The Role of Expectation Formation In a Real Business Cycle Model

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Abstract

This paper looks into the business cycle and welfare implications of adaptive expectations in a real business cycle model with money. The result shows that the impulse responses of macroeconomic variables depend on the way how expectations are formed. Especially the peak responses of the variables like consumption, capital stock and labor productivity and the persistence of the responses are affected by the extent of adaptiveness of expectations. However, adaptive expectations do not have large welfare cost. Hence if there are small deliberation and/or information costs associated with unboundedly rational expectations, adaptive expectations can be a rational choice.
1. Introduction

Since the seminal contribution by Muth (1961) and then by Lucas (1972, 1973), rational expectations have been a key building block in modern economic research. As John Muth (1961) noted in his paper, rational expectations hypothesize that expectations should be the same as the predictions of relevant economic theory, which means that rational expectations cannot be defined properly before the underlying theory is laid out.

However, many authors have recently raised questions on the unbounded rationality of economic agents in economic models (see Conlisk (1996) and Sargent (1993) for surveys). Especially, survey data on expectations of inflation and other variables are used to test the implications of rational expectations hypothesis, namely (1) expectations are required to be unbiased, (2) available information has to be used efficiently, and (3) any errors in expectations should be orthogonal to the known information set. Holden, Peel and Thompson (1985) used many survey data sets including Livingston survey, the Survey of Consumer Finances by the Survey Research Center, University of Michigan, Carson and Parkin (1975) series obtained from the UK Gallup Poll, data from survey conducted by the Institute of Applied Economic and Social Research, University of Melbourne, etc. and found that the data including inflation, wage, interest rate and other variables commonly reject the unbiasedness, efficiency and/or orthogonality predictions of rational expectations.\footnote{See also Lovell (1985) for a survey. He and other authors used survey data sets like the Manufacturers’ Inventory and Sales Expectations Survey conducted by the Department of Commerce from late 1959 through 1976. As Holden et al., he also concluded that the rational expectations hypothesis is rejected by the survey data.}

Frenkel and Froot (1987) made use of exchange rate expectations survey polled by American Express Corporation, the Economist Financial Report, and Money Market Services, Inc. to test various hypotheses regarding exchange rate fluctuations. They found that the survey data reject the rational expectations hypothesis. Ito (1990) used the survey data collected by the Japan Center for International Finance in Tokyo to find that many individuals’ forecasts violate the rational expectations hypothesis. Sharir, Diamond and Tversky (1997) utilized a survey data to see if economic agents have money illusion and they found that they have indeed. Data from recent experimental asset markets favor adaptive over rational expectations (Smith, Suchanek and Williams (1998), Plott and Sunder (1988), Marimon
and Sunder (1993)). However, the experimenters found that experienced subjects move toward rational expectations. The evidence also suggests that expectations may not be rational, depending on experience, difficulty of the forecasting task etc.

Until Lucas popularized the hypothesis, most empirical economists had modelled expectations as adaptive and the most widely used adaptive form of expectations was Koyck transformation, which uses distributed lags (see Koyck (1954) and Lucas and Rapping (1969)). Recently, Baak (1997) and Nerlove and Fonari (1998) showed that adaptive expectations explain better the fluctuations in U.S. beef cattle supply. However, according to this form of expectations, the expectations on a variable depend only on the past realized values of the variable and thus this form of expectations has systematic expectational error. For example, suppose the monetary authority announces that it will increase the rate of money growth. Then it will affect the expectations immediately in the case of rational expectations but not have any effect on the expectations in the period of announcement in the case of adaptive expectations.

Because adaptive expectations involve systematic error, welfare loss is associated with them. This paper addresses this question in a real business cycle model with money. First, we ask how large welfare cost is associated with adaptive expectations. This question is potentially important since if the welfare cost is not large, economic agent may easily deviate from rational expectations in the presence of costs of information gathering, deliberation etc. Second, we ask how adaptive expectations affect the characteristics of business cycles. We compare the impulse responses in a model with adaptive expectations to those in a model with rational expectations. The result shows that the welfare costs of adaptive expectations are so small that deviating from rational expectations may be a rational choice in the presence of some costs of gathering information and/or of deliberating economic decisions. However, the business cycle characteristics depends critically on the way they form expectations. Especially the timing of peak response of some variables like consumption, capital stock and labor productivity are quite different in models with adaptive expectations than in the model with rational expectations. In addition, the persistence of impulse responses is affected by the degree of adaptiveness of expectations. In general, the more adaptive the expectations are, the more persistent are the impulse responses.

