Spinon-holon binding in t - J model

Tai-Kai Ng

Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay Road, Hong Kong

(Received 21 December 2004; published 27 May 2005)

Using a phenomenological model, we discuss the consequences of spinon-holon binding in the $U(1)$ slave-boson approach to t - J model in the weak-coupling limit. We find that spinon-holon binding produces a pseudogap normal state with a segmented Fermi surface and the superconducting state is formed by opening an “additional” d -wave gap on the segmented Fermi surface. The d -wave gap merges with the pseudogap smoothly as temperature $T \rightarrow 0$. The quasiparticles in the superconducting state are coupled to external electromagnetic field with a coupling constant of order $x^{\gamma/2}$ (x =hole concentration), where $0 \leq \gamma \leq 1$.

DOI: 10.1103/PhysRevB.71.172509

PACS number(s): 74.20.Mn, 74.25.Jb

The $U(1)$ slave-boson mean-field theory (SBMFT) of the t - J model has been used by many authors as a starting point for the theory of high- T_c superconductors.¹⁻⁴ With suitable refinements, the theory can explain a lot of the qualitative features of the cuprates.²⁻⁵ However, the theory does not produce a correct description of the low-energy quasiparticle properties in the underdoped regime. It predicts a very strong renormalization of quasiparticle charge in the superconducting state^{5,6} which is not observed experimentally.^{7,8} It has been suggested^{3,9} that the failure of SBMFT in describing quasiparticles is due to the lack of consideration of confinement between low-energy spinons and holons coming from strong gauge field fluctuations. This scenario has been studied phenomenologically in the $SU(2)$ formulation of the t - J model, where it was found that spinon-holon binding leads to formation of half-pocket (segmented) Fermi surfaces in the normal state,^{3,9} and a rather normal d -wave superconductor state. In this paper, we study spinon-holon binding in the $U(1)$ slave-boson formulation of the t - J model, assuming that an effective spinon-holon interaction U_o which is constant at distance range $d < l \sim \pi^{-1}x^{-1/2}$ (x =hole concentration) exists. Our goal is to understand how the quasiparticle properties in SBMFT are modified in the presence of this phenomenological interaction in the underdoped (small x) regime. We find that there exist two regimes in the x - U_o phase diagram. For small U_o (weak-coupling regime) Bose condensation of holes exists and the properties of the system can be studied in a small- x expansion, whereas Bose condensation of holes vanishes for large U_o (strong-coupling regime) and a new state which cannot be described by small- x expansion is formed. We shall concentrate on the quasiparticle properties in the weak-coupling regime in this paper.

We consider a model Hamiltonian on a two-dimensional square lattice, $H = H_{\text{MF}}^f + H_{\text{MF}}^b + H_c$, where

$$H_{\text{MF}}^f = \sum_{\vec{k}\sigma} \xi_{\vec{k}} f_{\vec{k}\sigma}^\dagger f_{\vec{k}\sigma} + \sum_{\vec{k}} [\bar{\Delta}^*(\vec{k})(f_{\vec{k}\uparrow}^\dagger f_{-\vec{k}\downarrow} - f_{\vec{k}\downarrow}^\dagger f_{-\vec{k}\uparrow}) + \text{H.C.}], \quad (1)$$

is the fermion (spinon) mean-field Hamiltonian in SBMFT. $\xi_{\vec{k}} = -[t\bar{b}^2 + (3J/8)\bar{\chi}]\gamma(\vec{k}) - \mu_f$, $\bar{\chi} = \langle \sum_{\sigma} f_{i\sigma}^\dagger f_{j\sigma} \rangle$

where $j = i + \hat{\mu}$ ($\mu = x, y$), and $\gamma(\vec{k}) = 2[\cos(k_x) + \cos(k_y)]$. $\Delta(\vec{k}) = (3J/4)\bar{\Delta}(\cos(k_x) - \cos(k_y))$ is a spinon pairing field where $\bar{\Delta} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$. The mean-field dispersion for the spinon is $E_f(\vec{k}) = \pm \sqrt{\xi_{\vec{k}}^2 + |\Delta(\vec{k})|^2}$.^{1,5}

