Influence of backward scattering on correlation exponents in a one-dimensional system

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Using bosonization and phase shift representation, we rigorously treat backward scattering of electrons on an impurity in a one-dimensional interacting spinless electronic system, and demonstrate that correlation exponents of the system depend on a phase shift induced by the backward scattering, and usual exponent duality between ultraviolet and infrared fixed points comes from the phase shift dependence of the correlation exponents.

I. INTRODUCTION

It is very important to rigorously treat backward scattering of electrons on an impurity in one-dimensional systems, because the x-ray absorption¹⁻⁴ and the impurity scattering⁵⁻¹⁴ in a Tomonaga-Luttinger liquid, edge state tunneling of the fractional quantum Hall effect at a quantum point contact,¹⁵⁻¹⁷ and quantum Brownian motion of an one-dimensional particle in a periodic potential,^{18,19} all involve the treatment of backward scattering term. For a repulsive electron interaction, the backward scattering term is relevant, and backward scattering potential is renormalized to infinity in the low energy limit. Therefore, usual perturbation expansion treatment of this term becomes impossible, and one needs to use renormalization group equations to compute some physical quantities. However, with different perturbation methods, one may obtain different results because of divergence of the renormalized backward scattering potential, so it is not surprising that there are debates on the low-energy behavior of electron density of states,^{5,14,20,21} the contribution of the backward scattering to the exponent of Fermi-edge singularity of x-ray absorption,^{1-4,14,22-24} and so on.

A spinless interacting electronic system can be generally described by the Hamiltonian $H = H_0 + H_{IM}$, where

$$\begin{aligned} H_0 = & -i\hbar v_F \int dx [\psi_R^\dagger(x) \partial_x \psi_R(x) \\ & - \psi_L^\dagger(x) \partial_x \psi_L(x)] + V \int dx \rho_R(x) \rho_L(x), \\ H_{IM} = & U_{2k_F} [\psi_R^\dagger(0) \psi_L(0) + \psi_L^\dagger(0) \psi_R(0)], \end{aligned} \quad (1)$$

where $\psi_R(x) [\psi_L^\dagger(x)]$ is the electron field that propagate to the right with wave vectors $\sim +k_F$, $\psi_L(x) [\psi_L^\dagger(x)]$ is the field operator of left propagating electrons with wave vectors $\sim -k_F$, $\rho_{R(L)}(x) = \psi_{R(L)}^\dagger(x) \psi_{R(L)}(x)$ are the electron density operators, and V describes density-density interaction with momentum transferring much smaller than k_F , as well as $U_{2k_F} = U(k=2k_F)$ is usual backward scattering potential of the electrons on a nonmagnetic impurity (barrier). In bosonization representation of the electron fields $\psi_{R(L)}(x)$, one can easily find that the backward scattering term has a

conformal dimension $\Delta = g$, where $g = [(2\pi\hbar v_F - V)/(2\pi\hbar v_F + V)]^{1/2}$. For repulsive electron interaction, $V > 0$, i.e., $g < 1$, the renormalized backward scattering potential $U_{2k_F}(l)$ is⁵ $U_{2k_F}(l) = e^{(1-g)l} U_{2k_F}$, where l is a scaling parameter of energy (momentum), in the low-energy limit, it goes to infinity, $U_{2k_F}(l \rightarrow \infty) \rightarrow \infty$. Therefore, usual expansion perturbation treatment of the backward scattering term is dangerous, it may give a wrong result. It is desirable to exactly treat the backward scattering.

In present paper, using phase shift representation, we can rigorously treat the backward scattering of impurity, and show the correct low energy behavior of the electron density of states and the physical explanation of exponent duality between ultraviolet and infrared fixed points. In Sec. II, we introduce the phase shift induced by the backward scattering, and give its renormalization group equation. In the phase shift representation, the action of the backward scattering to the system can be completely incorporated to the electron interaction term. In Sec. III, we calculate some correlation functions, and demonstrate that near the impurity there exists usual exponent duality between the ultraviolet and infrared fixed points. In Sec. IV, we demonstrate the reflection of electrons on the impurity without using any current-current correlation function. In Sec. V, we compare our method with previous ones, and give our conclusion.