In the next section of the paper, we describe the economic environment.
The third section defines the equilibrium, discusses the calibration of parameters and describes the solution technique. The fourth section presents the results from the simulation. Finally, section 5 concludes

2. The Economy

We consider an environment populated by a continuum of identical agents or households. Each agent is endowed with one unit of time per period, initial capital stock \( k_0 \) and initial money holding \( m_0 \).

Agents hold money because real money balances reduce the costs of consumption transaction, which will be made clear soon. Each agent maximizes his lifetime utility which assumed to be additively time separable:

\[
U = \tilde{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1-\sigma} \cdot \left( c_t^l \right)^{1-\sigma} \right] \right\},
\]

(1)

where \( c_t \) is the consumption and \( l_t \) is the leisure in period \( t \). Hence the hours of work can be obtained as \( n_t = 1 - l_t \). \( \beta \) is the utility discounting factor, and \( \nu \) and \( \sigma \) are preference parameters. Agents form expectations according to adaptive expectations formula, which will be clarified later, and we will let \( \tilde{E}_t \) denote the adaptive expectations conditional on the information available in period \( t \).

Agents maximize (1) subject to the following sequence of budget constraints:

\[
P_t(c_t + i_t + \psi_t) + m_{t+1} \leq W_t n_t + R_t k_t + m_t + \Gamma_t.
\]

(2)

The variables in the left-hand side of (2) are the household expenditures and those on the right-hand side are the available funds. \( i_t \) is the investments in period \( t \) and hence capital stock, \( k_t \), follows the following law of motion:

\[
k_{t+1} = (1 - \delta)k_t + i_t,
\]

(3)

where \( \delta \) is the rate of capital depreciation. \( \psi_t \) is the cost of transactions, which will be specified later and \( m_{t+1} \) is the money holdings in period \( t \).

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2We use the convention throughout that lower case letters denote individual choices and capital letters denote their aggregate per-capita counterparts.

3Note, however, that adaptive expectations do not utilize the available information fully.

4Aggregate per capita capital stock \( K_t \) follows an analogous law of motion.
which will be carried over to the period $t + 1$. $W_t$ and $R_t$ are the nominal wage rate and the nominal rental rate of capital, respectively, and $\Gamma_t$ is the lump sum transfer of money in period $t$ from the government, which can be written as:

$$\Gamma_t = (g_t - 1)M_t,$$

where $g_t$ is the gross growth rate of money between periods $t - 1$ and $t$. That is, the newly created money in period $t$ is transferred to the private agents in a lump sum way. We assume that the rate of money growth follows AR(1) process.

$$\ln(g_t) = \eta \ln(g_{t-1}) + \omega_t, \text{ where } \omega_t \sim i.i.d. N[(1 - \eta) \ln(g), \sigma_\omega^2]$$  \hspace{1cm} (5)

Here $g$ is the mean rate of money growth, which is assumed to be fixed, and $\sigma_\omega^2$ is the variance of the money growth innovation.

We assume that consumption transactions incur the following costs.

$$\psi_t = \phi \cdot c_t^\gamma \left( \frac{m_t + \Gamma_t}{P_t} \right)^{1-\gamma},$$  \hspace{1cm} (6)

where $\phi > 0$ and $\gamma > 1$. This specification of the transaction costs includes the cash in advance constraint as a special case\(^5\).

