A Real-time Sub-carrier Allocation Scheme for Multiple Access Downlink OFDM Transmission^s

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Abstract --In this paper, we propose sub-carrier allocation algorithms for a multiple access scheme in downlink OFDM transmission. Knowing the channel characteristics of all the users at the base station, the sub-carrier allocation algorithm assigns sub-carriers to the users in a way that the total transmit power is minimized. An optimal algorithm is presented to provide the best sub-carrier assignment. The complexity of the optimal solution renders it impractical for real-time application. To deal with the frequently changing channel characteristic of a fast time-varying multipath fading channel, a heuristic algorithm based on constructive assignment and iterative improvement is proposed which can give out a valid solution in real time. Experimental results show that the performance of this real-time algorithm is close to that of the optimal allocation.

transmission rate, have adopted OFDM for its ability to alleviate the effect of multipath fading. Besides, its multicarrier nature allows the use of adaptive modulation to enhance its performance significantly [1,2,3]. The idea of adaptive modulation in OFDM is as follows. Based on the channel characteristic, the number of bits to be transmitted, the level of modulation and the transmission power in each sub-carrier are selected in order to increase the transmission bit rate or reduce the required transmit power.

When OFDM is used with adaptive modulation over a frequency selective fading channel, a user usually can only use a small portion of the sub-carriers. It is mainly because the link between the base station and the *desired* mobile user experiences deep fade in the other sub-carriers. In a standard TDMA system employing OFDM with adaptive modulation, each user is allocated a fixed time slot and only that user can transmit in that time slot. These unused sub-carriers, as a result of adaptive modulation, are wasted and cannot be used by other users during that time slot. However, the sub-carriers that are in deep fade over the link between the BS and the designated mobile user may

I. INTRODUCTION

Orthogonal Frequency Division Multiplex (OFDM) has long been regarded as an efficient way to combat the inter-symbol interference (ISI). Proposed digital broadcasting systems such as Digital Video Broadcast (DVB) and Digital Audio Broadcast (DAB), which require high data

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not be in deep fade over the links between the BS and other mobile users. In fact, it is unlikely that a sub-carrier will be in deep fade over all the links as the fading statistics over these links are mutually independent. This motivates us to consider the multiple access schemes that users share the downlink transmission by adaptively using different sub-carriers, instead of using different time slots as in a TDMA system. This approach will allow all the sub-carriers to be used effectively and a sub-carrier is left unused only when it experiences deep fade in all the BS-to-mobile links. We apply this idea to the downlink transmission in a TDD system as the base station can estimate the instantaneous channel characteristics of all the BS-to-mobile links based on the received uplink transmissions.

In [4], a multiple access scheme that combined with adaptive modulation is proposed and it minimizes the overall transmission power by optimally allocating sub-carriers to users and number of bits to be send in each sub-carrier. In some situations, using sub-carrier allocation alone can already greatly enhance the performance and the use of adaptive modulation in these cases will be less useful. It is because by leaving out those worse sub-carriers, the channel characteristic of sub-carriers allocated to a user becomes less variant which renders the adaptive modulation less potential for improvement. Also, the complicated algorithm proposed in [4] is not suitable for the time varying channel as the required computation makes real-time implementation impractical. In this work, we propose a multiple access scheme that only use sub-carrier allocation. Optimal sub-carrier allocation algorithm is developed which results in minimum transmit power. Also real-time heuristic allocation algorithm which is close to the optimum while having much fewer computations is also

proposed. We compare the performance and complexity of these algorithms with other multiple access schemes to show the effectiveness and efficiency of the proposed scheme.

This paper is organized as follow. Section II gives the system model and formulates the optimization problem of sub-carrier allocation. Section III describes the optimal solution for the sub-carrier allocation problem. We then present in Section IV a heuristic allocation method that gives a close-to-optimal solution in real time. The comparisons of the performance between the proposed method and other existing static approaches are given in Section V. Section VI concludes the paper.

II. SYSTEM MODEL

The structure of a multiuser OFDM system using multicarrier allocation is shown in Fig. 1.

Fig.1. Block diagram of the OFDM system.

Let K be the number of users supported by the system and N be the number of sub-carriers. Also, let S_k denote the number of sub-carriers required by user k . Assume each sub-carrier has a bandwidth that is much smaller than the coherence bandwidth of the channel and the instantaneous channel gain of each sub-carrier for every user is known to the base station.

