Long-Term Electricity Contract Optimization with Demand Uncertainties

Pang Chan ^{a*}, Chi-Wai Hui ^a, Wenkai Li ^a

Haruo Sakamoto ^b, Kentaro Hirata ^b

Pu Li ^c

^a Department of Chemical Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

^b Process Development and Design Lab., Process Systems Engineering and Production Technologies, Research & Technology Development Division, Mitsubishi Chemical Group Science & Technology Research Center, Inc. 1, Toho-cho, Yokkaichi, Mie 510-8530, Japan

^c Institut für Prozess- und Anlagentechnik, Technische Universität Berlin, KWT 9 10623 Berlin, Germany

Abstract

This paper presents a study on selecting electricity contracts for a large-scale chemical production plant, which requires electricity importation, under demand uncertainty. Two common types of electricity contracts are considered, Time Zone (TZ) contract and Loading Curve (LC) contract. A multi-period linear probabilistic programming model is adopted for the contract selection and optimization. Hence, by using the probabilistic programming, a solution procedure is proposed that allow users to determine the best electricity contract according to their desired confident level of the uncertainties. In addition, due to the fact that the demand of product is uncertain, if one considers the overage and shortage of the products in the market as well, an interesting result can be obtained. The methodology is explained in the paper.

* Corresponding author 7004254@cc.m-kagaku.co.jp

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1. Introduction

Deterministic models are commonly used for production planning in which it is assumed that the prices, demand of products and the availabilities of raw materials, etc., are known. In fact, unlike process operations, market conditions in the future time horizon, in which process operation is planned, present uncertainties that play an important role in a planning process. Many studies on this topic have been conducted in the recent years [1].

Sources of uncertainty in production planning can be categorized as short-term or long-term [2] based on their time frames. Short-term uncertainty may include day-to-day processing variations, cancelled/rushed orders and equipment failure etc. Long-term uncertainty refers to raw material/final product unit price fluctuations, demand variations and production rate changes over longer time frames.

Nomenclature

Sets

$i, (i = 1, 2, \dots, 12)$	Month
$j, (j = 1, 2, 3)$	Shift (1: Day, 2: Night and 3: Mid-night)

Variables

y_{ij}	Amount of P produced at different i and j (t/shift)
q_{ij}	Amount of EL imported at different i and j (MW)
X_i	Demand, a random variable at month i (t)
x_i	Realized demand at month i (t)
P_i	$\sum_j y_{ij} \times SL_j$, Production target at month i (t)
S_D	Maximum electricity importation during the whole contract period at D shift (MW)
S_N	Maximum electricity importation during the whole contract period at N shift (MW)

S_M	Maximum electricity importation during the whole contract period at M shift (MW)
TEL	Annual electricity importation, $\sum_i \sum_j q_{ij} \times SL_j$ (MWH)
ELFix	Electricity fixed cost (MYEN)

Parameters

SL_j	Length of shift j (8 h/shift)
r	Product price \$0.035 (MYEN/t)
C_{ij}	Cost parameter of EL at different i and j (MYEN/MW)
q_{ijMin}	Lower bound of electricity importation in each shift (MW)
q_{ijMax}	Upper bound of electricity importation in each shift (MW)
a_D	Parameter corresponding to the max electricity importation at D shift. Value used in the case study: 14 (MYEN/MW)
a_N	Parameter corresponding to the max electricity importation at N shift. Value used in the case study: 3 (MYEN/MW)
a_M	Parameter corresponding to the max electricity importation at M shift. Value used in the case study: 1.5 (MYEN/MW)
a_T	Parameter corresponding to the total electricity importation. Value used in the case study: 0.0025 (MYEN/MWH)
α	Confidence Level
MA_j	Max. capacity of production at shift j (30 t per shift)
$ELMAX_j$	Maximum electricity importation limit at S_D , S_N and S_M (MW)
θ	Mean of demand at month i (t)
σ	Standard deviation of demand at month i (t)
ϕ	Probability density function
$g(x_i)$	Probability density function (pdf) of X_i , $g(x_i) = d\Phi(x_i)/dx_i$
Φ	Cumulative distribution function (cdf) of X
Φ^{-1}	Inverse of cumulative distribution function
z	$(P - \theta)/\sigma$

So far, very few applications of optimization under uncertainty have been made in chemical process operations [3], especially for the electricity contract optimization for industrial process operations. In fact, selecting a right electricity contract for a large-scale chemical production plant, which requires electricity importation, is very

important for minimizing overall energy cost. The amounts of steam and electricity usages in a chemical production site are fluctuated due to the uncertain market conditions. These fluctuations on one the hand can be adsorbed by the utility system, or on the other hand balanced by the importation of electricity.

