Dynamics of fermions coupling to a U(1) gauge field in the limit $e^2 \rightarrow \infty$

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In this paper we study the properties of a gas of fermions coupling to a U(1) gauge field at wave vectors $q < \Lambda \ll k_F$ at dimensions larger than 1, where $\Lambda \ll k_F$ is a high-momentum cutoff and k_F is the Fermi wave vector. In particular, we shall consider the $e^2 \rightarrow \infty$ limit, where charge and current fluctuations at wave vectors $q < \Lambda$ are forbidden. By generalizing a method reported in a previous paper [Phys. Rev. B **62**, 7019 (2000)], we show that the system can be described as a “marginal” Fermi liquid of spin 1/2; chargeless fermions with vanishing wave functions overlap with the bare fermions in the system.

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I. INTRODUCTION

In a previous paper¹ we studied a system of spinless fermions coupled to a longitudinal gauge field (Coulomb interaction) in dimensions larger than 1. Within a bosonization approximation, we show that in the limit $e^2 \rightarrow \infty$, the ground-state and low-energy excitations of the system can be described by a Fermi liquid with chargeless quasiparticles.¹ In this paper we shall generalize our previous approach to consider, in the $e^2 \rightarrow \infty$ limit, a system of spin $S=1/2$, charge e fermions minimally coupled to a U(1) gauge field ($A_0, \vec{A}$) in dimensions higher than 1. The gauge field dynamics is described by an effective long distance action $L_{gauge} = F_{\mu\nu}^2$, with a high-momentum cutoff $\Lambda \ll k_F$, where k_F is the Fermi wave vector. In this limit any nonuniform charge and current density fluctuations with wave vector $q < \Lambda$ in the system cost infinite energy and are forbidden. As a result, any physical state $|\psi\rangle$ that survives in this limit satisfies the constraints $\rho(\vec{q})|\psi\rangle=0$ and $\vec{j}(\vec{q})|\psi\rangle=0$, where $\rho(\vec{q})$ and $\vec{j}(\vec{q})$ are the charge and current density operators, respectively. The method we use is based on an idea originally proposed by Bohm and Pines,² and is similar in spirit to the field theory scheme proposed by Shanjar and Murthy³ for the fractional quantum Hall effect, though the details are different. Within the approximation effective low-energy Lagrangians that describe the chargeless particle-hole as well as single-particle excitations in the system are obtained. We shall see that the properties of the system are rather different from the case of fermions coupling only to longitudinal gauge fields, and correspond to a liquid of chargeless fermions with low-energy properties corresponding to a “marginal” Fermi liquid.

We shall work in the Coulomb gauge $\nabla \cdot \vec{A} = 0$, where the A_0 component of the gauge field is integrated out, resulting in an instantaneous Coulomb interaction $v(q) = 4\pi e^2/q^2$ at $q < \Lambda$ between fermions. The resulting Hamiltonian of the system is

$$H = \sum_{i\sigma} \frac{1}{2m} \left[\vec{p}_{i\sigma} - \frac{e}{c} \vec{A}(\vec{r}_{i\sigma}) \right]^2 + \frac{1}{2L^d} \sum_{\vec{q} \neq 0, q < \Lambda} v(q) \rho(\vec{q}) \rho(-\vec{q}) + H_{photon}, \quad (1)$$

where $\vec{p}_{i\sigma}$ and $\vec{r}_{i\sigma}$ are the momentum and position operators of the i th fermion with spin σ . $\rho(\vec{q}) = \sum_{\vec{k}\sigma} f_{\vec{k}+\vec{q}/2\sigma}^\dagger f_{\vec{k}-\vec{q}/2\sigma}$ is the density operator for the fermions, $f_{\vec{k}\sigma}^\dagger$ ($f_{\vec{k}\sigma}$) are fermion annihilation (creation) operators. L^d is the volume of the system, and c is the velocity of light,

$$H_{photon} = \sum_{\vec{q} \neq 0, q < \Lambda, \lambda} \omega_q \left(a_\lambda^\dagger(\vec{q}) a_\lambda(\vec{q}) + \frac{1}{2} \right), \quad (2)$$

where $a_\lambda^\dagger(\vec{q})$ ($a_\lambda(\vec{q})$) are photon annihilation (creation) operators with wave vector $\vec{q}$ and polarization λ . $\omega_q = cq$. We have set $\hbar=1$ to simplify the notation. Note that the Hartree ($\vec{q}=0$) interaction energy does not appear in the Hamiltonian as in typical Coulomb gas problems where the overall charge neutrality is maintained by the presence of a uniform background of opposite sign charges. After Fourier transforming and in second quantized form, we obtain

$$\begin{aligned} & \sum_{i\sigma} \frac{1}{2m} \left[\vec{p}_{i\sigma} - \frac{e}{c} \vec{A}(\vec{r}_{i\sigma}) \right]^2 \\ & \rightarrow \sum_{\vec{k}\sigma} \left(\frac{k^2}{2m} \right) f_{\vec{k}\sigma}^\dagger f_{\vec{k}\sigma} - \frac{e}{L^{d/2}} \sum_{\vec{q}\lambda} \vec{j}_p(\vec{q}) \cdot \vec{A}(\vec{q}, \lambda) \\ & + \sum_{\vec{q}\lambda} \left(\frac{e^2 n_0}{2m} \right) \vec{A}(\vec{q}, \lambda) \cdot \vec{A}(-\vec{q}, \lambda), \end{aligned} \quad (3a)$$

where $\vec{j}_p(\vec{q}) = \sum_{\vec{k}\sigma} (\vec{k}/m) f_{\vec{k}+\vec{q}/2\sigma}^\dagger f_{\vec{k}-\vec{q}/2\sigma}$ is the paramagnetic current operator, and

$$\vec{A}(\vec{q}, \lambda) = \left(\frac{2\pi}{\omega_q} \right)^{1/2} \vec{\xi}(\vec{q}, \lambda) [a_\lambda(\vec{q}) + a_\lambda^\dagger(-\vec{q})], \quad (3b)$$

where $\vec{\xi}(\vec{q}, \lambda)$ is the polarization vector of photons with a momentum $\vec{q}$ and a polarization mode λ . Note that all sums over $\vec{q}$'s are restricted to $\vec{q} \neq 0, |\vec{q}| < \Lambda$ in Eq. (3) and in all following equations. We have also replaced the fermion density operator $\rho(\vec{r})$ by its expectation value n_0 in the diamagnetic term in Eq. (3a). This approximation can be justified in

the $e^2 \rightarrow \infty$ limit, where density fluctuations of fermions are forbidden. We shall discuss this approximation more carefully in Sec. III.

