Improved Assessment of the Spectrum Efficiency of SDMA Systems

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Abstract—In this paper we present an assessment of the spectral efficiency of Spatial Division Multiple Access systems. Our analysis attempts to incorporate the effects of co-channel interference (CCI) for users sharing the same channel within a cell (intra-cell interference) as well as the CCI from other cells (inter-cell interference). To determine the outage probability we utilize exact results for combined Rayleigh and Log-normal shadowing in conjunction with a generic antenna pattern. Results are provided for the outage probability and spectral efficiency for various traffic loads and reuse distances.

I. INTRODUCTION

The use of adaptive antenna technology to allow users to share the same channel within a cell has become known as Spatial Division Multiple Access (SDMA). The potential advantage of using SDMA is that spectrum efficiency and capacity of existing systems could be increased [1]. SDMA can also be thought of as an extension to smart antenna systems with the reuse factors reduced to less than unity.

In this paper we present an assessment of the outage probability and spectral efficiency of various configurations of SDMA systems. The significance of our assessment is that we incorporate the effects of co-channel interference (CCI) for users sharing the same channel within a cell. We call this intra-cell interference and it results from the antennas of the SDMA system exhibiting non-zero side lobe levels. In general we can expect the side lobe level of any real antenna to be no more than 15 dB below the main beam and therefore significant interference between beams in an SDMA system is likely. In addition to the intra-cell interference we also include the effects of CCI from the surrounding cells in the cellular system and we refer to this as inter-cell interference.

In our analysis we utilize exact results for combined Rayleigh and Log-normal shadowing. To model the effects of the side lobe levels of the antenna system a generic antenna pattern is introduced in which the beamwidth and sidelobes can be specified as parameters. We consider the forward link only and do not include the effects of power control.

In section II we introduce the necessary preliminaries for our analysis. In section III we derive expressions for the outage probability and spectral efficiency. Finally in sections IV and V we present results and also conclude our paper.

II. System Model

Outage probability can be defined as the probability of failing to achieve adequate reception of a signal [2]. Under interference limited conditions we can compute outage probability by utilizing the co-channel interference probability. This is defined as

$$F(CI) = \sum_m F(CI|m)F_m(m) \quad (1)$$

where $F_m(m)$ is the probability that m co-channel interferers (including inter- and intra-cell) are active and $F(CI|m)$ is the corresponding conditional co-channel interference probability,

$$F(CI|m) = \text{prob} \{p_{i0}/p_{im} < z\} \quad (2)$$

where p_{i0} is the instantaneous power of the desired signal, p_{im} is the joint instantaneous interference power from m active channels, and z is the co-channel protection ratio [3].

If we write the joint interference signal as $p_{im} = \sum p_{ii}$ where p_{ii} is the instantaneous power from the i th interferer and assume combined Rayleigh and Log Normal fading for all the interferers then the co-channel interference probability can be expressed as [4]

$$F(CI|m) = 1 - \int_0^\infty dy_0 \frac{1}{\sqrt{\pi}} e^{-y_0^2} \left[\prod_{i=1}^m \frac{1}{\sqrt{\pi}} \int_{-\infty}^\infty \frac{p_{a0} e^{y_0 \sigma_i \sqrt{2}} e^{-y_i^2}}{p_{a0} e^{y_0 \sigma_i \sqrt{2}} + z p_{ai} e^{y_i \sigma_i \sqrt{2}}} dy_i \right] \quad (3)$$

where p_{a0} is the area mean power of the desired signal, p_{ai} and σ_i is the area mean and standard deviation of the i th interference signal respectively. This expression can be evaluated using the Hermite polynomial as described in [4].

To determine the area mean power from the desired and interfering signals we must consider the propagation loss and also the antenna pattern used in the SDMA system. Here we assume the propagation loss can be written in the general form

$$p_a = r_i^{-\gamma} \quad (4)$$

where r is the radius from the basestation and γ is the ground wave propagation path-loss slope which we take as $\gamma = 4$.

