Linear fracture mechanics for isothermal and adiabatic cracking in piezoelectric materials

Tong-Yi Zhang

Department of Mechanical Engineering
Center for Advanced Engineering Materials
Hong Kong University of Science and Technology
Clear Water Bay, Kowloon
Hong Kong

ABSTRACT

The isothermal full Gibbs function which is negative definite and the adiabatic internal energy which is positive definite are used to develop a linear fracture theory for isothermal and adiabatic cracking in a piezoelectric material, respectively. Both the full Gibbs function release rate and the internal energy release rate for crack propagation are evaluated and revealed to be positive functions of crack lengths and remote loads, even under pure electric loads only. When the electric permittivity inside the crack is treated as zero, each of the full Gibbs function release rate and the internal energy release rate has a compact form in terms of the mode II, I, III stress and electric displacement intensity factors.

Keywords: linear fracture mechanics, piezoelectric materials, isothermal cracking, adiabatic cracking

1. INTRODUCTION

Due to their intrinsic electro-mechanical coupling behavior, piezoelectric materials have been widely used in technology as sensors, transducers and actuators. The mechanical reliability of these materials becomes increasingly important as they are used in more and more sophisticated areas. Therefore, there has been tremendous interest in studying the fracture mechanics of piezoelectric materials¹⁻¹⁷. However, there is still confusion regarding the appropriate thermodynamic functions to be used to analyze the isothermal and adiabatic fracture processes. The electric enthalpy was used in previous analytic work^{5,8}, in order to maintain strains and electric field strengths as independent variables. Since the electric enthalpy is not positive definite, it may lead to a negative energy release rate, especially, the energy release rate is always negative under pure electric loading. To solve this problem, Park and Sun¹⁶ proposed that only the mechanical strain energy release rate should be used as a fracture criteria. From energy conservation point of view, however, the total energy in a domain containing a sample and its loading system must be conservative. For a reversible processing, i.e., for a Griffith crack, the surface energy of the new surfaces created during fracture must equal to the released total energy including both the mechanical and the electric energies. Our previous works^{9,17} have indicated that the total energy release rate is determined by the effective applied stress if the internal energy is used and the electric field inside a slit crack is considered. The crack width and length also have great effects on the energy release rate^{14,17}. Recently, Zhang and Tong¹⁸, and Zhang¹⁹ have used the full Gibbs function and the internal energy to systematically study isothermal and adiabatic cracking, respectively. Adiabatic cracking means during cracking there is no thermal exchange between the domain of interest and its surrounding, which is a good approximation for fast cracking. For a slow fracture processing, thermal flow occurs between the domain and its surrounding. If the cracking is extremely slow, the systems will remain at a constant temperature, and hence the fracture

processing is isothermal. In the present paper, we reviewed our previous work^{9,14,15,17,18,19} for isothermal and adiabatic cracking in piezoelectric materials.

2. ISOTHERMAL CRACKING

Following Parton and Kudryavtsev³, the full Gibbs function per unit volume, g is defined as

$$g = u - \sigma_{ij} \epsilon_{ij} - E_i D_i - Ts, \quad (1)$$

for which

$$dg = -\epsilon_{ij} d\sigma_{ij} - D_i dE_i - s dT, \quad (2)$$

where u and s are the internal energy and entropy per unit volume, respectively; T is the absolute temperature; $\mathbf{E}$ and $\mathbf{D}$ are the electric field strength and electric displacement vectors, respectively; and σ_{ij} , and ϵ_{ij} are the components of stress and strain tensors, respectively. At a constant temperature, Eq. (2) reduces to

$$dg = -\epsilon_{ij} d\sigma_{ij} - D_i dE_i, \quad (3)$$

The independent variables in Eq. (3) are the stresses and the electric field strengths. The constitutive equations are given by

$$\epsilon_{ij} = s_{ijkl}^{E,T} \sigma_{kl} + d_{kij}^T E_k, \quad D_i = d_{kij}^T \sigma_{kl} + \kappa_{ik}^{\sigma,T} E_k, \quad (4)$$