II. PHASE SHIFT REPRESENTATION OF BACKWARD SCATTERING

Now we use a new set of electron fields to re-write the Hamiltonian, so that we can diagonalize the backward scattering term, and use an unitary transformation to absorb it into the electron interaction terms by introducing a phase shift. To clearly showing our idea, we first consider wavefunction equations of the system at $V=0$,

$$-i\hbar v_F \partial_x \psi_R(x) + U_{2k_F} \delta(x) \psi_L(x) = E \psi_R(x),$$

$$i\hbar v_F \partial_x \psi_L(x) + U_{2k_F} \delta(x) \psi_R(x) = E \psi_L(x)$$

which can be rewritten as

$$-i\hbar v_F \partial_x \psi_{1(2)}(x) \pm U_{2k_F} \delta(x) \psi_{1(2)}(x) = E \psi_{1(2)}(x),$$

where $\psi_{1(2)}(x) = (1/\sqrt{2})[\psi_R(x) \pm \psi_L(-x)]$. In terms of these new electron fields, the backward scattering only induces a global phase shift $\delta_0 = U_{2k_F}/(\hbar v_F)$ on the left and right sides of the impurity, $\psi_{1(2)}(x) = e^{\mp i(\delta_0/2)\text{sgn}(x)} e^{iEx/(\hbar v_F)}$. However, for $V \neq 0$, the electron interaction term becomes very complicated because replacing $\psi_{R(L)}(x)$ by $\psi_{1(2)}(x)$ is a nonlocal transformation, on the other hand, the electron interaction will renormalize the phase shift δ_0 . In the bosonic representation²⁵⁻²⁷ of the electron fields $\psi_{1(2)}(x) = (D/2\pi\hbar v_F)^{1/2} \exp\{-i\Phi_{1(2)}(x)\}$, where $\partial_x \Phi_{1(2)}(x) = 2\pi\rho_{1(2)}(x)$, and D is bandwidth of the system, the Hamiltonian (1) can be written as

$$H_0 = \frac{\hbar v_F}{4\pi} \int dx \{ [\partial_x \Phi_+(x)]^2 + [\partial_x \Phi_-(x)]^2 \} + \frac{V}{4} \int dx \left\{ -\frac{1}{2\pi^2} \partial_x \Phi_+(x) \partial_x \Phi_+(-x), + \left(\frac{D}{\pi\hbar v_F} \right)^2 \sin[\sqrt{2}\Phi_-(x)] \sin[\sqrt{2}\Phi_-(-x)] \right\}, \quad (2)$$

$$H_{IM} = \frac{\hbar v_F \delta_0}{\sqrt{2}\pi} \int dx \delta(x) \partial_x \Phi_-(x),$$

where $\Phi_{+(-)}(x) = (1/\sqrt{2})[\Phi_1(x) \pm \Phi_2(x)]$. It becomes clear that the backward scattering only influences the boson field $\Phi_-(x)$, and the collective excitation modes described by $\Phi_+(x)$ is independent of the backward scattering. The H_0 part of the Hamiltonian is invariant under a renormalization group transformation, which can be easily proved by usual renormalization group method, whereas, the H_{IM} part will be renormalized under the renormalization group transformation because of the sine-type interaction term in Eq. (2). The only renormalizable quantity of the H_{IM} is the phase shift δ_0 , if we replace the phase shift δ_0 by a renormalized one $\delta(l)$, we can obtain a renormalized Hamiltonian

$$H^R = H_0 + H_{IM}|_{\delta_0 \rightarrow \delta(l)} \quad (3)$$

which is a set of Hamiltonians with index l (renormalization parameter), each of which describes the physics of the original Hamiltonian (1) or (2) in different low-energy scales. The Hamiltonian (3) can be directly obtained from the Hamiltonian (2) by using the usual renormalization group method (see below). Due to the phase shift δ_0 is only relevant to the boson field $\Phi_-(x)$, its renormalization comes from the sine-type interaction term, which can be treated perturbatively for small parameter $\gamma = V/(2\pi\hbar v_F)$. As usual renormalization procedure, the boson field $\Phi_-(x)$ can be divided into slow and fast components²⁸ $\Phi_-(x) = \Phi_-^<(x) + \Phi_-^>(x)$, where $\Phi_-^>(x)$ is composed of Fourier components with $e^{-dl}D < \hbar v_F p < D$. In terms of the fields $\Phi_-^>(x)$, the Hamiltonian only including the field $\Phi_-(x)$ can be written as $H' = H^< + H^>$, where

$$H^< = \frac{\hbar v_F}{4\pi} \int dx [\partial_x \Phi_-^<(x)]^2 + \frac{\hbar v_F \delta}{\sqrt{2}\pi} \int dx \delta(x) \partial_x \Phi_-^<(x) - \frac{V}{4} \left(\frac{D}{\pi\hbar v_F} \right)^2 \int dx \sin[\sqrt{2}\Phi_-^<(x)] \sin[\sqrt{2}\Phi_-^<(-x)],$$

$$H^> = \frac{\hbar v_F}{4\pi} \int dx [\partial_x \Phi_-^>(x)]^2 + \frac{\hbar v_F \delta}{\sqrt{2}\pi} \int dx \delta(x) \partial_x \Phi_-^>(x) - \frac{V}{\sqrt{2}} \frac{D}{\pi\hbar v_F} \int dx \Phi_-^<(x) \times \sin[\sqrt{2}\Phi_-^>(-x)] \cos[\sqrt{2}\Phi_-^>(x)].$$

