Fermi-liquid behavior and spin-charge separation in underdoped cuprates: Gaussian fluctuations in the $U(1)$ slave-boson mean-field theory of the t - J model

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The Gaussian fluctuations in slave-boson mean-field theory of the t - J model are analyzed in this paper in the low-doping regime where the superconducting to normal (pseudogap phase) transition is driven by vanishing of Bose-condensation amplitude. By eliminating the boson and constraint fields exactly in the linear-response regime, we show that the Gaussian theory describes an unconventional Fermi liquid superconductor where the superconducting to normal transition is a new type of phase transition characterized by a change of Landau parameter from $F_1 > -1$ in the superconducting phase to $F_1 = -1$ in the pseudo-gap phase, corresponding to a state with spin-charge separation. The consequences of this proposal are discussed.

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With accumulated experimental results, it is now believed that at low temperature, high- T_c superconductors are BCS-like d -wave superconductors close to the Mott insulator.¹⁻³ Strong electron correlation does not destroy Fermi-liquid behavior, and the ground state of the superconducting states remains Fermi-liquid- (superconductor)^{3,4} like. However, the situation is much less clear in the normal state (pseudogap regime) of underdoped cuprates where a d -wave BCS-like gap seems to survive, although the system is already a normal metal. Photoemission experiment indicates that a segmented Fermi surface seems to exist in this regime,^{1,5} which is hard to reconcile with a Fermi liquid description. A theoretical challenge is thus whether a phenomenological Fermi-liquid-type description can be found for the pseudogap phase, and determining what is the nature of the corresponding superconducting to normal transition.

The purpose of our paper is to address this question based on the $U(1)$ slave-boson mean-field theory (SBMFT) of the t - J model.^{6-9,11} SBMFT provides an unconventional description of the high- T_c cuprates where the pseudogap phase is described as a state with a d -wave spinon gap but superfluidity vanishes because of vanishing (slave)-boson condensation. This description is suggestive but not entirely satisfactory because of the explicit appearance of the slave degree of freedom in the theory. The description would be much more convincing if the physics could be expressed in terms of physical degrees of freedom, or quasiparticles only, without referring to slave particles.

A previous study¹⁰ indicates that at zero temperature, the slave degrees of freedom (constraint field and slave bosons) in SBMFT can be eliminated exactly in Gaussian theory, resulting in a linear transport equation for quasiparticles that has the same form as the transport equation for Fermi-liquid superconductors,¹⁴ with all Landau interaction parameters explicitly given, indicating that SBMFT describes a conventional Fermi-liquid superconductor at zero temperature.¹⁰ A generalization of this approach to finite temperatures shall provide us with valuable information about the behavior of the pseudogap phase. Our goal is to obtain a formulation of SBMFT in terms of physical particles only, and to see how the Landau Fermi-liquid description that comes out naturally in the superconducting state is modified in the pseudogap

state. We shall show that our result indicates a possible interpretation of the superconducting to pseudogap phase transition in terms of a temperature-dependent Landau parameter F_1 .

We consider a generalized t - J model on a square lattice^{6,12} that includes Coulomb interaction between charges.¹⁶ In the slave-boson representation, the Hamiltonian is

$$H = -t \sum_{\langle i,j \rangle \sigma} (b_i b_j^\dagger c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.}) - \mu \sum_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} + J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \frac{1}{2} \sum_{i,j} V(\vec{r}_i - \vec{r}_j) b_i^\dagger b_i b_j^\dagger b_j + \sum_i \lambda_i \left(b_i^\dagger b_i + \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma} - 1 \right), \quad (1)$$

where $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) and $b_i^\dagger$ (b_i) are the spin (fermion) and hole (boson) creation (annihilation) operators at site i , respectively. The electron annihilation operator is given by $f_{i\sigma} = c_{i\sigma} b_i^\dagger$ and $\vec{S}_i$ is the spin operator. The first term in the Hamiltonian represents electron hopping between nearest-neighbor pairs on a square lattice and μ is the chemical potential. The third and fourth terms represent Heisenberg exchange interaction between electron spins and repulsive Coulomb interaction between charges, respectively. The non-double occupancy constraint is enforced by the Lagrange multiplier field λ_i to the Hamiltonian.

