Simulation Estimation of Dynamic Switching Regression and Dynamic Disequilibrium Models – Some Monte Carlo Results

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Abstract

This article considers the estimation of dynamic exogenous switching regression models and dynamic endogenous switching models. With autocorrelation in disturbances or latent lagged dependent variables, likelihood functions of such models involve high dimensional integrals and a huge number of summations over unobserved regime paths. Simulated likelihood methods and simulated methods of moments are available. These approaches simulate both continuous and discrete latent dependent variables. By Monte Carlo experiments, it is found that the performances of various approaches depend crucially on how discrete state variables are simulated. The valuable approach is to simulate regime paths with regime probabilities based on the current and past sample information.

Key words: Simulated likelihood; Regime classification; Dynamic switching model; Markov switching model; Disequilibrium model

JEL classification: C15; C32

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1. Introduction

This article investigates econometric methods for the estimation of dynamic switching regression models and dynamic disequilibrium models. Earlier contributions on these models are the articles by Quandt (1974), Quandt and Ramsey (1978), Goldfeld and Quandt (1973), Fair and Jaffee (1972), Maddala and Nelson (1974), Goldfeld and Quandt (1975), and Kiefer (1978) among others. These models are used for the analysis of time series data. Due to computational complexity and the lack of computationally tractable statistical methods, those models have mostly been specified and estimated without dynamic structures. However, recent developments on simulation estimation methodologies may overcome such difficulties. It is the objective of this article to investigate possibly computationally tractable simulation methods for the estimation of such dynamic models.

Goldfeld and Quandt (1973) formulated switching regression models with dynamic structures where the regime switching process was modeled by a Markov chain. Under the assumption of no lagged latent dependent variables in regression equations, Cosslett and Lee (1985) suggested a recursive algorithm that is computationally tractable for the evaluation of the likelihood function. Subsequently, Hamilton (1989) introduced a filter algorithm (see also Kim 1994). The recursive algorithms in Cosslett and Lee and Hamilton provide efficient estimation and inference with the likelihood function. For more complicated models with lagged latent dependent variables, Cosslett and Lee have pointed out that the recursive algorithm originated in their article will not be applicable. So is the filtering algorithm in Hamilton (1989). When either serially correlated disturbances or lagged latent dependent variables appear in any switching regression equation of a model with m regimes and T periods, likelihood functions for such models involve T -fold integrals in addition to m^T -summations of regime paths. These likelihood functions are neither analytically nor computationally tractable with conventional quadrature methods.

Recent developments of simulation methods provide alternative approaches for handling multiple integrals (McFadden 1989; Pakes and Pollard 1989; Hajivassiliou and McFadden 1990; Borsch-Supan and Hajivassiliou 1993; Keane 1994, among others). For the static disequilibrium model, methods of simulated moments and simulated pseudo-likelihoods have been considered in Laroque and Salanie (1994). For dynamic disequilibrium models, simulated pseudo-likelihood methods have been proposed in Laroque and Salanie (1993) and methods for evaluating likelihood functions have been suggested in Hendry and Richard (1992). Lee (1994) has proposed a simulated likelihood approach for dynamic disequilibrium

models where only an own lagged dependent variable of first order can appear in each switching regression equation. Based on a ‘renewal’ property of such models, only latent continuous dependent variables but not regime paths need be simulated and the simulated likelihood is smooth. While the latter models are quite important (Quandt 1988), general models that allow higher order latent lagged variables and serially correlated disturbances do not possess the ‘renewal’ property. For the estimation of general models, only the simulated pseudo-likelihood approaches of Laroque and Salanie and simulated likelihood approaches of Hendry and Richard remain to be feasible, at least, in principle. Other simulation approaches are also available for the estimation of stochastic variance models, dynamic stochastic volatility models, and various nonlinear time series models (Shephard 1993, Danielsson and Richard 1993, Smith 1993, Gouriéroux et al. 1993, and others in a special issue on simulation techniques published by *Journal of Applied Econometrics*).

This article follows up the simulation estimation literature to investigate potentially computationally tractable simulated likelihood methods for the estimation of dynamic switching regression and disequilibrium models. We report our investigation and findings based on Monte Carlo experiments. Various possible approaches are compared in terms of computational tractability, finite sample performance of simulated estimators, and regime classification.¹

The organization of this article is as follows. In Section 2, we review some dynamic models. Likelihood functions for such models are derived. Several simulation algorithms are described in Section 3. Regime classification rules derived from simulated likelihood functions are suggested in Section 4. Monte Carlo experiments for a dynamic switching regression model are reported in Section 5. Section 6 reports Monte Carlo results for a dynamic disequilibrium model. We summarize our findings in Section 7.

2. Exogenous and Endogenous Switching Regression Models

Goldfeld and Quandt (1973) considered a situation in which observations, indexed by t , $t = 1, \dots, T$, are available on a dependent variable y_t and on a finite vector of independent variables x_t . The y_t is assumed to be generated by one of the m regression equations. The simplest model is specified when the probabilities of switching are independent of what state the system was in in the previous period and there are no serial correlations in regression equations. Estimation by the method of maximum likelihood

¹Hendry and Richard (1992) provided Monte Carlo evidences on the accuracy of a simulated likelihood function for a simple one parameter dynamic disequilibrium model. They evaluated the case of a time series with 50 periods based on 10,000 Monte Carlo drawings. They have investigated neither simulated likelihood estimators nor the issue of regime classification.

and methods of moments has been considered in Quandt and Ramsey (1978). By relaxing the simple switching mechanism, a Markov switching model was introduced in Goldfeld and Quandt (1973). Let I_t be the (latent) regime indicator at t , i.e., $I_t = i$ if the state is i at period t . By introducing a transition matrix Λ , the regime switching process is a Markov chain:

$$[P(I_{t-1} = 1), \dots, P(I_{t-1} = m)]\Lambda = [P(I_t = 1), \dots, P(I_t = m)], \quad (1)$$

where $P(I_t = i)$ is the probability in state i at t . Under these specifications and with unknown parameters suppressed, the log likelihood function is

$$L = \ln \left\{ \sum_{I_T=1}^m \cdots \sum_{I_1=1}^m \left[\prod_{t=1}^T f(y_t | I_t) \cdot \prod_{t=2}^T P(I_t | I_{t-1}) \cdot P(I_1) \right] \right\}, \quad (2)$$

where $P(I_t | I_{t-1})$ is the transition probability from state I_{t-1} at time $t-1$ to state I_t at time t and $f(y_t | I_t = i)$ is the probability density of y_t from the regression equation of regime i (Cosslett and Lee 1985). Direct evaluation of the likelihood function of (2) involves the sum of m^T terms and is computationally intractable except for small T . This difficulty, however, can be overcome by a recurrence algorithm in Cosslett and Lee (1985) and a filtering algorithm in Hamilton (1989).