In the transmitter side, the allocation algorithm assigns sub-carriers to each user according to the channel gains. The bit stream of each user is then transmitted out using the assigned sub-carriers. In each sub-carrier, the assigned bits are modulated

with a power level that can overcome the fading of

$$\sum_{n=1}^N \rho_{k,n} = S_k \quad \forall k \in \{1, 2, \dots, K\} \quad (2)$$

$$\sum_{k=1}^K \rho_{k,n} = 1 \quad \forall n \in \{1, 2, \dots, N\} \quad (3)$$

the channel. In addition to the power required to transmit the bits in the AWGN channel, power proportional to the inverse of the corresponding sub-carrier channel gain $\alpha_{k,n}$ is needed. The modulated signals of all the sub-carriers are then transformed into time domain using IFFT and cyclic prefix is inserted to eliminate the ISI. Dedicated sub-carriers are reserved for the broadcasting of the sub-carrier allocation information of which each receiver can retrieve to determine the corresponding sub-carriers for receiving the data.

In the receiver side, after removing the guard

$$P_T = \min \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} \frac{P}{\alpha_{k,n}^2}$$

$$\sum_{k=1}^K \rho_{k,n} = 1 \quad \forall n \in \{1, 2, \dots, N\}$$

$$\sum_{n=1}^N \rho_{k,n} = 1 \quad \forall k \in \{1, 2, \dots, K\}$$

interval and transforming the time domain samples into the frequency domain symbol, the user demodulates the sub-carriers prescribed by the allocation information to recover the original bit stream.

The goal of the sub-carrier allocation is to minimize the total transmit power of all the users while

satisfying the data transmission requirement for each user. Given P is the transmit power of a sub-carrier in the AWGN channel and $\rho_{k,n}$ is a binary assignment variable which is equal to 1 if sub-carrier n is assigned to user k or equal to 0 otherwise, the total transmission power is given by P_T . The objective is to find the values of the assignment variables, $\rho_{k,n}$, to minimize P_T while satisfying the following constraints,

Constraint (2) specifies that the total number of sub-carriers allocated to user k is S_k and constraint (3) specifies that each sub-carrier can only be allocated to one user.

III. OPTIMAL SUB-CARRIER ALLOCATION

The above optimization problem is similar to the classical assignment problem in the area of linear programming. Therefore, we can solve this problem by first transforming it into an equivalent assignment problem using the method described in [5]. Here each user is expanded to S_k sub-users and for every sub-user, only one sub-carrier will be allocated. The channel gain of the sub-user is equal to that of the original user. The optimization problem then becomes

The constraints are transformed to

Numerous methods (e.g. [7]) have been proposed to solve this classical assignment problem. One of

$$P_T = \min \sum_{n=1}^N \sum_{k=1}^K \rho_{k,n} \frac{P}{\alpha_{k,n}^2} \quad (1)$$

them is the famous Hungarian method proposed in [6] of which the computation complexity of the algorithm is $O(n^4)$.

In a time varying channel, the channel characteristic of users changes frequently. To cope with this

situation, we need a sub-carrier allocation algorithm that is fast enough to allocate the sub-carrier within the coherence time of the channel. The complexity of the above optimal algorithm is too large that an optimal solution may not be generated within the coherence time, especially for a fast-fading channel. Therefore, we propose a heuristic sub-carrier allocation scheme that satisfies the real time requirement.

IV. HEURISTIC SUB-CARRIER ALLOCATION

In an OFDM transmission system, the number of users and the variation of channel characteristic may change from time to time. Thus, it is better to have a sub-carrier allocation scheme that can cater for different range of channel coherence time. Also the algorithm is preferred to scale linearly with the number of users. One of the methods is to generate an initial allocation quickly and then keep on improving the allocation to achieve an optimal solution. By doing so, at least a valid allocation is achieved. The rest of the coherence time period can be used to further refine the solution. Here we propose a heuristic real-time algorithm which consists of two phases. First, an initial allocation is obtained via a constructive algorithm and then iterative improvement is done on the initial allocation to minimize the objective function.

A. Constructive initial allocation

First, we order the sub-carriers' channel gains for each user in descending order. Then, we consider the first sub-carriers in the ordered lists of all users, user by user. Suppose we come to consider the n^{th} sub-carrier of the k^{th} user. If user k 's sub-carrier requirement in (2) has not been satisfied and sub-carrier n has not been allocated to another user, then sub-carrier n is allocated to user k ; otherwise, we do nothing and skip to the next user. After that,

the process repeats with the next sub-carriers on the ordered lists. The algorithm continues until all the sub-carriers are allocated or all the users' requirements are satisfied. Fig. 2 shows an example of the algorithm. In this example, there are 8 sub-carriers and 3 users. We assume that both user 1 and user 2 need 3 sub-carriers and user 3 needs 2 subcarriers. The left portion of the figure shows the ordering of the sub-carriers of each user, with 1 representing the first and 8 the last in the ordered list. The sub-carrier assignment starts from user 1. The allocation of sub-carrier is shown in the right with '1' indicating the assignment.

