Interferometric straightness measurement system using triangular prisms

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Abstract. A new interferometric system for measuring straightness is developed. In the system, triangular prisms are used to generate two converging beams. A straightness reflector is used to reflect back the converging beams along their respective paths to form a combined beam. The combined beam is received by a detector system to count the interference fringes. Being different from the traditional straightness reflector, the new straightness reflector consists of not only reflecting surfaces but also refractive surfaces. The refractive surfaces cause a change in optical path difference (OPD) when a relative lateral displacement between the triangular prisms and the straightness reflector takes place. The relationship between the OPD and the lateral displacement is derived. The expected results agree with the results in the experiments. A straightness measurement with an accuracy of $0.1 \mu\text{m}$ is obtained over a travel range of 120 mm. © 1998 Society of Photo-Optical Instrumentation Engineers. [S0091-3286(98)00706-5]

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1 Introduction

In most modern machines, such as the CNC machine, many movements are involved with a table moving in a straight line along a guide. If the table does not move with high accuracy, the final product may not be of high accuracy. Therefore any lateral displacement of the table should be measured and controlled. The lateral error motion of a table along a straight line can be referred to as travel straightness.

Several straightness measurement systems have been developed.¹⁻⁸ A typical interferometric system is Hewlett-Packard's straightness interferometer system.^{1,2} In this system, a Wollaston prism is used to split a laser beam traveling along the principal axis into two diverging secondary beams. The two secondary beams deviate from the principal axis by an equal divergence angle α and travel to a straightness reflector. The straightness reflector reflects the secondary beams back along their respective paths. The measuring signals are produced when a lateral movement takes place between the Wollaston prism and the straightness reflector.

The optical resolution of the straightness interferometer system is dependent on the divergence angle α . The range of the interferometer for straightness measurement refers to the measurable distance of the moving table traveling along the principal axis. It is evident that the range of the respective interferometers is also limited by the angular deviation of the secondary beams. For example, to achieve a larger range the angular deviation should be very small, which results in a lower optical resolution. In addition, a larger

range or a larger angular deviation requires a larger straightness reflector, which is difficult to fabricate. Therefore, it is very important to obtain a balance among the optical resolution, the range, and the size of the straightness reflector by selecting an appropriate divergence angle α .

For a Wollaston prism, the divergence angle α can not be set freely because of fabrication problems, and Wollaston prisms are expensive. To overcome these problems, a holographic grating can be used to generate the two diverging beams.³ By selecting different grating pitches the divergence angle α can be set at any value. However, the low diffraction efficiency of the grating results in a low signal SNR.

The presented system uses triangular prisms to deviate the secondary beams by a certain angle α from the principal axis. This angle is related to both the acute angle and refractive index of the triangular prisms. Therefore any value of the angle α can easily be obtained by selecting an appropriate acute angle of the triangular prisms.

One of the advantages of using the triangular prisms is that not only the diverging beams but also the converging beams can be generated by different settings of the triangular prisms, which results in more flexible interferometer configurations. Especially, unlike the traditional straightness reflector, the straightness reflector in the present system consists of not only reflecting surfaces but also refractive surfaces. Simply put, the new straightness reflector also consists of triangular prisms. The refractive surfaces are the hypotenuse surfaces of the triangular prisms, which cause a change in optical path difference (OPD) when a relative lateral displacement between the triangular prisms and the straightness reflector takes place.

In addition, because a right-angle prism is arranged behind the triangular prisms in the straightness reflector, the

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Fig. 1 Diagram of the straightness interferometer system.

returned beams are insensitive to pitching movements and less sensitive to yawing movements of the movable table. Thus alignment of the components of the interferometer becomes easier.

2 Principle of Measurement

Figure 1 is a diagram of the straightness interferometer system using the triangular prisms. The system includes a laser, which provides a beam along the principal Y axis. The beam is split by a cube beamsplitter into two measuring beams. One of the beams remains on the Y axis, while the other is reflected by a reflector to become the beam parallel to the principal axis. Two triangular prisms 1 and 2 are placed in the paths of the beams. The beams transmit through the triangular prisms and undergo refraction at the hypotenuse surfaces $1c$ and $2c$ of the triangular prisms forming the converging secondary beams which diverge at angle α from the Y axis.

A straightness reflector is used to reflect the secondary beams back along their respective paths. The traditional straightness reflector includes two reflecting surfaces normal to the secondary beams, respectively. A newly designed straightness reflector includes refractive surfaces $3c$ and $4c$ and is placed in the path of the beams. In other word, the straightness reflector consists of two triangular prisms and a right-angle prism, as shown in Fig. 1. Having undergone refraction at the hypotenuse refractive surfaces $3c$ and $4c$, the converging secondary beams are rendered parallel to the Y axis. The beams are then reflected by the right-angle prism and are deviated a certain distance along the Z axis. The beams are refracted at the hypotenuse surfaces $3c$ and $4c$ again and reenter the triangular prisms 1 and 2. Finally, the two returned beams are combined in the cube beamsplitter.

