Surface states in two-dimensional metallodielectric photonic crystals studied by a multiple-scattering method

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We present a multiple-scattering method in conjunction with supercell calculations to study the electromagnetic surface states in two-dimensional metallodielectric photonic crystals. In the case of dielectric photonic crystals with cylindrical scatterers, this method is able to produce surface states inside a partial or full gap for p -polarized waves, similar to those found previously by using the method of plane-wave expansion. However, in two different systems of metallodielectric photonic crystals we have studied here, we do not find surface states. This is true for both perfect metal and metal with a Drude-like dielectric function. The absence of surface states may be attributed to the expulsion of electric field from the surface into the homogeneous medium due to the presence of metallic component, making the formation of surface unfavorable.

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I. INTRODUCTION

In the past few years, a great deal of effort has been devoted to the study of photonic crystals both theoretically and experimentally.^{1–9} Since a photonic crystal can have a spectral gap in which electromagnetic wave propagation is forbidden in all directions, it offers the possibility of controlling the flow of photons in a way analogous to electrons in a semiconductor. For instance, photonic crystals can suppress vacuum fluctuation and spontaneous emission, and can lead to interesting quantum electrodynamics effects.^{1–3} It has potential applications in quantum electronic devices, distributed-feedback mirrors, microwave antennae substrate, and its unusual optical properties can be exploited to control and guide the propagation of light.⁸ In fact, the prediction that photonic crystals would lead to efficient semiconducting lasers has also been a strong motivation for their development.

In real materials, photonic crystals are always finite in size and, therefore, surface states may exist.^{10–15} The existence of surface states can directly affect the performance and efficiency of photonic crystals in applications. Thus, it is important to study the surface states in photonic crystals. Meade *et al.*¹⁰ were the first to study and discuss theoretically the importance of surface modes of truncated photonic crystals in a three-dimensional (3D) dielectric diamond structure. They found that surface electromagnetic modes always exist for some termination position of the surface plane in structures supporting complete band gaps. Experimentally, the surface waves were detected at GHz frequencies in a 2D array of parallel alumina ceramic rods of circular cross section.¹¹ In this experiment, no evidence of surface modes was found for crystals terminated by complete rods for s -polarized waves. However, the surface modes were detected with the surface layer of cylinders cut in half (hemicylinders). Subsequently, the p -polarized surface states were studied in a dielectric photonic crystal of triangular lattice in two kinds of terminated surface with a complete cylinder surface layer.¹² Recently, the surface waves in 2D photonic crystals with square dielectric cylinders arranged in a square

lattice were investigated by Ramos-Mendieta and Halevi^{13,14} and Elson and Tran.¹⁵ They show that surface modes may appear in incomplete, as well as, complete gaps.

In the past, all the studies on surface states were done for dielectric photonic crystals and the theoretical method used was the plane-wave expansion. Recently, there has been increasing interest^{16–22} in metallodielectric photonic crystals due to the existence of large complete gaps in such systems. The natural question to ask is whether the existence of a metallic component in photonic crystal would favor the formation of surface states or not. In order to answer this question, we have generalized the multiple-scattering method in conjunction with supercell calculations in a slablike geometry to study the surface states in metallodielectric photonic crystals in two dimensions. Due to the existence of a metallic component, the plane-wave expansion converges slowly and becomes inefficient. This difficulty does not exist in the method of multiple scattering as the scattering properties of each scatterer (simple or complex) can be determined analytically. The multiple-scattering method [or the Korringa-Kohn-Rostoker (KKR) method] has been used to calculate band structures in an infinite periodic system.^{9,16}

Here we study the surface states in two different kinds of metallodielectric photonic crystals in a square lattice with cylindrical scatterers that are either pure metallic or of layered type with each metallic cylinder coated with a dielectric layer.