The firm in the economy produces output, $Y_t$, using the Cobb-Douglas production technology:

$$Y_t = A_t K_t^\theta N_t^{1-\theta}, \hspace{0.5cm} 0 \leq \theta \leq 1,$$

where $\theta$ is the share of capital in the production, $A_t$ is a productivity shock, $K_t$ is the per-capita capital stock and $N_t$ is the per-capita hours of work. It is assumed that the technology shock follows an AR(1) process:

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t, \text{ where } \varepsilon_t \sim i.i.d N(0, \sigma_\varepsilon^2),$$  \hspace{1cm} (8)

where $\sigma_\varepsilon^2$ is the variance of the technology shock innovation.

The firm solves a series of maximization problems in each period:

$$\max_{\{N_t, K_t\}} \left[ P_t \cdot \left( A_t K_t^\theta N_t^{1-\theta} \right) - W_t N_t - R_t K_t \right].$$

\(^5\)If $\gamma \to \infty$, (6) reduces to the cash-in-advance constraint.
The profit maximization by the firm implies the following conditions equating the marginal product of an input to its real price:

\[
W_t/P_t = (1 - \theta)A_t [K_t/N_t]^\theta
\]

(10)

\[
R_t/P_t = \theta A_t[K_t/N_t]^{\theta-1}.
\]

(11)

Note here that the firms’ problem is basically static.

Now we elaborate the way the agents form their expectations. Until the rational expectations were introduced by Lucas (1972, 1973), the most popular expectations formation was Koyck transformation (see Lucas and Rap- ping (1969) for example). That is, given the actual law of motion for money growth and technology shock, we assume that agents form their expectations using the Koyck transformation as follows.

\[
\frac{\tilde{E}_t(A_{t+1})}{\tilde{E}_{t-1}(A_t)} = \left[\frac{A_t}{\tilde{E}_{t-1}(A_t)}\right]^{\alpha_1}
\]

(12)

\[
\frac{\tilde{E}_t(g_{t+1})}{\tilde{E}_{t-1}(g_t)} = \left[\frac{g_t}{\tilde{E}_{t-1}(g_t)}\right]^{\alpha_1}
\]

(13)

Taking logarithm of these expectations formula, we can have the followings\(^{6}\).

\[
\ln\left(\tilde{E}_t(A_{t+1})\right) - \ln\left(\tilde{E}_{t-1}(A_t)\right) = \alpha_1 \left[\ln(A_t) - \ln\left(\tilde{E}_{t-1}(A_t)\right)\right]
\]

(14)

\(^{6}\)Using repeated substitution, we can have the expectations as the functions of the realizations of the shocks from the infinite past to the current period.

\[
\ln(A_t^\ast) = \alpha_1 \sum_{j=1}^{\infty} (1 - \alpha_1)^j \cdot \ln(A_{t-j})
\]

\[
\ln(g_t^\ast) = \alpha_2 \sum_{j=1}^{\infty} (1 - \alpha_2)^j \cdot \ln(g_{t-j})
\]
\[
\ln \left( \widetilde{E}_t(g_{t+1}) \right) - \ln \left( \widetilde{E}_{t-1}(g_t) \right) = \alpha_2 \left[ \ln(g_t) - \ln \left( \widetilde{E}_{t-1}(g_t) \right) \right]
\]  \hspace{1cm} (15)

To simplify the notation, we let \(X^e_t = \widetilde{E}_{t-1}(X_t)\) for any variable \(X_t\). Then we can rearrange the equations as follows.

\[
\ln(A^e_{t+1}) = \alpha_1 \ln(A_t) + (1 - \alpha_1) \ln(A^e_t)
\]  \hspace{1cm} (16)

\[
\ln(g^e_{t+1}) = \alpha_2 \ln(g_t) + (1 - \alpha_2) \ln(g^e_t)
\]  \hspace{1cm} (17)

Here we beg the question of why they form expectations in this way and we assume that the initial expectations are given as \(A^e_0\) and \(g^e_0\). They probably form the expectations in this way due to the fact that there are some psychological, deliberation and/or information costs. The sole reason to model expectations in this way is to see how much welfare gain rational expectations may have and then to look into the dependence of the characteristics of business cycle fluctuations on the way that they form expectations\(^7\).