The boson (holon) mean-field Hamiltonian is

$$H_{\text{MF}}^b = \sum_{\vec{q}} \epsilon(\vec{q}) b_{\vec{q}}^\dagger b_{\vec{q}} + \frac{U_h}{2} \sum_{\vec{q}} \Delta_h (b_{\vec{q}}^\dagger + b_{-\vec{q}})(b_{\vec{q}} + b_{-\vec{q}}^\dagger), \quad (2)$$

where we have introduced a short-ranged hole-hole repulsion term $U_h \sim t$ which is treated by usual Bogoliubov approximation¹⁰ and

$$\Delta_h = \bar{x}^2 + \frac{1}{V} \sum_{\vec{q}} \langle b_{\vec{q}} b_{-\vec{q}} \rangle, \quad (3)$$

where $\bar{x}$ is the hole Bose-condensation amplitude. Note that $\bar{x} < x$ in the presence of hole-hole and holon-spinon interactions. $\epsilon(\vec{q}) = -t\bar{\chi}\gamma(\vec{q}) + \mu_b$. The existence of Bose-condensation $\bar{x} > 0$ implies $\mu_b = t\bar{\chi}\gamma(0,0)$. The mean-field dispersion for the holon excitation is $E_h(\vec{q}) = \sqrt{\epsilon(\vec{q})^2 + 2\epsilon(\vec{q})U_h\Delta_h}$.⁵

We assume an effective spinon-holon interaction of form

$$H_c = U_o(x) \sum_{(|\vec{k} - \vec{k}'|_F, |\vec{q}|, |\vec{q}'| < \Lambda)} (f_{\vec{k}+\vec{q}\sigma}^\dagger b_{\vec{q}})(b_{\vec{q}'}^\dagger f_{\vec{k}+\vec{q}'\sigma}), \quad (4)$$

where we shall consider x -dependent spinon-holon interactions of form $U_o(x) \sim t/x^\eta$ for reasons which will become clear later. $\Lambda^{-1} \sim \pi\sqrt{x}$. We assume that the binding potential is effective only at a small range of momentum $\leq \Lambda$ around $\vec{q} = 0$ and around the spinon Fermi surface. Gaussian fluctuations above SBMFT result in an effective spinon-holon interaction with $U_o \sim t$.⁵ We shall examine the effect of spinon-holon binding for various values of η in the following.

The electron Green's function is computed in a generalized self-consistent Born approximation that involves self-consistent evaluation of the electron and boson Green's functions,

$$\bar{G}_c(k) = \begin{pmatrix} G_c(k) & F_c(k) \\ F_c^*(k) & -G_c(-k) \end{pmatrix}, \quad \bar{G}_b(q) = \begin{pmatrix} G_b(q) & F_b(q) \\ F_b^*(q) & G_b(-q) \end{pmatrix}, \quad (5)$$

where $k = (\vec{k}, i\omega)$. The self-consistent equation is

$$\bar{G}_c(k) = \bar{G}_c^{(0)}(k) + [U_o \bar{G}_c(k)], \quad (6a)$$

where

$$\bar{G}_c^{(0)}(k) = \begin{pmatrix} g_c(k) & f_c(k) \\ f_c^*(k) & g_c(-k) \end{pmatrix}, \quad (6b)$$

and

$$g_c(k) = xZ_g(T)g_f(k) + \frac{1}{V\beta} \sum_q g_f(k+q)G_b(q),$$

$$f_c(k) = xZ_f(T)f_f(k) - \frac{1}{V\beta} \sum_q f_f(k+q)F_b(q), \quad (6c)$$

where $g_f(k) = (i\omega + \xi_{\vec{k}})/(i\omega^2 - E_f(\vec{k})^2)$ and $f_f(k) = -\Delta_{\vec{k}}/(i\omega^2 - E_f(\vec{k})^2)$ are the mean-field normal and anomalous Green's functions of the spinons, respectively. $\sqrt{xZ_{g(f)}(T)}$ are effective Bose-condensation amplitudes in the normal and anomalous Green's functions and