In order to check the validity of our method, we have also studied the surface states in dielectric photonic crystals. For the case of a s -polarized wave, we do not find surface waves with a complete rod surface layer. This is consistent with the results of previous studies. For p -polarized waves, our multiple-scattering results show that the Bloch-type surface modes can exist in both complete and partial gaps in dielectric structures with a complete rod surface layer. This is also in agreement with previous finding on systems with square-shaped scatterers. However, for the metallodielectric photonic crystals we have studied here, we do not find the surface states. This is true for both “perfect” metal and metal modeled with a Drude-like dielectric function. The existence of a metallic component tends to expel the electric field from the

FIG. 1. Schematic picture of slablike photonic crystal structure in a square lattice. x and y describes the plane coordinate being set up. The cylinders are infinitely periodic in the y direction. The dashed frame represent the supercell.

surface into the homogeneous medium and make the formation of surface states unfavorable. Of course, the absence of surface states in the systems we have studied here does not imply the absence of surface states categorically in all metal-dielectric photonic crystals. In fact, we show explicitly in one case how surface states disappear gradually when metallic rods with increasing radii are inserted into a dielectric photonic crystal that supports surface states.

II. THEORY

We consider a 2D photonic crystal that is infinite in the y direction and of finite thickness in the x direction as is shown in Fig. 1. Since the system is periodic in the y direction, the problem can be reduced to a supercell calculation. A supercell is shown in the frame formed by dashed lines in Fig. 1. We consider only the propagation of electromagnetic waves perpendicular to the z axis and assume that all physical quantities are independent of z . In this case, the electromagnetic wave can be decomposed into s polarized (S wave) and p -polarized (P wave) waves. If we use u to represent the electric and magnetic fields in the z direction for S and P waves, respectively, the corresponding eigen equations take the forms

$$\nabla^2 u + k_0^2 \varepsilon(\vec{\rho}) u = 0, \quad (1)$$

and

$$\nabla \cdot \left(\frac{\nabla u}{\varepsilon(\vec{\rho})} \right) + k_0^2 u = 0, \quad (2)$$

where $k_0^2 = \omega^2/c^2$, ω is the wave frequency, $\varepsilon(\vec{\rho})$ is the dielectric constant in space, and c is the wave speed in vacuum.

The method of multiple scattering for a bulk system has been described in Refs. 9 and 16. Here we follow the notations in Ref. 9. Let us first consider the scattering of a particular cylinder j located at $\vec{\rho}_j = (\rho_j, \theta_j)$ under an external source $u_{inc}(\vec{\rho})$ at a fixed frequency $\omega = 2\pi f$. The incident waves come from both the radiation of other cylinders and the external source. The total field outside the cylinder j can be written as the sum of the incident and scattered fields, i.e.,

$$u(\vec{\rho}) = \sum_{m=-\infty}^{m=+\infty} [\alpha_m(j) J_m(k_0 \rho_j) + \tilde{B}_m(j) H_m^{(1)}(k_0 \rho_j)] e^{im\theta_{pj}}, \quad (3)$$

where $\vec{\rho}_{pj} = \vec{\rho} - \vec{\rho}_j = (\rho_{pj}, \theta_{pj})$ and $\vec{\rho} = (\rho, \theta)$ specify the position in a two-dimensional plane. $\alpha_m(j)$ and $\tilde{B}_m(j)$ represent the expansion coefficients of the incident and radiated waves at the j th cylinder, respectively. J_m and $H_m^{(1)}$ are, respectively, the Bessel function and the Hankel function of the first kind. For a layered scatterer with l coating layers, we denote the radius of the i th layer as r_i , $i=1, \dots, l$, $r_i < r_{i+1}$. The refractive index of the i th layer is represented by n_i , therefore, $k_i = k_0 n_i$. Using boundary conditions, we can derive the ratio $D_m(j) = \tilde{B}_m(j)/\alpha_m(j)$ for a l -layered cylinder in the following form

$$D_m = T_{21}^{(m)}/T_{11}^{(m)}, \quad (4)$$

where $T^{(m)}$ is a 2×2 transfer matrix, which can be determined from the products of l matrices:

$$T^{(m)} = T^{(l,m)} \dots T^{(2,m)} \cdot T^{(1,m)}, \quad (5)$$

here $T^{(i,m)}$ is also a 2×2 matrix,

$$T_{11}^{(i,m)} = e H_m^{(1)'}(k_{i+1} r_i) J_m(k_i r_i) - b J_m'(k_i r_i) H_m^{(1)}(k_{i+1} r_i), \quad (6)$$