Here we have neglected higher order terms of the field $\Phi_-^<(x)$ which does not contribute to renormalization group equation of the phase shift δ . After integrating out $\Phi_-^>(x)$ and rescaling momentum and energy, we can obtain a (perturbative) renormalization equation for the phase shift

$$\frac{d\delta(l)}{dl} = \frac{\gamma}{2} \sin[2\delta(l)], \quad \delta(l) = \arctan[e^{\gamma l} \tan(\delta_0)] \quad (4)$$

which is the same as that in Ref. 3 obtained when calculating the x-ray response function. Due to appearing the linear boson field term $\delta(x) \partial_x \Phi_-^>(x)$ in the Hamiltonian $H^>$, the boson field $\Phi_-^>(x)$ has a nonzero average value $\langle \Phi_-^>(x) \rangle = (\delta/\sqrt{2})\text{sgn}(x)$, therefore there is a $\sin(2\delta)$ form in the right-hand side of Eq. (4). For the case of $V=0$, we have the relation, $\delta(l) = \delta_0$, the phase shift is not renormalized, and the backward scattering potential only induces a global phase shift δ_0 , which is consistent with the above discussion. In the high-energy region, $\delta_0 \sim 0$, i.e., $\tan(\delta_0) \sim \delta_0$, and small γ ($g \simeq 1 - \gamma$), the renormalized phase shift can be rewritten as $\delta(l) = \arctan[U_{2k_F}(l)/(\hbar v_F)]$ which was used in our previous works.^{14,29} We see that for repulsive electron interaction $V > 0$ the infrared fixed point (zero temperature limit, i.e., $l \rightarrow \infty$) of the system corresponds to the phase shift $\delta^c = \delta(l \rightarrow \infty) = \pi/2$. It notes that the dimensionless parameter γ appears in the Eq. (4) which is obtained by one-loop approximation. Generally, only the dimensionless parameter g appear in physical quantities, not γ . Therefore, in low energy limit, a general expression of the renormalized phase shift is, $\delta(l) = \arctan[e^{(1/g-1)l} \tan(\delta_0)]$, where for small γ , $1/g - 1 \simeq \gamma$.

We use the renormalized Hamiltonian (3) to study the low-energy physical properties of the system, where all the quantities are finite in the low-energy limit. At $\delta(l=0) = \delta_0 \simeq 0$, it restores to the Hamiltonian (2), and at $\delta(l \rightarrow \infty) = \pi/2$, it is the Hamiltonian of the system at the infrared critical fixed point. In terms of the electron fields $\psi_{1(2)}(x)$, the backward scattering term becomes a simple potential scattering, and can be gauged away by an unitary transformation, with its effect on the system completely incorporated to the electron interaction term. In this way we can rigorously treat the backward scattering term. Following the procedures in Ref. 14, we consider the unitary transformation

$$U = \exp \left\{ i \frac{\delta}{2\pi} [\Phi_1(0) - \Phi_2(0)] \right\},$$

where $\delta \equiv \delta(l)$, and take the global gauge transformations

$$\psi_{1(2)}(x) = \bar{\psi}_{1(2)}(x) e^{i\theta_{1(2)}}, \quad \theta_1 - \theta_2 = \delta$$

the total Hamiltonian $\bar{H}^R = U^\dagger H^R U$ can be written as¹⁴

$$\begin{aligned} \bar{H}^R = & -i\hbar v_F \int dx [\bar{\psi}_R^\dagger(x) \partial_x \bar{\psi}_R(x) - \bar{\psi}_L^\dagger(x) \partial_x \bar{\psi}_L(x)] \\ & + \frac{V}{2} \int dx [\alpha \bar{\rho}_R(x) \bar{\rho}_R(-x) + \alpha \bar{\rho}_L(x) \bar{\rho}_L(-x) \\ & + 2\beta \bar{\rho}_R(x) \bar{\rho}_L(x)], \end{aligned} \quad (5)$$