	Sub-carriers	Sub-carriers
User 1	1 2 4 5 8 7 6 3	1 1 0 0 0 1 0 0
User 2	8 7 3 2 1 6 5 4	0 0 1 1 1 0 0 0
User 3	3 4 5 8 7 6 2 1	0 0 0 0 0 0 1 1
	Ordering	Assignment

Fig. 2. An example of sub-carrier allocation.

B. Iterative Improvement

The sub-carrier allocation obtained from the constructive initial allocation is likely to be trapped in a local minimum since a greedy strategy is used. It can be improved by iterative swapping of the sub-carrier between users to reduce the total transmit power. In each iteration, we try to swap the sub-carriers allocated between two users such that the power is reduced. To select the pairs of users and the corresponding sub-carriers to swap, we calculate a cost function for swapping two users. For each pair of users (i, j) , we calculate the power reduction factor, $p_{i,j}$ which is equal to the sum of $\Delta p_{i,j}$ and $\Delta p_{j,i}$ where $\Delta p_{i,j}$ is the maximum power reduction (over all the sub-carriers that have been allocated to user i) if a sub-carrier allocated to user i is re-allocated to user j instead. We also denote by n_{ij} the sub-carrier that maximizes the power

reduction. We sort all the power reduction factors in descending order and pick the first one in the list. If the first p_{ij} is positive, then we change the sub-carrier allocation by swapping sub-carrier n_{ij} from user i to user j and sub-carrier n_{ji} from user j to user i . We update the values of all p_{ij} 's after the swapping and then start another iteration. The iteration stops when all the power reduction factors are negative. That means we cannot reduce the power further by swapping the sub-carrier allocation of two users.

Fig. 3 An example of swapping.

The algorithm is illustrated in an example shown in Fig. 3. Each number in the matrix represents the channel gain $\alpha_{k,n}$ of each sub-carrier and a circle indicates that sub-carrier is assigned in the initial allocation. In this example, there are 3 pairs of users and for each pair, we find p_{ij} by finding the swapping that yield the largest reduction in power.

V. SIMULATION RESULTS

To show the effectiveness of the proposed algorithms, we compared the performance with other sub-carrier allocation methods. Here we consider two static sub-carrier allocation schemes. Both of them are based on the multiple access methods described in [3]. These are

OFDM-TDMA: Each user is assigned a TDMA time slot and only that user can use the sub-carriers within that time slot

OFDM-FDMA: Each user is assigned a band of sub-carriers and transmits using those sub-carriers in every OFDM symbol.

The time and sub-carrier assignments of the two OFDM schemes are illustrated in Figure 4.

Fig. 4. Different multiple access schemes.

We have simulated 10,000 sets of 3-path frequency selective Rayleigh fading channel using exponential power profile with 100ns root-mean-square (RMS) delay spread. The OFDM system has 128 sub-carriers and 5 users. The simulation is done on an Ultra Sparc I Workstation with 128MB memory. Figure 5 compares the SNR of different sub-carrier allocation schemes to achieve the same bit error rate (BER). It shows that the heuristic iterative improvement algorithm outperforms the static allocation schemes by at least 10dB, and it is very close to the optimal allocation. The average time to achieve an optimal allocation using Hungarian method is 612ms and that for the heuristic algorithm is 8.7ms. A speed up of about 70 times is achieved.

Figure 6 shows the bit SNR required to achieve a BER of 10^{-3} with respect to the average number of iterations carried out by the heuristic algorithm. The horizontal line represents the performance of the optimal sub-carrier allocation. It is shown that the performance of the heuristic algorithm converges very fast at the first few iterations and comes to a constant after 25 iterations. It means that it only

takes a few iterations for the algorithm to achieve a good allocation and hence can guarantee to satisfy the real-time requirement.

Fig. 5 Comparison of BER performance.

Fig. 6 Convergence of the proposed method.

VI. CONCLUSION

In this paper, we considered the multiple access

scheme for downlink OFDM transmission that

minimizes the transmit power. We explored the similarity between the sub-carrier allocation problem and the classical assignment problem and derived an optimal allocation method. To satisfy the time variation of fading channel, a real-time heuristic sub-carrier allocation method based on constructive assignment and iterative improvement is proposed. Simulation results show that the proposed allocation method provides a solution that is very close to the optimal solution.

VII. REFERENCES

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