$$T_{12}^{(i,m)} = e H_m^{(1)'}(k_{i+1} r_i) H_m^{(1)}(k_i r_i) - b H_m^{(1)'}(k_i r_i) H_m^{(1)}(k_{i+1} r_i), \quad (7)$$

$$T_{21}^{(i,m)} = -e J_m'(k_{i+1} r_i) J_m(k_i r_i) + b J_m'(k_i r_i) J_m(k_{i+1} r_i), \quad (8)$$

$$T_{22}^{(i,m)} = -e J_m'(k_{i+1} r_i) H_m^{(1)}(k_i r_i) + b J_m(k_{i+1} r_i) H_m^{(1)'}(k_i r_i), \quad (9)$$

where $e = k_{i+1}$, $b = k_i$ for S waves; $e = k_i$, $b = k_{i+1}$ for P waves.

If we expand the external field at the i th cylinder as

$$u_{inc}(\vec{\rho}) = \sum_{m=-\infty}^{m=+\infty} [\alpha_m^0(i) J_m(k_0 \rho_i)] e^{im\theta_{pi}}, \quad (10)$$

$\alpha_m(i)$ can be expressed as the sum of $\alpha_m^0(i)$ and contributions from other cylinders that depend linearly on $\tilde{B}_l(j)$. With the use of the Graf's addition theorem, a set of self-consistent equations for $\tilde{B}_m$ can be obtained

$$\begin{aligned} \tilde{B}_m(i) = & \alpha_m^0(i) D_m(i) + \sum_{j \neq i} \sum_{l=-\infty}^{\infty} \tilde{B}_l(j) D_m(i) \\ & \times e^{i(l-m)(\theta_{ij} + \pi)} H_{l-m}^{(1)}(k_0 \rho_{ij}), \end{aligned} \quad (11)$$

where $\vec{\rho}_{ij} = \vec{\rho}_j - \vec{\rho}_i = (\rho_{ij}, \theta_{ij})$.

To determine the eigenmodes, we have to turn off the external source and ask for the conditions of nontrivial solutions in Eq. (11). For a slablike photonic crystal, which possesses infinite periodicity in the y direction, we should use

Bloch-wave representation in the y direction and choose a supercell that contains all cylinders in the x direction as shown in Fig. 1. If we denote the μ th lattice site in the n th supercell as (n, μ) and $\tilde{k}$ as the Bloch wave vector in the y direction, in the absence of an external field, Eq. (11) is transformed into the form

$$B_m(\tilde{k}, \mu) = \sum_{\mu', m'} \tilde{G}_{\mu m, \mu' m'}(\tilde{k}) B_{m'}(\tilde{k}, \mu'), \quad (12)$$

with

$$\begin{aligned} \tilde{G}_{\mu m, \mu' m'}(\tilde{k}) = & D_m(\mu) (-1)^{m' - m} \left\{ \sum_n H_{m' - m}^{(1)}(k_0 |n\tilde{a}| \right. \\ & \left. + \tilde{\rho}_{\mu\mu'} |) e^{i(m' - m)\theta} e^{i\tilde{k}na} \right\} \\ & - D_m(\mu) (-1)^{m' - m} \\ & \times \{ \delta_{\mu\mu'} H_{m' - m}^{(1)}(k_0 | \tilde{\rho}_{\mu\mu'} |) e^{i(m' - m)\theta} \}, \end{aligned} \quad (13)$$

where $\tilde{\rho}_{\mu\mu'} = \tilde{\rho}_{\mu'} - \tilde{\rho}_{\mu}$ and a is the lattice constant. To evaluate the lattice sum in Eq. (13), we have generalized the method developed by Chin *et al.*²³ to the supercell geometry. A brief derivation is given in the Appendix. Equation (13) can be re-expressed as