where $\bar{\psi}_R(x) = [\bar{\psi}_1(x) + \bar{\psi}_2(x)]/\sqrt{2}$, $\bar{\psi}_L(-x) = [\bar{\psi}_1(x) - \bar{\psi}_2(x)]/\sqrt{2}$; $2\alpha = 1 - \cos(2\delta)$, $2\beta = 1 + \cos(2\delta)$; and $\bar{\rho}_{R(L)}(x) = \bar{\psi}_{R(L)}^\dagger(x) \bar{\psi}_{R(L)}(x)$. We have omitted the term $i(V/2)\sin(2\delta)\int_0^\infty dx [\bar{\psi}_L^\dagger(-x)\bar{\psi}_R(x) - \text{h.c.}][\bar{\rho}_R(-x) - \bar{\rho}_L(x)]$, because it has a conformal dimension $[\sim 1 + (g + 1/g)/2]$ larger than 2, and in the low-energy limit ($l \rightarrow \infty$), the phase shift δ goes to $\pi/2$, it becomes zero, therefore it only contributes high order correction to correlation functions. The Hamiltonian (5) describes the physical property of the original Hamiltonian (1) or (2) in different low energy scales presented by the renormalization parameter l [appearing in the phase shift $\delta \equiv \delta(l)$]. In the original Hamiltonian H^R (3) the impurity scattering term is obviously appearing, after the unitary and global gauge transformations, this term disappears in Eq. (5), but the electron interaction term is altered, which means that the impurity backward scattering alters the electron interaction. Physical explanation of the behavior is that the right- and left-moving electrons are partially reflected on the impurity site $x=0$ [see Eq. (13)], therefore the right- and left-moving electron wave functions are mixed, and the electron interaction is altered by the impurity backward scattering. This is the prominent character of the impurity backward scattering in one-dimensional system. Due to the electron interaction altered by the backward scattering, the low-energy physical properties of the system is also changed, such as its transport property and the electron density of states near the impurity.

From Eq. (5), we see that the backward scattering alters the effective electron interaction which determines correlation exponents of correlation functions (but density-density correlation function) of the system, therefore, it will influence the correlation exponents. To prove it, we write out the action of the system according to Eq. (5)

$$\begin{aligned} S = & \int_0^{1/T} d\tau \int dx \left\{ \bar{\psi}_R^\dagger(x, \tau) \mathcal{D}_R \bar{\psi}_R(x, \tau) + \bar{\psi}_L^\dagger(x, \tau) \mathcal{D}_L \bar{\psi}_L(x, \tau) \right. \\ & + \frac{V}{2} [\alpha \bar{\rho}_R(x, \tau) \bar{\rho}_R(-x, \tau) + \alpha \bar{\rho}_L(x, \tau) \bar{\rho}_L(-x, \tau) \\ & + 2\beta \bar{\rho}_R(x, \tau) \bar{\rho}_L(x, \tau)] - i\phi_R(x, \tau) \\ & \times [\bar{\rho}_R(x, \tau) - \bar{\psi}_R^\dagger(x, \tau) \bar{\psi}_R(x, \tau)] - i\phi_L(x, \tau) \\ & \times [\bar{\rho}_L(x, \tau) - \bar{\psi}_L^\dagger(x, \tau) \bar{\psi}_L(x, \tau)] \left. \right\}, \end{aligned} \quad (6)$$

where $\mathcal{D}_{R(L)} = \partial_\tau + i\hbar v_F \partial_x$, and T is temperature (choosing $k_B = 1$). We have introduced the Lagrangian multipliers (boson fields) $\phi_{R(L)}(x, \tau)$ to decouple the four-electron interaction by adding the constraint conditions $\bar{\rho}_{R(L)}(x, \tau) = \bar{\psi}_{R(L)}^\dagger(x, \tau) \bar{\psi}_{R(L)}(x, \tau)$. Here only quadratic form of the electron fields $\bar{\psi}_{R(L)}(x, \tau)$ appears in Eq. (6). After integrating out the auxiliary fields $\phi_{R(L)}(x, \tau)$ and electron fields $\bar{\psi}_{R(L)}(x, \tau)$, we can obtain the effective action^{14,30}

$$\begin{aligned} S_{\text{eff}}[\bar{\rho}] = & \frac{T}{L} \sum_{p, \omega} \left\{ \frac{1}{4A_R} \bar{\rho}_R(-p, -\omega) \bar{\rho}_R(p, \omega) \right. \\ & + \frac{1}{4A_L} \bar{\rho}_L(-p, -\omega) \bar{\rho}_L(p, \omega) + \frac{V}{2} [\alpha \bar{\rho}_R(-p, -\omega) \\ & \times \bar{\rho}_R(-p, \omega) + \alpha \bar{\rho}_L(-p, -\omega) \\ & \times \bar{\rho}_L(-p, \omega) + 2\beta \bar{\rho}_R(-p, -\omega) \bar{\rho}_L(p, \omega)] \left. \right\}, \end{aligned} \quad (7)$$

where $A_{R(L)} = (1/4\pi)[p/(p \mp i\omega)]$ (choosing $\hbar = v_F = 1$). According to this effective action (7), we can obtain the collective excitation spectrums of the system

$$\epsilon_\pm = \left[1 - \left(\frac{V}{2\pi\hbar v_F} \right)^2 (\alpha \pm \beta)^2 \right]^{1/2} p \quad (8)$$

