Quantum coherence control of solid-state charge qubit by means of a suboptimal feedback algorithm

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The quantum coherence control of a solid-state charge qubit is studied by using a suboptimal continuous feedback algorithm within the Bayesian feedback scheme. For the coherent Rabi oscillation, the present algorithm suggests a simple bang-bang control protocol, in which the control parameter is modulated between two values. For the coherence protection of the idle state, the present approach is applicable to arbitrary states, including those lying on the equator of the Bloch sphere which are out of control in the previous Markovian feedback scheme.

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I. INTRODUCTION

Being stimulated by the interest of solid-state quantum computation, in recent years the measurement problem of the solid-state qubit is becoming an intensive research subject. In particular, the setup of a charge qubit measured by a mesoscopic transport device, for instance, the quantum point contact (QPC), has received considerable attention.¹⁻⁴ Very recently, the mesoscopic QPC was experimentally demonstrated as a charge-sensitive electro-meter.⁵

Strikingly, rather than the projective strong measurement, the continuous weak measurement can be employed to realize some marvellous tasks, such as quantum state initialization, stabilization, and coherence protection, etc.⁶⁻⁸ The central idea is the measurement-result-based feedback control, which falls typically into two main categories, say, the Markovian scheme⁶ and the Bayesian scheme.^{9,10} In quantum optics, the study of quantum feedback control has been going on for more than a decade. However, in solid states it is a relatively new subject.¹¹⁻¹⁴ In this work, based on a suboptimal feedback algorithm,¹⁵ we study the problem of quantum coherence protection of a solid-state charge qubit which is subject to the continuous weak measurement by a mesoscopic QPC.

II. FORMALISM

To be specific, the charge qubit is assumed to be two coupled quantum dots (CQD's), occupied by a single electron,^{1,16,17} whose Hamiltonian reads $H_{qb} = \varepsilon(c_2^\dagger c_2 - c_1^\dagger c_1) + \Omega(c_1^\dagger c_2 + c_2^\dagger c_1)$. Here $c_i^\dagger$ (c_i) is the electron creation (annihilation) operator in the i th dot, ε and Ω are, respectively, the energy offset of the two local dot-states and their coupling strength. At zero-temperature and in the limit of large bias voltage across the QPC, the measurement back-action on the qubit is described by the following master equation ($\hbar=1$):¹⁷

$$\dot{\rho}(t) = -i[H_{qb}, \rho(t)] + \mathcal{D}[T + \mathcal{X}n_1]\rho(t), \quad (1)$$

where $n_1 = c_1^\dagger c_1$ is the occupation-number operator of the first dot, the parameters T and $\mathcal{X}$ are given by $|T|^2 = 2\pi|T|^2 g_L g_R eV \equiv D_2$, and $|T + \mathcal{X}|^2 = 2\pi|T + \mathcal{X}|^2 g_L g_R eV \equiv D_1$.

Here, the tunneling amplitudes through QPC, T , and $\mathcal{X}$, are assumed to be independent of the reservoir states, and real for simplicity; g_L and g_R are the density of states for the left and right reservoirs; $V = (\mu_L - \mu_R)/e$ is the bias voltage across the QPC. The superoperator in Eq. (1) is defined as $\mathcal{D}[r]\rho = \mathcal{T}[r]\rho - \mathcal{A}[r]\rho$, where $\mathcal{T}[r]\rho = r\rho r^\dagger$, and $\mathcal{A}[r]\rho = (r^\dagger r \rho + \rho r^\dagger r)/2$. Intuitively, corresponding to the qubit electron in either the first or the second dot, the QPC measurement current of $I_1 = eD_1$ or $I_2 = eD_2$ is expected. However, for continuous weak measurement, owing to the qubit electron jumps between the two dots, the continuous output current $I(t)$ is a complicated stochastic random quantity. But, by using $I(t)$ one can design a proper feedback Hamiltonian $H_{fb}(t)$ to maintain or improve the quantum coherence of the qubit. Conditioned on $I(t)$ and involving $H_{fb}(t)$, Eq. (1) is unravelled as¹⁷⁻¹⁹

$$\begin{aligned} d\rho_c(t) = & -i[H_{qb}, \rho_c]dt + \sum_{j=x,y,z} \gamma_j \mathcal{D}[\sigma_j]\rho_c dt + \mathcal{D}\left[\frac{\mathcal{X}}{2}\sigma_z\right]\rho_c dt \\ & + \mathcal{H}\left[\frac{\mathcal{X}}{2}\sigma_z\right]\rho_c dW(t) - i[H_{fb}(t), \rho_c]dt. \end{aligned} \quad (2)$$