$$[U_o G_c(k)] = \bar{\theta}(\vec{k}) \frac{U_o}{V\beta} \sum_{|\vec{q}| < \Lambda} g_f(k+q)G_b(q),$$

$$[U_o F_c(k)] = \bar{\theta}(\vec{k}) \frac{U_o}{V\beta} \sum_{|\vec{q}| < \Lambda} f_f(k+q)F_b(q), \quad (6d)$$

where $\bar{\theta}(\vec{k}) = \theta(\Lambda - |\vec{k} - \vec{k}_F|)$. The boson Green's functions are given by

$$\bar{G}_b^{-1}(q) = \begin{pmatrix} i\Omega - \epsilon_{\vec{q}} - \Sigma_b(q) & \Delta_b(q) \\ \Delta_b^*(q) & -i\Omega - \epsilon_{-\vec{q}} - \Sigma_b(-q) \end{pmatrix}, \quad (7a)$$

where

$$\Sigma_b(q) = U_h \Delta_h + \frac{U_o}{V\beta} \sum_{|\vec{k}| - k_f < \Lambda} G_c(k)g_f(k+q),$$

$$\Delta_b(q) = U_h \Delta_h - \frac{U_o}{V\beta} \sum_{|\vec{k}| - k_f < \Lambda} F_c^*(k)f_f(k+q). \quad (7b)$$

The system is in a superconducting state if both the *electron* normal and anomalous Green's functions are nonzero, and is in the normal state if only the normal Green's function is nonzero. Self-consistent determination of the Green's functions is not performed in the previous $SU(2)$ theory.^{3,9} We find that self-consistent determination of electron and boson Green's function is important for identifying the strong and weak coupling regimes whereas self-consistency for the spinon Green's function does not affect the qualitative properties of the system. The spinon Green's function is not

evaluated self-consistently here for simplicity.

Equations (4)–(10) can be solved in a small- x expansion when $\eta < 1/2$. In this limit, we may perform a small $\vec{q}, \Omega$ expansion of the holon Green's function self-energies to obtain

$$G_b(q) = \frac{\bar{\Omega} + \bar{\epsilon}_{-\vec{q}}}{(\bar{\Omega} - \bar{\epsilon}_{\vec{q}})(\bar{\Omega} + \bar{\epsilon}_{-\vec{q}}) + |\Delta_b(0)|^2},$$

$$F_b(q) = \frac{-\Delta_b^*(0)}{(\bar{\Omega} - \bar{\epsilon}_{\vec{q}})(\bar{\Omega} + \bar{\epsilon}_{-\vec{q}}) + |\Delta_b(0)|^2}, \quad (8)$$

where $\bar{\Omega} = Z_b^{-1}\omega$, $\Delta_b(0) = \Delta_b(\vec{q}=0, \Omega=0)$, $Z_b^{-1} = 1 - (\partial \Sigma_b(\vec{q}=0, \omega)/\partial \omega)_{\omega=0}$, and $\bar{\epsilon}_{\vec{q}} = \epsilon_{\vec{q}} + (\partial \Sigma_b(\vec{q}, \omega=0)/\partial(\vec{q}^2))_{\vec{q}=0}\vec{q}^2 \sim t_{\text{eff}}\vec{q}^2 - \mu_h$. A self-consistent evaluation of the self-energies gives $\partial \Sigma_b/\partial \omega \sim (U_o^2/J)\bar{x}$, $\partial \Sigma_b/\partial(\vec{q}^2) \sim U_o$ and $\Delta_b(0) \sim U_h \Delta_h - (U_o^2/J)\bar{x}^{3/2}$. Therefore, $t_{\text{eff}} \sim t + U_o$ and $Z_b \sim 1$ for $\eta < 1/2$ at small x . At zero temperature, the boson occupation number is a sum of two terms, $x = \bar{x} + x_{nbc}$, where $x_{nbc} \sim \sqrt{\Delta_b(0)\bar{x}}/t_{\text{eff}}$ is the density of uncondensed bosons arising from interactions. With Eqs. (3) and (8), we obtain a self-consistent equation for $\Delta_b(0)$ of form