where $s_{ijkl}^{E,T}$ are the isothermal elastic compliances at a constant electric field strength; $\kappa_{ik}^{\sigma,T}$ are the isothermal dielectric permittivities at a constant stress; d_{kij}^T are the isothermal piezoelectric moduli. There is a relation between the isothermal piezoelectric moduli d_{kij}^T and isothermal piezoelectric constants e_{mij}^T :

$$d_{mij}^T = e_{mkl}^T s_{kl ij}^{E,T}, \quad e_{mij}^T = d_{mkl}^T c_{kl ij}^{E,T}, \quad (5)$$

where $c_{kl ij}^{E,T}$ are the isothermal elastic constants at a constant electric field strength. The full Gibbs function per unit volume at a constant temperature can be evaluated from Eqs. (3) and (4). It is given by

$$g = -\frac{1}{2} s_{ijkl}^{E,T} \sigma_{kl} \sigma_{ij} - d_{kij}^T E_k \sigma_{ij} - \frac{1}{2} \kappa_{ki}^{\sigma,T} E_k E_i = -\frac{1}{2} \epsilon_{ij} \sigma_{ij} - \frac{1}{2} D_i E_i. \quad (6)$$

Eq.(6) shows that the full Gibbs function density can be apparently divided into two terms: mechanical one and electrical one. It was proved that the full Gibbs function density is negatively defined¹⁸.

The governing equations are given by:

$$\epsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad E_i = -\phi_{,i}, \quad \sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad (7)$$

with boundary conditions

$$\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{t}, \quad (\mathbf{D}^1 - \mathbf{D}^2) \cdot \mathbf{n} = -q, \quad (\mathbf{E}^1 - \mathbf{E}^2) \times \mathbf{n} = 0, \quad (8)$$

where u_i is a component of the displacement vector, ϕ is the electric potential, q is the density of free charge on the surface between the two media, $\mathbf{t}$ is the traction along the boundary, and $\mathbf{n}$ is the unit vector normal to the boundary.

For an isothermal process, the statement of the total virtual work for the entire domain, Π , takes the following form¹⁵

$$\int_{\Gamma} (u_i \delta t_i - D_i \delta \phi n_i) d\Gamma + \int_{\Pi} \delta g d\Pi = 0, \quad (9)$$

where Γ denotes the boundary of the domain.

Consider a pre-cracked piezoelectric sample. Following Rice's treatment²⁰ of elastic fracture mechanics, we define an isothermal potential energy P as

$$P = \int_{\Gamma} (u_i t_i - D_i \phi n_i) d\Gamma + \int_{\Pi} g d\Pi, \quad (10)$$

and the J-integral as

$$J = \frac{\partial P}{\partial A}, \quad (11)$$

where A is the area of crack surface. Like the full Gibbs function, both the potential energy and the J-integral can be apparently divided into two parts, the mechanical and the electric, as follows:

$$P = P_m + P_e, \quad P_m = \int_{\Gamma} u_i t_i d\Gamma - \int_{\Pi} (\int \varepsilon_{ij} d\sigma_{ij}) d\Pi, \quad P_e = \int_{\Gamma} -D_i \phi n_i d\Gamma - \int_{\Pi} (\int D_i dE_i) d\Pi, \quad (12)$$

$$J = J_m + J_e, \quad J_m = \frac{\partial P_m}{\partial A}, \quad J_e = \frac{\partial P_e}{\partial A}. \quad (13)$$

It should be pointed out that the piezoelectricity couples the mechanical and electric fields. This means that the mechanical field is caused by both mechanical and electric loads, as is the electric field.