Compared to Eq. (1), in addition to the feedback term (the fifth one), we have included here the possible noise term (the second one) due to external environment, which would spoil the quantum coherence and needs to be eliminated by the feedback control strategy. In Eq. (2), the superoperator “ $\mathcal{H}$ ” is defined by $\mathcal{H}[r]\rho = r\rho + \rho r^\dagger - \rho \text{Tr}[r\rho + \rho r^\dagger]$. $dW(t)$ is the Weiner increment, which is a Gaussian-white-noise stochastic variable, having the property of $E[dW(t)] = 0$ and $E[dW(t)dW(\tau)] = \delta(t - \tau)dt$, with $E[\dots]$ the ensemble average over a large number of stochastic realizations. The Weiner increment $dW(t)$ is associated with the output current $I(t)$ in terms of the relation,

$$I(t) - \bar{I} = \Delta I(\rho_{11}^c - \rho_{22}^c)/2 + \sqrt{S_0/2}\xi(t), \quad (3)$$

where $\xi(t)dt = dW(t)$, $\bar{I} = (I_1 + I_2)/2$, $\Delta I = I_1 - I_2$, and $S_0 = 2e\bar{I}$. $\rho_{ij}^c(t) = \langle i|\rho_c(t)|j \rangle$ is the density matrix element over the qubit basis states. Based on the output current $I(t)$, there are two

typical schemes to design the feedback Hamiltonian, i.e., the so-called Markovian “ $I(t)$ -based” feedback,⁶ and the Bayesian “ $\rho_c(t)$ -based” feedback.^{9,10} In the following, the second one will be implemented to protect the qubit coherent evolution and its idle state, on the basis of a suboptimal feedback algorithm.¹⁵

III. FEEDBACK CONTROL OF THE RABI OSCILLATION

For simplicity, consider the symmetric qubit ($\varepsilon=0$). The “desired” pure-state evolution to be protected is

$$|\psi_d(t)\rangle = \cos(\Omega t)|1\rangle - i \sin(\Omega t)|2\rangle, \quad (4)$$

where $|1\rangle$ and $|2\rangle$ are the dot states. The basic idea of Bayesian feedback control is to carry out first an *estimate* of the qubit state $\rho_c(t)$ based on the noisy current $I(t)$, then compare it with the desired state $\psi_d(t)$ and design the feedback Hamiltonian using the calculated difference. In Ref. 11 the *estimated* quantity is the *relative phase* between the superposed states “ $|1\rangle$ and $|2\rangle$,” i.e., $\phi(t) = \arctan(2 \operatorname{Im} \rho_{12}^c(t) / [\rho_{11}^c(t) - \rho_{22}^c(t)])$, which is compared with $\phi_0 = 2\Omega t \pmod{2\pi}$ defined from the desired state. Then, the phase “error” $\Delta\phi(t) = \phi(t) - \phi_0$ is used to design the feedback Hamiltonian, i.e., $H_{fb}(t) \propto \Delta\phi(t)$. In a certain sense, this scheme may be inconvenient to be implemented in practice, since the feedback Hamiltonian is a complicated stochastic function of time. In the following, we propose an alternative scheme based on a suboptimal algorithm.¹⁵

For real-time feedback control, each feedback step acts only for an infinitesimal time, “ Δt .” The suboptimal algorithm is based on the principle that the state evolution in each infinitesimal time step will maximize the fidelity of the estimated state with the desired (target) state. The state evolution in the presence of feedback is governed by Eq. (2). As far as the term related to the feedback Hamiltonian is concerned, the final state $\rho_c(t + \Delta t)$ is given by

$$\begin{aligned} \rho_c(t + \Delta t) = & \rho_c(t) - i[H_{fb}, \rho_c(t)]\Delta t \\ & - \frac{1}{2}[H_{fb}, [H_{fb}, \rho_c(t)]](\Delta t)^2 + \dots \end{aligned} \quad (5)$$