To make the problem stationary, we use the following changes of variables by dividing nominal quantities by aggregate stock of money \(M_{t+1}\). That is, we define the following.

\[
\frac{\bar{P}_t}{P_t} = \frac{P_t}{M_{t+1}}, \quad \frac{\bar{W}_t}{W_t} = \frac{W_t}{M_{t+1}}, \quad \frac{\bar{R}_t}{R_t} = \frac{R_t}{M_{t+1}}
\]

Using these changes of variables, we can rewrite (2) and (6) as follows.

\[
c_t + i_t + \psi_t + m_{t+1} \leq \left( \frac{\bar{W}_t}{P_t} \right) n_t + \left( \frac{\bar{R}_t}{P_t} \right) k_t + \frac{\bar{m}_t + (g_t - 1)}{g_t P_t}
\]  \hspace{1cm} (18)

\[
\psi_t = \phi \cdot c_t^\gamma \cdot \left( \frac{\bar{m}_t + g_t - 1}{g_t P_t} \right)^{1-\gamma}
\]  \hspace{1cm} (19)

Note also that (10) and (11) can be rewritten as follows.

\[
\frac{\bar{W}_t}{P_t} = (1 - \theta)A_t[K_t/N_t]^\theta
\]  \hspace{1cm} (20)

\(^7\)In addition, it is acknowledged that there can be numerous ways of expectation formation which are not rational. However, since the Koyck transformation has been the most popular econometric formulation in the literature, we use it in the text.
Once the equilibrium is obtained, we can recover the variables without hat through reverse changes of variables. Aggregating (18) and (19) and using (20) and (21), we can have the following resource constraint.

\[ C_t + I_t + \Psi_t = A_t K_t^{\theta} N_t^{1-\theta} \]  

(22)

\[ \Psi_t = \phi \cdot C_t^\gamma \cdot \left( \frac{1}{P_t} \right)^{1-\gamma} \]  

(23)

(22) is the per capita resource constraint, which may be considered to be the market clearing condition in the goods market, and (23) is the per capita costs of consumption transactions.

3. Equilibrium, Calibration and Simulation

The state of the economy can be defined as follows.

\[ s_t = (1, \ln(A_t), \ln(A_t^r), \ln(g_t), \ln(g_t^e), k_t, K_t, \bar{m}_t)' \]  

(24)

Let \( V(s_t) \) denote the value function in period \( t \). Then the agent’s lifetime utility maximization problem can be expressed using Bellman equation as follows:

\[ V(s_t) = \max_{s.t.} \left\{ \frac{1}{1-\sigma} \left[ \epsilon_t (1 - n_t)^{1-\sigma} \right]^{1-\sigma} + \beta \mathbb{E}_t[V(s_{t+1})] \right\} \]  

(25)

The problem is standard except that the expectations are formed in an adaptive way.

3.1 Equilibrium

The equilibrium in the economy can be defined using the Bellman equation (25) and the firm’s profit maximization problem (9).

Definition: An equilibrium is a pair of allocations:

\[ \{ c_t, n_t, i_t, \bar{m}_{t+1}, k_t, \psi_t, Y_t, C_t, N_t, K_t, \Psi_t \}_{t=0}^\infty \]
prices \( \{ \tilde{P}_t, \tilde{W}_t, \tilde{R}_t \}_{t=0}^{\infty} \) and value function \( V(s_t) \) such that, given the initial values \( k_0, m_0, A_0, A^c_0, g_0 \) and \( g^c_0 \),

(i) \( \{ c_t, n_t, i_t, m_{t+1}, k_{t+1}, V(s_t) \}_{t=0}^{\infty} \) solves the representative agent’s utility maximization problem, (25), given the prices and the per capita quantities.

(ii) Given the prices, \( \{ Y_t, N_t, K_t \}_{t=0}^{\infty} \) solves the producer’s profit maximization problem, (9), and

(iii) For each \( t \), the prices solve:

\[
C_t + I_t + \Psi_t = Y_t, \quad n_t = N_t, \quad k_t = K_t,
\]

when \( c_t = C_t, i_t = I_t, \tilde{m}_t = 1 \) and \( \psi_t = \Psi_t \).

