finite system, the phase is well correlated throughout the system whenever $T \ll T_{2D}/\ln N$ [[16], p. 451], which ensures that L_ϕ is greater than the system size. This indicates that persistent currents are possible in such a finite 2D system whenever the temperature is sufficiently low.

Next, we introduce the atomic system and laser configuration, which are depicted in Fig. 1.

The four-level system can be chosen by optical coupling from the internal electronic states of, e.g., $5S_{1/2} \leftrightarrow 5P_{1/2}$ transitions in ⁸⁷Rb atoms. Three hyperfine states, $|1\rangle$, $|2\rangle$, and $|3\rangle$, are coupled to the excited state $|0\rangle$ by three circularly polar-

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FIG. 1. (Color online) Level scheme and laser configuration of a “tripod” system created in ^{87}Rb atoms.

ized Laguerre-Gaussian (LG) beams. The corresponding Rabi frequencies are denoted $\Omega_j(\vec{r}) \equiv \tilde{\Omega}_j(\rho) e^{il_j\varphi} e^{ik_jz}$, where $\vec{r}=(x,y,z)$ is the position of the center of mass, z is the coordinate along the laser axis, $\rho=\sqrt{x^2+y^2}$, $\tan \varphi=y/x$, l_j is an integer ($j=1,2,3$), and k_j is a wave vector. We assume these laser beams are tuned to achieve electromagnetically induced transparency [25].

For simplicity, we initially neglect interactions between atoms (such interactions are considered later). The total atomic Hamiltonian is $H=T+V+W$, where $T=p^2/2M$ is the center-of-mass kinetic energy, $V(\vec{r})=V_0|0\rangle\langle 0|+V_1|1\rangle\langle 1|+V_2|2\rangle\langle 2|+V_3|3\rangle\langle 3|$ is a trapping potential, and $W(\vec{r})$ is the atom-laser interaction. If $\{|\eta(\vec{r})\rangle\}$ is any orthonormal basis in the “internal” space spanned by $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$, we can write

$$H = \sum_{\eta'} \sum_{\eta} |\eta'\rangle H_{\eta'\eta} \langle \eta|, \quad (1)$$

where the center-of-mass operator $H_{\eta'\eta} \equiv \langle \eta' | H | \eta \rangle$ is

$$H_{\eta'\eta} = \frac{1}{2M} (\delta_{\eta'\eta} p^2 + \vec{p} \cdot \vec{A}_{\eta'\eta} + \vec{A}_{\eta'\eta} \cdot \vec{p}) + U_{\eta'\eta}. \quad (2)$$

Here $\vec{A}_{\eta'\eta}(\vec{r}) \equiv -i\hbar \langle \eta' | \vec{\nabla} | \eta \rangle$ is an effective vector potential acting on the center of mass, while

$$U_{\eta'\eta}(\vec{r}) = \langle \eta' | V | \eta \rangle + \langle \eta' | W | \eta \rangle + \frac{\hbar^2}{2M} \langle \vec{\nabla} \eta' | \vec{\nabla} \eta \rangle \quad (3)$$

is an effective scalar potential. The basis $\{|\eta(\vec{r})\rangle\}$ is usually chosen to diagonalize the atom-laser interaction W , so that $\langle \eta' | W | \eta \rangle = \delta_{\eta'\eta} W_{\eta}(\vec{r})$.

In the rotating-wave approximation, this interaction is [19] $W = -\hbar \sum_{i=1}^3 (\Omega_i |0\rangle\langle i| + \Omega_i^* |i\rangle\langle 0|)$. The eigenstates of W include two dark states [19] $|a\rangle = (\Omega_2 \Omega_3 |1\rangle - \Omega_1 \Omega_3 |2\rangle) / (\Theta_2 \tilde{\Omega}_3)$ and $|b\rangle = e^{i(k_1+k_2)z} (\Omega_1^* \Omega_3 |1\rangle + \Omega_2^* \Omega_3 |2\rangle - \Theta_2^2 |3\rangle) / (\Theta_2 \Theta_3)$, where $\Theta_j = (\sum_{i=1}^3 |\Omega_i|^2)^{1/2}$. The dark states have internal energy $W_a(\vec{r}) = W_b(\vec{r}) = 0$ and are well separated from the other two “bright” states. We are therefore justified in invoking an *adiabatic* approximation [19] in which all motion is confined to the dark subspace spanned by $\{|a\rangle, |b\rangle\}$.

