Parametric study on multiple tuned mass dampers for buffeting control of Yangpu Bridge

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Abstract

A preliminary strategy of using multiple tuned mass damper (MTMD) system on buffeting control of the Yangpu Bridge are presented in this paper. The MTMD system consists of a set of TMDs which are attached to the center region of the bridge's main span and are symmetrical about the center of the main span as well as about the central line along the bridge span. It is found that the control efficiency of the MTMD system is sensitive to its frequency characteristics, namely, the central frequency ratio and the frequency bandwidth ratio. On the other hand, the damping ratio of the TMDs has less significant effects on the control efficiency. A level-type of TMD has been proposed which requires less space for installation as compared to the traditional hanging type of TMD. A total of seven MTMD systems which corresponding to seven different mass ratios are designed. Anyone of these seven MTMD systems can be used for the buffeting control of the Yangpu Bridge depending on the required control efficiency and the available budget.

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1. Introduction

Nowadays, many bridges with significant span lengths have been completed or are being constructed in the world, such as the Akashi Kaikyo Bridge (center span 1990 m) in Japan, the Normandy Bridge (856 m) in France, the Tsingma Bridge (1377 m) in Hong Kong, the Jiangyin Bridge (1385 m) and the Yangpu Bridge (602 m). The latter two are both in China. With the increase of the bridge span, research on controlling buffeting response of these bridges has been a problem of great concern, especially for those bridges located in the regions where typhoon often occurs.

All the theoretical analyses, experimental research and practical uses have proven that the tuned mass damper (TMD) is an effective device for controlling structural vibration. The TMD has many advantages, such as compactness, reliability, efficiency and low cost. Ever since Den Hartog [1] proposed an optimal design for a TMD's properties under harmonic condition, the TMD has been extended to control vibration of structures under various types of external force conditions [2,3,4].

It is well known that the performance of a TMD is sensitive to the frequency ratio between the TMD and the structure. Slight deviation of the frequency ratio from its design value, either it is due to drift of the TMD's frequency or the structural frequency, would render drastic deterioration of the TMD's performance.

Igusa and Xu [5] proposed a new concept of multiple tuned mass damper (MTMD) for controlling structural vibration with variable frequencies. The basic idea is to use a large number of small TMDs whose natural frequencies are distributed around the dominant natural frequency of a structure so as to have a TMD system with more robust performance. The model characteristics and efficiency of the MTMD was studied by Abe and Fujino [6] using the perturbation technique. The dynamic characteristics and performance of the MTMD

system under random loading were further analyzed by Kareem and Kline [7]. Recently, the MTMD system has been extended to control structures with closely spaced natural frequencies [8] using the perturbation technique.

For a long-span cable-stayed bridge under wind excitation, the buffeting response is usually contributed from its first vertical bending mode [9]. Normally, the vertical vibration frequencies of a bridge tend to increase due to the action of the flutter derivatives H_3^* and H_4^* [10], and tend to decrease due to vehicle loads. To design a TMD system under this situation is an interesting problem worth considering.

Per request of the Yangpu Bridge Management Department, a preliminary strategy for the buffeting control of the Yangpu Bridge using a MTMD system is proposed in this paper. A total of seven sets of MTMD systems corresponding to different mass ratios between the MTMD and the bridge are studied. The optimal properties for these seven MTMD systems are determined numerically. Due to some practical constraints, a level-type TMD is proposed which requires less space as compared to the traditional hanging-type TMD.

2. Basic theory of buffeting control using MTMD

2.1. Equations of motion and frequency response functions

Let it be assumed that a MTMD system is used to control the buffeting response of cable-stayed bridge which mainly comes from its first symmetric vertical mode. The bridge is excited by fluctuating wind forces and its dominate response is in the vertical direction described by $y(x,t)$. The MTMD system consists of a set of n TMDs attached to the center region of the bridge's main span, where the mode shape amplitude of the first symmetric mode is the

largest (see Fig.1). The TMDs are symmetrical about the center of the main span as well as about the center line along the bridge span. The damping ratios for all the TMDs are assumed to be the same as ζ_t .

To ensure the symmetry of the TMDs installed on a bridge, the number of TMD sets on each side of the bridge should be odd, and the total number of the TMDs will be even, such as 10, 14, 18, 22, etc.