The condition (i) requires the consumer’s utility maximization and the condition (ii) is for the producer’s profit maximization. The condition (iii) is the market clearing condition in good and factor market. Note that although we have introduced the adaptive expectations, the problem is still recursive, and the equilibrium prices and quantities in period \( t \) are functions of state in period \( t \).

### 3.2 Calibration

Our urgent goal is to evaluate the effects of adaptive expectations on the business cycles and their implication on welfare. To achieve this goal, we proceed as in the real business cycle literature by assigning values to the parameters of the model based on the National Income and Product Accounts (NIPA) and other features of the U.S. economy. We then simulate these model economies to obtain the quantitative implications.

We set the parameter values as follows. We interpret a period in this model as a quarter which dictates that we set \( \beta \) to be .99. This means that the annual real interest rate in the steady state is four percent (see Prescott (1986)). We assume that \( \alpha = 0.33 \), which implies that the hours of work are roughly one third of the total endowment of time and that \( \sigma = 2 \), which has been used many times in the real business cycle literature including Stockman and Tezar (1994). We borrow the parameter values for the cost of transactions from Marshall (1992) and set the value as: \( \gamma = 2.79 \) and \( \phi = 9.234 \cdot \gamma/10^3 \). Capital stock is assumed to be depreciated at the rate of 10% per annum, i.e. \( \delta = 0.025 \). We use \( \theta = 0.36 \), which has been used in most of real business cycle models (see Kydland and Prescott (1982), Hansen (1985) and Cho and Cooley (1995)) and is based on the NIPA using
a somewhat narrow definition of capital. For the baseline simulations the
standard deviations of the shock innovations are set as $\sigma_\epsilon = 0.008$ and $\sigma_\omega = 0.009$, and the persistence parameters of the shocks are set as: $\rho = 0.95$ and $\eta = 0.49$. These numbers are used by Cooley and Hansen (1989) and Cho and Cooley (1995) in a similar environment. The mean gross growth rate of money, $g$, is assumed to be 1.03. The mean growth rate of money determines the long run inflation in this economy and welfare costs are associated with the growth rate of money larger than $\beta$. We vary the values of expectations parameters $\alpha_1$ and $\alpha_2$ and see how they affect the welfare and fluctuations characteristics.

3.3 Steady State
The steady state can be obtained from the following conditions.

\[(1 - \theta)\nu AK^{\theta}N^{-\theta}(1 - N) = (1 - \nu)C \left\{ 1 + \phi \gamma C^{-\gamma - 1} \left( \frac{1}{P} \right)^{1-\gamma} \right\} \]  
(27)

\[\beta \left\{ \theta AK^{\theta - 1}N^{1-\theta} + (1 - \delta) \right\} = 1 \]  
(28)

\[\phi(1 - \gamma)C^{\gamma} \left( \frac{1}{P} \right)^{1-\gamma} + \frac{1}{P} = \frac{\beta}{gP} \]  
(29)

\[C + I + \phi C^{\gamma} \left( \frac{1}{P} \right)^{1-\gamma} = AK^{\theta}N^{1-\theta} \]  
(30)

\[I = \delta K, \]  
(31)

where we used the fact that $\widehat{M} = 1$. The variables without time subscript denote the steady state values of the counterparts with time subscript. The meaning of these conditions is not unusual. (27) equates the marginal utility of leisure to the marginal utility of wage, i.e. marginal product of labor in utility terms. (28) balances the marginal value of accumulating one unit of capital to the marginal cost of it. (29) equates the marginal cost of holding money to its marginal benefit. (30) is the resource constraint and (31) is the steady state requirement.
3.4 Simulation

The simulation method is not very different from the one in Cho and Cooley (1995). It is based on linear quadratic approximation of the objective function around the steady state, which was originally developed by Kydland and Prescott (1982) and used by many authors including Hansen (1985), Cooley and Hansen (1989). The key difference between the method here and the one used by these authors is in the fact that the expectations are adaptive, which will be made clear in a moment. To apply the method, we have to solve the budget constraint for consumption and substitute in the period utility function and the resulting utility function is approximated by a quadratic function. Using the quadratic utility function, we can have the following linear-quadratic Bellman equation, which approximates the original problem (25).