$$\Delta_b(0) \sim a\bar{x}^{(1/2)(1+\eta)}\sqrt{\Delta_b(0)} + U_h\bar{x} - b\bar{x}^{3/2-2\eta},$$

where $a \sim U_h$, $b \sim t^2/J$ are numerical factors. It is easy to see that $\Delta_b(0) \sim \bar{x}$ for $\eta < 1/4$ when holon-holon repulsion dominates, and $\Delta_b(0) \sim \bar{x}^{3/2-2\eta}$ for $\eta > 1/4$ where spinon-holon interaction dominates. Correspondingly, $x_{nbc} \sim \bar{x}^{1+\eta/2}$ for $\eta < 1/4$ and $x_{nbc} \sim \bar{x}^{1+(1/2)(1/2-\eta)}$ for $\eta > 1/4$. In particular, $x_{nbc} > \bar{x}$ for $\eta > 1/2$ where Bose-condensation vanishes and our small- x expansion breaks down. In this case, a new state that cannot be described by SBMFT as a starting point is formed. The corresponding $x-U_o$ diagram has a phase boundary at $U_c \sim 1/x^{1/2}$. For $U_o < U_c$, the system is in the weak-coupling regime with nonzero Bose-condensation amplitude $\bar{x} \neq 0$, whereas Bose condensation vanishes for $U_o > U_c$. We shall examine the quasiparticle properties in the weak-coupling regime in the following.

First, we consider zero temperature. In this case, $xZ_g(0) = xZ_f(0) = \bar{x} \sim x$ and the electron Green's functions can be written as

$$G_c(k) \sim G_{\text{inc}}(k) + \frac{\bar{x}(\omega + \bar{\xi}_{\vec{k}})}{\omega^2 - \bar{E}_{\vec{k}}^2} + O(x^{\gamma/2}),$$

$$F_c(k) \sim -\frac{\bar{x}\Delta_{\vec{k}}}{\omega^2 - \bar{E}_{\vec{k}}^2} + O(x^{\gamma/2}), \quad (9)$$

where $\gamma = 1 - \eta$ for $\eta < 1/4$ and $\gamma = 3(1/2 - \eta)$ for $\eta > 1/4$, $\bar{\xi}_{\vec{k}} = \xi_{\vec{k}} + U_o\bar{x}$ and $\bar{E}_{\vec{k}}^2 = \xi_{\vec{k}}^2 + \Delta_{\vec{k}}^2$. G_{inc} is a smooth background coming from convolution of the spinon and holon Green's functions. The quasiparticle behavior is determined by the terms with weight $\sim \bar{x}$. For $|\omega| \sim \bar{E}_{\vec{k}} \lesssim U_o\bar{x}$, the quasiparticle term is of order $O(1)$ and is larger than the $O(x^{\gamma/2})$ term for $\eta < 1/2$. This is also the energy range where the spinon-

holon bound states are stable. The bound states become unstable and decay into separate spinons and holons at $\bar{E}_{\vec{k}} \geq U_o \bar{x}$.

At temperatures $T > T_c \sim T_{BE}$ where Bose-condensation vanishes, the anomalous Green's function is zero. For T just above T_{BE} , we may consider the bosons to be “almost Bose-condensed”³ where $Z_g(T \geq T_{BE}) \sim 1$ and $Z_f(T \geq T_{BE}) \sim 0$. We obtain with this approximation

$$G_c(k) \sim G_{inc}(k) + \frac{x(\bar{\omega} + \bar{\xi}'_{\vec{k}})}{\bar{\omega}^2 - \bar{E}'_{\vec{k}}^2} + O(x^{\gamma/2}), \quad (10)$$

where $\bar{\xi}'_{\vec{k}} = \xi_{\vec{k}} + U_o x/2$, $\bar{\omega} = \omega - U_o x/2$, and $\bar{E}'_{\vec{k}}^2 = \xi_{\vec{k}}'^2 + \Delta_{\vec{k}}^2$. Notice that for each momentum $\vec{k}$, there exist two branches of quasiparticles with energies $\pm \bar{E}$, both in the superconducting and the normal states.