It is assumed that the frequency of the central TMD, namely TMD1 in Fig. 1, is the lowest, and the frequencies of the other TMDs increase with an equal frequency increment towards both ends (TMDk's) of the MTMD system. The ratio between the MTMD system's frequency-bandwidth and the structural frequency (hereafter referred to as frequency-bandwidth ratio), γ_{fw} , is defined as

$$\gamma_{fw} = \frac{|f_k - f_1|}{f_s}, \quad (1)$$

where f_1 and f_k are the frequencies of TMD1 and TMDk, respectively, the former is the smallest frequency and the latter is the largest one in the MTMD system; $k=(n+2)/4$; n is the total number of the TMDs; f_s is the natural frequency of the bridge's first symmetric vertical mode. Another important factor describing the frequency characteristics of the MTMD system is the ratio between the MTMD's central frequency, f_m , and f_s . It is hereafter referred to as the central frequency ratio, γ_{cf} , and is defined as

$$\gamma_{cf} = \frac{f_m}{f_s}, \quad (2)$$

where f_m can be calculated by

$$f_m = \frac{1}{k} \sum_{i=1}^k f_i, \quad (3)$$

Together, these two parameters, γ_{fw} and γ_{cf} , completely describe the frequency characteristics of the MTMD, so they are taken as the basic design variables for the MTMD system.

Once the above two parameters are optimally determined, the frequency increment of the MTMD, Δ , and the frequency of i th TMD (see Fig.1) can be found, respectively, as

$$\Delta = \frac{|f_k - f_1|}{k-1} = \frac{\gamma_{fw} \times f_s}{k-1} \quad (4)$$

and

$$f_i = f_m - \frac{\Delta}{k} \sum_{j=1}^{k-1} j + (i-1)\Delta \quad (i=1, 2, \dots, k). \quad (5)$$

Based on the theory of MTMD and the buffeting analysis method proposed by Scanlan and Gade [11], the differential equations governing the motions of the bridge deck and the TMDs expressed in the matrix form are derived as

$$[M]\{\ddot{\xi}\} + [C]\{\dot{\xi}\} + [K]\{\xi\} = \{F\}, \quad (6)$$

where

$$\{\xi(t)\} = \{Y_1(t), Y_{t1}(t), \dots, Y_{tn}(t)\}^T; \quad (7)$$

$$[M] = \begin{bmatrix} 1 & & & \\ & \mu_1 & & \\ & & \ddots & \\ & & & \mu_n \end{bmatrix}; \quad (8)$$

$$[K] = \begin{bmatrix} \omega_s^2 + \sum_{i=1}^n K_i & -K_1 & \dots & -K_n \\ -K_1 & K_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ -K_n & 0 & 0 & K_n \end{bmatrix}; \quad (9)$$

$$[C] = \begin{bmatrix} 2\zeta_{sT}\omega_s + \sum_{i=1}^n C_i & -C_1 & \dots & -C_n \\ -C_1 & C_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ -C_n & 0 & 0 & C_n \end{bmatrix}; \quad (10)$$

$$\{F\} = \{F_1^*, 0, \dots, 0\}^T; \quad (11)$$

$$K_i = \mu_i \omega_i^2; \quad C_i = 2\mu_i \omega_i \zeta_i; \quad F_1^* = F_L / M_s; \quad (12) \sim (14)$$

$Y_1(t)$ is the generalized vertical coordinate of the first symmetric vertical mode of the bridge;

$Y_{ti}(t)$ is the first mode generalized vertical coordinate of i th TMD; ω_s and ω_i are the

circular frequencies of the bridge and the i th TMD, respectively; the aerodynamically modified

damping ratio of the structure, ζ_{sT} , which is the sum of the structural damping ratio, ζ_s , and the aerodynamic damping, is expressed as

$$\zeta_{sT} = \zeta_s - \rho B^2 H_1^* G_1 / (2M_s), \quad (15)$$

with

$$G_1 = \int_0^L \phi_1^2(x) dx; \quad (16)$$

where ζ_i and ω_i are the damping ratio and the circular frequency of the i th TMD, respectively; $\mu_i = M_{ti} / M_s$ is the generalized mass ratio between the i th TMD and the structures; L is the total length of the bridge; $\phi_1(x)$ is the first mode vertical bending shape function of the bridge; M_{ti} and M_s are the first mode generalized masses of the i th TMD and the bridge, respectively, which can be calculated using the following equations:

$$M_{ti} = m_{ti} \times \phi_1^2(x_i) \quad (17)$$

and

$$M_s = \int_{\text{struct}} m(x) \phi_1^2(x) dx, \quad (18)$$

where m_{ti} and $m(x)$ are the mass of the i th TMD and the mass of the bridge structure per unit length. In Eq. (14), F_L is the first mode generalized buffeting lift force, which can be

expressed based on the Scanlan method as

$$F_L = \rho U B \int_0^{L_m} \left\{ -C_L u(x, t) - \frac{1}{2} \left[C'_L + \frac{A}{B} C_D \right] w(x, t) \right\} \phi_1(x) dx, \quad (19)$$

where ρ , U and B are the air density, the mean wind speed and the bridge deck width, respectively; C_L , C'_L and C_D are the aerodynamic coefficients; $u(x, t)$ and $w(x, t)$ are the longitudinal and the vertical components of the fluctuating wind speed, respectively; L_m is the main span length of the bridge.

The first mode frequency response function of the bridge can be obtained from the Eq. (6) and is shown as follows

$$H_y(\omega) = \frac{1}{H_1 + H_2 i}, \quad (20)$$

where

$$H_1 = -\omega^2 + \omega_s^2 + \sum_{i=1}^n \frac{\mu_i \omega_i^2 \omega^2 (\omega^2 - \omega_i^2 - 4\zeta_t^2 \omega^2)}{(\omega_i^2 - \omega^2)^2 + 4\zeta_t^2 \omega_i^2 \omega^2}; \quad (21)$$

$$H_2 = 2\zeta_{sT} \omega_s \omega + \sum_{i=1}^n \frac{2\mu_i \omega_i \omega^5 \zeta_t}{(\omega_i^2 - \omega^2)^2 + 4\zeta_t^2 \omega_i^2 \omega^2}. \quad (22)$$

The frequency response function of the i th TMD is

$$H_{ti}(\omega) = H_y(\omega) \times \frac{\omega^2}{(\omega_i^2 - \omega^2) + 2\zeta_t \omega_i \omega i} \quad (23)$$

2.2. Buffeting response of the bridge and the TMDs

From Eq. (6), the formulas for the RMS values of the buffeting displacements of the bridge deck and the TMDs can be derived and are shown, respectively, as follows

$$\sigma_y(x) = \left[\int_0^\infty S_y(\omega) d\omega \right]^{\frac{1}{2}} = \left[\frac{\phi_1^2(x)}{M_s^2} \int_0^\infty |J(\omega)|^2 |H_y(\omega)|^2 S_F(\omega) d\omega \right]^{\frac{1}{2}} ; \quad (24)$$

$$\sigma_{ti}(x_i) = \left[\int_0^\infty S_{ti}(\omega) d\omega \right]^{\frac{1}{2}} = \left[\frac{\phi_1^2(x_i)}{M_s^2} \int_0^\infty |J(\omega)|^2 |H_{ti}(\omega)|^2 S_F(\omega) d\omega \right]^{\frac{1}{2}} , \quad (25)$$

where $|J(\omega)|^2$ is so called “joint acceptance function” and is expressed as

$$|J(\omega)|^2 = \int_0^{L_m} \int_0^{L_m} \phi_1(x_1) \phi_1(x_2) \exp(-c|x_1 - x_2| / L_m) dx_1 dx_2 \quad (26)$$

and

$$S_F(\omega) = (\rho UB)^2 \left[C_L^2 S_u(\omega) + \frac{1}{4} \left(C_L' + \frac{A}{B} C_D \right)^2 S_w(\omega) \right]; \quad (27)$$

$S_u(\omega)$ and $S_w(\omega)$ are the power spectra of $u(x,t)$ and $w(x,t)$, and can be assumed to be of Kaimal's and Panofsky's forms [11] , respectively; also, $c = \frac{7L_m\omega}{2U\pi}$.

2.3. Optimization of the TMDs' parameters

Defining a reduction ratio, η , as

$$\eta = \frac{(\sigma_y)_w - \sigma_y}{(\sigma_y)_w} \times 100\% \quad (28)$$

where $(\sigma_y)_w$ and σ_y are the maximum RMS values of the buffeting response of the bridge deck without and with control, respectively. The optimal set of the MTMD's properties can be defined as those properties that maximize the reduction ratio.

For practical application of a MTMD system, there exist three possible physical constraints where the first and the second are the static strength and the fatigue strength for the springs of the TMDs, respectively, and the third is the space limitation for the TMDs' motions on the bridge deck. These constraints have been theoretically studied and the constraint equations have been established by Gu and Xiang (1992).