It is noted that one collective excitation spectrum depends on the phase shift δ , and another one is independent of it. This can be easily understood that the interaction terms in the Hamiltonian (5) can be rewritten as

$$\begin{aligned} & \frac{V}{4} \{ [\bar{\rho}_R(x) + \bar{\rho}_L(-x)][\bar{\rho}_R(-x) + \bar{\rho}_L(x)] \\ & - \cos(2\delta) [\bar{\rho}_R(x) - \bar{\rho}_L(-x)][\bar{\rho}_R(-x) - \bar{\rho}_L(x)] \}, \end{aligned}$$

therefore ϵ_+ and ϵ_- are the collective excitation spectrums of the fields $\bar{\rho}_R(x) + \bar{\rho}_L(-x)$ and $\bar{\rho}_R(x) - \bar{\rho}_L(-x)$, respectively. In terms of these fields, we have two interaction strength parameters V and $V \cos(2\delta)$, which determine correlation exponents of the system.

III. CALCULATION OF CORRELATION FUNCTIONS

In the phase shift representation, the renormalized phase shift δ reflects the influence of the backward scattering on the system. The general effective action (7) which depends on the phase shift δ can be used to compute correlation functions of the system, we therefore can clearly show the influence of the backward scattering on their correlation exponents. According to the bosonization representation of the electron fields $\bar{\psi}_{R(L)}(x, \tau)$

$$\begin{aligned} \bar{\psi}_{R(L)}(x, \tau) = & \left(\frac{D}{2\pi\hbar v_F} \right)^{1/2} \\ & \times \exp \left\{ \pm \frac{2\pi T}{L} \sum_{\omega, p \neq 0} \frac{e^{-i(px - \omega\tau)}}{p} \bar{\rho}_{R(L)}(p, \omega) \right\} \end{aligned}$$

any field operator $A[\bar{\psi}_{R(L)}]$ can be represented by the density fields $\bar{\rho}_{R(L)}(x, \tau)$. For example, the field $\bar{\psi}_R^\dagger(x, \tau)\bar{\psi}_L(x, \tau)$ can be written as

$$\begin{aligned} \bar{\psi}_R^\dagger(x, \tau)\bar{\psi}_L(x, \tau) = & \frac{D}{2\pi\hbar v_F} \exp\left\{-\frac{2\pi\tau}{L}\right. \\ & \times \sum_{\omega, p \neq 0} \frac{e^{-i(p x - \omega \tau)}}{p} \\ & \left. \times [\bar{\rho}_R(p, \omega) + \bar{\rho}_L(p, \omega)]\right\}. \end{aligned}$$

Therefore, with the general effective action S_{eff} [Eq. (7)] we can calculate the correlation function of the field $A[\bar{\psi}_{R(L)}]$

by taking the functional average over the density fields $\bar{\rho}_{R(L)}(x, \tau)$

$$\begin{aligned} G_A(x, \tau; x', \tau') = & \frac{1}{\Omega} \int D\bar{\rho}_R D\bar{\rho}_L A^\dagger[\bar{\psi}_{R(L)}(x, \tau)] \\ & \times A[\bar{\psi}_{R(L)}(x', \tau')] e^{-S_{\text{eff}}[\bar{\rho}]} \end{aligned} \quad (9)$$

where $\Omega = \int D\bar{\rho}_R D\bar{\rho}_L \exp\{-S_{\text{eff}}[\bar{\rho}]\}$. Using Eq. (9), we compute the Green function $G_{R(L)}(x, \tau; x', \tau')$ of the electron $\bar{\psi}_{R(L)}(x, \tau)$ which can be used to calculate the electron density of states, and the correlation function $G_{RL}(x, \tau; x', \tau')$ of the field $\bar{\psi}_R^\dagger(x, \tau)\bar{\psi}_L(x, \tau)$ which can be used to calculate tunneling conductance of the system. By a direct calculation, we obtain (using real time, $\tau = it$, $\Delta t > 0$)