Importing electricity from an external supplier requires a long-term contract that normally covers a period of one to three years. Several types of electricity supply contracts are designed by the suppliers that target for different type of customers. The contract fee often depends on the overall of annual consumption, the maximum consumption rates at the peak, medium and low demand periods and the emergency backup, etc. In the existence of uncertain factors such as product demands, the determination of a long-term electricity supply contract and its values are definitely affected. This paper is going to present a mathematical model and a systematic procedure for selecting an optimal electricity long-term contract under uncertainty. In this model, the demand of product is assumed to be uncertain. By considering the overage and shortage of the products in the market, an interesting result is found for decision-making.

Notice that the overage products refer to the products that cannot be absorbed by the market and disposed eventually; while the shortage products are not compensated from the outside environment in order to fulfil the market demand. No inventory is provided for holding such kind of unsold products.

2. Probabilistic approach

There have been two general stochastic approaches [4] to solve uncertain problems: one is the two-stage stochastic programming and the other is chance constrained programming (also known as probabilistic approach). In authors [5] argued that the choice of the appropriate method is content-dependent, with no single theory being sufficient to model all kinds of uncertainty. It is noticed that with both solution strategies there have been, until now, no suitable approaches to the nonlinear problems. In this paper, the probabilistic approach is mainly concentrated and elaborated in the next section.

From the application point of view, the method of chance constrained programming to linear, nonlinear and dynamic problems has been developed and applied to many disciplines like finance and management [6, 7] in recent years. However few

applications have been made in chemical process operations [3]. Usually, the probabilistic approach seeks to satisfy the constraints involved by a pre-determined confidence level, denoted as α ($0 \leq \alpha \leq 1$), based on the known probability density or cumulative distribution of the uncertain variables [4, 6]. Therefore, this approach can provide essential information on the economical achievements as a function of the desired confidence level. For complex plant operations under multiple uncertainties, the sources of risk that have the most significant impacts on the profitability can be identified [8, 9].

Using the probabilistic approach, a stochastic programming problem under chance constraints is formulated for optimization. In many cases, it can be relaxed to an equivalent deterministic LP or NLP problem. The essential challenge lies in the computation of the probabilities of holding the constraints. Its unique feature is that the resulting solution ensures the probability of satisfying constraints. The solution leads to an expected optimal value of the objective function by searching for the decision in a feasible region at a given confidence level. In addition, the term -- Confidence Level can be defined into two ways: the first one is related to the constraint that the amount of the product being produced can satisfy the market demand. For example, we have 90% confidence to fulfil the market demand for the produced product. The second definition is related to the constraint that the amount of the generated product can be sold to the market. For example, we have confidence to sell out 90% of product. Different people have different interpretation to this term; users should make clear before using it. In this work, we adopt the former definition throughout the paper. Since α has to be defined by the decision-marker, it is possible to select different confidence levels and make a compromise between the objective value and the risk of constraint violation.

3. A Case study

3.1. Problem Definitions

A motivating example shown in Figure 1 is used for illustrating the problem and the proposed solution method. In this problem, process “A” is going to produce product by mainly consuming electricity. Each ton of product being produced consumes 850

kW of electricity. The price of raw material is negligible in this example. The product demand from the market is under uncertain.

3.2. Uncertain market demand

In this case, the objective is to maximize the profit. However, due to the uncertain market demand, i.e. the demand of product is an uncertain variable, X , cannot be determined a priori. This results in an overage of product if the production rate is overestimated or a shortage of product if the production rate is underestimated. Both situations will lead to a profit reduction or even a negative profit for the production plant. Fortunately, this unhealthy condition can be remedied by taking the demand uncertainty into consideration. Let assume the uncertainties of product demands having a normal distribution. As shown in Table 1, the distribution is described with four seasons: Winter, Spring, Autumn and Summer, each season has its demand distribution, while the demand distribution in autumn and spring are the same. Figure 2 represents the density function of the uncertain variable in different seasons.

3.3. Electricity contract and cost calculation

As the production of product requires electricity, based on the demand distribution, the overall electricity imported from a power plant can be estimated. Actually, this is an important step to have a general idea regarding the amount of electricity being consumed before signing a contract with the power plant. In general, there are two common types of electricity contracts provided by the power plant [10], which are Time Zone (TZ) contract and Loading Curve (LC) contract, users can base on contractual information provided by the electricity suppliers or their estimated electricity purchase amount to choose an optimal contract in order to minimize the electricity cost. The characterization of the two kinds of contracts is as follows:

TZ Contract: Electricity supply is classified into several types (e.g. peak, normal and off peak) according to the demand in different time shifts and periods. Simple regulations or restrictions are employed in the contract to encourage the users to consume more electricity during the off peak periods, such as different prices, intake maximum and/or minimum values at different time periods.