$$\begin{aligned} \tilde{G}_{\mu m, \mu' m'}(\tilde{k}) = & D_m(\mu) (-1)^{m' - m} \left\{ \frac{2\pi}{a} \sum_n \tilde{F}_{m' - m}(k_0, -\tilde{k} \right. \\ & \left. + \frac{2\pi}{a} n, \tilde{\rho}_{\mu\mu'} \right\} - D_m(\mu) (-1)^{m' - m} \\ & \times \left\{ \delta_{\mu\mu'} \delta_{mm'} \left[H_1^{(1)}(k_0 \rho_m) \right. \right. \\ & \left. \left. + \frac{2i}{\pi k_0} \right] / J_1(k_0 \rho_m) \right\}, \end{aligned} \quad (14)$$

where

$$\tilde{F}_l(k_0, k_y, \tilde{\rho}_{\mu\mu'}) = \int_{-\infty}^{\infty} dk_x P_l(k_0, k) e^{-ik_x \rho_{\mu\mu'} x} e^{-ik_y \rho_{\mu\mu'} y}, \quad (15)$$

with

$$P_l(k_0, k) = \frac{(i)^{l+1} k_0 J_{l+1}(k \rho_m) e^{il\theta_k}}{\pi^2 k J_{l+1}(k_0 \rho_m) (k_0^2 - k^2 + i\eta k_0)}, \quad (16)$$

where $\tilde{k} = (k, \theta_k)$ and η is an infinitesimal positive number. ρ_m is a parameter introduced for the calculation of lattice sum. To integrate Eq. (16), we split the factor $1/(k_0^2 - k^2 + i\eta k_0)$ into a principal part and a δ function in the imaginary part. The real part is integrated numerically, whereas the imaginary part can be integrated analytically. From the Appendix, it is easy to see that $\tilde{G}_{\mu m, \mu' m'}(\tilde{k})$ is independent of the choice of the parameters ρ_m . We have checked this

FIG. 2. (A) Calculated photonic band structure for a square lattice of dielectric cylinder with radius $R=0.38$ and $\varepsilon=11.4$ in air for p wave. (B) Band projection along the Γ - Y direction and surface mode dispersion (dotted line) of slablike photonic crystal with the thickness of ten layers. Attention is drawn to the lower two projected bands. The dashed line is the lightline.

point numerically. However, in the calculation, it is preferable to choose $\rho_m = 1$ as was pointed out in Ref. 23. For each Bloch wave vector, the roots of the following secular equation represent both bulk states and surface states:

$$\det[\delta_{\mu\mu'} \delta_{mm'} - \tilde{G}_{\mu m, \mu' m'}(\tilde{k})] = 0. \quad (17)$$

In our calculation, we have truncated the lattice sum $\sum_n \tilde{F}_{m' - m}$ in Eq. (14) at $n_0 = 20$. This ensures a high accuracy of our calculations. It will be shown at the end of the next section that the percentage error introduced in eigenfrequencies is less than 0.1%.

III. NUMERICAL RESULTS AND DISCUSSION

We first study the case of dielectric cylinders with the dielectric constant $\varepsilon = 11.4$. For S waves, it is known that surface states normally do not exist when the surface is terminated with a complete cylinder of circular cross section.¹¹ To test our method, we have solved Eq. (17) in a ten-layer sample with cylinder radius $R = 0.38$ (in units of lattice constant a). For each Bloch wave vector, the roots of Eq. (17) can be grouped into quasicontinuous bands that are separated by gaps. The projected bands so obtained coincide with the results of band-structure calculations of an infinite system. This verifies that our Eq. (14) is correct and accurate. However, we cannot find split-off modes inside the gaps that represent the surface states. This is in agreement with the theoretical calculation and experiment measurements of Ref. 11 that surface states do not exist in the surface terminated by complete rods for S waves.