During experiments, the generalized displacement, Δ , force, F , charge, Q , and voltage, V , should be monitored to measure the isothermal J-integral. The magnitude of the full Gibbs function of a piezoelectric sample under mechanical and electric loads is the sum of the area under the generalized displacement-force curve and that under the charge-voltage curve. P_m equals the area between the displacement-force curve and the displacement axis and P_e equals the area between the charge-voltage curve and the charge axis. Using multi-samples with identical geometry but different crack lengths, the J-integral can be measured experimentally. Take two samples as an example. One sample has a crack length a and the other has a crack length $a + \Delta a$. Figures 1a-d show ΔP_m and ΔP_e . Then the J-integral is approximately determined by

$$J \approx \frac{\Delta P_m + \Delta P_e}{B\Delta a} \quad \text{or} \quad J = \lim_{\Delta a \rightarrow 0} \frac{\Delta P_m + \Delta P_e}{B\Delta a}, \quad (14)$$

where B is the sample thickness. Although the Δ - F and Q - V curves are drawn in a linear manner in Fig. 1a-d, the method proposed here should hold for nonlinear cases.

Fig. 1a. Isothermal displacement-force plot for two pre-cracked samples under the same force.

Fig. 1b. Isothermal displacement-force plot for two pre-cracked samples under the same displacement.

Fig. 1c. Isothermal charge-voltage plot for two pre-cracked samples under the same voltage.

Fig. 1d. Isothermal charge-voltage plot for two pre-cracked samples under the same charge.

3. ADIABATIC CRACKING

The total differential of the internal energy per unit volume, u , is³

$$du = \sigma_{ij} d\epsilon_{ij} + E_i dD_i + T ds. \quad (15)$$

For an adiabatic process, there is no heat flow, and thus Eq. (15) reduces to

$$du = \sigma_{ij} d\epsilon_{ij} + E_i dD_i. \quad (16)$$

The independent variables in Eq. (16) are the strains and the electric displacements. The stresses and electric field strengths are given by the following constitutive equations:

$$\sigma_{ij} = c_{ijkl}^{D,s} \varepsilon_{kl} - h_{ijk}^s D_k, \quad E_i = -h_{ikl}^s \varepsilon_{kl} + \beta_{ik}^{e,s} D_k, \quad (17)$$

where $c_{ijkl}^{D,s}$ are the adiabatic elastic moduli at constant electric displacements; $\beta_{ik}^{e,s}$ are the adiabatic dielectric permittivities at constant strains; and h_{ijk}^s are the adiabatic piezoelectric moduli. The internal energy per unit volume for an adiabatic process can be evaluated from Eqs. (16) and (17). It is given by

$$u = \frac{1}{2} c_{ijkl}^{D,s} \varepsilon_{kl} \varepsilon_{ij} - h_{kij}^s D_k \varepsilon_{ij} + \frac{1}{2} \beta_{ki}^{e,s} D_k D_i = \frac{1}{2} \varepsilon_{ij} \sigma_{ij} + \frac{1}{2} D_i E_i. \quad (18)$$

Apparently, the internal energy density can be divided into two parts: mechanical and electrical. It was proven also that the internal energy density is positively defined¹⁹. Integrating the internal energy density over the entire domain yields the internal energy which, of course, has a positive value.

For an adiabatic process, the statement of the total virtual work for the entire domain, Π , takes the following form:

$$\int_{\Gamma} (t_i \delta u_i - \phi n_i \delta D_i) d\Gamma - \int_{\Pi} \delta u d\Pi = 0. \quad (19)$$

The adiabatic potential energy P for a pre-cracked sample is defined as

$$P = \int_{\Gamma} (t_i u_i - \phi D_i n_i) d\Gamma - \int_{\Pi} u d\Pi. \quad (20)$$

Similarly, the potential energy can be apparently divided into two parts, the mechanical and the electric, as follows:

$$P = P_m + P_e, \quad P_m = \int_{\Gamma} t_i u_i d\Gamma - \int_{\Pi} (\int \sigma_{ij} d\varepsilon_{ij}) d\Pi, \quad P_e = \int_{\Gamma} -\phi D_i n_i d\Gamma - \int_{\Pi} (\int E_i dD_i) d\Pi. \quad (21)$$

The J-integral can also be apparently divided into the mechanical and the electric parts as given by Eq. (13). It should be pointed out again that the piezoelectricity couples the mechanical and electric fields. This means that the mechanical field is caused by both mechanical and electric loads, as is the electric field.