LC Contract: Besides different electricity types, the LC contract has more restrictions on the purchase of customers, so that the electricity company can plan its operation easier. The customers are requested to follow the upper and lower importation limits at all time shifts, the annual overall purchasing amount, and some other constraints for the contracted year. If any of these restrictions is violated, penalties will be charged on the customers. The customers however are rewarded by a lower electricity tariff and are guaranteed by the contracted demand.

For the calculation of the electricity contract fee, the cost is divided into two categories, variables cost and fixed cost. Different kinds of variable cost calculation method are adopted in current commercial practice, and they can be briefly classified into three types. First, the variable cost only has one fixed value. The variable cost only depends on the intake amount and the fixed cost parameter. Second, the electricity company defines a cost profile, which divides the electricity purchased amount into different ranges, and each range has its own cost parameter. The variable cost will then be calculated by the purchased amount multiplying with its corresponding cost parameter. Third, the cost parameters vary according to the time shifts (D: Day, N: Night & M: Mid-night). Thus, the variable cost is calculated by the total importing amount and the time period of the importation. In this paper, both contracts are using the third type to calculate the variable cost.

Besides the variable cost, fixed cost also takes a very important position in the electricity supply contract. Fixed cost is a fixed annual fee that the plant site has to pay every year, which depends on the maximum importation peak on the contracted period. Allowing a higher maximum import level, the production site might have a higher flexibility for production and/or maintain its production even during utility equipment failure. On the other hand, a higher maximum importation rate increases the fixed cost. The profit gain due to higher productivity may not be able to compensate the additional payment of the electricity contract. Thus, careful consideration of the electricity contract is necessary.

For the TZ contract, the contract fee contains the variable cost only; the calculation method is already mentioned above by using the third method. The upper bound of 25 MW importation rate in each shift is applied for the whole contracted period. Notice that there is no lower bound of annual importation for the TZ contract.

For the LC contract, the upper bound of importation rate is different in different seasons and time shifts. The contract fee contains two parts, variable cost and fixed cost. The fixed cost calculation follows the Eq. (1) stated below.

Those parameters and conditions of the two contracts offered by the power plant are shown in the Table 2, 3 and 4.

$$ELFix = a_D S_D + a_N S_N + a_M S_M + a_T TEL \quad (1)$$

In brief summary, the equations to calculate the contract fee of TZ and LC contracts are shown as follows:

$$ELcost_{TZ} = \sum_{ij} [(Amount\ of\ EL\ imported)_j \times (cost\ parameter)_j]_i \quad (1a)$$

$$ELcost_{LC} = \sum_{ij} [(Amount\ of\ EL\ imported)_j \times (cost\ parameter)_j]_i + ELFix \quad (1b)$$

Let us use two simple examples to show the calculation of the electricity cost based on the different contract types (TZ or LC), and see which contract is optimal. The electricity prices are provided in Table 2.

Example 1: The annual electricity importation and the upper bound at each shift are shown Table 5. Then the electricity cost can be calculated as follows:

$$\begin{aligned} ELcost_{TZ} &= 500*0.022+1000*0.011+2000*0.0077 \\ &=37.4\ MYEN \end{aligned}$$

$$\begin{aligned} ELcost_{LC} &= (500*0.018+1000*0.006+2000*0.003) \\ &\quad + (0.3*14+0.4*3+0.7*1.5+3500*0.0025) \\ &=36.2\ MYEN \end{aligned}$$

It means selecting the LC contract is better than TZ contract.

Example 2: The annual electricity importation and the upper bound at each shift are shown Table 6. Then the electricity cost with different contracts can be calculated as follows:

$$\begin{aligned} \text{ELcost}_{\text{TZ}} &= 300*0.022+700*0.011+1500*0.0077 \\ &= 25.85 \text{ MYEN} \end{aligned}$$

$$\begin{aligned} \text{ELcost}_{\text{LC}} &= (300*0.018+700*0.006+1500*0.003) \\ &\quad + (0.3*14+0.4*3+0.7*1.5+2600*0.0025) \\ &= 27.05 \text{ MYEN} \end{aligned}$$

In this case TZ contract would be better!

These two examples demonstrate the choice of an electricity contract depends on the annual electricity importation. The customers should have a good estimation of electricity amount to be consumed during the contracted period before making a decision.

However, it might turn out that the contract being chosen may not be the best due to the uncertain factors. An uncertain demand, one of the uncertain factors, definitely affects the rate of electricity importation during the contracted period. Hence, the customers might suffer from the high electricity costs if the demand uncertainty is neglected.