For P waves, the situation is different. Figure 2(A) displays the bulk band structures for P waves in a square lattice with $R = 0.38$ and $\varepsilon = 11.4$. In Fig. 2(B), we plot the band projection along the y direction corresponding to Fig. 2(A). Note that the vertical scale is different for Fig. 2(A) and Fig. 2(B). Each shaded region is bounded by two curves that

FIG. 3. (A) Calculated photonic band structure for a square lattice of dielectric cylinders with radius $R=0.38$ and $\varepsilon=17.9$ in air for p wave. (B) Surface mode dispersion (dotted line) of slablike photonic crystal along Γ -X direction with the thickness of ten layers. The bulk gap, as shown in (A), is indicated between the dot-dashed lines. The dashed line is the lightline.

correspond to $\tilde{k}_x=0$ and π/d . It should be mentioned that $\tilde{k}_x$ and $\tilde{k}_y$ are symmetric in square lattice and $\tilde{k}_y$ also represents the Bloch wave vector $\tilde{k}$ in Eq. (14). The dotted line on the right-hand side of the lightline (the dashed line in the gap) in Fig. 2(B) represents the dispersion curve of surface modes in a ten-layer sample. This kind of mode represents a Bloch-type wave that is caused by the interference effects near the surface of the photonic crystal. The origin of this surface mode is different from the well-known surface polaritons in a metal surfaces that is a result of a negative dielectric constant. It should be mentioned that the surface modes obtained here are doubly degenerate due to the existence of two surfaces in a slablike geometry. When the layer number is reduced to three or four, the degeneracy will split as the modes at two interfaces couple to each other.

When the dielectric constant of a cylinder increases, complete gaps can also appear for P waves. In Fig. 3(A), we plot the bulk band structures for P waves in square lattice with $R=0.38$ and $\varepsilon=17.9$ (e.g., GaSb cylinder). A complete photonic band gap opens up between the first and second bands. Inside this gap, we find surface modes in a slab of ten layers as shown by the dotted line in Fig. 3(B). Thus, our results for dielectric photonic crystals are consistent with the previous calculations of square cylinders that surface modes can occur in both incomplete and complete band gaps.^{10–15}

To study the surface states in metallodielectric photonic crystals, we consider two different systems. In the first system, we insert a metallic rod in the center of each dielectric cylinder, resulting in “coated cylinder” photonic crystals. For the metallic component, we first take the dielectric constant as $\varepsilon=-1000$ for simplicity. This is close to the case of perfect metal. In the case of $R=0.38$ and $\varepsilon=11.4$, a complete gap between the third and fourth bands can be found for P waves when the radius of metallic insertion is between $r=0.12$ and 0.23 . Within this range, we do not find any surface state. As an example, in Fig. 4(A) we plot the band

FIG. 4. (A) Calculated photonic band structure for a square lattice of coated cylinders with dielectric cylinder radius $R=0.38$ and an inner “perfect” metal cylinder of radius $r=0.18$. The dielectric constant of a dielectric and perfect metal cylinder are taken as 11.4 and -1000 , respectively. (B) Surface mode dispersion (no solution in gap region) of slablike photonic crystal along Γ -X direction with the thickness of ten layers. The bulk gap, as shown in (A), is indicated between the horizontal dot-dashed lines. The dashed line is the lightline.

structures when the inner radius $r=0.18$. A complete band gap opens up in the normalized frequency range $0.4328 \leq \omega a/2\pi c \leq 0.4582$ as indicated by two dash-dotted lines. The result of surface state calculation is shown in Fig. 4(B). We do not find any surface state to the right of the lightline (the dashed line) inside the gap. The absence of any surface state may be attributed to the following reason. Since the existence of a surface state requires electromagnetic energy localization in the high-dielectric region near surface layers,^{10,11,13} adding a metallic component in the dielectric cylinder tends to expel the field away from the surface into the homogeneous medium and, hence, hinder the formation of surface states. This effect is also expected for 3D photonic crystals. The phenomenon of field energy repulsion in such coated cylinders has been studied recently in bulk photonic crystals.²⁴

The absence of any surface state in the above metallodielectric photonic crystals does not depend on the choice of a dielectric constant for the metallic component. In fact, we have also adopted the Drude-like dielectric constant for the metallic component, i.e., $\varepsilon=1-f_p^2/f^2$, where f_p is the plasma frequency. The value of f_p is taken as 3600 THz.^{17,20} When the lattice constant is $0.5 \mu\text{m}$, the results are shown in Figs. 5(A) and 5(B) in the normalized units. Again, we do not find surface states inside the gap. It should be pointed out that although the absorption is not included in our calculation, nevertheless, its inclusion is not expected to affect our results. For bulk photonic crystals, it has been shown in Refs. 20 and 21 that the presence of absorption will only affect the band structures slightly, as long as the frequency is not close to the plasma frequency.