A lateral movement along the X axis between the triangular prisms and the straightness reflector will cause a change in the OPD between the two measuring arms. A detector system is positioned in the path of the combined beam to detect the change of light intensity caused by the change of the OPD.

When the straightness reflector travels along the Y axis, with reference to Fig. 1, there is no change in the OPD between the two measuring arms. However, a lateral movement along the X axis will increase the optical path length in one measuring arm and decrease the optical path length in the other measuring arm. Therefore, the change of the

Fig. 2 Diagram of the optical paths arrangement in one of the measuring arms.

interference fringes are produced by the change of the OPD of the two measuring beams. The detector system produces signals from the interference fringes to give a reading of straightness error.

It can be observed that the term "triangular" applies not only to prisms that are literally triangular in shape. For example, an acute angle of the triangle may be cut off to simplify manufacturing, leaving a four-sided prism. Such prisms are also considered "triangular" prisms.

Figure 2 shows one arm of the optical path arrangement in Fig. 1. Although the straightness reflector consists of a triangular prism and a right-angle reflector, it can be simply redrawn as a triangular prism with a reflecting surface on the bottom of the right-angle face. This is so because the simulation results will be the same for calculating the change in optical path length caused by a relative lateral displacement between the triangular prism and the straightness reflector. The two hypotenuse surfaces of the triangular prisms should be parallel to each other. The straightness reflector is moved along the X axis by a small distance x , which causes the optical path length to change by a distance b in the air and a height difference h in the straightness reflector. From the law of refraction and using geometry, the relation between a lateral motion x and an optical path length change Δ can be derived as follows:

$$n \sin \theta = \cos \beta, \quad (1)$$

$$a = x \sin \theta, \quad (2)$$

$$b = a / \sin \beta, \quad (3)$$

$$h = b \sin (\theta + \beta), \quad (4)$$

$$\Delta = 2(nh - b) = 2x \cos (\theta + \beta) = 2x \sin \alpha, \quad (5)$$

where n is the index of refraction of the triangular prisms and the straightness reflector, θ is one of the acute angles of the triangular prism and the straightness reflector, β is the angle between the refractive direction of the beam and the hypotenuse of the triangular prism, and α is the divergence angle of the beams deviated from the principal axis.

According to Eq. (5), we can define $\Delta = kx$. Characterized by the optical resolution of the system, the slope k is related to the acute angle θ of the triangular prisms. Figure

3 Graph of slope k varying with angle θ of the triangular prism.

shows the graph of slope k varying with the angle θ . The graph shows an improvement in the optical resolution of the interferometer when a larger angle is selected. For $n = 1.510$ and $\theta = 15^\circ$, the optical path length change Δ induced by the relative lateral displacement x can be expressed by $\Delta = 0.2785x$, where $k = 0.2785$.

Because a reverse optical path length change occurs in the other arm, the change of the OPD between the two measurement arms will be 2Δ .

The phase change between the two measuring beams will be

$$2\pi \text{OPD} / \lambda = 4\pi \Delta / \lambda = 8\pi x \sin \theta / \lambda, \quad (6)$$

where λ is the wavelength of the laser. When the two measuring beams interfere, the resultant intensity is given by

$$1 + B \cos(\delta + \delta_0), \quad (7)$$

where A and B are constants, and δ_0 is the phase difference at the starting position. When a relative lateral displacement takes place, the phase difference between two coherent waveforms changes progressively, then the intensity of the resultant interference waveform will vary cyclically through a series of light and dark fringes. These variations are counted and used to compute the change in the difference between the two path lengths. Thus, the lateral displacement can be derived from the intensity I using Eqs. (7).

The range of the interferometer $Y_{\max}$ is determined by the following equation:

$$x = S / \tan \alpha, \quad (8)$$

where S is the size of the straightness reflector, as shown in Fig. 2. It is evident that a small angular deviation α will result in a larger traveling range.

In practice, two triangular prisms 1 and 2 may be movable components mounted on the moving table. The

straightness reflector and the other components may be fixed components. Also, the components of a cube beamsplitter, a reflector, and two triangular prisms 1 and 2 may be integrated into a fixed component and mounted on a fixed base. The straightness reflector is a movable component and can be mounted on the moving table.

When the movable component moves along the Y axis, the lateral movement or the straightness error in the X axis will be checked. The movement or the straightness in the Z axis can also be checked by turning the related components by a 90° angle.

3 Experiments

The straightness interferometer system is comprised of a laser source, a beamsplitter, a plane reflector, two triangular prisms, a straightness reflector, and a detector system. The triangular prisms and the straightness reflector have been fabricated with $\theta = 15^\circ$. The glass material is K9 and the index of refraction n is 1.510 at wavelength $\lambda = 632.8 \text{ nm}$. The divergence angle of the secondary beams α is about 8° .

To calibrate the interferometer system, a piezoelectric transducer (PZT) linear positioner is used to produce a precise lateral displacement. The PZT linear positioner (Model: P-854, produced by the Physik Instrumente Co.) gives an accurate displacement up to $0.3 \mu\text{m}$ for each 1-V input. A manual travel range of 18 mm can also be produced by the device. The straightness reflector is mounted on the PZT linear positioner. An inductive transducer is used to monitor the movement.