As it was noted in Goldfeld and Quandt (1973), error terms in regression equations might likely be autocorrelated. Denote the dependent variable in regime i at time t by y_{it}^* with a superscript $*$ to distinguish it from the observed y_t . The observed y_t relates to the underlying regression equations by $y_t = y_{it}^*$ if $I_t = i$. A first-order Markov process, $u_{it} = \rho_i u_{i,t-1} + \epsilon_{it}$, will imply that $y_{it}^* = \rho_i y_{i,t-1}^* + (x_t - \rho_i x_{t-1})\beta_i + \epsilon_{it}$. For dynamic models, one may introduce both latent and observed lagged dependent variables in switching equations. At each t , only one of y_{it}^* , $i = 1, \dots, m$, can be observed and the remaining ones are not. Corresponding to the value of I_t , let z_t^* be the $(m-1)$ -dimensional subvector consisting of unobservable dependent variables. Let $\bar{y}_t = (y_1, \dots, y_t)$, $\bar{I}_t = (I_1, \dots, I_t)$, and $\bar{z}_t^* = (z_1^*, \dots, z_t^*)$ for simplicity. Let $h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*)$ be the joint density of $\bar{y}_T$, $\bar{I}_T$ and $\bar{z}_T^*$. The likelihood function is

$$h(\bar{y}_T) = \sum_{I_T=1}^m \cdots \sum_{I_1=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) dz_1^* \cdots dz_T^*. \quad (3)$$

The T summations sum up all the likelihood components over the m^T regime paths. The integrals are integrated over all latent continuous dependent variables for each regime path. The recurrence algorithm in Cosslett and Lee (1983) and Hamilton's filter can not be applied because they rely on the Markov property of both regime switching and the observed dependent variables. The Markov property does not

hold in the presence of latent lagged dependent variables. For such dynamic models, one has to face both the m^T -summation problem and the high dimensional integrals problem.

The formulation of regime switching in the above models is known as exogenous switching. There are endogenous switching models where the regime switching mechanism involves dependent variables of regression equations. An important class of endogenous switching models consists of disequilibrium models. Quandt (1988, p.134) formulated a dynamic disequilibrium model of the form: $y_{1t}^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^* - y_{t-1}) + \epsilon_{1t}$, $y_{2t}^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^* - y_{t-1}) + \epsilon_{2t}$, and $y_t = \min\{y_{1t}^*, y_{2t}^*\}$. This model captures dynamic adjustments of unsatisfied demand or supply over time. The regime switching process is endogenously determined as $I_t = 1$ if and only if $y_{2t}^* > y_{1t}^*$. For this model, the auxiliary latent variable z_t^* is that $z_t^* = y_{2t}^*$ if $I_t = 1$ and $z_t^* = y_{1t}^*$ if $I_t = 2$. This z_t^* represents the unobserved quantity (the long side of the market) at t . The likelihood function is

$$h(\bar{y}_T) = \sum_{I_T=1}^2 \cdots \sum_{I_1=1}^2 \int_{y_T}^{\infty} \cdots \int_{y_1}^{\infty} h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) dz_1^* \cdots dz_T^*. \quad (4)$$

Integrals in likelihood functions of endogenous switching models have usually restricted ranges. On the contrary, ranges of integrals for exogenous switching models are unrestricted. Detailed surveys on these models can be found in Quandt (1988).

3. Simulation Estimation Approaches

Consider first an exogenous dynamic switching model. The likelihood function has the general form in (3). We follow Hendry and Richard (1992) to consider simulation techniques based on factorizations of the sequential joint density of the observables and latent dependent variables. Let the past information set generated by $\bar{y}_{t-1}$, $\bar{I}_{t-1}$ and $\bar{z}_{t-1}^*$ be denoted by $\mathcal{J}_{t-1}$. A recurrence relation of special interest is

$$h(\bar{y}_t, \bar{I}_t, \bar{z}_t^*) = g(z_t^* | I_t, y_t, \mathcal{J}_{t-1}) P(I_t | y_t, \mathcal{J}_{t-1}) f(y_t | \mathcal{J}_{t-1}) \cdot h(\bar{y}_{t-1}, \bar{I}_{t-1}, \bar{z}_{t-1}^*). \quad (5)$$

The $g(z_t^* | I_t, y_t, \mathcal{J}_{t-1})$ is the density of the unobserved dependent variables at t given the past information and the knowledge which states have or have not been occupied at time t . The additional conditioning on y_t is just to capture the possible contemporaneous correlation of y_t with z_t^* . The second component is a conditional probability of regime occurrence given the past $\mathcal{J}_{t-1}$ and currently available sample y_t at t . The $f(y_t | \mathcal{J}_{t-1})$ is the density of the sample y_t given the past $\mathcal{J}_{t-1}$. The last two components can be derived from a model specification of $P(I_t | \mathcal{J}_{t-1})$ and $f(y_t | I_t = i, \mathcal{J}_{t-1})$ where $i = 1, \dots, m$. It is obvious

that

$$f(y_t | \mathcal{J}_{t-1}) = \sum_{i=1}^m f(y_t | I_t = i, \mathcal{J}_{t-1}) P(I_t = i | \mathcal{J}_{t-1}). \quad (6)$$

By the Bayes rule,

$$P(I_t | y_t, \mathcal{J}_{t-1}) = f(y_t | I_t, \mathcal{J}_{t-1}) P(I_t | \mathcal{J}_{t-1}) / f(y_t | \mathcal{J}_{t-1}). \quad (7)$$

Under the decomposition in (5), the likelihood function can be rewritten as

$$\begin{aligned} h(\bar{y}_T) &= \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{t=2}^T f(y_t | \mathcal{J}_{t-1}) g(z_t^* | I_t, y_t, \mathcal{J}_{t-1}) \\ &\quad \cdot P(I_t | y_t, \mathcal{J}_{t-1}) \cdot f(y_1) g(z_1^* | I_1, y_1) P(I_1 | y_1) \prod_{t=1}^T dz_t^*. \end{aligned} \quad (8)$$

An alternative decomposition is

$$h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) = g(z_T^* | I_T, y_T, \mathcal{J}_{T-1}) f(y_T | I_T, \mathcal{J}_{T-1}) P(I_T | \mathcal{J}_{T-1}) \cdot h(\bar{y}_{T-1}, \bar{I}_{T-1}, \bar{z}_{T-1}^*). \quad (9)$$

With this recurrence formulae, the likelihood function can be rewritten as

$$\begin{aligned} h(\bar{y}_T) &= \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{t=2}^T f(y_t | I_t, \mathcal{J}_{t-1}) g(z_t^* | I_t, y_t, \mathcal{J}_{t-1}) \\ &\quad \cdot P(I_t | \mathcal{J}_{t-1}) \cdot f(y_1 | I_1) g(z_1^* | I_1, y_1) P(I_1) \prod_{t=1}^T dz_t^*. \end{aligned} \quad (10)$$

The decomposition in (9) seems natural as it translates the original model formulation in terms of density and probability functions. These recurrence relations and the corresponding likelihood functions of (8) and (10) motivate simulation algorithms for evaluating the likelihood function. The approaches will simulate both latent continuous dependent variables and discrete dependent variables.