We have also studied the surface states for the case of pure metallic cylinders. For various cylinder radii up to $R=0.45$, we do not find any surface state. A typical result is

FIG. 5. (A) Calculated photonic band structure for a square lattice of coated cylinders with dielectric cylinder radius $R=0.38$ and an inner Drude-like metal cylinder of radius $r=0.18$. The dielectric constant of dielectric region is taken as 11.4. (B) Surface mode dispersion (no solution in gap region) of slablike photonic crystal along Γ -X direction with the thickness of ten layers. The bulk gap, as shown in (A), is indicated between the horizontal dot-dashed lines. The dashed line is the lightline.

shown in Fig. 6 for metallic cylinders of radii $R=0.42$, where the solid and dotted lines denote the results of perfect metal and Drude-like metal, respectively. Here, we have used the same dielectric constants for metal as adopted in Figs. 4 and 5.

Finally, it should be pointed out that the absence of any surface state in the metallodielectric photonic crystals we have studied above does not imply the nonexistence of surface states in all metallodielectric photonic crystals. For in-

FIG. 6. (A) Calculated photonic band structure for a square lattice of metal cylinders of radius $R=0.42$. The dielectric constant of a perfect metal cylinder is taken as -1000 (solid lines) and Drude-like behavior (dotted lines). (B) Surface mode dispersion (no solution in gap region) of slablike photonic crystal along Γ -X direction with the thickness of ten layers. The bulk gap, as shown in (A), is indicated between the dashed lines ($\epsilon=-1000$) and the horizontal dot-dashed lines (Drude-like). The long dashed line is the lightline.

FIG. 7. Surface mode dispersion (dotted line) of slablike photonic crystal along Γ -X direction with the thickness of ten layers. The bulk gap, as shown in Fig. 3(A). The dot-dashed line is the result without an inner metal rod; the dotted line is the result with an inner metal rod $r=0.07$, which the bulk gaps are indicated between the solid lines and long-dashed lines, respectively. The dashed line is the lightline.

stance, in dielectric photonic crystals that can support surface states, the insertion of a tiny fraction of a metallic component should not destroy the surface state suddenly. To study how the surface state disappears as the metallic volume fraction increases, we consider the system shown in Fig. 3. After the insertion of a metallic cylinder at the center of each dielectric cylinder, our calculations show that the surface state and gap position of the system change very slightly from that shown in Fig. 3 when the inner metal radius r is less than 0.01. Noticeable changes are found when $r>0.01$. In Fig. 7, we show the results for $r=0.07$ as an example. The surface state shifts upward from the dot-dashed line to the dotted line, and the band edges also shift from the solid line to the long-dashed line. If r is increased to 0.09, the surface state disappears completely, while the gap position becomes $0.2468 \leq \omega a/2\pi c \leq 0.2518$.

In addition, we would like to discuss the accuracy of the results presented above. Since the accuracy of our results depends very much on how accurately we evaluate the lattice sum in Eq. (14), here, by choosing different truncations n_0 in the series, we estimate explicitly the percentage error involved in a typical lattice sum and show the reliability of our calculations. In Table I below, we list values of the lattice sum of a particular set of parameters, i.e., $\omega a/2\pi c=0.25$, $m'=-3$, $m=1$, $\rho_{\mu,\mu'}=0$ at various truncations n_0 . In our calculations we have chosen $n_0=20$. However, from Table I we find the difference between the values of the lattice sum truncated at $n_0=20$ and $n_0=40$ is only about 0.12%. To see how such a small difference could affect the results of our calculations, we have recalculated the surface states shown in Fig. 7 by using $n_0=40$. We find that the error introduced by the truncation in eigenfrequencies is generally less than

TABLE I. The comparison of values in the lattice sum with a particular set of parameters, $\omega a/2\pi c=0.25$, $m'=-3$, $m=1$, and $\rho_{\mu,\mu'}=0$ at different truncations $n=0$.