4. Production planning under uncertainty

Assuming that this is a 12-month, $i = 1, \dots, 12$, planning period production with 3 operating shifts, $j = D, N, M$ in a day (8 hours each shift). The objective of this study is to generate a production plan for maximum profit with considerations of uncertain market demand and electricity prices together with the electricity contract information.

Table 7 shows the amount of product that satisfies the market demand at the predefined confidence level. Take March at $\alpha = 0.3$ as an example to demonstrate the calculation of the data in Table 7. Using the mean and standard deviation of March distribution, a cumulative density function curve can be obtained and shown in

Figure 3. At $\alpha = 0.3$, the product amount that satisfies the market demand is determined, which is 9905.61 t. This method can be applied for finding other production targets at different confidence levels.

4.1. Mathematical Formulation

The optimization problem is formulated with two different stochastic constrained models. The first model is used for *Electricity Cost* calculation and the second is for *Revenue* calculation. The electricity cost calculation is based on the defined confidence level; optimization is required to find out the optimal imported electricity amount in each shift. After that the revenue calculation can then be easily calculated based on the optimal imported electricity amount. There is no optimization required for revenue calculation. Let us first define the objective function in a mathematical way:

$$\text{Obj. function} \quad \text{Max} \quad \text{Profit} = (\text{Revenue} - \text{Electricity cost})$$

The solutions for calculating the Revenue and Electricity cost are explained in detail in the following two sections. The electricity cost calculation is discussed first.

4.1.1. The electricity cost

To model the calculation of the electricity cost, let q_{ij} and y_{ij} denote the positive variables of electricity input and product output in different months (i) and shifts (j), respectively. C_{ij} and SL_j represent the electricity price and shift length. ELFix shown in Eq. (1) is comprised of some terms, let S_D , S_N and S_M denote the maximum electricity demand at D, N and M shifts corresponding to their parameters a_D , a_N and a_M . S_D , S_N and S_M are positive variables to be determined by the model. TEL stands for the total electricity importation during the contracted period corresponding to the parameter, a_T . The model is implemented in the framework of GAMS [11] and solved it with the solver OSL [12]. In summary, the electricity cost corresponding to a predefined confidence level can be obtained by solving the following model:

$$\text{Min} \quad \text{Electricity cost} = \sum_i \sum_j (C_{ij} q_{ij}) \times SL_j + \text{ELFix} \quad (2.1)$$

$$\text{s.t} \quad q_{ij} = 0.85 y_{ij} \quad (2.2)$$

$$q_{ij\text{Min}} \leq q_{ij} \leq q_{ij\text{Max}} \quad (2.3)$$

$$\Pr \left(\sum_j (y_{ij} \times SL_j) \geq X_i \right) \geq \alpha \quad (2.4)$$

$$y_{ij} \leq MA_j \quad (2.5)$$

$$TEL \geq 100,000 \quad (2.6)$$

$$S_D, S_N, S_M \leq ELMAX_j \quad (2.7)$$

$$q_{ij}, y_{ij} \geq 0 \quad (2.8)$$

$$i = 1, \dots, 12 \text{ and } j = 1, \dots, 3$$

Eq. (2.1) minimizes the variable and fixed cost of electricity. Eq. (2.2) enforces the material balance, it means that 850 kW of electricity required producing one ton of product per hour; while Eq. (2.3) ensures that the supply of electricity at each shift would not under/exceed the assigned lower/upper bound ($q_{ij\text{Min}}/q_{ij\text{Max}}$) value by the contract (note that different contracts have different $q_{ij\text{Min}}$ and $q_{ij\text{Max}}$ values). Eq. (2.4) determines the production rate of $\sum_j (y_{ij} \times SL_j)$ at each month, which fulfils the market demand by the defined confidence level (i.e. the probability of production rate greater than the demand should be equal or greater than the defined confidence level). Eq. (2.5) ensures that the production rate in each shift should not exceed the maximum plant capacity, MA_j . Eq. (2.6) restricts the plant to import at least 100,000MWH during the whole contract period. This constraint is applied for the LC contract only (note that $TEL = \sum_i \sum_j q_{ij} \times SL_j$). Eq. (2.7) restricts the electricity purchase below the upper limit for electricity importation at each shift, $ELMAX_j$.