We assume a tight confinement along the laser axis (z) to “freeze” the atomic motion in that direction—i.e., we

consider a quasi-2D system. In the x - y plane, the trap potential $V(\rho)$ has the following matrix elements in the dark subspace:

$$V_{aa} = V_1 \cos^2 \phi + V_2 \sin^2 \phi, \quad (4a)$$

$$V_{ab} = (V_1 - V_2) \cos \theta \cos \phi \sin \phi, \quad (4b)$$

$$V_{bb} = V_1 \cos^2 \theta \sin^2 \phi + V_2 \cos^2 \theta \cos^2 \phi + V_3 \sin^2 \theta, \quad (4c)$$

where $\tan \phi = \tilde{\Omega}_1(\rho) / \tilde{\Omega}_2(\rho)$ and $\tan \theta = \Theta_2(\rho) / \tilde{\Omega}_3(\rho)$. We choose a harmonic-oscillator potential $V_1 = V_2 \equiv \bar{V} = \frac{1}{2} M \omega^2 \rho^2$, but let $V_3 = \bar{V} + \delta$, where the constant shift δ can be generated by a slight detuning. In this case, $V_{ab} = 0$, and atoms in the two dark states feel different trap potentials, with the difference given by $\Delta V = V_{aa} - V_{bb} = \delta \sin^2 \theta$. The purpose of δ is to open a small gap between the dark states, as discussed further in the context of many-body effects below.

In general, both ϕ and θ are functions of ρ . Here we choose three LG doughnut beams (i.e., the radial quantum numbers are zero). The first two beams have the same waist radius ($\nu_1 = \nu_2$) but carry opposite OAM ($l_2 = -l_1 \equiv l \neq 0$), while the third one has a larger waist radius ($\nu_3 > \nu_1$) and carries zero OAM ($l_3 = 0$, i.e., a usual Gaussian beam). With these laser beams, ϕ is a constant and $\tan \theta = \alpha \rho^{|l|} e^{-\rho^2/2\nu^2}$, where α is a constant determined by the laser intensities and $\nu^{-2} = \nu_1^{-2} - \nu_3^{-2}$.

In the dark subspace, the nonvanishing in-plane components of the vector potential are

$$A^\varphi = \frac{\hbar}{\rho} \begin{pmatrix} \lambda & \gamma \cos \theta \\ \gamma \cos \theta & -\lambda \cos^2 \theta \end{pmatrix}, \quad (5)$$

in which $\lambda = l \cos 2\phi$ and $\gamma = l \sin 2\phi$. It is remarkable that atoms in $|a\rangle$ experience an *ideal* flux tube with zero radius—i.e., the magnetic flux through any circle enclosing the z axis is $h\lambda$, and there is no return flux [26] in the region $z \neq 0$. For atoms in $|b\rangle$, however, the flux through a circle centered on the z axis is $-h\lambda / [1 + \tan^2 \theta(\rho_c)]$, which depends on the radius ρ_c of the circle. In an experiment, the laser parameters can be chosen so that the condition $\tan^2 \theta \ll 1$ (i.e., $|\Omega_1|^2 + |\Omega_2|^2 \ll |\Omega_3|^2$) is satisfied for arbitrary ρ . Under this assumption, the approximation $\cos \theta \approx 1$ holds, and the EMF for atoms in $|b\rangle$ can be approximated by an ideal flux tube with a flux opposite to that for atoms in $|a\rangle$. We use this approximation for the rest of the paper.

Let us now study the center-of-mass motion in the gauge potentials. We initially use the approximation $\sin \theta \approx 0$ to neglect the difference between the trap potentials felt by atoms in the two dark states, and write $V_{bb} \approx V_{aa} = \bar{V}$. Substituting Eqs. (5) and (2), we find that the Hamiltonian describing the quasi-2D motion is

$$H_{aa} = T + \bar{V} + \frac{\hbar^2}{2M\rho^2} \left(l^2 - i2\lambda \frac{\partial}{\partial \varphi} \right), \quad (6a)$$

$$H_{bb} = T + \bar{V} + \frac{\hbar^2}{2M\rho^2} \left(l^2 + i2\lambda \frac{\partial}{\partial \varphi} \right), \quad (6b)$$

$$H_{ab} = \frac{\hbar^2}{2M\rho^2} \left(-i2\gamma \frac{\partial}{\partial \varphi} \right). \quad (6c)$$

Eigenenergies and eigenfunctions of this Hamiltonian can be obtained analytically. Here we limit our discussion to the special class of solutions for which the coupling (6c) vanishes, so that atoms in $|a\rangle$ and $|b\rangle$ are independent and feel opposite vector potentials $+\hbar\lambda/\rho$ and $-\hbar\lambda/\rho$, respectively. This occurs when the wave function is independent of φ . In this case, there are two eigenstates $\Psi^{(a)} \equiv \psi_{(0)}^{(1)}$ and $\Psi^{(b)} \equiv \psi_{(1)}^{(0)}$, where $\psi(\rho) = (\pi|l|!)^{-1/2} (M\omega/\hbar)^{(|l|+1)/2} \rho^{|l|} \times \exp(-M\omega\rho^2/2\hbar)$. If initially atoms are prepared in $\Psi^{(a)}$ or $\Psi^{(b)}$, they will move stably and independently in these states. Hereafter, we assume this initial condition.