n_0	$\text{Re } \Sigma_n \tilde{F}_{m'-m}$	$\text{Im } \Sigma_n \tilde{F}_{m'-m}$
1	-8.197 239e-17	-0.191 244 39
5	-1.599 801e-16	-0.373 250 42
10	-1.591 081e-16	-0.371 204 56
15	-1.588 139e-16	-0.370 518 45
20	-1.586 725e-16	-0.370 187 65
25	-1.585 905e-16	-0.369 996 97
30	-1.585 380e-16	-0.369 874 60
35	-1.585 018e-16	-0.369 790 30
40	-1.584 756e-16	-0.369 728 93

0.1%. For example, the surface state, shown in Fig. 7 without metal rods, at $k_y=0.6$ changes from $\omega a/2\pi c=0.25311$ at $n_0=20$ to 0.25328 at $n_0=40$, and at $k_y=0.7$, it changes from $\omega a/2\pi c=0.26032$ to 0.26046.

IV. CONCLUSION

We have generalized the multiple-scattering method in conjunction with supercell calculations to investigate the surface states in two-dimensional metallodielectric photonic crystals with cylindrical scatterers. In the case of dielectric photonic crystals, we find surface states for P waves in both incomplete and complete gaps. This is consistent with results of previous studies based on the plane-wave expansion. However, in two different kinds of metallodielectric photonic crystals we have studied, no surface state is found. The absence of any surface state may be caused by the field energy expulsion from the surface into the homogeneous medium due to the presence of metallic cylinders. This makes the formation of surface states unfavorable. A similar effect is expected in 3D photonic crystals that contain a metallic component.

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APPENDIX: LATTICE SUMS IN A SLABLIKE CYLINDER PHOTONIC CRYSTAL

In order to calculate the lattice sums in Eq. (13), i.e.,

$$\sum_n H_l^{(1)}(k_0|n\vec{a}+\vec{\rho}_{\mu\mu'}|)e^{il\theta}e^{i\vec{k}na}-\delta_{\mu\mu'}H_l^{(1)}(k_0|\vec{\rho}_{\mu\mu'}|)e^{il\theta}, \quad (\text{A1})$$

we start with the following relation:

$$H_0^{(1)}(k_0\rho)=\frac{i}{\pi^2}\int_{-\infty}^{\infty}\frac{e^{i\vec{k}\cdot\vec{\rho}}}{k_0^2-k^2}d\vec{k}. \quad (\text{A2})$$

From the above equation, we can obtain

$$\begin{aligned} & \sum_n H_0^{(1)}(k_0|\vec{\rho}-\vec{\rho}_{\mu\mu'}-n\vec{a}|)e^{i\vec{k}na} \\ &= \frac{i}{\pi^2} \sum_n \int_{-\infty}^{\infty} \frac{d\vec{k} e^{i\vec{k}\cdot\vec{\rho}} e^{-i\vec{k}\cdot\vec{\rho}_{\mu\mu'}} e^{i(\vec{k}-k_y)na}}{k_0^2-k^2}, \\ &= \sum_{l=-\infty}^{\infty} \frac{i^{l+1}}{\pi^2} \left\{ \sum_n \int_{-\infty}^{\infty} \frac{d\vec{k} e^{-i\vec{k}\cdot\vec{\rho}_{\mu\mu'}} e^{i(\vec{k}-k_y)na}}{k_0^2-k^2} \right. \\ & \quad \left. \times J_l(k|\vec{\rho}|) e^{il\theta_{\vec{k}}} \right\} e^{-il\theta}. \end{aligned} \quad (\text{A3})$$

We expand the left-hand side in Eq. (A3) by using Graf's addition theorem,²⁴ and obtain the following equation:

$$\begin{aligned} & \sum_n H_0^{(1)}(k_0|\vec{\rho}-\vec{\rho}_{\mu\mu'}-n\vec{a}|)e^{i\vec{k}na} \\ &= \sum_{l=-\infty}^{\infty} \left\{ H_0^{(1)}(k_0|\vec{\rho}-\vec{\rho}_{\mu\mu'}|) \delta_{l,0} \right. \\ & \quad + \sum_{n \neq 0} H_l^{(1)}(k_0|\vec{\rho}_{\mu\mu'} \\ & \quad \left. + n\vec{a}|) e^{il\theta_{\rho_{\mu\mu'}+na}} J_l(k_0|\vec{\rho}|) e^{i\vec{k}na} \right\} e^{-il\theta}. \end{aligned} \quad (\text{A4})$$