Next we introduce the bosonization procedure, where the dynamics of the system are described by Wigner function operators¹ $\rho_{\vec{k}\sigma}(\vec{q}) = f_{\vec{k}+\vec{q}/2\sigma}^+ f_{\vec{k}-\vec{q}/2\sigma}$. We shall work in the path-integral formulation, where the Wigner operators are introduced in the imaginary-time action of the system through Lagrange multiplier fields,¹

$$S = \int_0^\beta d\tau \left[\sum_{\vec{k}\sigma} f_{\vec{k}\sigma}^+(\tau) \left(\frac{\partial}{\partial \tau} + \frac{k^2}{2m} - \mu \right) f_{\vec{k}\sigma}(\tau) \right. \\ \left. - \sum_{\vec{k}\sigma, \vec{q}} i \lambda_{\vec{k}\sigma}(\vec{q}, \tau) [\rho_{\vec{k}\sigma}(\vec{q}, \tau) - f_{\vec{k}+\vec{q}/2\sigma}^+(\tau) f_{\vec{k}-\vec{q}/2\sigma}(\tau)] \right. \\ \left. + \frac{1}{L^d} \sum_{\vec{q}, \vec{k}, \vec{k}'} v(\vec{q}) \rho_{\vec{k}}(\vec{q}, \tau) \rho_{\vec{k}'}(-\vec{q}, \tau) \right. \\ \left. - \frac{\sqrt{2}e}{L^{d/2}} \sum_{\vec{k}, \vec{q}, \vec{\lambda}} \left(\frac{\vec{k} \cdot \vec{A}(\vec{q}, \vec{\lambda}, \tau)}{m} \right) \rho_{\vec{k}}(\vec{q}, \tau) \right] + S'_{\text{photon}}, \quad (4)$$

where μ is the chemical potential; and

$$S'_{\text{photon}} = S_{\text{photon}} + \sum_{\vec{q}, \vec{\lambda}} \left(\frac{e^2 n_0}{2m} \right) \vec{A}(\vec{q}, \vec{\lambda}, \tau) \cdot \vec{A}(-\vec{q}, \vec{\lambda}, \tau),$$

where S_{photon} is the pure photon action, $\rho_{\vec{k}}(\vec{q}) = (1/\sqrt{2}) \sum_{\sigma} \rho_{\vec{k}\sigma}(\vec{q})$ are *density* Wigner function operators, and $\lambda_{\vec{k}\sigma}(\vec{q})$'s are Lagrange multiplier fields introduced to enforce the constraint $\rho_{\vec{k}\sigma}(\vec{q}) = f_{\vec{k}+\vec{q}/2\sigma}^+ f_{\vec{k}-\vec{q}/2\sigma}$. For later convenience we shall also introduce *spin* Wigner operators $\sigma_{\vec{k}}(\vec{q}) = (1/\sqrt{2}) \sum_{\sigma} \sigma \rho_{\vec{k}\sigma}(\vec{q})$.

The photon action S'_{photon} can be diagonalized easily to obtain

$$S'_{\text{photon}} = \sum_{\vec{q}, \vec{\lambda}, i\omega_n} (-i\omega_n + \Omega_q) b_{\vec{\lambda}}^+(\vec{q}, i\omega_n) b_{\vec{\lambda}}(\vec{q}, i\omega_n), \quad (5a)$$

where $b(b^+)$'s are photon eigenmodes with eigenfrequencies $\Omega_q = \sqrt{\omega_p^2 + \omega_q^2}$, where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the fermion plasma frequency. The vector field $\vec{A}(\vec{q}, \vec{\lambda})$ can be written in terms of these eigenmodes as

$$\vec{A}(\vec{q}, \vec{\lambda}) = \left(\frac{2\pi}{\Omega_q} \right)^{1/2} \vec{\xi}(\vec{q}, \vec{\lambda}) [b_{\vec{\lambda}}(\vec{q}) + b_{\vec{\lambda}}^+(-\vec{q})]. \quad (5b)$$

To derive an action in terms of Wigner operator fields, we first integrate out the fermion fields $f(f^+)$'s to obtain an action in terms of $\rho_{\vec{k}\sigma}(\vec{q})$ and $\lambda_{\vec{k}\sigma}(\vec{q})$ fields. The resulting action is then expanded in powers of $\lambda_{\vec{k}\sigma}(\vec{q})$ fields. The

$\lambda_{\vec{k}\sigma}(\vec{q})$ fields can be integrated out if we keep only terms to second order (Gaussian approximation), resulting in quadratic action S in terms of $\rho_{\vec{k}}(\vec{q})$, $\sigma_{\vec{k}}(\vec{q})$ and photon fields only.¹ We obtain $S = S_{\rho-p} + S_{\sigma}$, where

$$S_{\rho-p} = \frac{1}{2} \sum_{\vec{k}, \vec{k}', \vec{q}, i\omega_n} \left[- \frac{1}{\chi_{0\vec{k}}(\vec{q}, i\omega_n)} (\delta_{\vec{k}, \vec{k}'} + \frac{2v(\vec{q})}{L^d}) \right. \\ \left. \times \rho_{\vec{k}}(\vec{q}, i\omega_n) \rho_{\vec{k}'}(-\vec{q}, -i\omega_n) \right. \\ \left. - \frac{\sqrt{2}e}{L^{d/2}} \sum_{\vec{k}, \vec{q}, \vec{\lambda}, i\omega_n} \left(\frac{\vec{k} \cdot \vec{A}(\vec{q}, \vec{\lambda}, i\omega_n)}{m} \right) \right. \\ \left. \times \rho_{\vec{k}}(\vec{q}, -i\omega_n) + S'_{\text{photon}} \right] \quad (6a)$$

and

$$S_{\sigma} = - \frac{1}{2} \sum_{\vec{k}, \vec{q}, i\omega_n} \frac{1}{\chi_{0\vec{k}}(\vec{q}, i\omega_n)} \sigma_{\vec{k}}(\vec{q}, i\omega_n) \sigma_{\vec{k}}(-\vec{q}, -i\omega_n), \quad (6b)$$

where

$$\chi_{0\vec{k}}(\vec{q}, i\omega_n) = \frac{n_{\vec{k}-\vec{q}/2} - n_{\vec{k}+\vec{q}/2}}{i\omega_n - \frac{\vec{k} \cdot \vec{q}}{m}},$$

and $n_{\vec{k}} = \theta(-\xi_{\vec{k}})$ at zero temperature is the free fermion occupation number. $S_{\rho-p}$ and S_{σ} can be expressed in terms of canonical boson fields by introducing¹

$$\rho_{\vec{k}}(\vec{q}, i\omega_n) = \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \{ \theta[\Delta_{\vec{k}}(\vec{q})] b_{\vec{k}}^+(\vec{q}, i\omega_n) \\ + \theta[-\Delta_{\vec{k}}(\vec{q})] b_{\vec{k}}(-\vec{q}, -i\omega_n) \}, \\ \sigma_{\vec{k}}(\vec{q}, i\omega_n) = \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \{ \theta[\Delta_{\vec{k}}(\vec{q})] s_{\vec{k}}^+(\vec{q}, i\omega_n) \\ + \theta[-\Delta_{\vec{k}}(\vec{q})] s_{\vec{k}}(-\vec{q}, -i\omega_n) \}, \quad (7)$$

where $\Delta_{\vec{k}}(\vec{q}) = n_{\vec{k}-\vec{q}/2} - n_{\vec{k}+\vec{q}/2}$. Putting Eq. (7) back into $S_{\rho-p}$, after some straightforward manipulations, we obtain