The fidelity of this state with the target state is defined by

$$\begin{aligned} \langle \psi_d | \rho_c(t + \Delta t) | \psi_d \rangle = & \langle \psi_d | \rho_c(t) | \psi_d \rangle - i \langle \psi_d | [H_{fb}, \rho_c(t)] | \psi_d \rangle \Delta t \\ & - \frac{1}{2} \langle \psi_d | [H_{fb}, [H_{fb}, \rho_c(t)]] | \psi_d \rangle (\Delta t)^2 + \dots \end{aligned} \quad (6)$$

To optimize the fidelity, one should maximize the coefficient of Δt , which is the dominant term. Similar to other control theories, the maximization must be subject to certain constraints, such as the restriction on the maximum eigenvalue of H_{fb} , the sum of the norms of the eigenvalues, or the sum of the squares of the eigenvalues, etc. Physically, these constraints stem from the limitation of the feedback strength or finite Hamiltonian resources. Here we adopt the last type of constraint, namely,

FIG. 1. The ensemble evolution of $\rho_{11}(t) = E[\rho_{11}^c(t)]$ for different feedback strengths $\mu=0$ (gray solid curve); 0.001 (black solid curve); 0.1 (dashed curve).

$$\operatorname{Tr}[H_{fb}^2] = \sum_n \lambda_n^2(H_{fb}) \leq \mu. \quad (7)$$

Under this constraint, the feedback Hamiltonian can be constructed as¹⁵

$$H_{fb} = ic[|\psi_d(t)\rangle\langle\psi_d(t)|, \rho_c(t)], \quad (8)$$

where $c = \sqrt{\frac{\mu}{2(a-b^2)}}$, with $a = \langle \psi_d(t) | \rho_c^2(t) | \psi_d(t) \rangle$, and $b = \langle \psi_d(t) | \rho_c(t) | \psi_d(t) \rangle$. Substituting Eq. (4) into Eq. (8) yields

$$H_{fb} = F\sigma_x, \quad (9)$$

with

$$F = \begin{cases} \sqrt{\frac{\mu}{2}}, & \Delta\phi < 0, \\ -\sqrt{\frac{\mu}{2}}, & \Delta\phi > 0. \end{cases} \quad (10)$$

In this case, the suboptimal feedback control resembles a simple *bang-bang* control, where the feedback parameter between two values is determined by the phase error $\Delta\phi$ as defined above. However, for other control purpose such as the protection of an arbitrary idle state which is to be studied soon, the suboptimal feedback scheme does not fall into the category of the “bang-bang” control, where the feedback Hamiltonian is more complicated.

Now we present a Monte Carlo simulation for the feedback effect based on the above algorithm. In the simulation the coupling strength between the qubit and the detector is characterized by the measurement-induced dephasing rate, which is assumed $\Gamma_d = \mathcal{X}^2/2 = 0.25 \Omega$. In terms of the ensemble-averaged quantity $\rho_{11}(t) = E[\rho_{11}^c(t)]$, Fig. 1 shows that the simple feedback designed above can really eliminate the back-action of the detector, where the ideal Rabi oscillation is eventually recovered from the damped one by increasing the feedback strength. The result in Fig. 1 is carried out for the case of the ideal detector and in the absence of an external noisy environment, i.e., by setting $\gamma_j = 0$.