For the antenna pattern we assume the SDMA system will be implemented using a switched beam approach and introduce the generic pattern shown in figure 1. The angle θ_m specifies the beamwidth of the main lobe implying there can be $K = 2\pi/\theta_m$ positions for the main lobe. The angle θ_s specifies the beamwidth side lobes while the parameter α determines the relative side lobe level compared to the main beam. Typically α will be between 10 dB and 15 dB. For antenna side lobes the area mean power (in dB) thus becomes

$$p_a = \alpha r_i^{-\gamma} \quad (5)$$

Fig. 1. Generic Antenna Pattern

We assume a regular cell size structure and introduce the distance R as the radius of a cell and D as the distance between the centers of the neighboring co-channel cells. The reuse distance R_u can then be defined as

$$R_u = \frac{D}{R} \quad (6)$$

With this notation we can express the required cluster size to meet the reuse distance as

$$C = R_u^2/3 \quad (7)$$

For a particular cell we assign N channels to meet the offered traffic A and blocking probability B so that a total of CN channels are needed in the network. Using this notation we can define spectral efficiency as [3]

$$E = \frac{A}{NC} \text{ Erlang/channel/cell} \quad (8)$$

For SDMA systems we also use the integer parameter P to denote the number of beams reused in a cell. This implies that for a SDMA system without sectoring the effective number of channels available in a cell is PN and consequently we can expect the spectral efficiency to be P times greater for SDMA systems. However, associated with this increase there will a greater outage probability.

III. Analysis

To estimate the outage probability using (1) and (3) we need to determine the area mean power of the desired and interfering signals.

Because we are only considering the forward link of the SDMA system we select the worse case position for the mobile station (cell edge) and assume the average desired power is $p = R^{-\gamma}$.

The interferers can be divided into inter-cell and intra-cell interferers. In general we can expect up to 6 first tier cells to cause inter-cell interference since we are using a regular reuse structure. In our analysis we assume that the main beam of the inter-cell interferers is most dominant and discard the effects of the sidelobes. The intra-cell interference will be governed by the antenna pattern and number of reused beams and we take account of both the main and side lobes of these interferers.

We consider two types of system. One in which handovers are implemented at the basestation so that no more than one main beam can be allocated to a particular direction at each basestation. The other is where handovers are not implemented so that more than one main beam can be allocated to a particular direction. We consider this situation first.

III.A Without Basestation Handovers

When we do not consider handovers at the basestation we can expect each of the 6 cells causing inter-cell interference to have up to P beams allocated in the direction of the desired user. Therefore there can be up to $6P$ inter-cell interferers, each with average power $p = D^{-\gamma}$.

Similarly for intra-cell interference we can expect up to $P - 1$ intra-cell interferers when handovers are not implemented. The average power level of the intra-cell interferer however will depend on whether a sidelobe or main lobe is directed in the same angle as the beam serving the desired user. When the interference is from a side lobe we can expect the interference power to be $p = \alpha R^{-\gamma}$ and we define this as *side* interference. If the interference is from a main lobe the interference power will be $p = R^{-\gamma}$ and we define this as *main* interference.

Since we have divided the interference into three different types we find it convenient to rewrite (1) as

$$F(CI) = \sum_{n=0}^{6P} \sum_{p+q=0}^{P-1} F(CI|n, p, q) F_{n,p,q}(n, p, q) \quad (9)$$

where the index n refers to the inter-cell interferers while the index p and q refer to the main and side intra-cell interference respectively and $m = n + p + q$. Because the inter-cell and intra-cell interference are independent we can rewrite the probability $F_{n,p,q}(n, p, q)$ as

$$F_{n,p,q}(n, p, q) = F_n(n) F_{p,q}(p, q) \quad (10)$$

where n again represents the number of inter-cell interferers and p and q refer to the number of main and side

lobes respectively causing intra-cell interference so that in (3) $m = n + p + q$.

To determine the exact form for the various terms $F_n(n)$ and $F_{p,q}(p, q)$ we find it useful to define the following probabilities

$$P(\mathcal{M}) = \frac{\theta_m}{2\pi} \quad (11)$$

$$P(\mathcal{S}) = \frac{2\theta_s}{2\pi} \quad (12)$$

$$P(\mathcal{B}) = \frac{A}{PN} \quad (13)$$

where $P(\mathcal{M})$ is the probability that the main beam will be positioned in at a particular direction, $P(\mathcal{S})$ is the probability that a side lobe will be positioned in at a particular direction while $P(\mathcal{B})$ is the probability that a **beam** will be active.