$$\begin{aligned}
S_{\rho-p} = & \frac{1}{2} \sum_{\vec{k}, \vec{q}, i\omega_n} \left(-i\omega_n + \frac{|\vec{k} \cdot \vec{q}|}{m} \right) b_{\vec{k}}^+(\vec{q}, i\omega_n) b_{\vec{k}}(\vec{q}, i\omega_n) + \frac{1}{2} \sum_{\vec{q}, \lambda, i\omega_n} (-i\omega_n + \Omega_q) b_{\lambda}^+(\vec{q}, i\omega_n) b_{\lambda}(\vec{q}, i\omega_n) \\
& + \frac{1}{L^d} \sum_{\vec{k}, \vec{k}', \vec{q}, i\omega_n} v(q) \sqrt{|\Delta_{\vec{k}}(\vec{q}) \Delta_{\vec{k}'}(\vec{q})|} \theta[\Delta_{\vec{k}}(\vec{q})] \theta[\Delta_{\vec{k}'}(\vec{q})] \\
& \times [b_{\vec{k}}^+(\vec{q}, i\omega_n) b_{\vec{k}'}(\vec{q}, i\omega_n) + b_{\vec{k}}^+(\vec{q}, i\omega_n) b_{-\vec{k}'}^+(-\vec{q}, -i\omega_n) + b_{-\vec{k}}(-\vec{q}, -i\omega_n) b_{\vec{k}'}(\vec{q}, i\omega_n) \\
& + b_{-\vec{k}}(-\vec{q}, -i\omega_n) b_{-\vec{k}'}^+(-\vec{q}, -i\omega_n)] - \frac{1}{L^{d/2}} \sum_{\vec{k}, \vec{q}, \lambda, i\omega_n} m_{\vec{k}}(\vec{q}, \lambda) \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \theta[\Delta_{\vec{k}}(\vec{q})] [b_{\vec{k}}^+(\vec{q}, i\omega_n) - b_{-\vec{k}}(-\vec{q}, -i\omega_n)] \\
& \times [b_{\lambda}(\vec{q}, i\omega_n) + b_{\lambda}^+(-\vec{q}, -i\omega_n)]
\end{aligned} \tag{8a}$$

and

$$S_{\sigma} = \frac{1}{2} \sum_{\vec{k}, \vec{q}, i\omega_n} \left(-i\omega_n + \frac{|\vec{k} \cdot \vec{q}|}{m} \right) s_{\vec{k}}^+(\vec{q}, i\omega_n) s_{\vec{k}}(\vec{q}, i\omega_n), \tag{8b}$$

where $m_{\vec{k}}(\vec{q}, \lambda) = (2e/m)(\pi/\Omega_q)^{1/2} \vec{k} \cdot \vec{\xi}(\vec{q}, \lambda)$. $S_{\rho-p}$ describes a coupled system of (fermion) *density* particle-hole pair excitations and photons, whereas S_{σ} describes a system of free (fermion) *spin* particle-hole excitations. The density- and spin-particle-hole pair excitations are described by boson fields $b(b^+)_{\vec{k}}(\vec{q})$ and $s(s^+)_{\vec{k}}(\vec{q})$, respectively, satisfying typical boson commutation rules $[b(s)_{\vec{k}}(\vec{q}), b(s)_{\vec{k}'}^+(\vec{q}')] = \delta_{\vec{k}, \vec{k}'} \delta_{\vec{q}, \vec{q}'}$ and $[b(s)_{\alpha}, b(s)_{\beta}^+] = [b(s)_{\alpha}^+, b(s)_{\beta}^+] = 0$, etc. In this form the dynamics of the original fermion system is described completely in terms of charge- and spin- boson fields (bosonized).

Note that we have restricted ourselves to the Gaussian approximation in deriving $S_{\rho-p}$ and S_{σ} . Higher-order interaction terms between bosons will appear in a cumulant expansion of the $\lambda_{\vec{k}}(\vec{q})$ fields.⁴ The convergence of the cumulant expansion is formally controlled in our model by the small parameter $\epsilon \sim (\Lambda/k_F)$, where a term of order $[b^m b^{+(n-m)}]$ in the cumulant expansion has to leading order a term $\sim \epsilon^{2(d-1) \times (n-1)}$ in each increasing order of the cumulant expansion.^{5,1} Note, however, that the smallness of ϵ does not guarantee that all calculated physical quantities will converge uniformly in the cumulant expansion. In particular, we shall see that infrared divergence appears in higher-order expansion of the transverse gauge field.^{10,11} We shall discuss the consequences of the infrared divergences in more detail in Sec. IV.

It is easy to show that the Gaussian approximation is essentially the same as usual random-phase-approximation theory for interacting fermions,^{1,6} except the additional assumption that particle-hole excitations can be treated as independent bosons in bosonization theory.¹ Note that particle-hole pairs are not all *independent* in a fermion system because of the Pauli exclusion principle. The assumption of independent bosons is a major approximation in the Gaussian theory in two and three dimensions. This is in contrast to

what happens in one dimension, where the entire particle-hole excitation spectrum can be represented rigorously by bosons⁷ when the fermion spectrum is linearized near the Fermi surface.

II. DENSITY PARTICLE-HOLE EXCITATION SPECTRUM IN BOSONIZATION THEORY

The eigenstates and eigenvalue spectrum described by the action $S_{\rho-p}$ can be obtained by diagonalizing the bosonized action [Eq. (8a)] using a generalized Bogoliubov transformation. For each wave vector $\vec{q}$ we introduce the Bogoliubov transformations⁸

$$\begin{aligned}
b_{\vec{k}}(\vec{q}) &= \sum_{k'} [\alpha_{kk'}^> \gamma_{k'}(\vec{q}) + \beta_{kk'}^> \gamma_{-k'}^+(-\vec{q})], \\
b_{-\vec{k}}(-\vec{q}) &= \sum_{k'} [\alpha_{kk'}^< \gamma_{-k'}(-\vec{q}) + \beta_{kk'}^< \gamma_{k'}^+(\vec{q})],
\end{aligned} \tag{9}$$

where $k = \vec{k}, \lambda (k' = \vec{k}', \lambda')$, and the $\gamma(\gamma^+)_{k}(\vec{q})$ operators are required to diagonalize the Hamiltonian, i.e.,

$$H_{\rho-p} = \sum_{\vec{k}} E_{\vec{k}}(\vec{q}) \gamma_{\vec{k}}^+(\vec{q}) \gamma_{\vec{k}}(\vec{q}) + E_G, \tag{10}$$

where $E_{\vec{k}}(\vec{q})$ are the eigenenergies and E_G is the ground-state energy of the system. The eigenstates may represent dressed particle-hole excitations ($k = \vec{k}$) or dressed photons ($k = \lambda$). Additional collective modes may appear in the system, and are also included in the sum $\sum_{\vec{k}}$. The matrix elements α and β satisfy the orthonormality conditions

$$\begin{aligned}
\sum_{k''} [\alpha_{kk''}^{>(<)} \alpha_{k'k''}^{>(<)*} - \beta_{kk''}^{>(<)} \beta_{k'k''}^{>(<)*}] &= \delta_{kk'}, \\
\sum_{k''} [\alpha_{kk''}^{<} \beta_{k'k''}^{>} - \beta_{kk''}^{<} \alpha_{k'k''}^{>}] &= 0.
\end{aligned} \tag{11}$$