The transition from superconducting to normal state as the temperature rises from 0 to T_c can also be studied in the almost Bose-condensed approximation where we approximate $Z_g(T) \sim 1$ and $1 > Z_f(T) > 0$. With this, we obtain after some algebra,

$$G_c(k) \sim G_{inc}(k) + \frac{x}{4} \sum_{i=1,2} \left(\frac{u_i(k)}{\omega - E_i(\vec{k})} + \frac{v_i(\vec{k})}{\omega + E_i(\vec{k})} \right) + O(x^{\gamma/2}) \quad (11)$$

where $E_{1(2)}^2(\vec{k}) = Z_f \Delta_{\vec{k}}^2 + (E_{\vec{k}}^c + (-)(U_o x/2))^2$, $E_{\vec{k}}^c = \sqrt{\bar{\xi}'_{\vec{k}}^2 + (1 - Z_f) \Delta_{\vec{k}}^2}$, and $u(v)_1(\vec{k}) = (1 + \bar{\xi}'_{\vec{k}}/E_{\vec{k}}^c) \{1 + (-)[(\bar{\xi}'_{\vec{k}} + U_o x/2)/E_1(\vec{k})] + (-)(1 - Z_f)[\Delta_{\vec{k}}^2/(E_{\vec{k}}^c E_1(\vec{k}))]\}$, $u(v)_2(\vec{k}) = (1 - \bar{\xi}'_{\vec{k}}/E_{\vec{k}}^c) \{1 + (-)[(\bar{\xi}'_{\vec{k}} + U_o x/2)/E_2(\vec{k})] - (+)(1 - Z_f) \times [\Delta_{\vec{k}}^2/(E_{\vec{k}}^c E_2(\vec{k}))]\}$. A corresponding expression also exists for $F_c(k)$.

We observe that four branches of quasiparticles exist at intermediate temperature $0 < T < T_c$. However $u_2 = v_2 = 0$ at $T = 0$ and $v_1 = u_1 = 0$ at $T \geq T_c$, showing that two of the four branches of quasiparticles vanish at these temperatures. These results together suggest a rather unconventional picture for the low-temperature behavior of high- T_c cuprates in our theory. The normal state is a pseudogap state with two branches of quasiparticles where the tunneling density of states (DOS) has a d -wave like gap in the spectrum. Notice that unlike the d -density wave state,¹¹ the present pseudogap state does not break translational symmetry. The normal density of states computed by Eq. (10) for three different dopings $x = 0, 0.05, 0.1$ with $J = 1$, $t = 3$, and $U_h = U_o = 4$ and energy resolution $\Delta\omega \sim 0.15J$ is shown in Fig. 1. The cutoff in momentum space is implemented by introducing a cutoff factor e^{-q^2/Λ^2} in the numerical integrations. We note that the energy resolution in our calculation is not high enough to provide quantitative information of the system. Nevertheless, the shifting of DOS into quasiparticle spectral weight as x increases with a d -wave gap structure is clear from the figure. This global feature is in agreement with tunneling measurements.¹² The DOS in the superconducting state is similar to the normal state except for the sharper d -wave gap structures and that the superconducting DOS minimum is

Electron Density of States

FIG. 1. Tunneling density of states for $x=0, 0.05, 0.1$.

pinned at zero energy, whereas it is pinned at an energy $E \sim U_o x/2$ in the normal state.

Correspondingly, the position of the Fermi surface in the normal state defined by $\bar{E}'_{\vec{k}} = -U_o x/2$ is downshifted from the nodal point $\bar{E}'_{\vec{k}} = 0$, resulting in formation of segmented Fermi surface below the nodal points as in $SU(2)$ theory.^{3,9} We show in Fig. 2 the electron normal state occupation number computed at $x=0.1$. The electron occupation number exhibits “pocket” structures although the discontinuity across the Fermi surface occurs only on the inner side of the pocket (Fermi arc), in agreement with photoemission experiments.¹³