We can then write the term $F_n(n)$ as [3]

$$F_n(n) = \binom{6P}{n} (P(\mathcal{M})P(\mathcal{B}))^n (1 - P(\mathcal{M})P(\mathcal{B}))^{6P-n} \quad (14)$$

by making use of the binomial probability distribution.

The term $F_{p,q}(p, q)$ in (10) cannot be split further since the interference from the main and side lobes is not independent and so we must consider the joint probability function.

Using the notation introduced we first consider the simple example of when $P = 2$ (two beams) so that only 3 combinations of $p + q$ exist. We can tabulate the probabilities associated with the $F_{p,q}(p, q)$ as follows:

$$F_{p,q}(0, 0) = P(\overline{\mathcal{M}})(P(\mathcal{B}) + P(\overline{\mathcal{B}})) + P(\mathcal{M})P(\overline{\mathcal{B}}) + P(\overline{\mathcal{S}})(P(\mathcal{B}) + P(\overline{\mathcal{B}})) + P(\mathcal{S})P(\overline{\mathcal{B}}) \quad (15)$$

$$F_{p,q}(1, 0) = P(\mathcal{M})P(\mathcal{B}) \quad (16)$$

$$F_{p,q}(0, 1) = P(\mathcal{S})P(\mathcal{B}) \quad (17)$$

where the overline is used to denote the compliment set.

For more than 2 beams the combinations of the various side and main lobes become large and we must resort to a general formula. We provide this for the situation when handovers are implemented in the next section.

III.B With Basestation Handovers

When handovers are implemented, a basestation will not allocate more than one main in a particular direction to avoid significant intra-cell interference. Some decrease in spectral efficiency will result.

In this situation the formulas provided in the previous section can be simplified. In the case of inter-cell interference the number of possible interferes will reduce from $6P$ to just 6 and the associated formula becomes

$$F_n(n) = \binom{6}{n} (P(\mathcal{M})P(\mathcal{B}))^n (1 - P(\mathcal{M})P(\mathcal{B}))^{6-n} \quad (18)$$

Similar simplification occurs for the intra-cell interferers. Because only one main beam will be allocated to a particular direction we set $p = 0$ and therefore discard the possibility of main beam intra-cell interference. In this case we can write a general form for $F_{p,q}(0, q)$ by using the binomial distribution and writing it as

$$F_{p,q}(0, q) = \binom{P-1}{q} (P(\mathcal{S})P(\mathcal{B}))^q (1 - P(\mathcal{S})P(\mathcal{B}))^{P-1-q} \quad (19)$$

Using (3), (9), (18) and (19) we can therefore determine an estimate of the outage probability for a particular antenna pattern, fading and SDMA system.

IV. RESULTS

Using the analysis in the previous section we can provide results of outage and efficiency for various configurations of SDMA systems. In addition we compare the results for SDMA with and without handovers with those from a conventional omni-directional system without SDMA and also with results that consider inter-cell interference only.

In our system we assume that the main beamwidth θ_m is 10 degrees (so that there are 36 beams) and that the beamwidth of the sidelobes θ_s is 10 degrees. We set $\alpha = 10$ dB so that the sidelobe interference will be 10 dB less than that from the main lobe and use a protection ratio z of 8 dB. In (3), the combined Rayleigh and Log-Normal co-channel interference probability, we assume $\sigma_i = 6$ dB.

The first set of results we present are the outage probability as a function of reuse distance and these are presented in figure 2 and figure 3 for when 2 beams are reused per cell ($P = 2$). When SDMA is implemented and only inter-cell interference is accounted for the outage probability is always less than that from a conventional system without SDMA because the narrow beams in the SDMA reduce the interference. The difference between SDMA results with and without intra-cell interference is most significant when the re-use distance becomes large. We should also note that there is only a small difference between the outage with and without handovers.