Solving the resulting Bogoliubov equations, we find that there exist two kinds of solutions to these equations: (i) a

particle-hole continuum, with $E_{\vec{k}}(\vec{q}) = |\vec{k} \cdot \vec{q}|/m$; and (ii) collective modes, including (renormalized) photons with an energy $E_{\lambda}(\vec{q})$ determined by the eigenvalue equation $E_{\lambda}(\vec{q})^2 - \Omega_q^2 - (4\pi e^2/m^2)\chi_i[\vec{q}, E_{\lambda}(\vec{q})] = 0$, and plasmons, with energy $E_0(\vec{q})$ given by the eigenvalue equation $1 - v(q)\chi_0[\vec{q}, E_0(\vec{q})] = 0$, where

$$\chi_i(\vec{q}, \omega) = \frac{2}{L^d(d-1)} \sum_{\vec{k}} (\vec{k}_i \cdot \vec{k}_i) \chi_{0\vec{k}}(\vec{q}, \omega),$$

is the paramagnetic transverse current susceptibility, $\vec{k}_i = \vec{k} - \hat{q}(\vec{k} \cdot \hat{q})$, and

$$\chi_0(\vec{q}, \omega) = \frac{2}{L^d} \sum_{\vec{k}} \chi_{0\vec{k}}(\vec{q}, \omega)$$

is the Lindhard function. The energies of these collective modes are outside the particle-hole continuum. We now consider the solutions in more detail. First we consider the particle-hole excitations. After some lengthy algebra, we obtain

$$\begin{aligned} \alpha_{\vec{k}\vec{k}'}^{\geq} &= \delta_{\vec{k}\vec{k}'} + P \frac{\theta[\Delta_{\vec{k}}(\vec{q})]\theta[\Delta_{\vec{k}'}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})\Delta_{\vec{k}'}(\vec{q})|}}{L^d \left(\frac{|\vec{k}' \cdot \vec{q}|}{m} - \frac{|\vec{k} \cdot \vec{q}|}{m} \right)} \\ &\quad \times [2v_{eff}(q, |\vec{k}' \cdot \vec{q}|/m) - \vec{k}_i \cdot \vec{A}_{eff}(\vec{q}, \vec{k}')], \\ \beta_{\vec{k}\vec{k}'}^{\leq} &= \frac{-\theta[\Delta_{\vec{k}}(\vec{q})]\theta[\Delta_{\vec{k}'}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})\Delta_{\vec{k}'}(\vec{q})|}}{L^d \left(\frac{|\vec{k}' \cdot \vec{q}|}{m} + \frac{|\vec{k} \cdot \vec{q}|}{m} \right)} \\ &\quad \times [2v_{eff}(q, -|\vec{k}' \cdot \vec{q}|/m) + \vec{k}_i \cdot \vec{A}_{eff}^*(\vec{q}, \vec{k}')], \\ \alpha_{\lambda\vec{k}}^{\geq} &= -\frac{(|\vec{k}' \cdot \vec{q}|/m + \Omega_q) \frac{2e}{m} \left(\frac{\pi}{\Omega_q} \right)^{1/2} \theta[\Delta_{\vec{k}'}(\vec{q})]\sqrt{|\Delta_{\vec{k}'}(\vec{q})|}}{L^{d/2} (|\vec{k}' \cdot \vec{q}|/m)^2 - \Omega_q^2 - \frac{4\pi e^2}{m^2} \chi_i(\vec{q}, |\vec{k}' \cdot \vec{q}|/m)} \\ &\quad \times [\vec{\xi}_{\lambda}(\vec{q}) \cdot \vec{k}_i'], \\ \beta_{\lambda\vec{k}}^{\leq} &= \frac{(|\vec{k}' \cdot \vec{q}|/m - \Omega_q) \frac{2e}{m} \left(\frac{\pi}{\Omega_q} \right)^{1/2} \theta[\Delta_{\vec{k}'}(\vec{q})]\sqrt{|\Delta_{\vec{k}'}(\vec{q})|}}{L^{d/2} (|\vec{k}' \cdot \vec{q}|/m)^2 - \Omega_q^2 - \frac{4\pi e^2}{m^2} \chi_i(\vec{q}, -|\vec{k}' \cdot \vec{q}|/m)} \\ &\quad \times [\vec{\xi}_{\lambda}(\vec{q}) \cdot \vec{k}_i'], \end{aligned} \quad (12a)$$

for the particle-hole continuum spectrum $\vec{k}'$, where $v_{eff}(q, \omega) = v(q)/[1 - v(q)\chi_0(\vec{q}, \omega)]$ is the RPA effective interaction, and $\vec{A}_{eff}(\vec{q}, \vec{k}') = [(8\pi e^2/m^2)\vec{k}_i']/[(|\vec{k}' \cdot \vec{q}|/m)^2 - \Omega_q^2 - (4\pi e^2/m^2)\chi_i(\vec{q}, |\vec{k}' \cdot \vec{q}|/m)]$. We also obtain $\alpha(\beta)_{\lambda\vec{k}}^{\leq} = -\alpha(\beta)_{\lambda\vec{k}}^{\geq}$, and $\alpha(\beta)_{\vec{k}\vec{k}}^{\leq} = \alpha(\beta)_{\vec{k}\vec{k}}^{\geq}$.

For the longitudinal collective modes (plasmon), we obtain

$$\begin{aligned} \alpha_{\vec{k}0}^{\geq} &= \frac{1}{L^{d/2}} \frac{2\theta[\Delta_{\vec{k}}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})|}}{\left(E_0(\vec{q}) - \frac{|\vec{k} \cdot \vec{q}|}{m} \right) \left[-\frac{\partial \chi_0(q, \omega)}{\partial \omega} \right]_{\omega=E_0(\vec{q})}^{1/2}}, \\ \beta_{\vec{k}0}^{\leq *} &= -\frac{1}{L^{d/2}} \frac{2\theta[\Delta_{\vec{k}}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})|}}{\left(E_0(\vec{q}) + \frac{|\vec{k} \cdot \vec{q}|}{m} \right) \left[-\frac{\partial \chi_0(q, \omega)}{\partial \omega} \right]_{\omega=E_0(\vec{q})}^{1/2}}, \end{aligned} \quad (12b)$$

$$\alpha_{\lambda 0}^{\geq} = \beta_{\lambda 0}^{\leq *} = 0,$$

and $\alpha(\beta)_{\vec{k}0}^{\leq} = \alpha(\beta)_{\vec{k}0}^{\geq}$, $\alpha(\beta)_{\lambda 0}^{\leq} = 0$. Similarly, for the (renormalized) photon modes, we also obtain

$$\begin{aligned} \alpha_{\vec{k}\lambda}^{\geq} &= -\frac{\theta[\Delta_{\vec{k}}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})|}}{E_{\lambda}(\vec{q}) - \frac{|\vec{k} \cdot \vec{q}|}{m}} \left(\frac{\vec{k} \cdot \vec{\xi}_{\lambda}(\vec{q})C(q)}{L^{d/2}} \right), \\ \beta_{\vec{k}\lambda}^{\leq *} &= -\frac{\theta[\Delta_{\vec{k}}(\vec{q})]\sqrt{|\Delta_{\vec{k}}(\vec{q})|}}{E_{\lambda}(\vec{q}) + \frac{|\vec{k} \cdot \vec{q}|}{m}} \left(\frac{\vec{k} \cdot \vec{\xi}_{\lambda}(\vec{q})C(q)}{L^{d/2}} \right), \\ \alpha_{\lambda\lambda'}^{\geq} &= \frac{\delta_{\lambda\lambda'}}{[E_{\lambda}(\vec{q}) - \Omega_q]} \frac{e}{m} \left(\frac{\pi}{\Omega_q} \right)^{1/2} \chi_i[\vec{q}, E_{\lambda}(\vec{q})]C(q), \\ \beta_{\lambda\lambda'}^{\leq *} &= -\frac{\delta_{\lambda\lambda'}}{[E_{\lambda}(\vec{q}) + \Omega_q]} \frac{e}{m} \left(\frac{\pi}{\Omega_q} \right)^{1/2} \chi_i[\vec{q}, E_{\lambda}(\vec{q})]C(q), \end{aligned} \quad (12c)$$

where

$$C(q) = \left(\frac{\left(\frac{4\pi e^2}{m^2} \right)}{2E_{\lambda}(\vec{q}) - \left(\frac{4\pi e^2}{m^2} \right) \left[\frac{\partial \chi_i(\vec{q}, \omega)}{\partial \omega} \right]_{\omega=E_{\lambda}(\vec{q})}} \right)^{1/2},$$

and with $\alpha(\beta)_{\lambda\lambda}^{\leq} = \alpha(\beta)_{\lambda\lambda}^{\geq}$ and $\alpha(\beta)_{\vec{k}\lambda}^{\leq} = -\alpha(\beta)_{\vec{k}\lambda}^{\geq}$.