The value of the internal energy of a piezoelectric sample under mechanical and electric loads is the sum of the area under the generalized force-displacement curve and that under the voltage-charge curve. P_m equals the area between the force-displacement curve and the force axis and P_e equals the area between the voltage-charge curve and the voltage axis. Using multi-samples with identical geometry but different crack lengths, the adiabatic J-integral can be measured experimentally. Two samples are taken to demonstrate this method. One sample has a crack length a and the other has a crack length $a + \Delta a$. Figures 2a-d show ΔP_m and ΔP_e . Then the adiabatic J-integral is approximately determined using Eq. (14). As stated above for the

isothermal cracking, the method proposed here for the adiabatic cracking should also hold for nonlinear cases.

Fig.2a. Adiabatic force-displacement plot for two pre-cracked samples under the same displacement.

Fig.2b. Adiabatic force-displacement plot for two pre-cracked samples under the same force.

Fig.2c. Adiabatic voltage-charge plot for two pre-cracked samples under the same charge.

Fig.2d. Adiabatic voltage-charge plot for two pre-cracked samples under the same voltage.

4. THE FULL GIBBS FUNCTION ENERGY RELEASE RATE FOR ISOTHERMAL CRACK PROPAGATION

The anisotropic stress and electric fields can be determined by solving a standard eigen-equation^{18,19}

$$N\zeta_{\alpha} = p_{\alpha}\zeta_{\alpha}, \quad \text{with} \quad \zeta = \begin{pmatrix} A \\ L \end{pmatrix} \quad (22)$$

where N is a 8×8 matrix depended on materials constants and sample orientation, p_{α} and ζ_{α} are, respectively, the eigen-value and eigen-vector. Moreover, the matrix B ,

$$B = iAL^{-1} = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_{44} \end{pmatrix}, \quad (23)$$

is a Hermitian matrix, where $\mathbf{B}_1$ is the 3x3 upper left-hand block and B_{44} is the lower right-hand element. For stable materials Lothe and Barnett²¹ showed that

$$\mathbf{B}_2 = \overline{\mathbf{B}}_3^T, \quad \mathbf{B}_1 \text{ is positive definite, but } B_{44} < 0. \quad (24)$$

If the dielectric constant inside the crack is treated as zero, then, the full Gibbs function release rate for isothermal crack propagation can be explicitly expressed in terms of the mode II, I and III stress and electric displacement intensity factors in a very compact form

$$G^* = \sum_{i,j=1}^3 K_i (B_{ij} + \overline{B}_{ij}) K_j - K_4 (B_{44} + \overline{B}_{44}) K_4, \quad (25)$$

where $(K_1, K_2, K_3, K_4) = (K_{II}, K_I, K_{III}, K_D)$. Eq. (25) indicates that the full Gibbs function release rate is positive definite.

In order to compare the current results with those obtained by Park and Sun¹⁶, the PZT-4 ceramics are used here as a model material. The material constants are given below:

Elastic constants (10^{10} N / m^2):

$$c_{11} = 13.9, \quad c_{12} = 7.78, \quad c_{13} = 7.43, \quad c_{33} = 11.3, \quad c_{44} = 2.56;$$

Piezoelectric constants (C / m^2):

$$e_{31} = -6.98, \quad e_{33} = 13.84, \quad e_{15} = 13.44;$$

Dielectric constants (10^{-9} F / m):

$$\kappa_{11} = 6.00, \quad \kappa_{33} = 5.47;$$

where N is the force in Newtons, C denotes the charge in Coulombs, and F stands for the capacitance in Farad. Since the open mode fracture is most of interest and danger, we just calculate the full Gibbs energy release rate for a mode I crack where only the mechanical and electric loadings along the x_2 direction are considered.