3. Application of MTMD in buffeting control of the Yangpu Bridge

The Yangpu Bridge, which is a cable-stayed bridge with a composite deck and a center span of 602m, is located in Shanghai, China. Both the theoretical and experimental studies indicated

that the bridge may vibrates rather strongly under the action of natural wind (Gu and Xiang 1993). The inclined cable plans of the bridge make the frequency of the first torsional mode much higher than that of the first vertical bending mode. Furthermore, the vertical buffeting response is much larger than the torsional response. Figure 2 shows the vertical bending displacements of the bridge deck (Gu and Xiang 1993). In light of this condition, it is proposed that the vertical buffeting responses should be controlled by using some mechanical countermeasure. The MTMD system is finally chosen as the device due to its convenience, low cost and effectiveness for the variable frequency of the controlled mode.

Both the experimental and the numerical calculations show that the vibration frequency, which is denoted here as f_v , of the bridge varies in the neighborhood of the first vertical bending natural frequency due to the action of wind and the vehicle loads. Based on the flutter derivatives obtained from the wind tunnel tests, the vibration frequency is calculated to be as large as 1.05 times than the first vertical frequency of the bridge for the wind speeds in the practical range. On the other hand, the maximum reduction of the first vertical frequency is about ten percent due to the vehicle loads. So the variable range of the first vertical bending vibration frequency of the bridge is between $(0.9f_s)$ and $(1.05f_s)$. This frequency variation would be taken into consideration in the analysis of the TMDs and can be quantified in the following non-dimensional form,

$$\gamma_{dr} = \frac{f_v}{f_s}, \quad (29)$$

which is hereafter referred to as the drifting frequency ratio.

3.1. Total mass of the MTMD system and mass distribution

For the practical application, larger TMD's mass would give a better control performance, so we expect the total mass of the TMDs to be as large as possible. However, the total mass of the TMDs shall also be limited according to the required reduction ratio and the available budget. As a preliminary plan of the MTMD design for the buffeting control of the Yangpu Bridge, 7 sets of MTMDs are designed and one of these will be selected for the practical use. These 7 MTMDs have the total masses of 30, 50, 75, 100, 125, 150 and 200 tons, corresponding to the generalized mass ratios of about 0.3%, 0.5%, 0.75%, 1.0%, 1.25%, 1.5% and 2.0%, respectively.

In the practical application, after the frequency for each individual TMD has been determined, the designers are free to combinatorially adjust the mass and the stiffness of the TMD in order to provide the required frequency value. Basically, three methods are possible:

- (1). The mass of each TMD is the same while the stiffness varies;
- (2). The generalized mass of each TMD is the same while the stiffness varies;
- (3). The stiffness of each TMD is the same, while the mass varies.

Of the three methods, the third one is the most applicable in the practical situation because the stiffness of each TMD is difficult to adjust, while adjusting the mass of each TMD is much easier.

Figure 3 shows the reduction ratios for the vertical buffeting response at the main span center of the bridge as functions of the drifting frequency ratio, γ_{df} , for the total generalized mass ratio of 1.0% using these three methods. It is seen from the figure that there is little difference among these three curves, so the third method is adopted in the following analyses.

3.2. Number of TMDs

Generally speaking, if only a few TMDs are used, The MTMD system might not be robust enough to handle the frequency variation due to the wind and vehicle effects. On the other hand, the cost of the MTMD system would increase significantly as the number of the TMDs increase for a given total mass of the MTMD system. As a result, a proper number of the TMDs should first be chosen before optimization analysis on the parameters of the TMDs can be conducted. Figure 4 shows the typical calculated results of the reduction ratio versus the number of TMDs for the total generalized mass ratio of 1%. It is seen that the reduction ratio seems to converge as the number of TMD reaches beyond fourteen. Similar trends can be seen on the results of the other six MTMD systems. So the number of TMDs used are determined to be fourteen for all the 7 MTMD systems.

3.3. Selection of frequency distribution

The frequency distribution of the MTMDs is a main factor that affects the control efficiency. As stated above, two factors, γ_{cf} and γ_{fw} , completely describe the MTMD system's frequency characteristics. These two factors will be numerically optimized in the following.