Next we examine the solutions of the bosonized Hamiltonian in the $e^2 \rightarrow \infty$ limit. First we consider the collective modes. Using the results that $\chi_0(\vec{q}, \omega) \rightarrow n_0 q^2/m\omega^2$, and $\chi_i(\vec{q}, \omega) \rightarrow 2\langle \epsilon \rangle n_0 q^2/d\omega^2$ in the limit $\omega \gg k_F q/m$, where n_0 is the fermion density⁹ and $\langle \epsilon \rangle$ is the average kinetic energy per fermion in the free-fermion ground state, it is easy to see that, as $e^2 \rightarrow \infty$, the collective mode frequencies $E_0(q)$ and $E_{\lambda}(q)$ all approach the plasma frequency $\omega_P = (4\pi n_0 e^2/m)^{1/2} \rightarrow \infty$, and are outside the physical spectrum in this limit.

Despite the vanishing of collective excitations in the physical spectrum, the particle-hole excitation spectrum with

excitation energies $|\vec{k} \cdot \vec{q}|/m$ survives in bosonization theory in the limit $e^2 \rightarrow \infty$. In this limit $v_{eff}(q, |\vec{k}' \cdot \vec{q}|/m) \rightarrow -1/\chi_0(q, |\vec{k}' \cdot \vec{q}|/m)$ and $\vec{k} \cdot \vec{A}_{eff}(\vec{q}, \vec{k}') \rightarrow -2\vec{k}_t \cdot \vec{k}'_t/[mn_0 + \chi_t(q, |\vec{k}' \cdot \vec{q}|/m)]$. The coefficients $\alpha_{\vec{k}\vec{k}'}$ and $\beta_{\vec{k}\vec{k}'}$ remain regular, indicating that the particle-hole excitation spectrum survives under effect of confinement. It is straightforward to examine the charge and (transverse) current fluctuations carried by the (eigen)-particle-hole excitations by examining the commutators $[\rho(\vec{q}), \gamma_{\vec{k}}(\vec{q}')] = [\vec{j}_t(\vec{q}), \gamma_{\vec{k}}(\vec{q}')] = 0$, where

$$\vec{j}_t(\vec{q}) = \sum_{\vec{k}\sigma} \frac{e\vec{k}_t}{m} \rho_{\vec{k}\sigma}(\vec{q}) - (L^{d/2}) \sum_{\lambda} \frac{e^2 n_0}{m} \vec{A}(-\vec{q}, \lambda)$$

is the transverse current operator. Using Eqs. (7) and (12a) and typical boson commutation rules, it is straightforward to show that

$$[e\rho(\vec{q}), \gamma_{\vec{k}}(\vec{q}')] = \delta_{\vec{q}\vec{q}'} \theta[\Delta_{\vec{k}}(\vec{q})] \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \times \frac{-e}{1 - v(q)\chi_0(\vec{q}, |\vec{k} \cdot \vec{q}|/m)} \quad (13)$$

and

$$[\vec{j}_t(\vec{q}), \gamma_{\vec{k}}(\vec{q}')] = \delta_{\vec{q}\vec{q}'} \frac{-\sqrt{2}e}{m} \theta[\Delta_{\vec{k}}(\vec{q})] \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \times \left(\frac{\frac{|\vec{k} \cdot \vec{q}|}{m} - (cq)^2}{\frac{|\vec{k} \cdot \vec{q}|}{m} - \Omega_q^2 - \frac{4\pi e^2}{m^2} \chi_t\left(\vec{q}, \frac{|\vec{k} \cdot \vec{q}|}{m}\right)} \right), \quad (14)$$

both vanish in the limit $e^2 \rightarrow \infty$.

The gradual vanishing of the commutators as e^2 goes to infinity indicates that the charge and current fluctuations carried by the particle-hole excitations are projected out gradually as e^2 increases, resulting in *chargeless* particle-hole excitations in the confinement limit $e^2 \rightarrow \infty$. Note that the confinement state analytically continues to the usual Fermi liquid state, and that there is no phase transition in between. The Gaussian theory thus suggests that the confinement state of a gas of fermions is a fermion liquid state formed by spin $S=1/2$ *chargeless* quasiparticles. Note that this is a rather unusual fermion liquid state, since bare fermions in the system carry spin $1/2$ and charge e , and the quasiparticles must have a vanishing overlap with bare fermions if they carry zero charge. The nature of the liquid of chargeless fermions will be examined in more detail in Sec. III, where we shall construct explicitly “chargeless” fermion operators that describe the dynamics of the system in the limit $e^2 \rightarrow \infty$. The limitations of Gaussian theory will become clear in the process.

III. SINGLE-PARTICLE PROPERTIES AND LOW-ENERGY EFFECTIVE LAGRANGIAN

To construct quasiparticle operators for a chargeless fermion liquid, we start from the equation of motion of the bare fermion operator $\psi_{\sigma}(\vec{r}) = (1/L^{d/2}) \sum_{\vec{k}} e^{i\vec{k} \cdot \vec{r}} f_{\vec{k}\sigma}$ at imaginary time,

$$\frac{\partial \psi_{\sigma}(\vec{r})}{\partial \tau} = [H, \psi_{\sigma}(\vec{r})] = \frac{1}{2m} \nabla^2 \psi_{\sigma}(\vec{r}) - \frac{1}{L^d} \sum_{\vec{q}} v(q) \rho(\vec{q}) e^{-i\vec{q} \cdot \vec{r}} \psi_{\sigma}(\vec{r}) - \frac{ie}{mL^{d/2}} \sum_{\vec{q}\lambda} e^{i\vec{q} \cdot \vec{r}} \vec{A}(\vec{q}, \lambda) \cdot \nabla \psi_{\sigma}(\vec{r}), \quad (15)$$

where we have replaced the fermion-density operator $\rho(\vec{r})$ by its expectation value n_0 in the diamagnetic term as we did in Secs. I and II in deriving Eq. (15). In the bosonization approximation, the operators $\rho(\vec{q})$ and $\vec{A}(\vec{q}, \lambda)$ can be decomposed as $\rho(\vec{q}) = \rho_{ph}(\vec{q}) + \rho_c(\vec{q})$, and $\vec{A}(\vec{q}, \lambda) = \vec{A}_{ph}(\vec{q}, \lambda) + \vec{A}_c(\vec{q}, \lambda)$, where $v(q)\rho_{ph}(\vec{q})$ and $e\vec{A}_{ph}(\vec{q}, \lambda)/c$ describe the coupling of the fermion to particle-hole excitations through the scalar and vector field fluctuations, respectively, whereas $v(q)\rho_c(\vec{q})$ and $e\vec{A}_c(\vec{q}, \lambda)/c$ describe the coupling of the fermion to collective modes (plasmons and photons). It is straightforward to obtain