In the next set of results (figure 4 and figure 5) we provide outage probability as a function of carried traffic for cluster sizes of 12 and 27. These cluster sizes were selected to achieve outage probabilities of less than 10% and 1% respectively for a conventional system without SDMA under the traffic load of 49.6 Erlangs. In the results we note that the lines are piecewise continuous and this arises because the number of channels required to meet a given traffic assumes integer values only.

In the final set of results (figure 6 and figure 7), spectral efficiency as a function of blocking probability is provided for a fixed outage probability of 5% and 3% respectively. In order to achieve the same outage probability for the omni-directional system and the SDMA system (with handoff), different reuse distances are used in the two systems. These results are provided so that a comparison

Fig. 2. Outage probability as a function of reuse distance when the traffic carried per cell is 6.61 Erlangs (for a blocking probability of 2% this implies 12 effective channels are required and the number of beams is $P = 2$ so that $N = 6$).

Fig. 3. Outage probability as a function of reuse distance when the traffic carried per cell is 49.6 Erlangs (for a blocking probability of 2% this implies 60 effective channels are required and the number of beams is $P = 2$ so that $N = 30$).

between the spectral efficiencies of the various systems can be calculated for a given outage probability and for a given blocking probability. For example, in figure 6, for a blocking probability of 2%, we get efficiencies of 0.0357 and 0.1966 for the omni-directional system (reuse distance = 6.8) and SDMA system (reuse distance = 4.1) respectively.

Under most circumstances, as expected, SDMA system has a higher spectral efficiency than the omni-directional one. But the advantage disappears under certain conditions. For example, as can be seen in figure 7, when the outage probability is equal to 3% and the blocking prob-

Fig. 4. Outage probability as a function of carried traffic for cluster size of 12 (for a blocking probability of 2% and the number of beams is $P = 2$).

Fig. 5. Outage probability as a function of carried traffic for cluster size of 27 (for a blocking probability of 2% and the number of beams is $P = 2$).

ability is equal to 10%, there is little gain for SDMA system over the omni-directional one. This can be explained by figure 2. While for omni-directional system, we can increase the reuse distance to meet the stringent outage probability requirement. For SDMA system, however, the effect on reducing the outage probability by increasing the reuse distance is much less since we are in a region where the intra-cell interference dominates. As a result, a larger reuse distance is needed, erasing the gain of using two beams in the same cell.

V. Conclusions

We have provided an analysis of the outage and ef-

Fig. 6. Spectral efficiency as a function of blocking probability for an outage probability of 5% when the number of effective channels is 12 (the number of beams is $P = 2$).

Fig. 7. Spectral efficiency as a function of blocking probability for an outage probability of 3% when the number of effective channels is 12 (the number of beams is $P = 2$).

efficiency of an SDMA system when both inter-cell and intra-cell interference are taken into account. The results demonstrate that even when intra-cell interference is accounted for, a multiple times increase in spectral efficiency (compared to a conventional system) is possible with similar overall outage probabilities when $P = 2$.

Improvements to the analysis may be performed. This includes incorporating coherent rather than incoherent analysis for interferers originating from the same cell. In addition the analysis of Rician rather than Rayleigh fading would be useful in systems with small cell sizes.

REFERENCES

[1] R. Rheinschmitt and M. Tangemann, "Performance of Sectorised Spatial Multiplex Systems," *IEEE Veh. Technol.*, pp.426-430, Atlanta, 1996.

[2] K. W. Sowerby and A. G. Williamson, "Outage Probability Calculations for Multiple Cochannel Interferers in Cellular mobile radio systems," *IEE Proceedings, Pt F*, Vol 3, No 135, pp. 208-215, June 1988

[3] R. Prasad and A. Kegel, "Improved Assessment of Interference Limits in Cellular Performance," *IEEE Veh. Technol.*, vol. VT-40, no. 2, pp. 412-419, May 1991.

[4] J-P M G Linnartz, "Exact Analysis of the Outage Probability in Multiple-user mobile radio," *IEEE Trans. Commun.*, vol. COM-40, no. 1, pp. 20-23, January 1992.

[5] L-C Wang and C-T Lea, "Incoherent Estimation of Co-channel Interference Probability for Microcellular Systems," *IEEE Veh. Technol.*, vol. VT-45, no. 1, pp. 164-173, Feb 1996.