Atoms in $\Psi^{(a)}$ and $\Psi^{(b)}$ carry persistent currents that circulate about the z axis. To show this, we start by defining the density operator $\varrho(\vec{r}, \vec{r}_0) = \delta(\vec{r} - \vec{r}_0)$, where $\vec{r}$ is the coordinate operator and $\vec{r}_0$ is a parameter. In the Heisenberg picture, ϱ satisfies the continuity equation $\partial\varrho/\partial t + \vec{\nabla}_{\vec{r}_0} \cdot \vec{J} = 0$, where $\vec{J}(\vec{r}, \vec{r}_0) = \frac{1}{2}(\varrho\vec{v} + \vec{v}\varrho) = \{\varrho, \vec{v}\}$ is the current-density operator and $\vec{v}(\vec{r}, \vec{p}) = [\vec{p} + \vec{A}(\vec{r})]/M$ is the velocity operator. The probability density and current density of a given state are obtained from the expectation values of these operators.

For the states $\Psi^{(a)}$ and $\Psi^{(b)}$, these expectation values are (relabeling $\vec{r}_0 \rightarrow \vec{r}$)

$$\varrho^{(a,b)}(\vec{r}) = |\psi|^2, \quad \vec{J}^{(a,b)}(\vec{r}) = \pm \hat{\phi} \frac{\hbar l}{M\rho} |\psi|^2 \cos 2\phi, \quad (7)$$

where the upper (lower) sign is for $\Psi^{(a)}$ ($\Psi^{(b)}$). The opposite flux-tube EMFs acting on $\Psi^{(a)}$ and $\Psi^{(b)}$ therefore generate currents that circulate in opposite directions about the z axis. The mass density and mass current of atoms in these states are obtained simply by multiplying Eqs. (7) by M . Note that the mass current vanishes when $|\phi| = \pi/4$ (i.e., when $|\Omega_1| = |\Omega_2|$).

Let us now consider the spin current in our system, where “spin” (denoted S_z) is defined as the total atomic angular momentum about the z axis (including electronic and nuclear angular momenta). In the Heisenberg picture, the spin-density operator $\varrho_s = \{\varrho, S_z\}$ satisfies the continuity equation

$$\frac{\partial \varrho_s}{\partial t} + \vec{\nabla}_{\vec{r}_0} \cdot \vec{J}_s = \varrho_\tau, \quad (8)$$

where $\vec{J}_s = \{\vec{J}, S_z\}$ is the spin current density, $\varrho_\tau = \{\varrho, \tau\}$ is the torque density, and $\tau = \frac{1}{i\hbar}[S_z, H]$ is the torque operator. If $[S_z, H] = 0$, spin density is conserved [i.e., the right-hand side of Eq. (8) is zero].

For the configuration in Fig. 1, atoms in $|1\rangle$ and $|2\rangle$ have the same spin polarization $\hbar$. When $\tan^2 \theta \ll 1$, the $|3\rangle$ component of $|b\rangle$ is negligible, and we can write the spin operator for the dark subspace as

$$S_z = \hbar \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}. \quad (9)$$

This commutes with the Hamiltonian (6), so spin density is conserved. For the states $\Psi^{(a)}$ and $\Psi^{(b)}$, the spin density and spin current are

FIG. 2. (Color online) Wave function $\psi(\rho)$ with or without particle-particle interaction (in the mean-field approximation), for 10^4 atoms. The unit of length is $\sqrt{\hbar/M\omega}$.

$$\varrho_s^{(a,b)}(\vec{r}) = \pm \hbar |\psi|^2 \cos 2\phi, \quad \vec{J}_s^{(a,b)}(\vec{r}) = \hat{\phi} \frac{\hbar^2 l}{M\rho} |\psi|^2. \quad (10)$$

We see that $\Psi^{(a)}$ and $\Psi^{(b)}$ have opposite spin polarizations, but because their mass currents are also opposite [see Eq. (7)], their spin currents circulate in the same direction. Note that the spin polarization and mass current both vanish when $|\phi| = \pi/4$, but the spin current is independent of ϕ . Thus, we can generate a “pure” persistent spin current that is unaccompanied by either spin polarization or mass current.