$$v(q)\rho_{ph}(q) = \sum_{\vec{k}} \sqrt{|\Delta_{\vec{k}}(\vec{q})|} \theta[\Delta_{\vec{k}}(\vec{q})] \times \left[v_{eff}\left(q, \frac{|\vec{k} \cdot \vec{q}|}{m}\right) \gamma_{\vec{k}}^+(\vec{q}) + v_{eff}\left(q, \frac{|\vec{k} \cdot \vec{q}|}{m}\right) \gamma_{-\vec{k}}(-\vec{q}) \right],$$

$$v(q)\rho_c(\vec{q}) = L^{d/2} \left(-\frac{\partial \chi_0(q, \omega)}{\partial \omega} \right)_{\omega=E_0(\vec{q})}^{-1/2} [\gamma_0^+(\vec{q}) + \gamma_0(-\vec{q})] \quad (16a)$$

and

$$\frac{e\vec{A}_{ph}(\vec{q}, \lambda)}{m} = \frac{-1}{\sqrt{2}L^{d/2}} \tilde{\xi}_{\lambda}(\vec{q}) \sum_{\vec{k}} \theta[\Delta_{\vec{k}}(\vec{q})] \times \sqrt{|\Delta_{\vec{k}}(\vec{q})|} [\tilde{\xi}_{\lambda}(\vec{q}) \cdot \vec{A}_{eff}(\vec{q}, \vec{k}) \gamma_{\vec{k}}(\vec{q}) - \tilde{\xi}_{\lambda}(\vec{q}) \cdot \vec{A}_{eff}^*(\vec{q}, \vec{k}) \gamma_{-\vec{k}}^+(-\vec{q})],$$

$$\frac{e\vec{A}_c(\vec{q}, \lambda)}{m} = \sqrt{2}C(q) \tilde{\xi}_{\lambda}(\vec{q}) [\gamma_{\lambda}(\vec{q}) + \gamma_{\lambda}^+(-\vec{q})]. \quad (16b)$$

In particular, we observe that in the $e^2 \rightarrow \infty$ limit, the interaction between fermion and particle-hole excitations in both longitudinal $[v(q)\rho_{ph}(q)]$ and transverse $[e\vec{A}_{ph}(\vec{q}, \lambda)/c]$

channels remains regular and finite, and divergences in the $e^2 \rightarrow \infty$ limit appear only through the interactions between fermion and collective excitations. Note that *infrared* divergences in the interaction between fermion and particle-hole excitations exist. However, they are not results of taking $e^2 \rightarrow \infty$, and are not considered here. We shall discuss the effects of infrared divergences in Sec. IV.

The divergence in interaction between fermions and collective modes suggests that we first have to eliminate these interactions to obtain dynamics of (physical) fermion operators in the limit $e^2 \rightarrow \infty$. To eliminate these interactions, we look for a canonical transformation for the fermion operator,⁹

$$\psi_{P\sigma}(\vec{r}) = e^{\phi(\vec{r})} \psi_{\sigma}(\vec{r}), \quad (17)$$

where $\phi(\vec{r})$ is chosen such that the singular interaction between fermion and collective excitations are eliminated. Assuming that $\phi(\vec{r})$ depends linearly on the collective mode operators $\gamma_{0(\lambda)}(\vec{q})$ and $\gamma_{0(\lambda)}^+(\vec{q})$'s, and that H can be replaced by the bosonized Hamiltonian [Eq. (10)] in evaluating the commutator $[H, e^{\phi}]$, we obtain $\phi(\vec{r}) = \phi_l(\vec{r}) + \vec{W}(\vec{r}) \cdot \nabla$, where

$$\begin{aligned} \phi_l(\vec{r}) &= \frac{1}{L^{d/2}} \sum_{\vec{q}} \frac{e^{-i\vec{q} \cdot \vec{r}}}{E_0(\vec{q}) \left[-\frac{\partial \chi_0(q, \omega)}{\partial \omega} \right]^{1/2}} [\gamma_0^+(\vec{q}) - \gamma_0(-\vec{q})], \\ \vec{W}(\vec{r}) &= \frac{\sqrt{2}i}{L^{d/2}} \sum_{\vec{q}\lambda} \frac{e^{i\vec{q} \cdot \vec{r}} C(q)}{E_{\lambda}(\vec{q})} [\gamma_{\lambda}^+(-\vec{q}) - \gamma_{\lambda}(\vec{q})] \vec{\xi}_{\lambda}(\vec{q}), \end{aligned} \quad (18)$$

where ϕ_l and $\vec{W}$ describe the dressing of fermion by (longitudinal) plasmon modes and (transverse) photon modes, respectively. Note that $[\phi_l(\vec{r}), \vec{W}(\vec{r}') \cdot \nabla'] = 0$, because of decoupling between transverse and longitudinal fluctuations.

Next we examine the commutation relations between $\psi_{P\sigma}(\vec{r})$ and density and (transverse) current operators. It is straightforward to show that

$$\begin{aligned} [e\rho(\vec{q}), \psi_{P\sigma}(\vec{r})] &= e \left(\frac{2\chi_0[q, E_0(\vec{q})]}{E_0(\vec{q}) \left[-\frac{\partial \chi_0(q, \omega)}{\partial \omega} \right]_{\omega=E_0(\vec{q})}} - 1 \right) \\ &\quad \times e^{i\vec{q} \cdot \vec{r}} \psi_{P\sigma}(\vec{r}) \end{aligned} \quad (19a)$$

and

$$\begin{aligned} [\vec{j}_t(\vec{q}), \psi_{P\sigma}(\vec{r})] &= \frac{ie}{m} \left(1 - \frac{2C(q)^2}{E_{\lambda}(\vec{q})} \{ \chi_t[\vec{q}, E_{\lambda}(\vec{q})] + n_0 m \} \right) \\ &\quad \times e^{i\vec{q} \cdot \vec{r}} \nabla_t \psi_{P\sigma}(\vec{r}), \end{aligned} \quad (19b)$$

where $\nabla_t = \nabla - \hat{q}(\hat{q} \cdot \nabla)$. It is easy to see that both commutators vanish in the limit $e^2 \rightarrow \infty$, when $E_{0(\lambda)}(\vec{q}) \rightarrow \omega_P \rightarrow \infty$. It is also easy to see that the $\psi_{P\sigma}$ operator has the same commutation relation with the spin-density operator $\sigma(\vec{q}) = \sum_{\vec{k}} \sigma_{\vec{k}}(\vec{q})$ as a bare fermion operator ψ_{σ} . These results

together imply that the dressed single-particle operators $\psi_{P\sigma}(\vec{r})$'s represent spin $S = 1/2$, ‘‘chargeless’’ fermions in the limit $e^2 \rightarrow \infty$.