At any possible value of the parameter vector, latent variables that need to be simulated at t are I_t and z_t^* . Since z_t^* is defined conditional on I_t , it is imperative that I_t be simulated first. Simulate I_t from a multinomial distribution with probabilities $P(i | y_t, \mathcal{J}_{t-1})$, $i = 1, \dots, m$. Given a simulated $\mathcal{J}_{t-1}$, $P(i | y_t, \mathcal{J}_{t-1})$ can be constructed from (6) and (7). Given simulated I_t , z_t^* can be simulated from the density $g(z_t^* | I_t, y_t, \mathcal{J}_{t-1})$. Given initial values, one simulates a path of regimes $(I_1, \dots, I_T)$ and corresponding latent variables $(z_1^*, \dots, z_T^*)$ for the whole sample periods. Such a simulation process can be independently repeated many times, say S . Let $I_t^{(s)}$ and $z_t^{*(s)}$ be, respectively, the simulated values of I_t and z_t^* at t from the s th simulation run. The likelihood function can then be simulated as

$$h_c(\bar{y}_T) = \frac{1}{S} \sum_{s=1}^S \prod_{t=2}^T f(y_t | \mathcal{J}_{t-1}^{(s)}) \cdot f(y_1). \quad (11)$$

This simulated function is an unbiased estimate of the likelihood function $h(\bar{y}_T)$ given the sample observations. It is a consistent estimator of $h(\bar{y}_T)$ if S goes to infinity. This is a simulated likelihood approach with simulated regime paths based on conditional sample probabilities (SLSP). This simulated likelihood function has an interesting property that it nests the likelihood function of the static model as a special case. In a static model, there are no latent lagged dependent variables in regression equations and the switching mechanism does not depend on previous states, i.e., $f(y_t|I_t, \mathcal{J}_{t-1}) = f(y_t|I_t)$ and $P(I_t|\mathcal{J}_{t-1}) = P(I_t)$. In this case, the simulated function $h_c(\bar{y}_T)$ reduces to $\prod_{t=1}^T f(y_t)$, which is exactly the likelihood for the classical static model.

For the likelihood in (10), I_t can be simulated with transition probability $P(I_t|\mathcal{J}_{t-1})$. With the simulated I_t and $\mathcal{J}_{t-1}$, z_t^* can be simulated with the density $g(z_t^*|I_t, y_t, \mathcal{J}_{t-1})$. The likelihood function can be simulated as $h_p(\bar{y}_T) = \frac{1}{S} \sum_{s=1}^S \prod_{t=2}^T f(y_t|I_t^{(s)}, \mathcal{J}_{t-1}^{(s)}) \cdot f(y_1|I_1^{(s)})$. This is a simulated likelihood with simulated regime paths based on transition probabilities. Based on the same likelihood function, there are other possible approaches. Instead of simulating I_t based on $P(I_t|\mathcal{J}_{t-1})$ (and z_t^* based on $g(z_t^*|I_t, \mathcal{J}_{t-1})$), it is possible to simulate I_t randomly. Importance sampling is also possible. These other approaches do not, however, perform well in our subsequent Monte Carlo studies.

For dynamic disequilibrium models, regime probabilities are endogenously determined. The integrals of the likelihood function have limits determined by censoring due to the observed quantity being the minimum of supply and demand. For the $g(z_t^*|I_t, y_t, \mathcal{J}_{t-1})$, the support of z_t^* is $[y_t, \infty)$. Thus with the decomposition in (5), the likelihood function is

$$\begin{aligned}
h(\bar{y}_T) &= \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{y_T}^{\infty} \cdots \int_{y_1}^{\infty} \prod_{t=2}^T f(y_t|\mathcal{J}_{t-1}) g(z_t^*|I_t, y_t, \mathcal{J}_{t-1}) \\
&\quad P(I_t|y_t, \mathcal{J}_{t-1}) \cdot f(y_1) g(z_1^*|I_1, y_1) P(I_1|y_1) \prod_{t=1}^T dz_t^* \\
&= \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{t=2}^T f(y_t|\mathcal{J}_{t-1}) g(z_t^*|I_t, y_t, \mathcal{J}_{t-1}) \\
&\quad P(I_t|y_t, \mathcal{J}_{t-1}) \cdot f(y_1) g(z_1^*|I_1, y_1) P(I_1|y_1) \prod_{t=1}^T dz_t^*. \tag{12}
\end{aligned}$$

The last expression in (12) does not have finite limits in its integrals as the limits are implicitly captured in the conditional density g . The simulation approaches described for the dynamic switching regression model are applicable to the dynamic disequilibrium model with the understanding that z_t^* is simulated from a truncated density $g(z_t^*|I_t, y_t, \mathcal{J}_{t-1})$ instead of an untruncated one. The simulation of truncated

random variable is usually more complicated. This SLSP approach for evaluating the dynamic disequilibrium model has first appeared in Hendry and Richard (1992), p.11.

4. Regime Classification

As sample separation information is not available, one may like to identify the underlying regime that generates the sample observation at any sampling period. Such an issue is in the domain of discrimination analysis. A classification rule is optimal if it minimizes the total probability of missclassification (assuming parameters are known). At t , y_t will be classified to regime i if $P(I_t = i|\bar{y}_T) = \max\{P(I_t = j|\bar{y}_T) : j = 1, \dots, m\}$. This classification rule is optimal (Anderson 1958). An optimal classification rule has been applied in Lee and Porter (1984) for a static switching regression model. For the Markov switching model, recursive smoothing algorithms have been proposed in Cosslett and Lee (1985), Hamilton (1989), and Kim (1994). For the Markov switching model, simulation is unnecessary for a smoothing algorithm. Simulation estimation is needed for a general dynamic switching model.