Following Ref. 6, we decouple various terms in the Hamiltonian as in SBMFT,

$$b_i b_j^\dagger c_{i\sigma}^\dagger c_{j\sigma} \rightarrow \langle b_i b_j^\dagger \rangle c_{i\sigma}^\dagger c_{j\sigma} + b_i b_j^\dagger \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle - \langle b_i b_j^\dagger \rangle \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle, \quad (2)$$

where $\langle \cdots \rangle$ denotes expectation value. $\langle b_i b_j^\dagger \rangle = \langle b \rangle^2 = x$ at zero temperature, where x = concentration of holes, but has to be determined self-consistently in general.⁶ In particular, $\langle b \rangle = 0$ but $\langle b_i b_j^\dagger \rangle \sim x \neq 0$ above T_c (pseudogap state). Similar to Ref. 10, we decouple $\vec{S}_i \cdot \vec{S}_j$ in terms of $\Delta_{ij} = c_{i\uparrow} c_{j\downarrow} - c_{i\downarrow} c_{j\uparrow}$ and $\chi_{ij} = \sum_{\sigma} c_{i\sigma}^\dagger c_{j\sigma}$, with $\langle \vec{S}_i \cdot \vec{S}_j \rangle \rightarrow \frac{3}{8} (\langle \chi_{ij} \rangle^2 + \langle \Delta_{ij} \rangle^2)$. The boson (density-density) interaction term is decoupled as $n_i n_j \rightarrow \langle n_i \rangle n_j + n_i \langle n_j \rangle - \langle n_i \rangle \langle n_j \rangle$, where $n_i = b_i^\dagger b_i$ (Ref. 16) and the Lagrange multiplier field is replaced by a number function,

$\lambda_i \rightarrow \langle \lambda_i \rangle$ in mean-field theory. The decoupling is a generalized Hartree-Fock approximation including slave and constraint fields.

A time-dependent SBMFT can be formulated by considering the Heisenberg equation of motion $\frac{\partial}{\partial t} \hat{O} = i[\hat{H}, \hat{O}]$ for quadratic operators $\hat{O}_2$'s. The equations of motion for $\langle \hat{O}_2 \rangle$ will generate quartic terms $\langle \hat{O}_4 \rangle$, which are then decoupled according to the above decoupling scheme. To construct transport equations, we apply the above procedure to the expectation values $\langle \Delta_{ij} \rangle$, $\langle \chi_{ij} \rangle$, and $\langle B_{ij} \rangle = \langle b_i b_j^\dagger \rangle$. The resulting equations of motion for these functions are linearized and Fourier transformed,¹⁰ resulting in a set of linear transport equations for the fluctuating variables $\rho_{\vec{k}}(\vec{q}, t) = \sum_{\sigma} \langle c_{\vec{k}+\vec{q}/2\sigma}^\dagger(t) c_{\vec{k}-\vec{q}/2\sigma}(t) \rangle$, $\Delta_{\vec{k}}(\vec{q}, t) = \langle c_{-\vec{k}-\vec{q}/2\uparrow}(t) c_{\vec{k}-\vec{q}/2\downarrow}(t) - c_{-\vec{k}-\vec{q}/2\downarrow}(t) c_{\vec{k}-\vec{q}/2\uparrow}(t) \rangle$, and $\phi_{\vec{k}}(\vec{q}, t) = \langle b_{\vec{k}+\vec{q}/2}^\dagger(t) b_{\vec{k}-\vec{q}/2}(t) \rangle$. An additional equation for the constraint field $\lambda_{\vec{q}}(t)$ is obtained by requiring that the density fluctuations of the fermion field are exactly balanced by the density fluctuations of the boson field, i.e., $[\sum_{\vec{k}} \rho_{\vec{k}}(\vec{q}, t)] + [\sum_{\vec{k}} \phi_{\vec{k}}(\vec{q}, t)] = 0$ for all wave vector $\vec{q}$ and time t in the linear-response regime.¹⁰