To show that $\psi_{P\sigma}(\vec{r})$ and $\psi_{P\sigma}(\vec{r}')$ represent independent physical fermionic excitations in the system when $\vec{r} \neq \vec{r}'$, we examine the commutation relation between the dressed fermion operators. We obtain

$$\{\psi_{P\sigma}(\vec{r}), \psi_{P\sigma'}^+(\vec{r}')\} \sim \frac{\sqrt{2}}{n_0(\pi|\vec{r}-\vec{r}'|)^{d-1}} \hat{O}(\vec{r}, \vec{r}')$$

in the limits $e^2 \rightarrow \infty$ and $|\vec{r}-\vec{r}'| \rightarrow \infty$, where

$$\begin{aligned} \hat{O} &= \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} \right) \cdot \{ \psi_{\sigma'}^+(\vec{r}') e^{\phi(\vec{r})} [\nabla \psi_{\sigma}(\vec{r})] e^{\phi^+(\vec{r}')} \\ &\quad - [\nabla \psi_{\sigma'}^+(\vec{r}')] e^{\phi(\vec{r})} \psi_{\sigma}(\vec{r}) e^{\phi^+(\vec{r}')} \}. \end{aligned}$$

Note that the same commutation relation is obtained if we kept only the contributions from longitudinal gauge field $\phi_l(\vec{r})$.¹ This is because the dominant contribution to the commutator comes from the nonzero commutation relation between the plasmon cloud around one chargeless fermion $[\phi_l(\vec{r})]$ and the other bare fermion $[\psi_{\sigma'}(\vec{r}')]$. The nonvanishing commutator between chargeless fermions separated by large distance reflects the nonlocal nature of the chargeless fermion operator. Fortunately, the power-law decay of the commutator between $\psi_{P\sigma}$ operators separated by large distances at dimensions larger than 1 indicates that they can be used as starting points to construct independent single-fermion operators when describing the dynamics of the system at long distances.¹

The relation between the fermion liquid formed by the chargeless and bare fermions can be seen by examining the relation between the two ground-state expectation values $\langle \psi_{\sigma}^+(\vec{r}) \psi_{\sigma}(\vec{r}') \rangle$ and $\langle \psi_{P\sigma}^+(\vec{r}) \psi_{P\sigma}(\vec{r}') \rangle$. To leading order in the projected Hilbert space,¹ we obtain

$$\begin{aligned} \langle \psi_{\sigma}^+(\vec{r}) \psi_{\sigma}(\vec{r}') \rangle &\sim e^{-2(\pi m e^2/n_0)^{1/2}} \\ &\quad \times \int_{L^{-1}}^{\Lambda} d^d q^{(1/q^2)} \{ 1 - \cos[\vec{q} \cdot (\vec{r}-\vec{r}')] \} \\ &\quad \times \langle \psi_{P\sigma}^+(\vec{r}) \psi_{P\sigma}(\vec{r}') \rangle. \end{aligned}$$

The singular behavior in the exponential factor comes from plasmon contributions; photons do not contribute to this order. Note that the average density of bare fermions and dressed fermions are the same, and that the Fermi-surface volume of the dressed fermions is exactly the same as that of the bare fermions. It is also easy to see that¹

$$\langle \psi_{P\sigma}^+(\vec{r}) \psi_{\sigma}(\vec{r}) \rangle \sim e^{-(\pi m e^2/n_0)^{1/2} [\int_{L^{-1}}^{\Lambda} d^d q^{(1/q^2)}]} \langle \psi_{\sigma}^+(\vec{r}) \psi_{\sigma}(\vec{r}) \rangle,$$

which vanishes in the $e \rightarrow \infty$ limit, indicating that the bare and chargeless fermions have zero wave-function overlap, in agreement with the marginal Fermi-liquid picture.

The bosonization result we obtained in Sec. II suggests that in the limit $e^2 \rightarrow \infty$ the low-energy physics of our system

may be described as a Fermi liquid of the chargeless fermions. To examine this picture, we consider the equation of motion for the chargeless fermion operators $\psi_{P\sigma}$, assuming that they can be treated as canonical fermions. It is straightforward to show that

$$\begin{aligned} \frac{\partial \psi_{P\sigma}(\vec{r})}{\partial \tau} = & \frac{1}{2m} (\nabla^2 \psi_{P\sigma}(\vec{r}) - \{\nabla^2 \phi(\vec{r}) - :[\nabla \phi(\vec{r})]^2:\} \psi_{P\sigma}(\vec{r}) \\ & - 2 \nabla \phi(\vec{r}) \cdot \nabla \psi_{P\sigma}(\vec{r})) \\ & - \frac{1}{L^d} \sum_q v(q) \rho_{ph}(\vec{q}) e^{i\vec{q} \cdot \vec{r}} \psi_{P\sigma}(\vec{r}) \\ & - \frac{i}{L^{d/2}} \sum_{\vec{q}\lambda} e^{i\vec{q} \cdot \vec{r}} \frac{e}{m} \vec{A}_{ph}(\vec{q}, \lambda) \cdot \nabla \psi_{P\sigma}(\vec{r}), \end{aligned} \quad (20)$$

where we have neglected constant energy terms coming from normal ordering of operators in deriving Eq. (20). Note that, although direct couplings between fermions and collective modes are eliminated in the equation of motion for $\psi_{P\sigma}(\vec{r})$, interactions between chargeless fermions and particle-hole excitations remain in the equation of motion. Moreover, an indirect coupling to collective modes is also generated from the fermion kinetic-energy term, as is in the similar ‘‘small polaron’’ problem.⁹ It is easy to see by direct power counting of e^2 in the $\phi(\vec{r})$ field that the coupling of the chargeless fermion to collective excitations through the kinetic-energy term is much weaker than the original fermion-plasmon and fermion-photon couplings. In particular, the self-energy correction of chargeless fermions from $\phi(\vec{r})$ fields remains finite in the $e^2 \rightarrow \infty$ limit.

It is interesting to note that if we neglect the indirect coupling of chargeless fermions through the kinetic-energy term to the collective modes, the equation of motion of the chargeless fermions can be reproduced by the effective action

$$\begin{aligned} S_{eff}(\psi_P, \psi_P^\dagger) = & \sum_\sigma \int_0^\beta d\tau d^d x \psi_{P\sigma}^\dagger(\vec{x}, \tau) \left[\frac{\partial}{\partial \tau} - \frac{\nabla^2}{2m} - \mu \right. \\ & \left. + i \vec{A}_{eff}(\vec{x}, \tau) \cdot \nabla + \phi_{eff}(\vec{x}, \tau) \right] \psi_{P\sigma}(\vec{x}, \tau), \end{aligned} \quad (21a)$$

where $\phi_{eff}(\vec{q}) \sim v(\vec{q}) \rho_{ph}(\vec{q})$ and $\vec{A}_{eff}(\vec{q}) \sim e \vec{A}_{ph}(\vec{q})/m$, with dynamics given by

$$\begin{aligned} S_{eff}(\phi) = & -\frac{1}{2} \sum_{\vec{q}, i\omega} \chi_0(\vec{q}, i\omega) |\phi_{eff}(\vec{q}, i\omega)|^2, \\ S_{eff}(\vec{A}) = & -\frac{1}{2} \sum_{\vec{q}, i\omega} \bar{\chi}_t(\vec{q}, i\omega) \vec{A}_{eff}(\vec{q}, \omega) \cdot \vec{A}_{eff}(-\vec{q}, -i\omega), \end{aligned} \quad (21b)$$

where $\bar{\chi}_t(\vec{q}, i\omega) = \chi_t(\vec{q}, i\omega) + mn_0$ is the *total* transverse current susceptibility of free fermions. Fermions interacting with gauge fields with effective interaction (21) were studied

in detail by a number of authors,^{10–13} and it is believed that interaction between fermions and transverse gauge field with effective dynamics (21) may lead to a breakdown of Fermi-liquid theory and the formation of a marginal Fermi liquid. If this is the case, the ground state formed by the chargeless fermions would itself be in a marginal Fermi-liquid state, and our system could be viewed as a ‘‘double’’ marginal Fermi-liquid state of the original fermions. This is in contrast to (Gaussian) bosonization theory, which predicts that the chargeless fermions should form a *Fermi-liquid* state. The failure of Gaussian theory in producing a marginal Fermi-liquid state shows the limitation of Gaussian theory.