We now consider the extent to which the above conclusions are modified by the inclusion of particle-particle interactions, which are generally important in ultracold atomic systems [17]. We may write the many-body Hamiltonian [16] in the dark subspace as

$$H = \sum_{\alpha, \alpha', \beta, \beta'} \int d^2r \left(\frac{\hbar^2}{2M} \vec{\nabla} \hat{\psi}_\alpha^\dagger \cdot \vec{\nabla} \hat{\psi}_\alpha + V(\rho) \hat{\psi}_\alpha^\dagger \hat{\psi}_\alpha \right. \\ \left. + \frac{1}{2} W_0 \hat{\psi}_\alpha^\dagger \hat{\psi}_{\alpha'}^\dagger \hat{\psi}_{\alpha'} \hat{\psi}_\alpha + \frac{1}{2} W_2 \hat{\psi}_\alpha^\dagger \hat{\psi}_{\alpha'}^\dagger \vec{S}_{\alpha\beta} \cdot \vec{S}_{\alpha'\beta'} \hat{\psi}_{\beta'} \hat{\psi}_\beta \right), \quad (11)$$

where $\hat{\psi}_\eta^\dagger(\vec{r})$ and $\hat{\psi}_\eta(\vec{r})$ are creation and annihilation operators for an atom in dark state $|\eta\rangle$ ($\eta = a, b$). The last two terms are the interaction energy, where W_0 and W_2 are interaction strengths and $\vec{S}$ denotes atomic angular momentum.

The W_2 term is spin dependent and will generally tend to mix $|a\rangle$ and $|b\rangle$ to achieve a lower interaction energy. For a ^{87}Rb cloud of $\sim 10^4$ atoms, the expectation value of the W_2 term is of the order of $5 \times 10^{-3} \omega$ (evaluated using the wave function ψ in Fig. 2 and values given in Ref. [27]), which can be much smaller than the difference $\Delta V = \delta \sin^2 \theta$ between the trap potentials felt by atoms in the two dark states [if we take $\delta \approx 5\omega$ and $\sin^2 \theta \approx 0.01$, then $\Delta V \approx 5 \times 10^{-2} \omega$, which is much larger than the W_2 term while remaining negligible compared to the frequency ω of the trap potential in Eq. (6)]. Therefore, the mixing between the two dark states can be suppressed by ΔV . In other words, if initially all atoms are prepared in state $\Psi^{(a)}$ or $\Psi^{(b)}$, they will continue to move stably in the same state. This is the justification for our previous use of the single-particle picture to calculate persistent

currents. One consequence of the interaction is that the explicit form of the wave function $\psi(\rho)$ will be modified (this has been calculated numerically and is shown in Fig. 2). However, since the qualitative properties of the currents do not depend on the explicit form of ψ , our main results in Eqs. (7) and (10) remain valid.

Next, we discuss how to detect the persistent currents in our system. First, we note that charge-coupled device images of the interference fringes between a rotating atomic cloud and a nonrotating one (or a rotating one with known OAM) can determine the OAM carried by cold atoms, as shown in Refs. [28,29]. Here we propose a method of detecting the persistent currents: by measuring the frequency shift due to the angular Doppler effect [30]. This phenomenon is analogous to the well-known translational Doppler effect, and refers to the fact that atoms rotating in an optical field with angular momentum experience an azimuthal shift in their resonance frequency [30].

We note that the $|1\rangle$ ($|2\rangle$) component of $\Psi^{(a)}$ or $\Psi^{(b)}$ carries a phase factor $e^{i\ell\varphi}$ ($e^{-i\ell\varphi}$), which corresponds to circular motion with angular frequency $\omega_r = \hbar/M\rho^2$ ($-\hbar/M\rho^2$) varying with ρ . Therefore, at a typical radius $\rho \sim \sqrt{\hbar/M\omega}$, the shift in the resonant frequency due to the angular Doppler

effect will be $\Delta\omega = \omega_r l_p = l_p l \omega$ ($-l_p l \omega$) [30,31], where $l_p \hbar$ is the orbital angular momentum carried by probing beams (such as LG beams) used to excite atoms in $|1\rangle$ ($|2\rangle$).

In summary, we have shown that persistent mass and spin currents—remarkable phenomena usually associated with quantum liquids or solid-state systems—can be generated in cold atoms using optically created effective magnetic fields. The two dark states of the system experience opposite EMFs in the form of flux tubes along the laser axis, thus giving rise to mass currents that circulate in opposite directions about the laser axis. Our proposed system has the advantage of a higher degree of controllability than the analogous persistent currents previously achieved in mesoscopic rings or superconducting rings. In particular, the mass current is a continuous function of the laser beam amplitudes and can be set to zero (by choosing equal amplitudes) in a system with zero spin polarization but nonvanishing spin current.

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