Structural Models
Loan and optionality

- A firm with risky assets $V$, which are financed by equity $S$ and one debt obligation maturing at time $T$ with face value (par value) $F$.

- The firm’s liabilities are viewed as contingent claims issued against the firm’s asset.

- Default occurs at debt maturity $T$ whenever the firm’s asset value falls short of debt value.

Reference
<table>
<thead>
<tr>
<th>Condition</th>
<th>Debt</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_T \leq F$</td>
<td>0</td>
<td>$V_T$</td>
</tr>
<tr>
<td>$V_T &gt; F$</td>
<td>$V_T - F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

value of equity at maturity

$= \max (V - F, 0)$

which is equivalent to the payoff of a call option with strike $F$. 
total firm assets = total debts + total equity

Payoff received by bondholder at maturity
= \min(V, F) = F - \max(F - V, 0)

corporate loan = Treasury bond + short a put
Assumptions in Merton’s firm value model are:

1. Liabilities of firm consist only of a single class of debts;
2. Debt has a zero coupon and no embedded option features;
3. Interest rate is constant;
4. Firm value process follows the geometric Brownian motion;
5. Bankruptcy is costless;
6. Strict priority of claims is preserved in bankruptcy;
7. Bankruptcy is triggered only at bond maturity.
firm value process:

\[ \frac{dV}{V} = \mu dt + \sigma_V dZ \]

Debt value \( D(V, t) \) satisfies the Black-Scholes equation:

\[ \frac{\partial D}{\partial t} + \frac{\sigma^2}{2} V^2 \frac{\partial^2 D}{\partial V^2} + rV \frac{\partial D}{\partial V} - rD = 0 \]

with auxiliary conditions: \( D(V, T) = \min(V, P) \) and \( D(0, t) = 0 \).

\[ D(V, t) = Fe^{-r(T-t)} - L(V, t) \]
where *expected loss*

\[
L(V, t) = Fe^{-r(T-t)} N \left( -\frac{\ln \frac{V}{F} + r(T-t) - \frac{\sigma_V^2}{2} (T-t)}{\sigma_V \sqrt{T-t}} \right) - VN \left( -\frac{\ln \frac{V}{F} + r(T-t) + \frac{\sigma_V^2}{2} (T-t)}{\sigma_V \sqrt{T-t}} \right)
\]

1st term = present value of par times risk neutral probability of default
2nd term = expected recovery in the event of default
Write the expected loss as

\[
N(-d_2) \left[ F e^{-r(T-t)} - \frac{N(-d_1)}{N(-d_2)} V \right],
\]

where \( \frac{N(-d_1)}{N(-d_2)} \) is considered as the *expected discounted recovery rate*.

\( D_t = \) present value of par

- default probability \( \times \) expected discounted loss given default

where

\[
\text{default probability} = N(-d_2).
\]

Further, we define loss given default (LGD) by

\[
\text{Expected loss} = \text{default probability} \times \text{loss given default}.
\]
Numerical example

Data
\( V_t = 100, \sigma_V = 40\%, \ell_t = \) quasi-debt-leverage ratio = 60\%,
\( T - t = 1 \) year and \( r = \ln(1 + 5\%) \).

Calculations
1. Given \( \ell_t = \frac{Fe^{-r(T-t)}}{V} = 0.6, \)
   then \( F = 100 \times 0.6 \times (1 + 5\%) = 63. \)
2. Discounted expected recovery value
   \[ = \frac{N(-d_1)}{N(-d_2)} V = \frac{0.069829}{0.140726} \times 100 = 49.62. \]
4. Cost of default = value of the put
   \[ = 14.07\% \times 10.38 = 1.46; \]
   value of credit risky bond is given by 60 – 1.46 = 58.54.
Yield, \( Y_t = -\frac{1}{T-t} \ln \frac{D_t}{P} = r - \frac{1}{T-t} \ln \left[ \frac{1}{\ell_t} N(-d_1) + N(d_2) \right] \)

where \( \ell_t = \frac{F e^{-r(T-t)}}{V} \) = quasi-debt ratio.

Yield spread = \( Y_t - r = -\frac{1}{T-t} \ln \left[ \frac{1}{\ell_t} N(-d_1) + N(d_2) \right] \)

Yield spread is an increasing function of \( \ell_t \) and \( \sigma_V \).

Standard deviation, \( \sigma_D = \frac{N(-d_1)}{N(-d_1) + \ell_t N(d_2)} \sigma_V \).
increasing leverage
Terms structures of credit spreads

- Downward-sloping for highly leveraged firms.
- Humped shape for medium leveraged firms.
- Upward-sloping for low leveraged firms.

Possible explanation
- For high-quality bonds, credit spreads widen as maturity increases since the upside potential is limited and the downside risk is substantial.

Remarks
Most banking regulations do not recognize the term structure of credit spreads. When allocating capital to cover potential defaults and credit downgrades, a one-year risky bond is treated the same as a ten-year counterpart.
Empirical observations on credit spreads

- Default premiums are shown to be inversely related to firm size from empirical studies (not reflected in Merton’s model).

- When maturity is approached, the credit spread either tends to zero (for medium- to low-leveraged firms) or tends to infinity (for high-leveraged firms).

- Observed credit spreads are systematically higher than the model credit spreads (under realistic firm value volatilities).
Impact of various bond indenture provisions on risky debt valuation

1. *Inter-temporal default* (safety covenants)
   If the firm value falls to specified level, the bondholders are entitled to force the firm into bankruptcy and obtain the ownership of the assets.

2. *Subordinated bonds*
   Payments can be made to the junior debt holders only if the full promised payment to the senior debt holders has been made.