3.3.1. Central frequency ratio γ_{cf}

Analysis of the central frequency ratio is conducted for all the 7 MTMD systems. Figure 5 shows the reduction ratios as a function of the central frequency ratio γ_{cf} and the drifting frequency ratio γ_{df} for the total generalized mass ratios of 0.5%, 1.0% and 1.5%. The vertical dashed lines, e. g., $\gamma_{df}=0.9$ and 1.05 for the left and right dashed lines, respectively, indicate the range of the potential vibration frequency drifting due to the effects of wind and

traffic loads. The reduction ratio is higher at the left vertical dashed line while is lower at the right one for smaller values of γ_{cf} (such as $\gamma_{cf}=0.92$). As γ_{cf} value increases, the reduction ratio increases at the left while decreases at the right. In addition, in spite of the variance of the reduction ratio at the two dashed lines, the maximum values within the potential vibration frequency range are almost the same for different values of γ_{cf} . Figure 6 shows the reduction ratios versus the central frequency ratio γ_{cf} for the drifting frequency ratio γ_{sf} equal to 0.9 and 1.05, respectively. It is seen that the two curves cross at the central frequency ratios γ_{cf} equal to approximately 0.96 for all three mass ratios of 0.5%, 1.0% and 1.5%, respectively. These results seem to suggest that the performance of the MTMD systems with $\gamma_{cf} = 0.96$ are more robust for the drifting frequency ratio range between 0.9 and 1.05.

3.3.2. Frequency bandwidth ratio γ_{fw}

Figure 7 shows the reduction ratio versus the drifting frequency ratio and frequency bandwidth ratio γ_{fw} for the mass ratios of 0.5%, 1.0% and 1.5%, respectively. It is seen that the reduction ratio curves of larger frequency bandwidth ratios are flatter than those of small values of γ_{fw} . Also, larger frequency bandwidth ratios lead to the reduction ratios larger at both sides of the potential range of the vibration frequency and smaller at the drifting frequency ratio of about 1.0. With the decrease of the frequency bandwidth ratio, the maximum reduction ratio rises up. When the value of frequency bandwidth further reduces, the value of the reduction ratio at the center of the potential vibration frequency increases slowly while at the two edges of the frequency range the reduction ratio decreases faster. It is obvious that there exists an optimal value of the frequency bandwidth ratio among these values. To obtain this optimal value, a weighted mean reduction ratio $\bar{\eta}$ is defined as,

$$\bar{\eta} = A\eta_1 + B\eta_2 + C\eta_c, \quad (30)$$

where η_1 and η_2 are the reduction ratios for $\gamma_{df}=0.9$ and 1.05, respectively; η_c is the maximum reduction ratio; A, B and C are the weighting factors, which are assumed to be 0.25, 0.25 and 0.5 roughly considering the probability of appearance for the vibration frequency.

Figure 8 shows the values of weighted reduction ratio versus the frequency bandwidth ratio for the mass ratios of 0.5 %, 1.0% and 1.5%, respectively. From this figure the optimal frequency bandwidth ratio for each mass ratio can easily be determined based on the maximum weighting mean reduction ratio. The optimal frequency bandwidth ratio for the 7 sets of MTMD systems can be found in Table 1.

3.4. Selection of damping ratio

The damping ratios for all the TMDs for anyone of the 7 sets of MTMD systems are assumed to be the same, as stated above. The analysis shows that the optimal damping ratios for the 7 sets of MTMD systems vary between 0.02 and 0.04. The reduction ratios, however, are not very sensitive to the MTMDs' damping ratio within the potential range of the vibration frequency, so the detailed results are not shown here. The optimal damping ratios can also be found in Table 1.

3.5. Optimal parameters for MTMDs

Base on the above analysis, the optimal parameters of the MTMD systems are finally

determined. The results are listed in Table 1. For the practical implementation of the MTMD system, the total mass of the MTMD can first be chosen from the 7 sets of MTMD systems based on the required reduction ratio and the allocated budget for these devices, and then the parameters, e.g., the central frequency ratio and the frequency bandwidth ratio, etc., of the MTMDs can be determined from Table 1.