$$\begin{aligned} G_{R(L)}(x, t; x', t') = & \prod_{j=1}^2 \frac{1}{(\Delta x \mp \alpha_j \Delta t + i\eta)^{1/2}} \prod_{j=1}^2 [(\Delta x - \alpha_j \Delta t + i\eta)(\Delta x + \alpha_j \Delta t - i\eta)]^{-(g_j + 1/g_j - 2)/8} \\ & \times \left[\frac{(x+x' - \alpha_1 \Delta t + i\eta)^2 (x+x' + \alpha_1 \Delta t - i\eta)^2}{(2x+i\eta)(2x-i\eta)(2x'+i\eta)(2x'-i\eta)} \right]^{-(1/g_1 - g_1)/16} \\ & \times \left[\frac{(x+x' - \alpha_2 \Delta t + i\eta)^2 (x+x' + \alpha_2 \Delta t - i\eta)^2}{(2x+i\eta)(2x-i\eta)(2x'+i\eta)(2x'-i\eta)} \right]^{(1/g_2 - g_2)/16}, \end{aligned} \quad (10)$$

$$\begin{aligned} G_{RL}(x, t; x', t') = & \prod_{j=1}^2 [(\Delta x - \alpha_j \Delta t + i\eta)(\Delta x + \alpha_j \Delta t - i\eta)]^{-g_j/2} \\ & \times \left[\frac{(\Delta x - \alpha_1 \Delta t + i\eta)^2 (\Delta x + \alpha_2 \Delta t - i\eta)^2}{(2x+i\eta)(2x-i\eta)(2x'+i\eta)(2x'-i\eta)} \right]^{g_1/4} \left[\frac{(\Delta x - \alpha_2 \Delta t + i\eta)^2 (\Delta x + \alpha_2 \Delta t - i\eta)^2}{(2x+i\eta)(2x-i\eta)(2x'+i\eta)(2x'-i\eta)} \right]^{-g_2/4}, \end{aligned}$$

where $\Delta x = x - x'$, $\Delta t = t - t'$, $\alpha_1 = \sqrt{1 - \gamma^2}$, $\alpha_2 = \sqrt{1 - \gamma^2 \cos^2(2\delta)}$, $g_1 = g$, $g_2 = g(\delta) = [1 - \gamma \cos(2\delta)]^{1/2} / [1 + \gamma \cos(2\delta)]^{1/2}$, and η is a momentum cutoff factor proportional to $1/D$. For simplicity, we consider the asymptotic behavior of the correlation functions (10) for the two limits of near ($x \sim 0$) and far away from ($x \rightarrow \infty$) the impurity. According to Eq. (10), we have the relations

$$\begin{aligned} G_{R(L)}(x, t; x, 0) \sim & \begin{cases} (t - i\eta)^{-[1/g + g(\delta)]/2}, & x \rightarrow 0, \\ (t - i\eta)^{-[g + g(\delta) + 1/g + 1/g(\delta)]/4}, & x \rightarrow \infty, \end{cases} \\ G_{RL}(x, t; x, 0) \sim & \begin{cases} (t - i\eta)^{-2g(\delta)}, & x \rightarrow 0, \\ (t - i\eta)^{-(g + g(\delta))}, & x \rightarrow \infty. \end{cases} \end{aligned} \quad (11)$$

The power-law form of the $G_{R(L)}(x, t; x, 0)$ and $G_{RL}(x, t; x, 0)$ depends on the distance from the impurity site $x = 0$, because the electron interaction is non-local and the system does not have the translation invariance. According to Eq. (8), there exist two collective excitation spectrums $\epsilon_\pm$ of the electron density fields $\bar{\rho}_{R(L)}(x, \tau)$ in the system. In the bosonization representation of the electron fields $\bar{\psi}_{R(L)}(x, \tau)$, any field operator can be represented by the electron density fields. Therefore, any correlation exponent can only be a function

of the dimensionless interaction strength parameters g and $g(\delta)$. On the other hand, it is reasonable that the parameter $g(\delta)$ appears in the correlation exponent because the backward scattering alters the electron interaction which determines the correlation exponents of the system. The usual duality of the correlation exponents¹⁹ can be easily seen from the parameter $g(\delta)$. In ultraviolet fixed point, i.e., $\delta = 0$, we have $g(\delta = 0) = g$, and in the infrared critical fixed point, $\delta = \pi/2$, we have $g(\delta = \pi/2) = 1/g$. The change of the parameter $g(\delta)$ from g to $1/g$ derives from that the backward scattering alters the electron interaction. However, it is very important to note that the phase shift δ appearing in the exponents is a renormalized quantity which depends on the low energy scale of the system such as the frequency ω or temperature, and the asymptotic behavior in Eq. (11) are only for space coordinate x , in the low-energy (long time) limit, according to the expression of the renormalized phase shift (4), we have $\delta = \pi/2$, the electron Green function $G_{R(L)}(x, t; x, 0)$ is not influenced by the backward scattering as $\{x, t\} \rightarrow \infty$, but the correlation function $G_{RL}(x, t; x, 0)$ is drastically changed by the backward scattering as $\{x, t\} \rightarrow \infty$. The influence of the backward scattering on the correlation exponents can be understood on another aspect that in one-

dimensional spinless electron system the primary excitation mode is the charge density wave, whereas the backward scattering has a pinning effect on the charge density wave. The pinning strength increases as temperature decreases, at the zero-temperature limit, the charge density wave is completely pinned at the impurity site. However, as far away from the impurity, the influence of the pinning effect on the electron propagators is weak, but not zero, the reason is that the pinning effect provides an effective boundary condition for the electron wave functions, which is consistent with our calculation (13).