To quantitatively characterize the effect of feedback, the so-called synchronization degree is introduced as¹¹ $D = 2\langle \operatorname{Tr} \rho_c \rho_d \rangle - 1$. For instance, the complete synchronization is characterized by $D=1$, indicating that the desired state is perfectly protected by the feedback. In Fig. 2 we plot the

FIG. 2. Synchronization degree D as a function of feedback strength μ , for several magnitudes of dephasing due to the environment: $\gamma_z=0$ (solid curve); $0.05\Gamma_d$ (dashed curve); and $0.25\Gamma_d$ (dashed-dotted curve).

synchronization degree versus the feedback strength μ , in the presence of external “ σ_z ” dephasing with rates $\gamma_z=0$, $0.05\Gamma_d$, and $0.25\Gamma_d$, respectively. In a certain sense, the “ σ_z ” type error caused by the external environment can be regarded as the consequence of a nonideal detector since the observable is also σ_z . Figures 1 and 2 show that the fidelity of such feedback can be arbitrarily close to 100%, in the case of using an ideal detector and in the absence of external noises. Nevertheless, it decreases in the case of a nonideal detector and/or in the presence of significant interaction with the external environment, as well as in the case of finite bandwidth of the line which carries signal from the detector. This limitation also existed in the scheme of Ref. 11.

IV. PROTECTION OF THE IDLE STATE

Now we turn to the subject of using the above suboptimal feedback to protect the idle state of a qubit. This is essential in information storage, while the Rabi oscillation discussed above corresponds to information processing. In general, an arbitrary pure state of qubit can be expressed as

$$|\psi_T\rangle = \cos \frac{\theta}{2} |1\rangle + \sin \frac{\theta}{2} e^{i\phi} |2\rangle. \quad (11)$$

The goal of the feedback is to make it immune against the influence of environmental noise.

Following the same procedure used above, say, maximizing the “ Δt ” term in Eq. (6), and substituting Eq. (11) into Eq. (8), we obtain

$$H_{fb} = \lambda [c_1 \sigma_z + c_2 \sigma_x + c_3 \sigma_y], \quad (12)$$

with

$$\lambda = \sqrt{\mu/2(c_1^2 + c_2^2 + c_3^2)},$$

$$c_1 = 2\alpha\beta(\text{Re } \rho_{12}^c \sin \phi + \text{Im } \rho_{12}^c \cos \phi),$$

$$c_2 = (\beta^2 - \alpha^2)\text{Im } \rho_{12}^c - \alpha\beta(\rho_{11}^c - \rho_{22}^c)\sin \phi,$$

$$c_3 = \alpha\beta \cos \phi(\rho_{11}^c - \rho_{22}^c) - (\alpha^2 - \beta^2)\text{Re } \rho_{12}^c,$$

in which $\alpha = \cos \frac{\theta}{2}$, and $\beta = \sin \frac{\theta}{2}$.

FIG. 3. (a) 3D plot of the synchronization degree D of the feedback control for the measurement; (b) 2D plot of D of the feedback control for several magnitudes of dephasing due to the environment: $\gamma_z=0$ (solid curve); $0.05\Gamma_d$ (dashed curve); and $0.25\Gamma_d$ (dashed-dotted curve). The feedback strength is $\mu=0.1$.

So far the feedback Hamiltonian is constructed on the basis of maximizing the coefficient of Δt in Eq. (6). However, this algorithm does not work if the target state is the eigenstate of ρ_c , namely, $\rho_c |\psi_T\rangle = \lambda_T |\psi_T\rangle$. For instance, such a situation occurs when $\theta=0$ and π . In this case, the dominant “ Δt ” term in Eq. (6) vanishes, and one is forced to maximize the coefficient of $(\Delta t)^2$. Under the constraint on the Hamiltonian as described by Eq. (7), the explicit construction of the optimal H_{fb} reads¹⁵

$$H_{fb} = \sqrt{\mu/2}(|v_1\rangle\langle\psi_T| + |\psi_T\rangle\langle v_1|), \quad (13)$$

where $|v_1\rangle$ is the eigenvector with the largest eigenvalue of $\rho_c(t)$. Notice that for the idle target state $|1\rangle$ (or $|2\rangle$), the conditional evolution described by Eq. (2) will not generate quantum coherence between $|1\rangle$ and $|2\rangle$, because there is no coherent driving source. Thus the states $|1\rangle$ and $|2\rangle$ must be the eigenstates of $\rho_c(t)$, with eigenvalues $\rho_{11}^c(t)$ and $\rho_{22}^c(t)$, respectively. From Eq. (13), the concrete form of the feedback Hamiltonian is detailed as