Because the equations are linear, the boson variable $\phi_{\vec{k}}(\vec{q}, t)$ and constraint field $\lambda_{\vec{q}}(t)$ can be eliminated exactly from the coupled equations of motions.¹⁰ What is perhaps surprising is that the resulting transport equations for $\rho_{\vec{k}}(\vec{q}, t)$ and $\Delta_{\vec{k}}(\vec{q}, t)$ have exactly the form required by Fermi liquid theory in the $\vec{q} \rightarrow 0$ limit, if we interpret the Fourier transformed variables $\rho_{\vec{k}}(\vec{r}, t)$'s and $\Delta_{\vec{r}}(\vec{r}, t)$ as the local quasiparticle density and Cooper pair amplitudes in Fermi liquid theory, respectively,^{10,13,14} implying that SBMFT at the Gaussian level has the dynamical properties of a generalized Fermi liquid superconductor free of the slave degrees of freedom. The exact form of the equation of motion is rather complicated and we shall not write it down here. Interested readers can look at Ref. 10 for details.

The spectrum of the quasiparticle is given by the fermion mean-field (MF) Hamiltonian,⁶

$$H_{\text{MF}}^f = \sum_{\vec{k}\sigma} \xi_{\vec{k}} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} + \sum_{\vec{k}} [\bar{\Delta}^*(\vec{k}) (c_{\vec{k}\uparrow} c_{-\vec{k}\downarrow} - c_{\vec{k}\downarrow} c_{-\vec{k}\uparrow}) + \text{H.c.}], \quad (3)$$

where $\xi_{\vec{k}} = -(t\bar{B} + \frac{3J}{8}\bar{\chi})\gamma(\vec{k}) + \bar{\lambda} - \mu$, $\bar{B} = \langle B_{ii+\delta} \rangle$, and $\bar{\chi} = \langle \chi_{ii+\delta} \rangle$, where $\delta = \hat{x}, \hat{y}$ and $\gamma(\vec{k}) = 2[\cos(k_x) + \cos(k_y)]$. $\bar{\Delta}(\vec{k}) = \frac{3J}{4} \bar{\Delta}[\cos(k_x) - \cos(k_y)]$ represents the quasiparticle pairing field with $d_{x^2-y^2}$ symmetry, where $\bar{\Delta} = \langle \Delta_{i,i+\hat{x}} \rangle$. The mean-field dispersion for the quasiparticles is given by $E_f(\vec{k}) = \pm \sqrt{\xi_{\vec{k}}^2 + |\bar{\Delta}(\vec{k})|^2}$.

The quasiparticles interact with each other through a temperature, frequency, and wave-vector-dependent Landau interaction $f_{\vec{k}\vec{p}}(q)$ in the transport equation, with $H_{\text{int}} \sim \frac{1}{2} \sum_{\vec{k}, \vec{p}, \vec{q}, \omega} \rho_{\vec{k}}(\vec{q}, \omega) f_{\vec{k}\vec{p}}(q) \rho_{\vec{p}}(-\vec{q}, -\omega)$,¹⁰ where $q = (\vec{q}, \omega; T)$ and

$$f_{\vec{k}\vec{p}}(q) = -\frac{3J}{4} \sum_{\mu=\hat{x}, \hat{y}} \cos(k_\mu - p_\mu) + V(\vec{q}) - \frac{1}{\chi_h(q)} [1 - 2t\chi_h^c(\vec{k}; q)] \times [1 - 2t\chi_h^c(\vec{p}; q)] + (2t)^2 \chi_h^c(\vec{k}, \vec{p}; q), \quad (4)$$

where $\chi_h(q) = \sum_{\vec{k}} \chi_h(\vec{k}', q)$, $\chi_h^c(\vec{k}; q) = \sum_{\vec{k}'} \chi_h(\vec{k}', q) \cos(k_\mu - k'_\mu)$, and