The problem defined above is called stochastic probabilistic (or constrained) programming problem. To solve this problem with an available optimization routine, the probabilistic constraint must be relaxed by replacing Eq. (2.4) with the following (Please see the appendix part I):

$$\sum_j (y_{ij} \times SL_j) \geq \Phi^{-1}(\alpha) \quad (2.4a)$$

Hence, the only difference between the stochastic and relaxed programming is Eq. (2.4) and (2.4a), in which the stochastic constraint is relaxed to an equivalent deterministic form. The Φ^{-1} means the inverse of cumulative density function with $\Phi(\infty) = 1$ [13]. The meaning of Φ and Φ^{-1} would be explained in more detail in the following section.

4.1.2. The Revenue

The revenue corresponding to the same confidence level is calculated as follows:

$$\text{Revenue} = r \times \sum_i \int_0^\infty \min(P_i, x_i) g(x_i) dx_i \quad (3.1)$$

$$\text{s.t } P_i = \sum_j (y_{ij} \times SL_j) \quad (3.2)$$

$$i=1, \dots, 12 \text{ and } j=1, \dots, 3$$

r refers to the product price. P_i equals to the monthly production target. The demand, x , is assumed to have a continuous normal distribution that is described by two parameters, the mean θ and the standard deviation σ . Here $g(x)$ represents the probability density function (pdf). The cumulative distribution function (cdf) is the integral of the density function, i.e. $\int_0^\infty g(x) dx$. Hence, the expected demand (θ) can be calculated by multiplying the demand, x , in the integral. Hence, the term $\int_0^\infty xg(x) dx$ represents the expected demand in each month. However, by replacing it with $\int_0^\infty \min(P, x)g(x) dx$, it becomes the expected amount of product sold to the market, i.e. the smaller value among the demand and production rates would be selected for the integration. In the other words, products in overage or shortage are considered in the revenue calculation. In fact, this integral is based on the assumptions that the overage products that are not sold to the market would not be stored and

disposed eventually, while the shortage products would not be imported from outside companies to fulfil the market demand and no inventory holding as well. For other cases that do not apply for the above assumptions, the expected result is not valid.

Let take one simple example to illustrate the meaning of $\int_0^\infty \min(P, x)g(x)dx$.

Assume $x = 13000$ t and $P = 12000$ t. Since the demand is 13000 t, and the plant can only produce 12000 t, the actual amount of the product sold to the market is 12000 t only. Therefore, by multiplying the corresponding price, r , in the integral, the expected revenue can be calculated by using Eq. (3.1).

In addition, Eq. (3.1) is also related with Eq. (3.2), in which y_{ij} is already determined from the electricity cost calculation. This constraint is used to define the production target, P , in order to fulfil the market demand by the defined confidence level. Hence, by applying the basic probability techniques [14], Eq. (3.1) can be solved by available commercial routines, and the problem can be relaxed to the following deterministic form (Please see the appendix part II and III):

$$\text{Revenue} = r \times \sum_i \{ \theta_i \Phi(z_i) + P_i (1 - \Phi(z_i)) - \sigma_i \phi(z_i) \} \quad (4.1)$$

$$\text{s.t } P_i = \sum_j (y_{ij} \times SL_j)$$

$$\begin{aligned} &\text{where } z = (P - \theta) / \sigma \\ &i=1, \dots, 12 \text{ and } j=1, \dots, 3 \end{aligned}$$

where Φ , ϕ represent the cdf and pdf of the standard normal distribution, respectively. It is noted that Eq. (4.1) is obviously easier to solve than Eq. (3.1) since it does not have an integral term. The value of $\Phi(z)$, $\phi(z)$ can be easily obtained by calling standard routines. Hence, at different confidence levels, the expected revenue can be calculated. It is remind that the derivation of equations (2.4) and (3.1) can be found in the Appendix and the meaning of each symbol shown in these two models can be found in the Nomenclature.

Although individual models are set up for calculating the electricity cost and revenue, they can be combined as one model for optimization indeed. The revenue model,

however, doesn't involve the optimization skill; therefore it is possible to consider both models separately in order to make a clearer picture.

4.2. Results and Discussions

Table 8 and 9 show the results of revenue and electricity cost models.

Here we pick up $\alpha = 0.9$ as an example to illustrate the calculation of the expected revenue and electricity cost for TZ and LC contracts.

The Revenue: Using constrained equation (2.4a), by fixing the confidence level at 0.9, the production rate can be determined and shown in Table 10.