The probability $P(I_t|\bar{y}_T)$ is equal to $h(\bar{y}_T, I_t)/h(\bar{y}_T)$. For the dynamic switching regression model, $h(\bar{y}_T)$ is the likelihood function in (3) and

$$h(\bar{y}_T, I_t) = \sum_{I_1=1}^m \cdots \sum_{I_{t-1}=1}^m \sum_{I_{t+1}=1}^m \cdots \sum_{I_T=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) dz_1^* \cdots dz_T^*. \quad (13)$$

For simulation, it is illustrative to rewrite (13) in a slightly different way. Define dichotomous indicators $D_i(I_t)$, $i = 1, \dots, m$, such that $D_i(I_t) = 1$ if $I_t = i$; 0, otherwise. Then $h(\bar{y}_T, I_t = i) = h_{ti}(\bar{y}_T)$ where $h_{ti}(\bar{y}_T) = \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} D_i(I_t) h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) dz_1^* \cdots dz_T^*$. For the dynamic disequilibrium model, the $h(\bar{y}_T)$ is in (4) and $h_{ti}(\bar{y}_T) = \sum_{I_1=1}^m \cdots \sum_{I_T=1}^m \int_{y_T}^{\infty} \cdots \int_{y_1}^{\infty} D_i(I_t) h(\bar{y}_T, \bar{I}_T, \bar{z}_T^*) dz_1^* \cdots dz_T^*$. These $h_{ti}(\bar{y}_T)$, $i = 1, \dots, m$, can be simulated similarly to the simulation of likelihood functions. For the SLSP approach, $h_{ti}(\bar{y}_T)$ can be simulated as $h_{ti,c}(\bar{y}_T)$ where $h_{ti,c}(\bar{y}_T) = \frac{1}{S} \sum_{s=1}^S \prod_{r=1}^T D_i(I_r^{(s)}) f(y_r | \mathcal{J}_{r-1}^{(s)})$ and $P(I_t|\bar{y}_T)$ can be simulated as $P_c(I_t|\bar{y}_T)$ where $P_c(I_t|\bar{y}_T) = h_{ti,c}(\bar{y}_T)/h_c(\bar{y}_T)$. The indicator $D_i(I_t^{(s)})$ selects out simulated regime paths which go through the state i at t . The simulator $P_c(I_t|\bar{y}_T)$ lies properly between zero and one. With unknown parameters substituted by simulated estimates, our suggested classification rule is to classify y_t to regime i if $P_c(I_t = i|\bar{y}_T)$ is the largest probability among the m simulated conditional regime probabilities.

5. Monte Carlo Studies on Dynamic Switching Regression Model

For the dynamic model in the experiment, sample data are generated from a two-regimes dynamic

model with the Markov switching:

$$y_{1t}^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^* - y_{t-1}) + \sigma_1\epsilon_{1t}, \quad (14)$$

and

$$y_{2t}^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^* - y_{t-1}) + \sigma_2\epsilon_{2t}, \quad (15)$$

where ϵ_{1t} and ϵ_{2t} are normally distributed as $N(0, 1)$, independent of each other and serially uncorrelated. The y_{1t}^* and y_{2t}^* are, respectively, the latent dependent variables of regime 1 and regime 2. Regime occurrence is determined by a Markov Chain with two states as in (1). The transition matrix Λ is a 2×2 matrix with transition probabilities being p_{12} from regime 1 to regime 2 and p_{21} from regime 2 to regime 1. The stationary regime probability for regime 1 is $p_{1\cdot} = p_{21}/(p_{12} + p_{21})$. The exogenous variables x_{1t} and x_{2t} are independently generated from a normal $N(0, 2)$ random generator. The true parameters are $\beta_1 = -1$, $\beta_2 = 1$, $\lambda_1 = 0.5$, $\lambda_2 = 0.5$, $\sigma_1 = 1$ and $\sigma_2 = 1$. The transition probabilities are $p_{12} = 0.3$ and $p_{21} = 0.3$. The stationary regime probabilities $p_{1\cdot}$ and $p_{2\cdot}$ are therefore 0.5 for each regime. The stochastic processes are running after 20 periods before sample observations are taken.

As the initial y_{10}^* and y_{20}^* are unobservable for the sample, an initial value problem exists for our estimation methods. In all the following Monte Carlo results, our strategy is to arbitrarily set the initial values $y_{10}^* = y_{20}^* = y_0 (= 0)$. This may not be an issue if sample periods are not too short. Non-negativity constraints for σ_1 , σ_2 , p_{11} and p_{22} are imposed by the following reparameterizations: $\sigma_1 = \exp(\alpha_1)$, $\sigma_2 = \exp(\alpha_2)$, $p_{11} = 1/(1 + \exp(\omega_1))$ and $p_{22} = 1/(1 + \exp(\omega_2))$. As the magnitudes of λ_1 and λ_2 may theoretically be less than a unity (Quandt 1988), the transformations, $\lambda_1 = (1 - \exp(\gamma_1))/(1 + \exp(\gamma_1))$ and $\lambda_2 = (1 - \exp(\gamma_2))/(1 + \exp(\gamma_2))$, are also used. The parameters α_1 , α_2 , ω_1 , and ω_2 are unconstrained.

5.1 Simulated Likelihood Approaches

For the SLSP approach, the simulated likelihood is h_c in (11). For this particular two regimes model, the detailed steps are as follows. Throughout this section, $\phi(\cdot)$ and $\Phi(\cdot)$ denote, respectively, the standard normal density and probability functions.

Simulate a set of standard normal variables ϵ_t and a set of uniform random variables u_t on $[0, 1]$, $t = 1, \dots, T$, that are fixed and are invariant of parameters.

1. For period 1:

$$(a) \text{ Compute } R_1 = M_{11} + M_{21} \text{ where } M_{11} = \frac{1}{\sigma_1} \phi\left(\frac{y_1 - x_{11}\beta_1}{\sigma_1}\right) p_{1\cdot} \text{ and } M_{21} = \frac{1}{\sigma_2} \phi\left(\frac{y_1 - x_{21}\beta_2}{\sigma_2}\right) p_{2\cdot}.$$

Simulate a regime indicator I_1 such that $I_1 = 1$ with probability $M_{11}/(M_{11} + M_{21})$ using u_1 , i.e., $I_1 = 1$ if $u_1 < M_{11}/(M_{11} + M_{21})$; $I_1 = 2$, otherwise.

(b) Simulate $y_1^* = x_{21}\beta_2 + \sigma_2\epsilon_1$ if $I_1 = 1$, but simulate $y_1^* = x_{11}\beta_1 + \sigma_1\epsilon_1$ if $I_1 = 2$.

2. For period t : The I_{t-1} and y_{t-1}^* have been simulated in the previous steps. R_{t-1} has been evaluated.

Define $y_{1,t-1}^{(p)}$ and $y_{2,t-1}^{(p)}$ such that $y_{1,t-1}^{(p)} = y_{t-1}$ if $I_{t-1} = 1$; y_{t-1}^* , otherwise, and $y_{2,t-1}^{(p)} = y_{t-1}^*$ if $I_{t-1} = 1$; y_{t-1} , otherwise.

(a) Compute $R_t = (M_{1t} + M_{2t})R_{t-1}$ where $M_{1t} = \frac{1}{\sigma_1}\phi\left(\frac{y_t - x_{1t}\beta_1 - \lambda_1(y_{1,t-1}^{(p)} - y_{t-1})}{\sigma_1}\right)p_{I_{t-1}1}$, and $M_{2t} = \frac{1}{\sigma_2}\phi\left(\frac{y_t - x_{2t}\beta_2 - \lambda_2(y_{2,t-1}^{(p)} - y_{t-1})}{\sigma_2}\right)p_{I_{t-1}2}$. Simulate I_t such that $I_t = 1$ with probability $M_{1t}/(M_{1t} + M_{2t})$ using u_t .