\[
s_t' \cdot V^Q \cdot s_t = \max \left\{ (u'_t \ U'_t \ s'_t) \cdot Q \cdot \begin{pmatrix} u_t \\ U_t \\ s_t \\ s_{t+1} \end{pmatrix} + \beta \cdot E_t \left( s_{t+1}' \cdot V^Q \cdot s_{t+1} \right) \right\}
\]

s.t. (3), (5), (8), (16), (17),
and the law of motion for \( K_t \), given \( A_0, \ A_0^c, \ g_0 \) and \( g_0^c \).

where \( V^Q \) is a symmetric matrix and the matrices in (32) are defined as \( u'_t = (i_t, n_t, \bar{m}_{t+1}) \) and \( U'_t = (I_t, N_t, \bar{P}_t) \). Now the constraints in the problem (32) can be expressed using matrices as follows.

\[
s_{t+1} = \Lambda_1 \cdot s_t + \Lambda_2 \cdot u_t + \Lambda_3 \cdot U_t + \Lambda_4 \cdot \epsilon_{t+1},
\]

where the coefficient vectors are defined as:

\[
\Lambda_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 & 1 - \alpha_1 & 0 & 0 & 0 & 0 \\
0 & 0 & \eta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_2 & 1 - \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 - \delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 - \delta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\(^8\text{An excellent survey of the method can be found in Hansen and Prescott (1995).}\)
Note that if the expectations are rational, we have the following expectations.

\[ E_t(s_{t+1}) = \Lambda_1 \cdot s_t + \Lambda_2 \cdot u_t + \Lambda_3 \cdot U_t + E(\epsilon_{t+1}), \]  

(34)

where \( E_t(\cdot) \) and \( E(\cdot) \) are the conditional and unconditional mathematical expectations operator. However, since the expectations are formed according to the adaptive expectations formula, we have the following.

\[ \tilde{E}_t(s_{t+1}) = \Lambda_5 \cdot s_t + \Lambda_2 \cdot u_t + \Lambda_3 \cdot U_t, \]  

(35)

where the coefficient matrix \( \Lambda_5 \) is defined as follows.

\[ \Lambda_5 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 & 1 - \alpha_1 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 & 1 - \alpha_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_2 & 1 - \alpha_2 & 0 & 0 \\
0 & 0 & 0 & \alpha_2 & 1 - \alpha_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 - \delta & 0 \\
0 & 0 & 0 & 0 & 0 & 1 - \delta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \]

Note here that

\[ \tilde{E}_t(s_{t+1}) = s_{t+1}^\tau = \begin{pmatrix}
1, \ln(A_{t+1}^\tau), \ln(A_{t+1}^\tau), \ln(g_{t+1}^\tau), \ln(g_{t+1}^\tau), k_{t+1}, K_{t+1}, m_{t+1}
\end{pmatrix} \]

(36)

and \( \ln(A_{t+1}^\tau) \) and \( \ln(g_{t+1}^\tau) \) are following (16) and (17) respectively. In other words, adaptive expectations alter the coefficient matrix in the law of motion.

In order to solve the problem (32), we need to evaluate the expectations \( \tilde{E}_t(s_{t+1}^\tau \cdot V^Q \cdot s_{t+1}) \). However, adaptive expectations formula does not have
a general rule of forming expectations of the moment of a random variable higher than the first. That is, if we are to use the adaptive expectations in a model, we have to postulate how each moment of a random variable has to be computed. For simplification, we assume the following:

$$\tilde{E}_t(X_{t+1} \cdot Y_{t+1}) = \tilde{E}_t(X_{t+1}) \cdot \tilde{E}_t(Y_{t+1})$$  \hspace{1cm} (37)

for any random variable $X_{t+1}$ and $Y_{t+1}$. Then we can have the following.

$$\tilde{E}_t(s'_{t+1} \cdot V^Q \cdot s_{t+1}) = s'_{t+1} \cdot V^Q \cdot s_{t+1}$$  \hspace{1cm} (38)

Now we can evaluate the expectations in the Bellman equation using (35) and solve for the decision rules and the value of the lifetime utility.