The electron density of states can be obtained by the formula

$$D_{R(L)}(x, \omega) = \int \frac{dt}{2\pi} e^{i\omega t} G_{R(L)}(x, t; x, 0). \quad (12)$$

Near the impurity, in the high-energy region, the phase shift $\delta \sim 0$, the electron density of states $D_{R(L)}(0, \omega) \sim \omega^{(g+1/g)/2-1}$ is similar to that of the impurity-free case. In the low-energy region, the phase shift $\delta \sim \pi/2$, the electron density of states $D_{R(L)}(0, \omega) \sim \omega^{1/g-1}$ is drastically suppressed by the backward scattering for strong electron interaction. This can be explained that in the low-energy region, the electrons are nearly completely reflected by the impurity, due to the strong repulsive interaction of the scattered electrons with coming in electrons, there is little net electrons near the impurity. On the other hand, far away from the impurity, the electron density of states in the low-energy region is $D_{R(L)}(x \rightarrow \infty, \omega) \sim \omega^{(g+1/g)/2-1}$, independent of the backward scattering of electrons on the impurity.

IV. REFLECTION OF ELECTRONS ON THE IMPURITY

In previous work,⁵ due to the renormalized backward scattering potential goes to infinity in the low-energy limit, i.e., $U_{2k_F}(l \rightarrow \infty) \rightarrow \infty$, the authors conjectured that the infrared critical fixed point of the system corresponds to that the infinite quantum wire is broken at the impurity site $x=0$, and the electrons are completely reflected on the impurity. The transport property of the system in the low-energy regime is determined by the electron tunneling between two sides of the impurity. Now in the phase shift representation, we give a rigorous proof without using the Hamiltonian $\bar{H}^R$ and current-current correlation function. To this purpose, we only need the relations between the electron field $\psi_{R(L)}(x)$ and $\bar{\psi}_{R(L)}(x)$:

$$\begin{aligned} U^\dagger \psi_R(x) U &= \frac{1}{2} e^{i\theta_1} e^{i(\delta/2)\text{sgn}(x)} \{ [1 + e^{-i\delta[1+\text{sgn}(x)]}] \bar{\psi}_R(x) \\ &\quad + [1 - e^{-i\delta[1+\text{sgn}(x)]}] \bar{\psi}_L(-x) \}, \\ U^\dagger \psi_L(x) U &= \frac{1}{2} e^{i\theta_1} e^{-i(\delta/2)\text{sgn}(x)} \{ [1 - e^{i\delta[1-\text{sgn}(x)]}] \bar{\psi}_R(-x) \\ &\quad + [1 + e^{i\delta[1-\text{sgn}(x)]}] \bar{\psi}_L(x) \}. \end{aligned} \quad (13)$$

The $U^\dagger \psi_{R(L)}(x) U$ represents the scattered right(left)-moving electron field, whereas the $\bar{\psi}_{R(L)}(x)$ is the renormalized right(left)-moving electron field. According to Eq. (13), we

can easily obtain the reflection $\mathcal{R}$ and transmission $\mathcal{T}$ rates of electrons on the impurity, $\mathcal{R} = \sin^2(\delta)$, and $\mathcal{T} = \cos^2(\delta) = 1 - \mathcal{R}$. For the impurity-free case, i.e., $\delta=0$, it is trivial, $\mathcal{T} = 1$, and $\mathcal{R}=0$. However, at the infrared critical fixed point, the phase shift $\delta^c = \delta(l \rightarrow \infty) = \pi/2$, we have that $\mathcal{T}=0$, and $\mathcal{R}=1$, the electrons are completely reflected on the impurity. Away from the infrared critical fixed point, the electrons can tunnel through the impurity. This rigorously proves the conjecture in Ref. 5 that at zero temperature ($l \rightarrow \infty$) electrons are completely reflected on the impurity. Generally, it is also consistent with that one uses an open boundary condition at the impurity site to study the low-energy transport property of a system with a non-magnetic or magnetic impurity scattering.^{20,31}