$\chi_h^c(\vec{k}, \vec{p}; q) = \sum_{\vec{k}'} \chi_h(\vec{k}', q) \cos(k_\mu - k'_\mu) \cos(p_\mu - k'_\mu)$, where $\chi_h(\vec{k}', q) = (n_{\vec{k}', -\vec{q}/2}^b - n_{\vec{k}', +\vec{q}/2}^b) / (\omega - \epsilon_{\vec{k}', +\vec{q}/2} + \epsilon_{\vec{k}', -\vec{q}/2})$. $n_{\vec{k}}^b$ are (slave) boson occupation numbers and $\epsilon_{\vec{k}} = -t\bar{\chi}\gamma(\vec{k}) + \bar{\lambda}$ is the mean-field boson dispersion. $V(\vec{q})$ is the Fourier transform of density-density interaction $V(\vec{r})$.¹⁰ Notice that there is no definite relation between quasiparticle effective mass and Landau interaction here because of the absence of translational invariance.¹³ We find that the Fermi liquid form of transport equation and Landau interaction does not change when one goes from the superconducting state to the normal state, and the transition is reflected *only* in the change in the boson response functions χ_h , χ_h^c , and χ_h^{cc} in Eq. (4).

To make the physics more tractable, we consider the limit $|\vec{q}| \ll \pi/2$ and $k_B T \ll t\bar{\chi}$. In this limit, $\epsilon_{\vec{k}} \rightarrow t\bar{\chi}\vec{k}^2$, $n_{\vec{k}}^b \rightarrow n_{|\vec{k}|}^b$, $\bar{B} \rightarrow x$, and we obtain after some algebra

$$f_{\vec{k}\vec{p}}(q) \rightarrow -\frac{t_{\text{eff}}}{\bar{\chi}} \sum_{\mu} (1-z) \cos(k_\mu) \cos(p_\mu) + V(\vec{q}) + 2t \sum_{\mu} [\cos(k_\mu) + \cos(p_\mu)] + \frac{t_{\text{eff}}}{\bar{\chi}} \sum_{\mu, \nu} \left(\frac{1}{t_{\text{eff}} \bar{\chi}} \chi_{h\mu\nu}(q) - \delta_{\mu\nu} \right) \sin(k_\mu) \sin(p_\nu), \quad (5)$$

where $t_{\text{eff}} = 3J\bar{\chi}/4 + 2tx$ and $z = 2tx/t_{\text{eff}}$,¹⁰

$$\chi_{h\nu\mu}(q) = \left[\chi_{hl}(q) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + \chi_{ht}(q) \frac{q_\mu q_\nu}{q^2} \right] = (2t\bar{\chi})^2 \sum_{\vec{k}} k_\mu k_\nu \chi_h(\vec{k}, q) + 2t\bar{\chi} x \delta_{\mu\nu},$$

is the free boson current-current response function, and $\chi_{hl}(q)$ and $\chi_{ht}(q)$ are the corresponding longitudinal and transverse components. Notice that the bosons contribute only to the current-current interaction between quasiparticles in this limit.

To study the effect of bosons and the superconducting-normal transition, we approximate the $\cos(k_\mu)$ terms by their average values on the Fermi surface, defined by $\xi_{\vec{k}} = 0$.¹⁰ The approximation keeps the boson contributions exact but simplifies further analysis. We obtain

$$f_{\vec{k}\vec{p}}(q) \rightarrow U_0(q) + \frac{t_{\text{eff}}}{\bar{\chi}} \sum_{\mu, \nu} \left[F_{1l}(q) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + F_{1t}(q) \frac{q_\mu q_\nu}{q^2} \right] \sin(k_\mu) \sin(p_\nu), \quad (6)$$

where

$$U_0(q) = V(\vec{q}) + aJ + bt, \quad F_{1l(t)}(q) = \chi_{hl(t)}(q) / (t_{\text{eff}} \bar{\chi}) - 1, \quad (7)$$

where a, b are constants coming from averaging the $\cos(k_\mu)$ terms on the Fermi surface.¹⁰ U_0 is the usual Landau interaction that couples to density, whereas F_{1l} and F_{1t} couples to longitudinal and transverse currents, respectively. Notice that

$F_{1l}(q)$ and $F_{1r}(q)$ are identical in the limit $\vec{q} \rightarrow 0$, $\omega \neq 0$, but are different in general.