Once the quadratic value function for each value of the expectations parameters is obtained, we evaluate the lifetime utility using the steady state value of the state variables as the initial state. Suppose $U_a$ denote the lifetime utility defined in (32) from a model with an adaptive expectations. Then we obtain the welfare changes by comparing $U_a$ and the steady state utility level in the following way.

$$\frac{1}{(1 - \beta)(1 - \sigma)} \cdot \{ [C - \Delta C_a][N(1 - \nu)]^{1 - \sigma} \}^{1 - \sigma} = U_a,$$  \hspace{1cm} (39)

The following is a justification for the assumption. Adaptive expectations implies the following.

$$\tilde{E}_t \left[ \tilde{E}_{t+1} (X_{t+2}) \right] = \tilde{E}_t \left[ \alpha X_{t+1} + (1 - \alpha) \tilde{E}_t (X_{t+1}) \right] = \alpha \tilde{E}_t (X_{t+1}) + (1 - \alpha) \tilde{E}_t (X_{t+1}) = \tilde{E}_t (X_{t+1}) \cdot \tilde{E}_t (X_{t+1}) \cdot \tilde{E}_t (X_{t+1}) \cdot \tilde{E}_t (X_{t+1}) \cdot \tilde{E}_t (X_{t+1}),$$

where we assume that $\tilde{E}_t \left[ \tilde{E}_t (X_{t+1}) \right] = \tilde{E}_t (X_{t+1})$. That is, the expectations on the distant future are static. (37) implies an analogous static expectations in terms of product of two variables.

$$\tilde{E}_t \left[ \tilde{E}_{t+1} (X_{t+2} \cdot Y_{t+2}) \right] = \tilde{E}_t (X_{t+1} \cdot Y_{t+1})$$

In the case of rational expectations, the following holds.

$$E_t(X_{t+1} \cdot Y_{t+1}) = E_t(X_{t+1}) \cdot E_t(Y_{t+1}) + Cov_t(X_{t+1}, Y_{t+1})$$

If we compare the expression to (37), we may argue the assumption does not have a term corresponding to covariance term. However, since the two types of expectations formation are not analogous, we cannot know whether the covariance term is ignored or not.
where $\Delta C_a$ denotes the welfare changes due to uncertainty and adaptive expectations in terms of steady state consumption. Note that the cost involves two components: one is associated with the aggregate uncertainty and the other with adaptiveness of the expectations. Aggregate uncertainty involves welfare loss even under the rational expectations. Hence to obtain the welfare loss associated purely with the adaptiveness of expectations, we calculate the welfare loss, $\Delta C_r$, associated with aggregate uncertainty under rational expectations. Now the welfare cost associated with the adaptive expectations can be obtained as: $\Delta C_a - \Delta C_r$. We report $\Delta C_a - \Delta C_r$ as the percentage of the steady state output $Y$.

4. Results

The model is constructed to look at the consequence of expectations which are not rational. As was emphasized in the introduction, two key concerns are welfare loss associated with their adaptiveness and resulting changes in the business cycle characteristics. The results can be summarized as follows.

4.1. Welfare Cost of Adaptive Expectations

We first look at the welfare cost of adaptive expectations in the form of Koyck transformation. The result is depicted in figure 1. Figure 1(A) shows the welfare loss associated with each value of $\alpha_1$, which is the adaptive expectation parameter for the technology shocks. The adaptive expectations parameter for the monetary shocks, $\alpha_2$, is set as 0.5. Figure 1(B) shows the welfare loss associated with each value of $\alpha_2$. In this case the value of $\alpha_1$ is set as 0.5. The result in figure 1 shows that the welfare cost of adaptive expectations is about 0.016% of steady state output per quarter. We have varied the value of the expectations parameters to obtain the changes in welfare costs. However, we have not found any noticeable changes in the welfare loss. In sum, adaptive expectations in the form of Koyck transformation do not imply large welfare cost and the cost does not respond to changes in the adaptive expectations parameters.