It is also interesting to note that the same form of low-energy effective Lagrangian [Eq. (21) for spinons is obtained from a typical treatment of gauge field models in the $e^2 \rightarrow \infty$ limit¹⁴ for lattice systems satisfying the constraint of no double occupancy, suggesting that spinons in the usual treatment of the t - J model are strongly related to our chargeless fermions. A detailed treatment of the t - J model, using our approach, will be reported in a different paper.

Finally we examine the validity of replacing the fermion density operator $\rho(\vec{r})$ by the average fermion density n_0 in the diamagnetic term when we derive the equation of motions for bare and chargeless fermions. We shall now show that this approximation is justified in the $e^2 \rightarrow \infty$ limit for the chargeless fermions $\psi_{P\sigma}$. We look at the correction term in the equation of motion for chargeless fermions, $[\Delta H, \psi_{P\sigma}(\vec{r})]$, where

$$\Delta H = \frac{e^2}{2m} \int d^d r \vec{A}(\vec{r}) \cdot \vec{A}(\vec{r}) [\rho(\vec{r}) - n_0].$$

It is easy to show from Eqs. (16a), (19a), and (18) that $[\rho(\vec{q}), \psi_{P\sigma}(\vec{r})] \sim (1/e^2) \psi_{P\sigma}$, $e \vec{A}/m \sim e^{1/2}(\gamma_\lambda + \gamma_\lambda^\dagger)$, and $[e \vec{A}(\vec{r}), \psi_{P\sigma}(\vec{r})] \sim \nabla \psi_{P\sigma}$ in the $e^2 \rightarrow \infty$ limit. As a result we have

$$[\Delta H, \psi_{P\sigma}(\vec{r})] \sim [\vec{A}(\vec{r}) \cdot \nabla \psi_{P\sigma}(\vec{r})][\rho(\vec{r}) - n_0] + O\left(\frac{1}{e}\right).$$

The first term vanishes in the $e^2 \rightarrow \infty$ limit if the matrix elements $\langle n | (\rho(\vec{r}) - n_0) | m \rangle$ vanish faster than $1/e^{1/2}$ in the physical Hilbert space of interests. This will be the case if the physical Hilbert space is spanned by the chargeless fermion operators we construct, because of the commutation rule between density operator and chargeless fermions [Eq. (19a)].

IV. SUMMARY

In this paper we studied the low-energy properties of a gas of spin $S=1/2$ fermions interacting with a U(1) gauge field with high momentum cutoff $q < \Lambda \ll k_F$. In particular, we considered the $e^2 \rightarrow \infty$ limit where the gauge field becomes confining. We have analyzed the problem in two steps. Within the Gaussian approximation, we showed that a liquid-state solution is obtained where the particle-hole excitation spectrum of the system is always Fermi-liquid like, with the charge carried by the particle-hole excitation vanishing continuously in the $e^2 \rightarrow \infty$ limit. To examine the

above picture more carefully, in Sec. III we constructed $S=1/2$ chargeless fermionic operators that could be used as the starting point for constructing quasiparticles in the system. We found that the dynamics of these “physical” single-particle operators are governed by a Lagrangian very similar to the effective Lagrangian obtained in a conventional treatment of the gauge field model.¹⁴ The solution we obtained describes a kind of “double-marginal” Fermi-liquid state, where (i) the chargeless fermions have a vanishing wave-function overlap with bare fermions in the system, and (ii) they form a marginal Fermi-liquid state themselves.

It is useful to distinguish the effects of longitudinal and transverse gauge fields on our system, in particular how they contribute in the $e^2 \rightarrow \infty$ limit, to the construction of chargeless fermions and their effective dynamics. First we considered the longitudinal gauge field. The main effect of longitudinal gauge field appears through fermion-plasmon interaction, which is singular in the limit $e^2 \rightarrow \infty$. In a dimension $d=2$, there exists an additional infrared singularity associated with the response of plasmon field (orthogonality catastrophe effect) when charged particles are added to the system.¹⁵ The interaction between fermions and particle-hole excitations is finite and regular for any value of e^2 . The singular fermion-plasmon interaction and the infrared singularity at dimension $d=2$ are eliminated together with our canonical transformation [Eq. (17)],¹ and the remaining interaction between the chargeless fermions and longitudinal gauge fields is regular and introduces no further singularities in the chargeless fermion system.¹

However, the situation is quite different for the transverse gauge field. The singularity associated with the fermion-photon interaction in the limit $e^2 \rightarrow \infty$ is weaker than the corresponding singularity associated with fermion-plasmon interaction, as can be seen from a direct power counting of e^2 in ϕ_l and $\vec{W}$ fields. As a result, the canonical transformation [Eq. (17)] is dominated by the effect of plasmons in the limit $e^2 \rightarrow \infty$. On the other hand, the interaction between bare fermions and particle-hole excitations through transverse gauge field is infrared singular for any value of e^2 in both two and three dimensions,¹² and the singularity carries over to the effective interaction [Eq. (21)] between dressed fermions and the transverse gauge field. It is believed that the infrared singularity in the interaction between fermi-

ons and particle-hole excitations through a transverse gauge field will lead to the formation of a marginal Fermi-liquid state.^{10–13}

The possibility of the formation of a marginal Fermi-liquid state of the chargeless fermions points out the limitation of bosonization theory in the Gaussian approximation, where a Fermi-liquid-type particle-hole excitation spectrum is always obtained in the limit $e^2 \rightarrow \infty$. It is clear that higher-order terms in the cumulant expansion of the transverse gauge field have to be included in bosonization theory to obtain the correct marginal Fermi-liquid behavior,¹² and a complete theory where particle-hole and single-particle excitations are treated self-consistently is still missing.

An immediate question that accompanies this observation is that we have used the excitation spectrum in the Gaussian approximation to construct chargeless fermion operators in our paper. If the particle-hole excitation spectrum is incorrect, our results, based on chargeless fermion operators $\psi_{P\sigma}$, become questionable. Fortunately, although the particle-hole excitation spectrum in Gaussian theory is questionable, the plasmon and photon excitations we obtained are exact in the $e^2 \rightarrow \infty$ limits. In particular our canonical transformation [Eq. (17)] for the chargeless fermions $\psi_{P\sigma}$ involves only the collective mode operators, and is independent of the Gaussian approximation. However, it has to be noted that in deriving the effective action $S_{eff}(\psi_P, \psi_P^\dagger)$, which describes the scattering of chargeless fermions with particle-hole excitations, it is assumed that the particle-hole excitations are described correctly by Gaussian theory. Strong modifications may occur if the spectrum of the particle-hole excitation is modified strongly in the correct theory.

Another, more fundamental question, is how valid it is to treat our chargeless fermions as canonical fermions, given that they obey a rather nontrivial commutation relation. In particular, it is possible that the unusual commutation relation between fermions may lead to additional non-Fermi-liquid behavior. At present we have no answer to this question.

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