Table 1 Optimal Parameters of MTMD System

Total mass of MTMD (tons)	30	50	75	100	125	150	200
$\sum \mu_i$ (%)	0.30	0.50	0.75	1.0	1.25	1.50	2.0
$(\gamma_{cf})_{opt}$	0.96	0.96	0.96	0.96	0.96	0.96	0.96
$(\gamma_{fw})_{opt}$	0.15	0.15	0.175	0.175	0.20	0.20	0.25
$(\zeta_t)_{opt}$	0.02	0.03	0.03	0.03	0.03	0.03	0.04
$(\eta)_{opt}$ (%)	11.5	15.2	18.2	20.8	22.3	23.4	25.5

4. Physical style of the TMDs

The net height available where MTMD are to be mounted inside the bridge girder is 2.7 m. A sufficient space should be provided for both the static stretching and the vibration displacements of the TMDs. The static stretching for the spring of a classical hanging-type TMD under the action of the its own weight can be calculated as

$$\delta = \frac{g}{(2\pi f_t)^2}, \quad (31)$$

where g is the gravity acceleration; f_t is the TMD's frequency, which is roughly equal to the structural frequency. The first vertical bending frequency of the Yangpu Bridge is 0.286 Hz, so the static stretching value for the spring of the TMD will be about 3 m. Such a large static stretching might cause the spring to perform nonlinearly and also the TMD can not be fitted into the space available for installation. Even after the obtainable maximum prestress is added to the spring, the net height is still not sufficient for the TMDs. As a result, a lever-type TMD, which is schematically shown in Fig. 9, is proposed in the present paper. The static stretching of the spring of this so-called lever-type TMD is calculated to be l'/l times of that of the classical hanging-type TMD. If the value of l/l' is 4, the static stretching for the spring of the lever-type TMD would be about 70 cm. This enables the spring design of the TMDs possible. Detailed calculations on all 7 sets of the MTMDs systems also show that the constrained conditions, namely the static and the fatigue strengths of the springs and the space limitation inside the bridge deck for the installation of the TMDs can all be satisfied if the value of l/l' is set to 4.

In addition, the effects of the inertial forces along the horizontal rigid rod due to the curved motion of the TMD's mass has been taken into consideration in the design of the vertical rigid rods and the hinges. Analysis shows that these inertial forces have negligible effects on the control efficiency.

5. Conclusions

The MTMD system is an effective device for controlling buffeting response of bridges. In the present study, 7 sets of MTMD systems are studied and their optimal properties are numerically obtained. Anyone of these 7 sets of MTMD systems can be used for the buffeting control of the Yangpu Bridge. The numerical results suggest the following conclusions:

(1) In the practical design of the MTMD system, the stiffnesses of the springs of all the TMDs could be designed to be the same, while the mass of each TMD could be determined based on its frequency and stiffness. This method may simplify the design and implementation of the TMDs;

(2) The control efficiency is sensitive to the frequency characteristics of the MTMD system, namely the central frequency ratio and the frequency bandwidth ratio defined in this paper.

(3) The damping ratio of the TMDs has less significant effects on the control efficiency than the frequency characteristics. For simplification, the damping ratios of all the TMDs for anyone of the 7 sets of the MTMD systems are assumed to be the same;

(4) The lever-type TMD can make the design of the TMD possible, especially for the case when the structural frequency is low and the space available for the TMDs' installation is limited.

(5) The optimal parameters of the 7 sets of MTMD systems are obtained and presented. Anyone of these 7 sets of MTMD systems can be chosen for the practical installation.

In this study, the optimal frequency parameters, the central frequency ratio γ_{cf} and the frequency bandwidth ratio γ_{fw} have been determined based on some reasonable assumption. More accurate estimation of these two parameters can be obtained if the probability distribution of the drifting frequency ratio of the bridge can be prescribed.

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Fig. 1 Layout of the MTMDs on a bridge

Fig. 2 Buffeting response at the center of the main span of the Yangpu Bridge

Fig. 3 Comparison of the three possible methods for TMD design
 $(\Sigma\mu_i=1.0\%, \gamma_{cf}=0.96, \gamma_{fw}=0.75, \zeta_t=0.03, n=14)$

Fig. 4 The effects of number of TMDs on the reduction ratio
 $(\Sigma\mu_1=1.0\%, \gamma_{cf}=0.96, \gamma_{fw}=0.75, \zeta_t=0.03)$

Fig. 5 Reduction ratio versus drifting frequency ratio for various γ_{cf} values

Fig. 6 Reduction ratios at $\gamma_{df}=0.9$ and 1.05 versus central frequency ratio

Fig. 7 Reduction ratio versus γ_{df} for various γ_{fw} values
($\gamma_{cf}=0.96, \zeta_t=0.03$)

Fig. 8 Weighted mean reduction ratio versus γ_{fw} for Fig. 7

Fig. 9 Physical style of the level-type TMD