4.2 Adaptive Expectations and Aggregate Fluctuations

Although expectations parameters do not affect the welfare costs in a noticeable way, they affect the business cycle characteristics. Figure 2 shows the impulse responses when there is 1% increase in the technology shock innovation. In the figure, we have assumed that the expectation parameter on
money growth($\alpha_2$) is 0.5. When the value of $\alpha_1$ is 0, all variables respond more vigorously than in the case of rational expectations. At the peak response, output responds more by about 0.4% with adaptive expectations. The reason that the responses of output and labor are more vigorous in this case than in the case of rational expectations can be found in the fact that intertemporal substitution of labor takes place more vigorously with $\alpha_1$ being 0 than with rational expectations. In other words, if $\alpha_1 = 0$, expectations are purely static and hence they are not revised according to arrival of new information on the technology shocks. This means that technology shock innovations are recognized as purely temporary and hence that maximum intertemporal substitution takes place. On the other hand, if $\alpha_1$ is large, impulse responses become less vigorous under adaptive expectations than under rational expectations. With the value of $\alpha_1$ being 0.5 or 1.0, output and labor respond less. This can also be explained referring to the intertemporal substitution effect. For example, if $\alpha_1 = 1.0$, next period’s expected technology shock is the same as the one realized in this period, which means that this period’s shock is perceived as permanent. Hence intertemporal substitution takes place less and labor responds less. Furthermore the impulse responses become more persistent as $\alpha_1$ gets smaller.

Another notable changes in the impulse responses are in the peak responses. The peak responses of the macroeconomic variables under adaptive expectations do not coincide with those under rational expectations. Generally speaking, the peak responses are delayed as $\alpha_1$ gets smaller. The most conspicuous changes can be found in the impulse responses of consumption, capital stock and labor productivity. If $\alpha_1$ is 0, the impulse responses under adaptive expectations lag behind those under rational expectations. However, when $\alpha_1$ is 0.5 or 1.0, impulse responses under adaptive expectations lead those under rational expectations.

Impulse responses in the case of 1% increase in the money shock innovation are depicted in figure 3\textsuperscript{11}. Although the impulse responses in this case are very weak, we can differentiate the responses among the cases in the figure. If the value of $\alpha_2$ is 0, all variables are not responding to the monetary innovation. Since they expect in this case that there are no changes in the money growth up to the infinite future, the intertemporal and intratemporal substitution among consumption, investment and leisure, does not take place.

\textsuperscript{11}For comparison, the same scale as in figure 2 is used.
and thus aggregate demand does not change. As the value of $\alpha_2$ increases, impulse responses become larger, which is due to the fact that they expect the shock innovations more permanent with the value of $\alpha_2$. That is, if they perceive that the money shock innovation is permanent and hence expect higher inflation, they will substitute more among goods.

However, one puzzling fact out of figure 3 is the following. The key of the Lucas supply function (see Lucas (1973)) is in the expectational error and monetary shocks may have large real effect due to the error. Here the agents make the expectational error every period without any large real effect of the monetary shocks. Probably this can be explained as follows. The rational agents readily correct their mistake once they receive the relevant information and the correction may magnify the real effect. But the adaptive agent never correct their mistake and hence the misperception may not allow the monetary shocks to have significant real effect.

5. Conclusion

Adaptive expectations in the form of Koyck transformation are introduced in a real business cycle model with money. This form of adaptive expectations does not disturb the recursive nature of the problem and hence we can use the solution methods developed in the literature to simulate the model. But adaptive expectations imply that the expected future states cannot be obtained by mathematical expectations of the law of motion of the state of the economy. We found that the welfare cost of adaptive expectations is very small, which means that small deliberation and/or information cost may easily make agents deviate from rational expectations. However, the business cycle characteristics depend on how expectations are formed. Changes in the way of expectation formation affect magnitude, persistence and the peak responses of the impulses responses of macroeconomic variables, especially those of consumption, capital accumulation and labor productivity.

References


Figure 1. Welfare Cost of Adaptive Expectations

(A) Adaptive Expectations: Technology Shock

(B) Adaptive Expectations: Money Shock
Figure 2. Impulse Responses: Technology Shock
Figure 3. Impulse Responses: Monetary Shock