Then, by using the Eq. (4.1), the expected revenue can be calculated as follows:

$$\begin{aligned} \text{Revenue} &= 0.035 \times 4 \times \\ &\left[12000\Phi\left(\frac{12281.94-12000}{220}\right) + 12281.94\left(1 - \Phi\left(\frac{12281.94-12000}{220}\right)\right) - 220\phi\left(\frac{12281.94-12000}{220}\right) \right. \\ &\quad + 10000\Phi\left(\frac{10230.68-10000}{180}\right) + 10230.68\left(1 - \Phi\left(\frac{10230.68-10000}{180}\right)\right) - 180\phi\left(\frac{10230.68-10000}{180}\right) \\ &\quad \left. + 13000\Phi\left(\frac{13320.39-13000}{250}\right) + 13320.39\left(1 - \Phi\left(\frac{13320.39-13000}{250}\right)\right) - 250\phi\left(\frac{13320.39-13000}{250}\right) \right] \\ &= 4895.69 \text{ MYEN} \end{aligned}$$

The electricity cost: At $\alpha = 0.9$, the optimal annual electricity importation at each shift and the maximum electricity importation after optimization are shown in the Table 11.

Then the cost can be computed as follows:

$$\begin{aligned} \text{ELcost}_{\text{TZ}} &= 292*0.021+48540.74*0.011+73000*0.0077 \\ &= 1102.17 \text{ MYEN} \end{aligned}$$

$$\begin{aligned} \text{ELcost}_{\text{LC}} &= (9775.92*0.018+39056.31*0.006+73000*0.003) \\ &\quad + (7.18*14+15*3+25*1.5+121832.23*0.0025) \end{aligned}$$

$$=1116.85 \text{ MYEN}$$

Hence, Profit under TZ contract = $4895.69 - 1102.17 = 3793.52 \text{ MYEN}$

$$\text{Profit under LC contract} = 4895.69 - 1116.85 = 3778.84 \text{ MYEN}$$

The figures 4 and 5 indicate the electricity importation under the TZ and LC contracts at $\alpha = 0.9$. Both of them show the company should import less electricity at D and N shifts and the amount pointed to the upper bound at M shift. The reason is due to the high electricity price corresponding to D and N shifts, the company should avoid consuming electricity at these two shifts in order to minimize the electricity cost. Actually, this is a typical profile for the LC/TZ contract in the industrial practice. A similar result could be obtained at various confidence levels for both contracts.

Table 12 and figure 6 show the results of the two profits by applying the TZ and LC contracts. It can be seen that both contracts have the optimal points at one particular confidence level. It is interesting to point out that this result cannot be found from some conventional uncertainty studies. In conventional studies, if α is toward zero, the demand inequality of Eq. (2.4a) is meaningless, which means that the optimization can be made without this constraint. Then the profit remains maximal at α equals to 0. In other words, the Profit against confidence level curve is always descending.

However, this is only correct when we use the expectation of demand for calculating the revenue. If we use Eq. (4.1) to calculate the revenue instead of using expectation, the result shows a maximum point at one particular confidence level. Therefore, when we only adopt the TZ contract for the production planning, we should choose $\alpha = 0.7$ for the production target in order to maximize the profit. The same situation is applied for the LC contract, but the optimal confidence level is located at 0.4. It is noted that there is a considerable drop of profit at α from 0.9 to 0.9995. The reason is that, if the confidence level is increased from this range, more products may turn into the overage products and cannot be sold to the market eventually. Thus the profit decreases. This is the same logic for explaining the considerable change of profit at the lower confidence level.

However, it is interesting to note that if both contracts are taking into consideration simultaneously, there is an intersecting point. The two curves intersect at the $\alpha = 0.76$. Hence, if a desired confidence level chosen by the decision-marker is smaller than

0.76, the profit obtained under the LC contract is higher than the TZ contract and vice versa. It turns out that if a company has planned its production schedule with the confidence level lower than 0.76, there would be less than 76% probability for the products to satisfy the market demand, and then it is better to choose the LC contract; otherwise the TZ contract would be the wise choice for the confidence level above 0.76.

The selection of the right electricity contract is sensitive to the prices of electricity. Considering another set of electricity price for the TZ contract with D: 20000 YEN/MWH, N: 10500 YEN/MWH and M: 7500 YEN/MWH, in which the price at all shifts are decreased a little bit compared with those prices given in Table 2. The profit changes with confidence level are shown in figure 7. It is found that by decreasing the electricity price for the TZ contract, the profit definitely increases. However this shifts the intersecting point from the $\alpha=0.76$ to $\alpha=0.26$. This means that if the company has planned its production schedule with the confidence level lower than 0.26, then it is better to choose the LC contract; otherwise the TZ contract would be a better choice. The solution can provide different information for the decision-maker to analysis and choose the electricity contract based on the defined confidence level at different combination of contract values.

5. Conclusions

In this paper, the probabilistic programming model is used for selecting a right electricity importation contract under demand uncertainties. The selection of electricity contract and the optimization of contracted terms should be carefully studied for every individual consumer under his/her desired confidence level. Instead of letting the optimizer determine the selection of electricity contract, this paper presented an approach that allows the users to make decisions based upon the desired confidence level of satisfying the uncertain demands. A case study is used to illustrate the scope of the approach.