(b) Simulate y_t^* as follows:

- i. If $I_t = 1$, simulate $y_t^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^{(p)} - y_{t-1}) + \sigma_2\epsilon_t$.
- ii. If $I_t = 2$, simulate $y_t^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^{(p)} - y_{t-1}) + \sigma_1\epsilon_t$.

The above recurrence algorithm can be repeated many times with independently simulated sets of ϵ and u . With S simulation runs, denote $R_{t,j}$ the value of R_t for the j th simulation run. The likelihood function can be simulated as $L_c = \ln\left(\frac{1}{S}\sum_{j=1}^S R_{T,j}\right)$. Other simulated likelihood approaches can similarly be described.²

5.2 Simulated Pseudo Maximum Likelihood Approach

There are several versions of simulated pseudo maximum likelihood (SPML) approaches introduced in Laroque and Salanie (1993). The one recommended is an approach which uses both simulated first and second order moments in a normal pseudo-likelihood function. Simulate standard normal variables ϵ_{1t} and ϵ_{2t} , $t = 1, \dots, T$, which are fixed for estimation. Based on (14) and (15), given possible values of parameters and initial values $y_{1,0}^* = y_{2,0}^* = y_0 = 0$, y_t can be recursively simulated. At t , given that I_{t-1} , $y_{1,t-1}^*$, $y_{2,t-1}^*$ and y_{t-1}^* have been simulated, simulate I_t conditional on I_{t-1} with the transition probabilities p_{12} and p_{21} . Simulate $y_{1t}^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^* - y_{t-1}^*) + \sigma_1\epsilon_{1t}$ and $y_{2t}^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^* - y_{t-1}^*) + \sigma_2\epsilon_{2t}$. Set $y_t^* = y_{1t}^*$ if $I_t = 1$; otherwise, set $y_t^* = y_{2t}^*$. Denote the simulated value y_t^* from the s th run by $y_t^{*(s)}$. With a total of S runs, the conditional mean and variance of y_t can then be simulated by the empirical moments: $E_t(\theta) = \frac{1}{S}\sum_{s=1}^S y_t^{*(s)}$ and $V_t(\theta) =$

²Interested readers can consult Lee (1995) for details.

$\frac{1}{S} \sum_{s=1}^S \left(y_t^{*(s)} - E_t(\theta) \right)^2$, where θ denotes the parameter vector of the model. The SPML estimator is the minimizer of $\frac{1}{2T} \sum_{t=1}^T \left[(y_t - E_t(\theta))^2 / V_t(\theta) + \ln V_t(\theta) \right]$. It is possible to generalize this approach by using more moment equations. Let $V_{Gt}(\theta) = \frac{1}{S} \sum_{s=1}^S (y_t^{*(s)} - E_t(\theta), y_{t-1}^{*(s)} - E_{t-1}(\theta))' (y_t^{*(s)} - E_t(\theta), y_{t-1}^{*(s)} - E_{t-1}(\theta))$. A generalized pseudo-likelihood approach (GSPML) can use the pseudo-likelihood $\frac{1}{2(T-1)} \sum_{t=2}^T [(y_t - E_t(\theta), y_{t-1} - E_{t-1}(\theta)) V_{Gt}^{-1}(\theta) (y_t - E_t(\theta), y_{t-1} - E_{t-1}(\theta)) + \ln \det V_{Gt}(\theta)]$.³

5.3 Results of Monte Carlo experiments

The Markov switching model with (14) and (15) has been estimated by the various simulation methods. We have experimented with different sample sizes T (length of time series) and different numbers of simulation runs S . For each case, the number of replications is 400. The optimization subroutine used for estimation is based on the Powell method (see Press et al. 1986, p.299). A static version (setting $\lambda_1 = \lambda_2 = 0$) is first estimated and its estimates are used as initial starting values for the Powell subroutine.⁴ We report the empirical mean (Mean) and standard deviation (SD) of our estimates for the eight unknown parameters in the model. We report also the number of convergent replications for each case (# conv) and the average maximized logarithmic simulated likelihood values ($\ln(\text{lk})$). All the computations are performed in either SUN SPARC 10 or 20 models 50 and 60, or IBM RS/6000 model 580 workstations. The program language is Fortran 77.

Table 1 reports results for the SLSP approach. We have experimented with T from 60 to 200 and S from 50 to 1000 or 2000. For the case with $T = 60$, about 15 replications among the total 400 replications fail to converge. For $T = 100$, the number of failure is only 1 to 4 cases. With $T = 200$, all replications converge. For all these cases, the estimates of β_1 and β_2 are nearly unbiased. Some biases are in the estimates of σ_1 and σ_2 for $T = 60$. These biases decrease as T increases. These estimates are nearly unbiased for sample of size 200. There are obvious downward biases for the estimates of λ_1 and λ_2 and upward biases for the estimates of p_{12} and p_{21} . As sample size increases, these biases tend to reduce. Such biases still exist for $T = 200$, but are not large. The SD of estimate reduces as sample size increases. The rate of reduction seems to be compatible with the usual $\sqrt{T}$ -rate. The estimates are, in general, not sensitive to the number of simulation runs S . The biases of the estimates of λ 's and p 's tend to reduce with larger S but the chances are small. In terms of CPU time cost for one replication, it takes on average

³This way of using cross moments of lagged variables has been recommended to me by Professor G. Laroque in personal communication.

⁴Similar results are obtained from different starting values.

29.55 seconds for the case with $T = 60$ and $S = 100$ in an IBM RS/6000 Model 580 workstation. The CPU time increases to 50.16 seconds for $T = 100$ and $S = 100$; 93.21 seconds for $T = 200$ and $S = 100$; and 973.15 seconds for $T = 200$ and $S = 1000$. The CPU time is approximately linear in T or S .⁵

Table 2 reports results on regime classification based on the SLSP simulation method and its estimates in Table 1. Each sample observation is classified to be generated either from regime 1 or from regime 2. Since data are artificially generated, underlying regimes are available. The percentage of overall correct classification as well as separate percentages for each regime are reported. The summary statistics and frequency distributions of correct classification are slightly sensitive to S . Classification accuracy improves slightly as S increases. On average (Mean), 72% of regimes are correctly classified. The statistics of correct overall classification have smaller deviations (SD) than the ones of the separate regime classifications. In terms of frequency distributions for $T = 60$, 50% of the replications have regimes correctly classified by more than 74% of periods; 90% of the replications have regimes correctly classified by more than 60% of periods. The latter percentage tends to be larger as T increases. With $T = 200$, 90% of the replications have regimes correctly classified by more than 67% of periods.⁶ As T increases, the distributions of correct classifications have similar means but variances are reduced.