To understand the consequence of the Landau interaction (6), (7), we compute the density-density and current-current response functions. They are given by the standard Fermi-liquid expressions^{10,13,14}

$$\chi_d(\vec{q}, \omega; T) = \frac{\chi_{0d}(\vec{q}, \omega; T)}{1 - \left[U_0(\vec{q}) + \left(\frac{F_{1l}(q)}{1 + F_{1l}(q)} \right) \frac{\omega^2}{t_{\text{eff}} \bar{\chi} \vec{q}^2} \right] \chi_{0d}(\vec{q}, \omega; T)} \quad (8)$$

and

$$\chi_r(\vec{q}, \omega; T) = \frac{\chi_{0r}(\vec{q}, \omega; T)}{1 - \left(\frac{F_{1r}(q)}{1 + F_{1r}(q)} \right) \frac{\chi_{0r}(\vec{q}, \omega; T)}{(t_{\text{eff}} \bar{\chi})}} \quad (9)$$

where $\chi_{0d}(q)$ and $\chi_{0r}(q)$ are the mean-field density-density and current-current response functions for a BCS superconductor^{10,14} in the absence of Landau interactions, respectively. It is straightforward to show that at $T \rightarrow 0$, $\bar{\chi} = 4t\bar{\chi}$,¹⁰ $F_{1l}(q) \rightarrow z - 1$, $F_{1r}(q) \rightarrow z\omega^2/(\omega^2 - \epsilon_q^2) - 1$, and the response functions reduce to the ones obtained in Ref. 10.

The response functions can be understood in the language of $U(1)$ gauge theory. In the small- x limit, $\chi_{hl}(q)^{-1}$ and $\chi_{ht}(q)^{-1}$ are both of order x^{-1} and we can neglect terms of order $O(1)$ compared with those in Eqs. (8) and (9). Using Eq. (7) and the identity $\chi_{hl}(q) = \omega^2 \chi_h(q)/\vec{q}^2$, it is easy to see that

$$\chi_{d(t)}(\vec{q}, \omega; T) \rightarrow \frac{\chi_{0d(t)}(\vec{q}, \omega; T) \chi_{hd(t)}(\vec{q}, \omega; T)}{\chi_{0d(t)}(\vec{q}, \omega; T) + \chi_{hd(t)}(\vec{q}, \omega; T)},$$

where the usual Ioffe-Larkin result is recovered.^{7,8,17} Recall that in this language, the normal (pseudogap) state is described by a state with zero Bose condensation, where $\chi_{ht}(0,0) \rightarrow 0$ because of gauge invariance and the corresponding superfluid density $\rho_s = \chi_r(0,0)$ vanishes.^{7,8,15} In our corresponding transport equation description, the vanishing of superfluid density is characterized by $1 + F_{1r}(0,0) \rightarrow 0$ [Eq. (7)]. Recall that $1 + F_{1r}(q)$ renormalizes (charge) current carried by quasi-particles in Fermi liquid theory,¹⁴ and $1 + F_{1r}(0,0) \rightarrow 0$ implies that quasiparticle carries no current in the long-wavelength and zero-frequency limit $(\vec{q}, \omega) \rightarrow (0,0)(t_{\text{eff}} \bar{\chi} |\vec{q}| \gg \omega)$. The loss of superfluidity in the pseudogap state is thus not due to renormalization of superconducting gap $\bar{\Delta} \rightarrow 0$ in this scenario, but due to vanishing of current carried by quasiparticles. Superconductivity “decouples” from a transverse electromagnetic field in the dc response of the pseudogap phase as a result, and the system becomes a normal metal.

Our analysis thus suggests that within the framework of Fermi liquid theory, an unconventional phase where (charge) current is decoupled from quasiparticles may exist. The phase can be identified naturally as a phase with spin-charge separation as suggested in gauge theory approaches,^{7-9,18} and our analysis provides a “Fermi-liquid” definition for this

FIG. 1. Schematic phase diagram of the phenomenological Fermi-liquid superconductor. Below T^* , $\Delta \neq 0$. Above T_b , $1 + F_1 = 0$; below it, $1 + F_1 \sim x$. Region (I) is the SC phase, (II) is the KT phase, and (III) is the spin-charge separated phase. The paraconductivity regime lies in region (II).

spin-liquid phase. Notice that in our description, spin-charge separation occurs rigorously only in the limit $(\vec{q}, \omega) \rightarrow (0,0) \times (t_{\text{eff}} \bar{\chi} |\vec{q}| \gg \omega)$ and remains coupled at any finite $(\vec{q}, \omega)$. In this sense, the state we obtained here can be characterized as a critical state with “marginal” spin-charge separation and the pseudogap phase is a “marginally” spin-charge separated state on top of a BCS superconductor. Notice that a “marginally” spin-charge separated state on top of a normal metal may also exist. In fact, it has been suggested that the normal state in optimally doped cuprates is an example of such a state.⁷ The charge dynamics of this state is subtle because of its critical nature and will be discussed separately.