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Appendix

Part I.

From Eq. (2.4), the production rate greater than the demand should be equal or greater than the predefined confidence level. To describe the random variables, X_i , it is assumed that the pdf of each variable is known.

$$\Pr\left(\sum_j (y_{ij} \times SL_j) \geq X_i\right) \geq \alpha$$

$$\Pr\left(X_i \leq \sum_j (y_{ij} \times SL_j)\right) \geq \alpha$$

$$\Phi\left(\sum_j (y_{ij} \times SL_j)\right) \geq \alpha$$

$$\sum_j (y_{ij} \times SL_j) \geq \Phi^{-1}(\alpha)$$

$$i = 1 \dots 12$$

Part II.

In Eq. (3.1), the value of the expected revenue with production target P_i needs to be evaluated, this is derived in the follows:

$$\begin{aligned} & r \times \sum_i \int_0^\infty \min(P_i, x_i) g(x_i) dx_i \\ &= r \times \sum_i \left(\int_0^{P_i} x_i g(x_i) dx_i + \int_{P_i}^\infty P_i g(x_i) dx_i \right) \end{aligned} \quad (A.1)$$

$$= r \times \sum_i \left(\int_0^\infty x_i g(x_i) dx_i - \int_{P_i}^\infty x_i g(x_i) dx_i + \int_{P_i}^\infty P_i g(x_i) dx_i \right) \quad (A.2)$$

$$= r \times \sum_i \left(\theta_i - \int_{P_i}^\infty x_i g(x_i) dx_i + \int_{P_i}^\infty P_i g(x_i) dx_i \right) \quad (A.3)$$

$$= r \times \sum_i \left(\theta_i - \int_{P_i}^\infty (x_i - P_i) g(x_i) dx_i \right) \quad (A.4)$$

$$= r \times \sum_i \left(\theta_i - [(\theta_i - P_i)(1 - \Phi(z_i)) + \sigma_i \phi(z_i)] \right) \quad (A.5)$$

$$= r \times \sum_i \left(\theta_i \Phi(z_i) + P_i (1 - \Phi(z_i)) - \sigma_i \phi(z_i) \right) \quad (A.6)$$

Part III.

From Eq. (A.4) to Eq. (A.5) can be derived by considering the following term:

$$\begin{aligned}
& \sum_i \left(\int_{P_i}^{\infty} (x_i - P_i) g(x_i) dx_i \right) \\
&= \sum_i \left(\int_{P_i}^{\infty} (x_i - \theta_i) g(x_i) dx_i + \int_{P_i}^{\infty} (\theta_i - P_i) g(x_i) dx_i \right) \\
&= \sum_i \left(\int_{P_i}^{\infty} (x_i - \theta_i) \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(x_i - \theta_i)^2}{2\sigma_i^2}} dx_i + (\theta_i - P_i)[1 - \Phi(z_i)] \right) \\
&= \sum_i \left(\int_{z_i}^{\infty} y_i \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} dy_i \sigma_i + (\theta_i - P_i)[1 - \Phi(z_i)] \right) \quad y_i = \frac{x_i - \theta_i}{\sigma_i} \quad \& \quad dx_i = \sigma_i dy_i \\
&= \sum_i \left(\sigma_i \int_{z_i}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} d\left(\frac{y_i^2}{2}\right) + (\theta_i - P_i)[1 - \Phi(z_i)] \right) \\
&= \sum_i \left(\sigma_i \frac{1}{\sqrt{2\pi}} e^{-\frac{y_i^2}{2}} \Big|_{y_i=z_i} + (\theta_i - P_i)[1 - \Phi(z_i)] \right) \\
&= \sum_i (\sigma_i \phi(z_i) + (\theta_i - P_i)[1 - \Phi(z_i)]) \\
&= \sum_i ((\theta_i - P_i)[1 - \Phi(z_i)] + \sigma_i \phi(z_i))
\end{aligned}$$

i = 1...12

The terms in Eq. (A.6) can be calculated with an available commercial routine.

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Fig. 1. Process network for the motivating example.

Fig. 2. Density function of the uncertain variable in the case study.

Fig. 3. The cumulative density function at March.

Fig. 4. Electricity importation Vs. Month of TZ contract.

Fig. 5. Electricity importation Vs. Month of LC contract.

Fig. 6: The profit Vs. Confidence level.

Fig. 7: Profit Vs. Confidence level with the second set of electricity price for the TZ contract.