By comparing Eq. (A3) with Eq. (A4), we find

$$\begin{aligned} & H_0^{(1)}(k_0|\vec{\rho}-\vec{\rho}_{\mu\mu'}|) \delta_{l,0} + \sum_{n \neq 0} H_l^{(1)}(k_0|\vec{\rho}_{\mu\mu'} \\ & \quad + n\vec{a}|) e^{il\theta_{\rho_{\mu\mu'}+na}} J_l(k_0|\vec{\rho}|) e^{i\vec{k}na} \\ &= \frac{i^{l+1}}{\pi^2} \sum_n \int_{-\infty}^{\infty} \frac{d\vec{k} e^{-i\vec{k}\cdot\vec{\rho}_{\mu\mu'}} e^{i(\vec{k}-k_y)na}}{k_0^2-k^2} J_l(k|\vec{\rho}|) e^{il\theta_{\vec{k}}}. \end{aligned} \quad (\text{A5})$$

Following Ref. 23, we introduce a parameter ρ_m and multiply both sides of Eq. (A5) by ρ^{l+1} . Then, integrating over ρ from 0 to ρ_m for Eq. (A5) and using the following relation:

$$\int_0^{\rho_m} \rho^{l+1} J_l(k'\rho) d\rho = \rho_m^{l+1} J_{l+1}(k'\rho_m)/k', \quad (\text{A6})$$

$$\int_0^{\rho_m} \rho^{l+1} Y_l(k'\rho) d\rho = \rho_m^{l+1} \frac{Y_{l+1}(k'\rho_m)}{k'} + \frac{2^{l+1}(l)!}{\pi k'^{l+2}}, \quad (\text{A7})$$

here Y_l is a Bessel function of the second kind, we obtain

$$\begin{aligned} & \sum_n H_l^{(1)}(k_0 | n\vec{a} + \vec{\rho}_{\mu\mu'} |) e^{il\theta} e^{ikna} - \delta_{\mu\mu'} H_l^{(1)}(k_0 | \vec{\rho}_{\mu\mu'} |) e^{il\theta} \\ &= \frac{i^{l+1} 2k_0}{a J_{l+1}(k\rho_m) \pi} \sum_n \int_{-\infty}^{\infty} \frac{dk_x e^{-ik_x \rho_{\mu\mu'} x} e^{-ik_y \rho_{\mu\mu'} y} J_{l+1}(\sqrt{k_x^2 + k_y^2} \rho_m) e^{il\theta_k}}{(k_0^2 - k_x^2 - k_y^2) \sqrt{k_x^2 + k_y^2}} - \delta_{\mu\mu'} \delta_{l,0} \left[H_1^{(1)}(k_0 \rho_m) + \frac{2i}{\pi k_0} \right] \Big/ J_1(k_0 \rho_m), \end{aligned} \quad (\text{A8})$$

$$= \frac{2\pi}{a} \sum_n \tilde{F}_l \left(k_0, -\tilde{k} + \frac{2\pi}{a} n, \vec{\rho}_{\mu\mu'} \right) - \delta_{\mu\mu'} \delta_{l,0} \left[H_1^{(1)}(k_0 \rho_m) + \frac{2i}{\pi k_0} \right] \Big/ J_1(k_0 \rho_m). \quad (\text{A9})$$

Here, $\tilde{F}_l$ has been given in Eq. (15). By substituting the above equation into Eq. (13), we can obtain Eq. (14).

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¹See, for example, *Photonic Band Gaps and Localization*, edited by C.M. Soukoulis (Plenum, New York, 1993); J. Opt. Soc. Am. B **10**, 208 (1993); *Photonic Band Gap Materials*, edited by C.M. Soukoulis (Kluwer, Dordrecht, 1996).

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