It is interesting to raise a more general question: within the theoretical framework of Fermi liquid superconductor, how many ways can a superconducting to normal transition occur? From Eq. (9) (see also Ref. 14), we observe that the superfluid density $\rho_s = \chi_r(\vec{q} \rightarrow 0, 0)$ may vanish either when (i) $\chi_{0r}(\vec{q} \rightarrow 0, 0) \rightarrow 0$ or (ii) $F_{1r}(0,0) \rightarrow -1$. Situation (i) corresponds to the usual BCS mean-field transition or may occur because of strong phase fluctuation in the order-parameter wave function.¹⁹ Situation (ii) corresponds to vanishing of current carried by quasiparticles, and our analysis suggests that this may be what is happening in high- T_c cuprates. There seem to be no other possibilities within the framework of Fermi-liquid superconductor.

The Fermi-liquid spin-charge separation phenomenology we developed can readily explain several unusual properties of the pseudogap phase. Notice that in our description, the zero-temperature superfluid density is given by $\rho_s(T=0) = \rho_s^{(0)} \frac{m}{m^*} (1 + F_1)$, where $1 + F_1 \sim x$ is the hole concentration and $m/m^* \sim 1$, $\rho_s^{(0)}$ is the superfluid density in BCS theory. This result immediately explains a qualitative feature that is unaccountable by the ordinary BCS theory: the separation of the pseudogap temperature T^* and the normal-superconducting transition temperature T_c . As has been pointed out by Emery and Kivelson,¹⁹ the pseudogap temperature ($\sim$ BCS mean-field transition temperature T_{MF}) and T_c will be separated if we identify the normal-superconducting transition as a Kosterlitz-Thouless (KT) transition where $T_c \sim T_{KT} \sim \frac{\hbar^2}{4m^*} \rho_s$.²⁰ $T_c \sim x$ will be separated from T^* if $1 + F_1 \sim x$ in the underdoped regime.

However, as has been pointed out by a number of authors,^{3,9} the KT transition picture alone cannot explain a number of properties of underdoped cuprates, including the appearance of a Fermi arc observed in photoemission experiment in the normal state,¹⁻³ and the narrowness of the paraconductivity regime,^{3,9} where, for example, the Nernst signal is strong.²¹ These features can be explained if we take the spin-charge separation transition into account. Assuming that the spin-charge separation transition occurs at a temperature $T_c^* \sim x$, the superfluid density would vanish at T_c^* and as a result the KT transition will occur at a lower temperature $T_c \sim \alpha T_c^*$, where $\alpha < 1$ is a numerical factor of order $O(1)$ in SBMFT of the t - J model. As a result, there are actually three phases below T^* in our picture. This scenario is shown schematically in the phase diagram Fig. 1. The transition between regions (I) and (II) is the superconducting-normal (KT) transition, whereas the transition between regions (II) and (III) describes the spin-charge separation transition, smeared out

by the KT transition into a crossover. The appearance of a Fermi arc/pocket in the quasiparticle spectrum in the spin-charge separated phase has been shown in Ref. 22, and the picture together with the Ioffe-Larkin expression for conductivity ($\sigma^{-1} = \sigma_f^{-1} + \sigma_b^{-1}$) provide also a natural explanation of the narrowness of the paraconductivity regime. In the vortex-liquid (KT) phase [region (II)], $\sigma_b \rightarrow \infty$, so $\sigma \sim \sigma_f$ is the vortex-liquid conductivity; in the spin-charge separated regime (III), $\sigma_b(\sim x) \ll \sigma_f(\sim 1-x)$, so $\sigma \sim \sigma_b$, the conductivity is dominated by the hole conductivity $\sim$ conductivity of charged boson gas, and the paraconductivity of the fermion component becomes invisible. The detailed transport behavior of the pseudogap phase in the spin-charge separation picture will be discussed in a separate paper.

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