We define the total charge operator of the system as $Q(t) = e \int dx [\rho_R(x, t) + \rho_L(x, t)]$, the current operator is $I(t) = \partial_t Q(t)$. With the equation, $\partial_t \rho_{R(L)}(x, t) = (i/\hbar) [H, \rho_{R(L)}(x, t)]$, the current operator is $I(t) = (ieU_{2k_F}/\hbar) \{ \cos(2\delta) [\bar{\psi}_L^\dagger(0, t) \bar{\psi}_R(0, t) - \bar{\psi}_R^\dagger(0, t) \bar{\psi}_L(0, t)] - i \sin(2\delta) [\bar{\rho}_R(0, t) - \bar{\rho}_L(0, t)] \}$. With this current operator, the tunneling conductance of the system can be calculated by the formula⁵

$$\begin{aligned} \sigma(\omega) \sim \int \frac{dt}{2\pi} \frac{\sin(\omega t)}{\omega} \{ \cos^2(2\delta) G_{RL}(0, t; 0, 0) - \sin^2(2\delta) \\ \times \langle [\bar{\rho}_R(0, t) - \bar{\rho}_L(0, t)] [\bar{\rho}_R(0, 0) - \bar{\rho}_L(0, 0)] \rangle \} \end{aligned} \quad (14)$$

which is composed of two parts, one is coming from the quasiparticle tunneling, and another from the charge density fluctuation. In the high-energy region, the phase shift $\delta \sim 0$, the quasiparticle tunneling conductance is $\sigma(\omega) \sim \omega^{2(g-1)}$, whereas in the low-energy region near the infrared critical fixed point, $\delta \sim \pi/2$, it is $\sigma(\omega) \sim \omega^{2(1/g-1)}$. These conclusions are consistent with the previous calculations.^{5,15,20} The charge density fluctuation may contribute a term proportional to^{13,32} ω^2 to the tunneling conductance (14). However, just as shown in Refs. 32 and 13, the ω^2 term appearing in the tunneling conductance is not universal, it strongly depends on a low-energy scale ω_c . For a finite quantum wire, it can be chosen as¹³ $\omega_c \sim 1/L$, where L is the length of the wire.

V. DISCUSSION AND CONCLUSION

According to the Bethe solution of the one-dimensional Hubbard model,³³ the wave function of the system can be explicitly written as a product of a charge part and spin part in the large- U limit, whereas the quantum numbers describe the charge and spin degrees are not independent of each other. Using the relations among these quantum numbers, the authors in Ref. 34 introduced a general phase shift method to calculate the correlation functions of the Hubbard model in the large- U limit. This phase shift is coming from the constraint between the quantum number of the spin part and the quantum number of the charge part. Generally, it is consistent with our calculation where the backward scattering of the impurity alters the electron-electron interaction, therefore, the correlation exponents of the system depend on the phase shift induced by the backward scattering.

That the exponent of the tunneling conductance changes from $2(g-1)$ to $2(1/g-1)$ as from the high- to low-energy

region is agreement with the usual exponent duality between the ultraviolet and infrared fixed points. The character of the exponent duality of correlation functions is that the backward scattering of the impurity alters the electron-electron interaction. In our previous works,¹⁴ we only focus on the behavior of the system near the infrared critical fixed point. Now we clearly show the behavior of the renormalized phase shift δ and the evolution of the correlation exponents from the ultraviolet to the infrared fixed points, and rigorously demonstrate that electrons are completely reflected on the impurity site at the infrared fixed point. In the phase shift representation, the behavior of the system near the impurity is very clear.

Generally, the problem of single impurity scattering in Luttinger liquid modeled by a boundary sine-Gordon Hamiltonian can be exactly solved by Bethe ansatz.³⁵ In Ref. 36, with the Bethe solution of the boundary sine-Gordon Hamiltonian, the authors studied the exponent duality by calculating the ground state energy and the boundary state of the system, and confirmed that the exponent of the tunneling current has this duality between the ultraviolet fixed point and the infrared critical fixed point. It is completely consistent with our calculation. However, in general, it is difficult

directly to calculate correlation functions by the Bethe ansatz³⁵ and conformal field theory³¹ in which there does not exist a general fusion rule among primary fields. In our phase shift representation, it is clear that the electron fields are connected with the original ones by a simple unitary transformation, and the Hamiltonian near the infrared critical fixed point can be directly obtained by the original one at the ultraviolet fixed point, therefore one can easily calculate any correlation function of the system.

In summary, with the bosonization and phase shift representation, we have rigorously treated the backward scattering in the one-dimensional interacting spinless electron system, and demonstrated that the correlation exponents of the system depend on the phase shift δ induced by the backward scattering, we therefore can clearly show the evolution of the correlation exponents from the ultraviolet fixed point to the infrared critical fixed point.

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