First, the crack tip opening has to be considered because an electric field may cause the crack close for a mode I crack. In that case, the traction-free boundary condition is violated and, consequently, whole analysis collapses. When only $\sigma_{22,\infty}$ and $E_{2,\infty}$ are applied, most experiments are conducted in this way, the crack tip opening in the x_2 direction is given by

$$\Delta u_2 = \sqrt{2ar} \left[((\mathbf{B} + \overline{\mathbf{B}})_{22} + (\mathbf{B} + \overline{\mathbf{B}})_{24} (\mathbf{e} \mathbf{c}^{-1})_{22}) \sigma_{22,\infty} + (\mathbf{e} \mathbf{c}^{-1} \mathbf{e} + \kappa)_{22} E_{2,\infty} \right], \quad (26)$$

where $\mathbf{c}^{-1}$ is reciprocal tensor of the isothermal elastic tensor, r is the distance behind the crack tip, and a is the half crack length. For simplicity, a crack with a half crack length $a = 1 \text{ cm}$ is used in following calculations. Fig. 3 shows the crack tip opening region and closing region. Without any applied stress of $\sigma_{22,\infty}$, a positive electric field strength opens the crack, but a negative electric field strength closes it. If a positive electric field is higher enough, the crack still opens under a compressive load. On the other hand, the crack closes under a tensile load when a large negative electric field strength is applied. When

calculating the full Gibbs energy release rate, we have to make sure that the crack opening condition is still holding.

Fig. 3. Crack opening and closing regions as functions of the applied electric field strength and the applied stress.

Fig. 4. The full Gibbs energy release rate as a function of the applied stress for a given electric field strength. For the given field strength of $-1,000 \text{ V/cm}$, the crack closes when the applied tensile stress is smaller than 1.1 MPa . The transition point is marked by the solid circle.

Fig.4 shows the full Gibbs energy release rate as a function of the remote applied stress, $\sigma_{22,\infty}$ with a given remote applied electric field strength of $E_{2,\infty}$. The full Gibbs energy release rate is always positive, as shown in Fig.4, with or without the applied electric fields. Without the applied electric field, the full Gibbs energy release rate is directly proportional to the square of the applied stress, as in a pure elastic body. A positive applied electric field increases the full Gibbs energy release rate. For a given negative applied electric field strength of $E_{2,\infty} = -1,000 \text{ V/cm}$, the crack remains closed until $\sigma_{22,\infty}$ reaches 1.1 MPa . When the applied stress is higher than 1.1 MPa but smaller than 1.9 MPa , the full Gibbs energy release rate with the negative electric field strength is higher than that without it. The reason is the contribution from the electric field to the full Gibbs energy release rate is larger than the mechanical counterpart. When the applied stress is higher than 1.9 MPa , the full Gibbs energy release rate with the negative electric field is lower than that without it, as illustrated in Fig.4. This is because the mechanical field dominates in this region and the coupling term between the mechanical and electric fields is negative, reducing the full Gibbs energy release rate.

The variation of the full Gibbs energy release rate with the applied electric field is shown in Fig.5. Without considering the crack closing, all curves of the full Gibbs energy release rate versus the applied electric field strength are concave and positive even without mechanical loading, which is consistent with the previous work on mode III cracks^{9,17}. However, the curves of the energy release rate versus the electric

field have convex shapes if the electric enthalpy was used^{5,8,16}. Only a positive electric field can propagate the crack without any applied mechanical load because the crack closes under a pure negative electric field strength. For a given mechanical load of $\sigma_{22,\infty}=2$ MPa, the applied electric field strength has to be larger than -1,900 V/cm to keep the crack open. In this case, the full Gibbs energy release rate decreases with increasing the electric field strength, reaches a minimum value and then increases.