Finally, results from SPML approaches are reported in Tables 3.1 and 3.2. Except for the estimates of the β 's, the performance of the SPML estimates is poor. The estimates of λ s are severely biased downward and the estimates of p s are biased upward. The GSPML estimates improve upon the SPML estimates but their biases are still large. Large simulation runs to a thousand do not help much. They have also serious numerical problems. More than one third of the 400 replications do not converge or converge to boundaries of parameters with zero variances.⁷ The time costs of both the SPML estimates are also larger than the time costs of corresponding SLSP estimates with the same values of T and S .⁸

⁵Results from other simulated likelihood approaches which simulate latent states randomly or based on the Markov transition probabilities are rather poor. Biases were much greater under the random selection approach. The biases observed for the randomly selected regime approach were mitigated somewhat when Markov chain probabilities were used to simulate regime paths. On the other hand, the special approach which simulates only the continuous latent variables introduced in Lee (1994) provide slightly better results. These results are reported in Lee (1995).

⁶As the underlying regime probabilities p_1 and p_2 are both 0.5, random guesses would have a 50% correct classification rate.

⁷Estimates with zero variances are excluded in the reported results.

⁸The simulation of latent states in a SPML approach is based on the Markov transition probabilities (and exogenous variables). Even though the approach of simulating latent states conditional on current and past sample observations provides superior simulated likelihood estimates, such a simulation procedure can not be used for a SPML approach. A pseudo-likelihood approach is, in essential, searching for a nonlinear regression function for the observed dependent variable conditional on exogenous variables. A valid nonlinear regression function will not contain the dependent variable as an regressor. There are also technical difficulties to include observed lagged dependent variables as regressors in a SPML approach for the estimation of latent dynamic models (Laroque and Salanie 1993, p.S123).

Table 1.1

SMLE with Simulated Regime Paths Based on Conditional Prob.(SLSP)

T	S		Mean	SD		Mean	SD	# conv	ln(lk)
60	50	β_1	-1.0272	.2363	β_2	1.0202	.2598	384	-99.02
		λ_1	.3447	.2822	λ_2	.3458	.2628		
		σ_1	.9342	.2485	σ_2	.9552	.2829		
		p_{12}	.4036	.2049	p_{21}	.4128	.1892		
60	100	β_1	-1.0231	.2306	β_2	1.0211	.2757	384	-99.11
		λ_1	.3550	.2721	λ_2	.3380	.2669		
		σ_1	.9277	.2294	σ_2	.9595	.2891		
		p_{12}	.3919	.1944	p_{21}	.4044	.1806		
60	500	β_1	-1.0185	.2339	β_2	1.0235	.2594	385	-99.07
		λ_1	.3674	.2828	λ_2	.3406	.2812		
		σ_1	.9186	.2332	σ_2	.9431	.2707		
		p_{12}	.3978	.2011	p_{21}	.4077	.1867		
60	1000	β_1	-1.0236	.2413	β_2	1.0172	.2752	386	-99.23
		λ_1	.3611	.2749	λ_2	.3452	.2773		
		σ_1	.9159	.2313	σ_2	.9449	.2814		
		p_{12}	.3932	.2011	p_{21}	.4030	.1889		
100	50	β_1	-1.0257	.1778	β_2	1.0215	.1844	396	-169.14
		λ_1	.4041	.2079	λ_2	.3895	.1982		
		σ_1	.9843	.1961	σ_2	.9948	.2142		
		p_{12}	.3788	.1583	p_{21}	.3777	.1384		
100	100	β_1	-1.0241	.1818	β_2	1.0221	.1835	397	-168.91
		λ_1	.4137	.2000	λ_2	.4056	.1913		
		σ_1	.9857	.1904	σ_2	.9791	.2064		
		p_{12}	.3751	.1540	p_{21}	.3863	.1452		
100	500	β_1	-1.0258	.1782	β_2	1.0226	.1785	399	-168.80
		λ_1	.4285	.2068	λ_2	.4207	.1869		
		σ_1	.9753	.1932	σ_2	.9687	.2013		
		p_{12}	.3656	.1552	p_{21}	.3709	.1417		
100	1000	β_1	-1.0223	.1709	β_2	1.0217	.1786	396	-168.68
		λ_1	.4301	.1958	λ_2	.4418	.1830		
		σ_1	.9726	.1869	σ_2	.9597	.1966		
		p_{12}	.3577	.1503	p_{21}	.3715	.1460		

Notes: CPU time per one replication

SLSP: T=60, S=100, 29.55 seconds; T=100, S=100, 50.16 seconds.

Table 1.2

SMLE with Simulated Regime Paths Based on Conditional Prob.(SLSP)

T	S		Mean	SD		Mean	SD	# conv	ln(lk)
200	50	β_1	-1.0179	.1236	β_2	.9997	.1225	400	-343.59
		λ_1	.4242	.1199	λ_2	.4155	.1154		
		σ_1	1.0213	.1350	σ_2	1.0395	.1422		
		p_{12}	.3796	.1033	p_{21}	.3687	.0952		
200	100	β_1	-1.0207	.1198	β_2	1.0059	.1252	400	-343.34
		λ_1	.4210	.1202	λ_2	.4229	.1134		
		σ_1	1.0200	.1322	σ_2	1.0363	.1481		
		p_{12}	.3752	.0989	p_{21}	.3648	.0955		
200	500	β_1	-1.0189	.1179	β_2	1.0024	.1204	400	-342.57
		λ_1	.4399	.1120	λ_2	.4374	.1020		
		σ_1	1.0085	.1295	σ_2	1.0204	.1331		
		p_{12}	.3703	.1006	p_{21}	.3580	.0887		
200	1000	β_1	-1.0193	.1176	β_2	1.0046	.1224	400	-342.40
		λ_1	.4414	.1194	λ_2	.4382	.1084		
		σ_1	1.0005	.1266	σ_2	1.0214	.1351		
		p_{12}	.3637	.1021	p_{21}	.3555	.0899		
200	2000	β_1	-1.0186	.1182	β_2	1.0042	.1160	400	-342.21
		λ_1	.4498	.1138	λ_2	.4465	.1106		
		σ_1	1.0010	.1242	σ_2	1.0103	.1332		
		p_{12}	.3644	.1004	p_{21}	.3498	.0873		

Notes: CPU time per one replication

SLSP: T=200, S=100, 93.21 seconds; T=200, S=1000, 973.15 seconds.

Table 2

Regime Classification - based on Simulated Paths with Conditional Prob.