$$H_{fb} = \begin{cases} \sqrt{\frac{\mu}{2}}\sigma_x, & \rho_{11}^c < \rho_{22}^c, \\ 0, & \rho_{11}^c > \rho_{22}^c, \end{cases} \quad (14)$$

for $\theta=0$ ($|\psi_T\rangle=|1\rangle$), and

$$H_{fb} = \begin{cases} 0, & \rho_{11}^c < \rho_{22}^c, \\ \sqrt{\frac{\mu}{2}}\sigma_x, & \rho_{11}^c > \rho_{22}^c, \end{cases} \quad (15)$$

for $\theta=\pi$ ($|\psi_T\rangle=|2\rangle$). Here we have used the property that if $|v_1\rangle$ in Eq. (13) is the target state, we should set $H_{fb}=0$ for that time step, since in this case the fidelity will not increase under the feedback action.

In the following numerical simulations, we first consider the problem of merely eliminating the back-action of an ideal detector (i.e., $\gamma_j=0$), in the absence of any other external influence of the environment. The result is shown in Fig. 3(a). It is observed that the synchronization degree is independent of the relative phase “ ϕ ” between the two superposed components [cf. Eq. (11)], but depends on “ θ ” symmetrically around “ $\theta=\pi/2$ ” (the minimal point). Since observing “ σ_z ” will induce dephasing of the target state, this

FIG. 4. 3D plot of the synchronization degree D of the feedback control for the σ_x type error. The parameters are feedback strength $\mu=0.1$ and noise strength $\gamma_x=0.05\Gamma_d$.

leads to the smaller synchronization degree for the more coherently superposed state. For $\theta=0$ and π , the corresponding target states are the eigenstates of the observable σ_z ; they are thus immune against the back-action of the detector, leading to $D=1$ and $H_{fb}=0$ according to the proposed feedback scheme. On the other hand, for $\theta=\pi/2$, the corresponding target state is $|\psi_T\rangle=\frac{1}{\sqrt{2}}(|1\rangle+e^{i\phi}|2\rangle)$, which is the most coherently superposed state. This state is destroyed most seriously by the detector's back-action, whose synchronization degree is thus minimal.

Now we consider the influence of the external environment. In reality, a possible error is the σ_z type, which causes similar pure dephasing as the detector's back-action does. As a consequence, the “ (θ, ϕ) ” dependence behavior of the synchronization degree shown in Fig. 3(b) is similar to Fig. 3(a), but it decreases with the dephasing strength. Another possible error is the σ_x type, which causes relaxation between the states $|1\rangle$ and $|2\rangle$. The feedback result is shown by Fig. 4, where the synchronization degree depends on both “ θ ” and

“ ϕ .” In particular, in contrast to the “ σ_z ” type error, here for the states $|\psi_T\rangle=|1\rangle$ and $|2\rangle$ (corresponding to $\theta=0$ and π) the feedback efficiency is worst, since they are the most seriously affected states by the “ σ_x ” type error. Note also that the QPC tunneling amplitude related to $|1\rangle$ is smaller than that related to $|2\rangle$, which leads to $D_{\theta=0}>D_{\theta=\pi}$.

V. CONCLUSION

To summarize, based on a suboptimal feedback algorithm we have presented a study on quantum coherence control of a solid-state charge qubit, for both its quantum evolution and idle state. For the coherent Rabi oscillation, our study leads to a simple bang-bang control feedback scheme, in terms of modulating a single control parameter by only two values, which differs from the existing Bayesian scheme¹¹ and may be easier to implement in some cases. For the coherence protection of the idle state, the present approach is applicable to *arbitrary states*, including those lying on the equator of the Bloch sphere which are *out of control* in the previous Markovian feedback scheme.²⁰ In addition to eliminating the measurement back-action, the proposed feedback scheme can also protect the qubit from environmental influence *to some extent*. However, as many other approaches, it cannot protect the qubit from *drastic influence* of the external environment. This drawback may be partially overcome by developing other *optimal* feedback algorithms, which is a promising but challenging task in this field.

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