To simply verify the performance of the straightness interferometer system, a ML10 laser unit is selected to build up the measurement system. The ML10 laser unit is the basis of the Renishaw Laser Interferometer System.⁹ It comprises a HeNe laser operating at a wavelength of $0.633 \mu\text{m}$, with a long-term stability of better than 1 in 10^7 . The unit also contains all the circuitry associated with the laser, not only that for the laser tube power supply and stability control, but also the optical detector and detection computation circuitry, including the fringe counter, interpolation, and signal strength functions.

In the experimental setup, the laser and detector shown in Fig. 1 are replaced by the ML10. The straightness reflector is mounted on the PZT linear positioner to give a precise lateral displacement. The ML10 laser unit converts the photoelectric signals to the readings of displacement. The data are displayed by using a software package and a computer notebook.

Note that the reading directly from the computer notebook display is for the linear distance measurement. For the measurement of the straightness, the reading data should be divided by a slope of k . A calibration curve of the interferometer system is shown in Fig. 4. The regression equation of the curve is $y = 0.277x - 1.1244$. The slope k is 0.277. Compared with the theoretical value of $k = 0.2785$ as calculated by Eq. (5), the experimental result agrees well with the expected result.

The resolution test results of the interferometric system are shown in Fig. 5. The incremental lateral displacements are generated by the PZT. The figure illustrates that the resolution of the interferometer system is better than 0.1

Fig. 4 Calibration curve of the interferometer system.

μm . Figure 6 shows the repeatability test results. The standard deviation σ is 36 nm, therefore the repeatability of the system is $3\sigma = 0.11 \mu\text{m}$.

Air turbulence is a very important factor affecting the accuracy of the system. An error caused by air distribution exists in the experimental room. Although the system is relatively less affected by the air turbulence of the static gradients of the air refractive index over small travel distances, the effect still exists because of the fact that the signal of beam is not stable and data change in the nanometer order when the PZT is not moving.

The interferometric system can be used to measure the travel straightness of a moving table. As shown in Fig. 1, the moving table is driven to travel along the Y direction. According to Eq. (8), the travel range of the moving table can reach 120 mm when S is 12 mm. The straightness reflector is mounted on the moving table. Because a right-angle prism is arranged behind the triangular prisms in the straightness reflector, the returned beams are insensitive to pitching movements and less sensitive to yawing movements of the movable table. The error motion or the travel straightness of the moving table can be displayed and recorded by the computer notebook. Figure 7 shows the measurement results. The straight line is a least-squares fitting line. It shows that the travel direction of the moving table is not exactly in the Y direction. However, the real deviation can be obtained by subtracting the data point from the point calculated by the solid straight line. The results are net straightness errors, as shown in Fig. 8. All the data in Fig. 8 should be divided by a slope $k = 0.277$ to obtain the actual straightness error of the moving table.

Fig. 5 Resolution test results of the system.

Fig. 6 Repeatability test results of the system.

4 Discussion of Errors

Here, we limit our discussion to the errors caused by the optical components of the system. Two important parameters of the straightness reflector and the triangular prism are the flatness of the hypotenuse refractive surface and the deviation of the angle θ . We assume that $\lambda = 633 \text{ nm}$, $\theta = 15 \text{ deg}$, $n = 1.510$. Then, $\beta = 66.99 \text{ deg}$.

According to Eq. (5), we can define

$$k = 2 \cos(\theta + \beta). \quad (9)$$

Now, differentiating Eqs. (9) and (1) with respect to θ , we obtain

$$\Delta k = 2 \sin(\theta + \beta) \left(\frac{n \cos \theta}{\sin \beta} - 1 \right) \Delta \theta. \quad (10)$$

If $\theta = 15 \text{ deg} \pm 5 \text{ min}$, $\Delta \theta = \pm 5 \text{ min} = \pm 0.00145 \text{ rad}$, then $\Delta k = \pm 0.00168$. Thus, slope $k = 0.2785 \pm 0.00168$. In the experiments, the regression slope k is 0.277 and is within the tolerance range.

The flatness error e of the hypotenuse refractive surface of the triangular prism will cause an optical path length change Δ in the measurement. Then the following equation can be derived:

$$\Delta = \frac{2e(1 - n \cos \alpha)}{\sin \beta} \quad (11)$$

Fig. 7 Travel straightness measurement results for a moving table

Fig. 8 Net straightness error of the moving table.

5 Conclusion

A new interferometric system for straightness measurement was developed. A newly designed straightness reflector is fabricated and gives good performance. The relationship between the OPD and the lateral displacement was derived. The expected result agrees with the result in the experiment. A straightness measurement with an accuracy of $0.1 \mu\text{m}$ is obtained over a travel range of 120 mm. Aside from measuring straightness, the system is also suitable for high-accuracy lateral vibration monitoring and for motion control systems.

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