T	S	Mean	SD	10%tile	25%tile	median	75%tile	90%tile
First regime								
60	50	.7133	.1608	.4800	.6036	.7419	.8333	.9032
60	100	.7222	.1629	.5000	.6071	.7433	.8430	.9211
60	500	.7300	.1654	.5000	.6216	.7576	.8571	.9333
60	1000	.7327	.1672	.5161	.6389	.7581	.8571	.9355
100	50	.7198	.1315	.5625	.6334	.7363	.8142	.8772
100	100	.7284	.1282	.5581	.6481	.7442	.8197	.8824
100	500	.7322	.1357	.5581	.6596	.7500	.8246	.8841
100	1000	.7444	.1315	.5714	.6699	.7558	.8443	.9000
200	50	.7095	.0848	.6009	.6531	.7097	.7688	.8086
200	100	.7119	.0872	.5974	.6547	.7245	.7684	.8166
200	500	.7218	.0871	.6211	.6667	.7297	.7756	.8295
200	1000	.7257	.0835	.6231	.6733	.7304	.7789	.8319
Second regime								
60	50	.7108	.1618	.5000	.6000	.7361	.8450	.8974
60	100	.7200	.1661	.5000	.6207	.7471	.8529	.9048
60	500	.7206	.1689	.4839	.6154	.7500	.8571	.9130
60	1000	.7246	.1724	.4615	.6207	.7586	.8649	.9143
100	50	.7418	.1309	.5357	.6458	.7375	.8056	.8679
100	100	.7148	.1391	.5366	.6486	.7381	.8125	.8667
100	500	.7348	.1431	.5500	.6538	.7555	.8382	.9000
100	1000	.7307	.1424	.5319	.6667	.7556	.8275	.8889
200	50	.7284	.0819	.6274	.6811	.7314	.7838	.8235
200	100	.7335	.0855	.6220	.6848	.7420	.7915	.8359
200	500	.7416	.0814	.6296	.6945	.7489	.8000	.8404
200	1000	.7406	.0821	.6405	.6923	.7500	.7974	.8359
overall classification								
60	50	.7202	.0853	.6000	.6667	.7167	.7833	.8333
60	100	.7307	.0847	.6167	.6833	.7333	.7833	.8333
60	500	.7350	.0847	.6333	.6667	.7500	.8000	.8333
60	1000	.7379	.0844	.6333	.6833	.7500	.8000	.8333
100	50	.7238	.0617	.6500	.6800	.7250	.7600	.8000
100	100	.7292	.0609	.6500	.6900	.7300	.7700	.8100
100	500	.7410	.0585	.6700	.7000	.7400	.7800	.8100
100	1000	.7451	.0615	.6700	.7100	.7400	.7900	.8200
200	50	.7211	.0417	.6700	.6900	.7200	.7500	.7750
200	100	.7254	.0426	.6750	.6950	.7250	.7550	.7800
200	500	.7342	.0439	.6800	.7050	.7300	.7650	.7900
200	1000	.7357	.0425	.6800	.7050	.7350	.7650	.7850

Table 3.1

Simulated Pseudo MLE

T	S		Mean	SD		Mean	SD	# conv
Simulated Pseudo MLE (SPML)								
60	100	β_1	-1.0289	.3141	β_2	1.0289	.3340	231
		λ_1	.1784	.3737	λ_2	.0994	.3201	
		σ_1	.9161	.5018	σ_2	.8890	.5145	
		p_{12}	.5586	.2300	p_{21}	.5470	.2080	
60	200	β_1	-1.0335	.3289	β_2	.9997	.3498	222
		λ_1	.1875	.3436	λ_2	.1294	.3374	
		σ_1	.9117	.4902	σ_2	.8826	.5930	
		p_{12}	.6001	.2266	p_{21}	.5729	.1953	
60	500	β_1	-1.0262	.3142	β_2	1.0484	.3540	239
		λ_1	.1519	.3543	λ_2	.1439	.3259	
		σ_1	.8405	.4914	σ_2	.9031	.4764	
		p_{12}	.5603	.2336	p_{21}	.5819	.2089	
60	1000	β_1	-1.0447	.3241	β_2	1.0418	.3402	253
		λ_1	.1814	.3341	λ_2	.1707	.3357	
		σ_1	.8667	.4665	σ_2	.8606	.5228	
		p_{12}	.5628	.2268	p_{21}	.5710	.2043	
100	100	β_1	-1.0226	.2457	β_2	1.0282	.2647	254
		λ_1	.1793	.3629	λ_2	.2142	.2754	
		σ_1	.9451	.4734	σ_2	.9170	.4365	
		p_{12}	.5662	.2065	p_{21}	.5522	.1956	
100	200	β_1	-1.0199	.2361	β_2	1.0393	.2472	247
		λ_1	.1732	.3652	λ_2	.1961	.3025	
		σ_1	.9125	.4267	σ_2	.9471	.4967	
		p_{12}	.5406	.2030	p_{21}	.5569	.1801	
100	500	β_1	-1.0257	.2396	β_2	1.0292	.2549	265
		λ_1	.2149	.3144	λ_2	.2162	.2765	
		σ_1	.8885	.4653	σ_2	.9527	.4558	
		p_{12}	.5607	.2051	p_{21}	.5632	.1825	
100	1000	β_1	-1.0158	.2400	β_2	1.0398	.2494	282
		λ_1	.2070	.3461	λ_2	.2106	.2972	
		σ_1	.9079	.4923	σ_2	.8924	.4575	
		p_{12}	.5280	.1903	p_{21}	.5556	.1691	

Notes: CPU time per one replication

SPML: T=100, S=500, 421.63 seconds; T=60, S=1000, 539.92 seconds.

Table 3.2

Simulated Pseudo MLE

T	S		Mean	SD		Mean	SD	# conv
Generalized Simulated Pseudo MLE (GSPML)								
60	100	β_1	-1.0011	.3005	β_2	1.0106	.3324	232
		λ_1	.2961	.3360	λ_2	.2515	.3156	
		σ_1	.8943	.5112	σ_2	.8626	.4722	
		p_{12}	.4930	.2134	p_{21}	.4971	.2027	
60	200	β_1	-1.0415	.3308	β_2	1.0131	.2916	238
		λ_1	.2786	.3209	λ_2	.2581	.2866	
		σ_1	.9058	.4372	σ_2	.8682	.4771	
		p_{12}	.5049	.2165	p_{21}	.4886	.1720	
60	500	β_1	-1.0267	.2910	β_2	1.0459	.3162	243
		λ_1	.2944	.3130	λ_2	.2907	.2874	
		σ_1	.8548	.4689	σ_2	.8547	.4907	
		p_{12}	.4730	.1939	p_{21}	.5103	.2000	
60	1000	β_1	-1.0203	.3001	β_2	1.0378	.3197	238
		λ_1	.2986	.2831	λ_2	.2932	.2755	
		σ_1	.8619	.4842	σ_2	.8733	.5255	
		p_{12}	.4761	.2008	p_{21}	.5040	.1908	
100	100	β_1	-1.0428	.2446	β_2	1.0093	.2545	238
		λ_1	.3580	.2592	λ_2	.3188	.2349	
		σ_1	.9822	.4265	σ_2	.8584	.4216	
		p_{12}	.5032	.1968	p_{21}	.4717	.1630	
100	200	β_1	-1.0128	.2178	β_2	1.0315	.2296	251
		λ_1	.3260	.2790	λ_2	.3387	.2452	
		σ_1	.8885	.4296	σ_2	.9143	.4892	
		p_{12}	.4576	.1752	p_{21}	.4936	.1743	
100	500	β_1	-1.0255	.2183	β_2	1.0256	.2374	271
		λ_1	.3677	.2527	λ_2	.3442	.2311	
		σ_1	.8926	.4264	σ_2	.8654	.4395	
		p_{12}	.4565	.1700	p_{21}	.4712	.1675	
100	1000	β_1	-1.0058	.2158	β_2	1.0418	.2380	275
		λ_1	.3562	.2565	λ_2	.3536	.2347	
		σ_1	.8673	.4379	σ_2	.9298	.4158	
		p_{12}	.4439	.1753	p_{21}	.4585	.1471	