Fig. 5. The full Gibbs energy release rate as a function of the applied electric field strength for a given stress. If no mechanical load is applied, the electric field strength must be positive to keep the crack open. For the applied tensile stress of 2 MPa, the electric field strength has to be higher than -1,900 V/cm to satisfy the crack opening condition. Two solid circles mark the two transition points.

Fig. 6. The applied stress as a function of the applied electric field strength for a given full Gibbs energy release rate.

Fig. 6 indicates that for a given full Gibbs energy release rate, the remote applied stress decreases with increasing electric field strength. The crack opening condition is satisfied for all the three curves. As can be seen in Fig. 6, in order to reach a certain level of driving force for crack propagation, a higher stress is required if the applied electric field strength is negative; the opposite is also true that a positive electric field reduces the applied stress. The results agree, at least quantitatively, with the experimental observations¹⁶.

If the dielectric constant inside the crack is not treated as zero, we may consider a piezoelectric sample as a capacitor. Its capacitance depends, to a large extent, on the crack width. As long as the crack width is assumed to be zero, the capacitance remains the same with or without a crack. That means a pure

electric load will not provide any driving force for crack propagation. The size effects have been simply discussed in a previous note¹⁴. A real crack actually has a finite width and a finite crack opening which depends on the geometry of the sample, the material properties and the loading conditions. Therefore the size effects could be very complicated and will be comprehensively studied later.

5. THE INTERNAL ENERGY RELEASE RATE FOR ADIABATIC CRACK PROPAGATION

The difference of the fracture mechanics for adiabatic cracking from that for isothermal cracking exists in materials properties, while the formulas have the similar forms^{18,19}. The materials properties for isothermal and adiabatic cracking are already introduced, respectively, in sections 2 and 3. Assuming all the material properties are constant, we have the following correlation between the isothermal and adiabatic properties:

$$c_{ijk}^{E,s} = c_{ijkl}^{E,T} + \frac{T\lambda_{ij}^E\lambda_{kl}^E}{\rho c^{\epsilon,E}}, \quad e_{ijm}^s = e_{ijm}^T - \frac{Tp_m^{\epsilon}\lambda_{ij}^E}{\rho c^{\epsilon,E}}, \quad \kappa_{mk}^{\epsilon,s} = \kappa_{mk}^{\epsilon,T} - \frac{Tp_m^{\epsilon}p_k^{\epsilon}}{\rho c^{\epsilon,E}}. \quad (27)$$

where λ_{ij}^E are the temperature stress coefficients at constant electric field strengths; p_m^{ϵ} are the pyroelectric coefficients which determine the temperature dependence of the electric displacement; $c^{\epsilon,E}$ is the specific heat at constant strains and electric field strengths; and ρ is the mass density of the material. Since the formulas for adiabatic cracking look the same as that for isothermal cracking, the equations given in section 4 for isothermal cracking can, in general, be used for adiabatic cracking. It should be kept in mind that the difference in materials properties may or may not change fracture behavior, which should be treated carefully for each individual piezoelectric material. The details were reported in References [18] and [19], respectively, for isothermal and adiabatic cracking.

6. CONCLUDING REMARKS

The present work used the full Gibbs energy and the internal energy to develop a linear fracture mechanics theory for isothermal cracking and adiabatic cracking in piezoelectric materials. The full Gibbs function is negative definite and the internal energy is positive definite, each of which leads to a positive energy release rate for crack propagation. If the dielectric constant of the crack is treated as zero, the release rate can be expressed in a compact form in terms of the four intensity factors. In this case the curves of the full Gibbs energy release rate versus the applied electric field strength are concave, while the electric enthalpy energy release rate versus the applied electric field strength are convex. For a given full Gibbs energy release rate, a positive electric field strength reduces the applied stress and a negative electric field increases it. Similar results are obtained for adiabatic cracking.

7. ACKNOWLEDGMENTS

The authors are grateful to Mr. Caifu Qian for his plotting the figures. Financial support from the Hong Kong Research Grants Council under RGC grant HKUST573/94E is gratefully acknowledged.