<table>
<thead>
<tr>
<th>claim</th>
<th>$V &lt; P$</th>
<th>$P \leq V \leq P + Q$</th>
<th>$V &gt; P + Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>senior bond</td>
<td>$V$</td>
<td>$P$</td>
<td>$P$</td>
</tr>
<tr>
<td>junior bond</td>
<td>$0$</td>
<td>$V - P$</td>
<td>$Q$</td>
</tr>
<tr>
<td>equity</td>
<td>$0$</td>
<td>$0$</td>
<td>$V - P - Q$</td>
</tr>
</tbody>
</table>

$P = \text{par value of senior bond}$

$Q = \text{par value of junior bond}$
Longstaff- Schwartz model (1995)

*Interest rate uncertainty*
Vasicek interest rate process: \( dr = a(c - r)dt + \sigma_r dZ_r \)

*Bankruptcy-triggering mechanism*
Threshold value \( \nu(t) \) for the firm value at which financial distress occurs: take \( \nu(t) = K = \text{constant} \).

* If a reorganization occurs during the life of the bond, the bondholder receives \( 1 - \omega \) times the par value at maturity.
Briys-de Varenne model (1997)

Take $\nu(t) = \alpha P \ B(r, t; T)$
where $B(r, t; T)$ is the default-free zero-coupon bond value,
$\alpha$ is a constant

Derivation from the strict priority rule
Write down of creditor claims

\[
D_t = f_1 \alpha P 1_{T_{V, \nu < T}} + F 1_{T_{V, \nu < T}, V_T \geq p} + f_2 V_T 1_{T_{V, \nu \geq T}, V_T < p}
\]

where $T_{V, \nu}$ is the first passage time of the firm value process $V$
to the barrier $\nu$.  

Zhou model (1997)

Jump-diffusion process for the firm value process

\[
\frac{dV}{V} = (\mu - \lambda m)dt + \sigma dZ + (\Pi - 1)dY
\]

where \(dY\) is a Poisson process with intensity parameter \(\lambda\);
\(\Pi > 0\) is the jump amplitude with expected value \(m + 1\);
\(\mu\) is the expected instantaneous rate of change of firm value.

- Allows for a jump process to shock the asset value process.
- Remedy the unrealistic phenomena of small short-maturity spreads in pure diffusion firm value process. Default may occur by surprise.
References


Dominant factors in structural models for risky debts

1. Issuer’s asset value process.
2. Issuer’s capital structure.
3. Loss given default.
4. Terms and conditions of the debt issue.
5. Default-free interest rate process.
6. Correlation between the default-free interest rate and asset value.

- Difficult to estimate the parameter values when implementing the models.
Bankruptcy resolution

Weiss analyzed 37 firms (1990, *J. of Fin. Econ.* p.285-314) for the *direct costs* and *violation of priority* of claims.

1. Direct costs encompass the legal and administrative fees, including the costs of lawyers, accountants, etc. They average 3.1% of the book value of debt plus market value of equity at the end of the fiscal year preceding bankruptcy – small impact on debt valuation.

2. Priority of claims is violated for 29 of the 37 firms studied.
   - The costly valuation hearings may make creditors approve a plan in which their priority is violated.
   - Tax-law cooperation of the equity holders is essential to preserve tax-loss carryforwards.
Strategic debt service

Strategic debt service may account for 30% to 40% of the premium on risky debt. Models are constructed to examine the effect on valuation of expanding the strategy space open to equity holders.

- Risk premium prior to bankruptcy may be significantly boosted by expectation of deviations from absolute priority.

- Debtholders in distressed firms are persuaded to accept concessions.

*Indirect bankruptcy costs are the unmeasurable opportunity costs*
1. Lost sales and a decline in the inventory value.
2. Increased operating costs.
3. Reduction in the firm’s competitiveness.
Recovery rates (LGD)


1. The highest average recoveries came from public utilities (70%) and chemical, petroleum and related products (63%).

2. The original rating of a bond issue as investment grade or below investment grade has virtually no effect on recoveries once seniority is accounted for.

3. Neither the size of the issue nor the time to default from its original date of issuance has any association with the recovery rate.
Recovery rates (cont’d)

4. Seniority does play the expected role
   • Senior secured debt averages about 58% of face value.
   • Senior unsecured, 48%.
   • Senior subordinate, 34%.
   • Junior subordinate, 31%.

Some data

<table>
<thead>
<tr>
<th>Industry</th>
<th>recovery rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public utilities</td>
<td>70.47%</td>
</tr>
<tr>
<td>Wholesale and retail trade</td>
<td>44%</td>
</tr>
<tr>
<td>Casino, hotel and recreation</td>
<td>40.15%</td>
</tr>
<tr>
<td>Financial institutions</td>
<td>35.69%</td>
</tr>
<tr>
<td>Seniority</td>
<td>recovery rate</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td><strong>Senior secured</strong></td>
<td></td>
</tr>
<tr>
<td>Investment grade</td>
<td>54.80%</td>
</tr>
<tr>
<td>Non-investment grade</td>
<td>56.42%</td>
</tr>
<tr>
<td><strong>Senior unsecured</strong></td>
<td></td>
</tr>
<tr>
<td>Investment grade</td>
<td>48.20%</td>
</tr>
<tr>
<td>Non-investment grade</td>
<td>48.73%</td>
</tr>
<tr>
<td><strong>Discount and zero coupon</strong></td>
<td></td>
</tr>
<tr>
<td>Investment grade</td>
<td>24.14%</td>
</tr>
<tr>
<td>Non-investment grade</td>
<td>24.42%</td>
</tr>
</tbody>
</table>