The superconducting transition at T_c is driven by opening an “additional” superconducting gap ($\sim Z_f^{1/2} \Delta_{\vec{k}}$) at the Fermi arc. The opening of the superconducting gap doubles the branches of quasiparticles at an intermediate temperature below T_c . However the superconducting order parameter merges with the pseudogap smoothly as temperature $T \rightarrow 0$, where two of the four branches of quasiparticles vanish. In Fig. 3, we show the electron DOS computed in our theory at $x=0.1$ for three different values of $Z_f=0.0$ (normal state),

FIG. 2. Electron occupation in the normal state for $x=0.1$ at the momentum region $0 \leq k_x, k_y \leq \pi$. The lines are equally spaced constant contours with $n_{\max} \sim 0.52$ and $n_{\min} \sim 0.31$.

FIG. 3. Electron density of states at $x=0$ for three different values of $z=0.0, 0.5, 1.0$.

$Z_f=0.5$ ($T>0$ superconducting state), and $Z_f=1.0$ ($T=0$ superconducting state). The gradual opening of gap at the Fermi arc ($Z_f=0.5$) and merging of superconducting gap with pseudogap at $T=0$ ($Z_f=1$) is clear from the figure. The smooth merging of the superconducting gap with the pseudogap reflects the common origin of the two gaps in our theory and would be absent in theories where the two gaps are of different origins.

A problem associated with the SBMFT approach to high- T_c superconductors is that the quasiparticle charge q_{eff} inferred from the temperature-dependent London penetration depth is of order $O(1)$ experimentally,⁷ but is of order x in SBMFT.^{5,6} A recent experiment also indicates that the simple quasiparticle picture for temperature-dependent London penetration depth¹⁴ may be violated at the extremely low-doping regime.⁸ In the following, we study how quasiparticles in our theory couple to an external electromagnetic field. The electromagnetic field $\vec{A}$ couples to electrons through the t term in

the t - J model. The paramagnetic coupling to electromagnetic field is in linear response,

$$\delta H_P = -2it \sum_{\vec{k}, \vec{q}} \sin(k_\mu) \left(\bar{x} \sum_{\sigma} f_{\vec{k}+\vec{q}\sigma}^{\dagger} f_{\vec{k}\sigma} + \bar{\chi} b_{\vec{k}+\vec{q}}^{\dagger} b_{\vec{k}} \right) A_{\mu}(\vec{q}),$$

where the first term represents coupling through spinons and the second term represents coupling through holons. The paramagnetic coupling of $\vec{A}$ to quasiparticles (=spinon-holon bound state in our theory) can be evaluated by examining the first-order change in electron Green's functions by δH_P . After some lengthy algebra we find that at zero temperature, the nodal quasiparticles in the superconducting state are minimally coupled to external magnetic field as in usual d -wave superconductors with coupling constant $q_{\text{eff}} \sim (U_o/J)(x_{ncb}/\bar{x}^{1/2}) \sim x^{\gamma/2}$. We note that $\gamma > 0$ in the weak-coupling regime ($1/2 > \eta > 0$) and q_{eff} vanishes as $x \rightarrow 0$, although the vanishing rate is slower than predicted in pure SBMFT. The quasiparticle charge becomes of order $O(1)$ only in the strong-coupling regime where Bose condensation vanishes.

Summarizing, we have examined the effect of spinon-holon binding on the quasiparticle properties in the $U(1)$ slave-boson approach to the t - J model. In the weak-coupling regime, we find that spinon-holon binding produces a segmented Fermi surface at normal state and a rather normal d -wave superconductor at $T=0$, in agreement with photoemission experiments.¹³ The theory predicts a rather non-trivial crossover behavior between $T=T_c$ to $T=0$ where four branches of quasiparticles exist in the intermediate state. This prediction is yet to be tested in photoemission or tunneling experiments. The problem of quasiparticle charge is not completely resolved in our theory. Our analysis suggests that quasiparticle charge would be of order one, only in the strong-coupling regime when Bose-condensation amplitude vanishes. A fully self-consistent treatment in this regime is missing and will be the subject of future investigations.

The author thanks P. A. Lee for interesting him in this investigation and for many useful comments. This work is supported by HKRGC through Grant No. 602803.

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