8. REFERENCES

1. W.F.J. Deeg. "The analysis of dislocation, crack, and inclusion problems in piezoelectric solids." *PhD Thesis*, Stanford University, 1980.
2. Y.E. Pak and G. Herrmann. "Conservation laws and the material momentum tensor for the elastic dielectric." *Int. J. Engng. Sci.* Vol. 24, pp. 1365-1374, 1986.
3. V.Z. Parton and B.A. Kudryavtsev. "Electromagnetoelasticity Piezoelectrics and Electrically Conductive Solids." Gordon and Breach Science Publishers, New York, 1988.
4. R.M. McMeeking. "Electrostrictive stresses near crack-like flaws." *J. Appl. Math. Phys.* Vol. 40, pp. 615-627, 1989.
5. P.E. Pak. "Crack extension force in a piezoelectric material." *J. Appl. Mech.* Vol. 57, pp. 647-653, 1990.
6. H. Sosa and Y.E. Pak. "Three-dimensional eigenfunction analysis of a crack in a piezoelectric material." *Int. J. Solids Structures*, Vol. 26, pp. 1-15, 1990.
7. H. Sosa. "Plane problems in piezoelectric media with defects." *Int. J. Solids Structures* Vol. 28, pp. 491-505, 1991.
8. Z. Suo, C.M. Kuo, D.M. Barnett, and J.R. Willis. "Fracture mechanics for piezoelectric ceramics." *J. Mech. Phys. Solids* Vol. 40, pp. 739-765, 1992.
9. T.-Y. Zhang and J.E. Hack. "Mode-III cracks in piezoelectric materials." *J. Appl. Phys.* Vol. 71, pp. 5865-5870, 1992.
10. H. Sosa. "On the fracture mechanics of piezoelectric solids." *Int. J. Solids Structures* Vol. 29, pp. 2613-2622, 1992.
11. P.E. Pak. "Linear electro-elastic fracture mechanics of piezoelectric materials." *Int. J. Fracture* Vol. 54, pp. 79-100, 1992.
12. Y.E. Pak and A. Tobin. "On electric field effects in fracture of piezoelectric materials." *Mech. Electromagn. Mater. Structures* Vol. AMD 161/MD 42, pp. 51-62, 1993.
13. Z. Suo. "Models for breakdown-resistant dielectric and ferroelectric ceramics." *J. Mech. Phys. Solids* Vol. 41, pp. 1155-1176, 1993.
14. T.-Y. Zhang. "Effect of sample width on the energy release rate and electric boundary conditions along crack surfaces in piezoelectric materials." *Int. J. Fracture* Vol. 66, pp. R33-R38, 1994.
15. T.-Y. Zhang. "J-integral measurement for piezoelectric materials." *Int. J. Fracture* Vol. 68, pp. R33-R40, 1994.
16. S. Park and C.-T. Sun. "Fracture criteria for piezoelectric ceramics." *J. Am. Ceram. Soc.*, Vol. 78, pp. 1475-1480, 1995.
17. T.-Y. Zhang and P. Tong. "Fracture mechanics for a mode-III crack in a piezoelectric material." *Int. J. Solids Structures* Vol. 33, pp. 343-359, 1996.
18. T.-Y. Zhang and P. Tong. "Analysis of elliptical cavities in piezoelectric solids: application to fracture mechanics." unpublished work, 1995.
19. T.-Y. Zhang. "Linear fracture mechanics for adiabatic cracking in piezoelectric materials." unpublished work, 1995.
20. J.R. Rice. "Mathematical analysis in the mechanics of fracture." *Fracture*, Vol. II, ed. H. Liebowitz, Academic Press, New York, pp. 191-311, 1968.
21. J. Lothe and D.M. Barnett. "Integral formalism for surface waves in piezoelectric crystals. Existence considerations." *J. Appl. Phys.* Vol. 47, pp. 1799-1807, 1975.