Table 1

The Mean of production target of product and its standard deviation

Month	Winter (Nov – Feb)	Spring & Autumn (Mar, Apr, Sep, Oct)	Summer (May – Aug)
Mean θ (t)	12000	10000	13000
Standard Deviation σ (t)	220	180	250

Table 2

Electricity price in LC and TZ contract

Shift	Electricity Price, C_{ij} (YEN/MWh)	
	LC contract	TZ contract
D	18000	21000
N	6000	11000
M	3000	7700

Table 3

Upper and lower bound of electricity importation in LC and TZ contract

Shift	Upper bound of electricity importation (MWh/shift)	
	LC contract	TZ contract
D	10	25
N	15	25
M	25	25

The lower bound of electricity importation in each shift is applied for the LC contract, which is equal to 0.5MW

Table 4

Fixed electricity cost calculation for the LC contract

S_D, S_N, S_M	Maximum of normal EL import at D, N and M shifts (MW).
a_D, a_N, a_M	Fixed cost parameter of the normal EL import at D, N and M shifts (Day: 14, Night: 3, Mid-night: 1.5, MYEN/MW).
TEL	Total electricity import (MWh), at least 100,000 MWh.
a_T	Cost parameter for total electricity import (0.0025 MYEN/MWh).

Table 5

Data for Example 1

Shift	Annual EL importation (MWh)	Max EL importation in each shift (MW)
D	500	0.3
N	1000	0.4
M	2000	0.7

Table 6

Data for Example 2

Shift	Annual EL importation (MWh)	Max EL importation in each shift (MW)
D	300	0.3
N	700	0.4
M	1500	0.7

Table 7

The Production Target of product at different confidence levels

Confidence level	Winter (Nov – Feb) (T)	Spring & Autumn (Mar, Apr, Sep, Oct) (T)	Summer (May – Aug) (T)
0.0005	11276.07	9407.63	12177.24
0.10	11718.06	9769.32	12679.61
0.20	11814.84	9848.51	12789.59
0.30	11884.63	9905.61	12868.90
0.40	11944.26	9954.40	12936.66
0.50	12000.00	10000.00	13000.00
0.60	12055.74	10045.60	13063.34
0.70	12115.37	10094.39	13131.10
0.80	12185.16	10151.49	13210.41
0.90	12281.94	10230.68	13320.39
0.9995	12724.02	10592.36	13822.70

*Notice that 0.0005 and 0.9995 are used as the minimum and maximum confidence levels instead of 0 and 1. Hence 99.9% cases would be covered.

Table 8

Expected Revenue with respect to confidence level

Confidence level	Expected Revenue (MYEN)
0.0005	4600.52
0.1	4779.07
0.2	4813.25
0.3	4834.96
0.4	4851.01
0.5	4863.70
0.6	4874.06
0.7	4882.68
0.8	4889.84
0.9	4895.69
0.9995	4899.99

Table 9

Electricity cost with respect to confidence level under LC and TZ contracts and the total electricity import

Confidence level	Electricity cost (MYEN)		EL imported (MW)
	LC contract	TZ contract	
0.0005	899.25	991.02	111727.19
0.10	989.79	1039.87	116167.77
0.20	1009.97	1050.56	117140.01
0.30	1025.91	1058.27	117841.07
0.40	1039.56	1064.86	118440.10
0.50	1052.32	1071.02	119000.00
0.60	1065.08	1077.18	119559.90
0.70	1078.73	1083.77	120158.93
0.80	1094.70	1091.48	120859.98
0.90	1116.85	1102.17	121832.23
0.9995	1218.04	1151.02	126272.86

Table 10

Product rate

α	Nov – Feb (t)	Mar, Apr, Sep, Oct (t)	May – Aug (t)
0.9	12281.94	10230.68	13320.39

Table 11

Amount of electricity to be purchased

Shift	TZ contract	LC contract	
	Annual EL Imported (MW)	Annual EL Imported (MW)	Max EL Imported (MW)
D	292.00	9775.92	7.18
N	48540.74	39056.31	15.0
M	73000.00	73000.00	25.0

Table 12

The profit Vs. Confidence level

Confidence level	Profit (MYEN)	
	LC contract	TZ contract
0.0005	3701.27	3609.50
0.10	3789.28	3739.20
0.20	3803.28	3762.69
0.30	3809.04	3776.69
0.40	3811.45 [#]	3786.15
0.50	3811.38	3792.68
0.60	3808.99	3796.88
0.70	3803.95	3798.91 [#]
0.80	3795.14	3798.36
0.90	3778.84	3793.52
0.9995	3681.95	3748.97

[#] The maximum profit obtained by the contract.