Notes: CPU time per one replication

GSPML: T=60, S=200, 157.84 seconds; T=100, S=200, 263.81 seconds; T=100, S=1000, 1387.50 seconds.

6. Monte Carlo Studies On Dynamic Disequilibrium Model

Sample data for the dynamic disequilibrium model are generated from equations:

$$y_{1t}^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^* - y_{t-1}) + \sigma_1\epsilon_{1t}, \quad (16)$$

$$y_{2t}^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^* - y_{t-1}) + \sigma_2\epsilon_{2t}, \quad (17)$$

and

$$y_t = \min(y_{1t}^*, y_{2t}^*), \quad (18)$$

where ϵ_{1t} and ϵ_{2t} are independent of each other and are i.i.d. $N(0,1)$ for all t . This model has six parameters. The true parameters are set to $\beta_1 = -1$, $\lambda_1 = 0.5$, $\sigma_1 = 1$, $\beta_2 = 1$, $\lambda_2 = 0.5$, and $\sigma_2 = 1$.

6.1 Simulation with Simulated Regime Paths based on Conditional Sample Probabilities

The simulated likelihood of the SLSP approach is h_c in (11). The detailed simulation steps for this special model are as follows.

Simulate two sets of uniform random variables u_t and v_t , $t = 1, \dots, T$, on $[0, 1]$, which are fixed and invariant of unknown parameters during estimation.

1. For period 1:

- (a) Compute $R_1 = M_{11} + M_{21}$ where $M_{11} = \frac{1}{\sigma_1}\phi\left(\frac{y_1 - x_{11}\beta_1}{\sigma_1}\right) \cdot \Phi\left(-\frac{y_1 - x_{21}\beta_2}{\sigma_2}\right)$, and $M_{21} = \frac{1}{\sigma_2}\phi\left(\frac{y_1 - x_{21}\beta_2}{\sigma_2}\right) \cdot \Phi\left(-\frac{y_1 - x_{11}\beta_1}{\sigma_1}\right)$.
- (b) Simulate regime indicator I_1 such that $I_1 = 1$ with the probability $M_{11}/(M_{11} + M_{21})$ using v_1 .
- (c) Simulate $y_1^* = x_{21}\beta_2 - \sigma_2\Phi^{-1}\left[u_1\Phi\left(-\frac{y_1 - x_{21}\beta_2}{\sigma_2}\right)\right]$ if $I_1 = 1$, but simulate $y_1^* = x_{11}\beta_1 - \sigma_1\Phi^{-1}\left[u_1\Phi\left(-\frac{y_1 - x_{11}\beta_1}{\sigma_1}\right)\right]$ if $I_1 = 2$.

2. For period t :

The I_{t-1} and y_{t-1}^* have been simulated in the previous steps. R_{t-1} has also been evaluated. Define $y_{1,t-1}^{(p)}$ and $y_{2,t-1}^{(p)}$ such that $y_{1,t-1}^{(p)} = y_{t-1}$ if $I_{t-1} = 1$; y_{t-1}^* , otherwise, and $y_{2,t-1}^{(p)} = y_{t-1}^*$ if $I_{t-1} = 1$; y_{t-1} , otherwise.

- (a) Compute $R_t = (M_{1t} + M_{2t})R_{t-1}$ where

$$M_{1t} = \frac{1}{\sigma_1}\phi\left(\frac{y_t - x_{1t}\beta_1 - \lambda_1(y_{1,t-1}^{(p)} - y_{t-1})}{\sigma_1}\right) \cdot \Phi\left(-\frac{y_t - x_{2t}\beta_2 - \lambda_2(y_{2,t-1}^{(p)} - y_{t-1})}{\sigma_2}\right),$$

and $M_{2t} = \frac{1}{\sigma_2} \phi \left(\frac{y_t - x_{2t}\beta_2 - \lambda_2(y_{2,t-1}^{(p)} - y_{t-1})}{\sigma_2} \right) \cdot \Phi \left(-\frac{y_t - x_{1t}\beta_1 - \lambda_1(y_{1,t-1}^{(p)} - y_{t-1})}{\sigma_1} \right)$. Simulate I_t such that $I_t = 1$ with probability $M_{1t}/(M_{1t} + M_{2t})$ using v_t .

(b) Simulate y_t^* as follows:

- i. If $I_t = 1$, simulate a truncated normal variable $\epsilon_t = - \left[u_t \Phi \left(-\frac{y_t - x_{2t}\beta_2 - \lambda_2(y_{2,t-1}^{(p)} - y_{t-1})}{\sigma_2} \right) \right]$ and simulate $y_t^* = x_{2t}\beta_2 + \lambda_2(y_{2,t-1}^{(p)} - y_{t-1}) + \sigma_2\epsilon_t$.
- ii. If $I_t = 2$, simulate $\epsilon_t = -\Phi^{-1} \left[u_t \Phi \left(-\frac{y_t - x_{1t}\beta_1 - \lambda_1(y_{1,t-1}^{(p)} - y_{t-1})}{\sigma_1} \right) \right]$ and $y_t^* = x_{1t}\beta_1 + \lambda_1(y_{1,t-1}^{(p)} - y_{t-1}) + \sigma_1\epsilon_t$.

With S simulation runs, denote $R_{t,j}$ the simulated value of R_t for the j th simulation run. The simulated likelihood function is $L_c = \ln \left(\frac{1}{S} \sum_{j=1}^S R_{t,j} \right)$.

6.2 Simulated Pseudo Maximum Likelihood Approach

The SPML approaches will simulate the model as in (16-18). At t , given that $y_{i,t-1}^*$ for $i = 1, 2$ and y_{t-1}^* have been simulated, simulate $y_{it}^* = x_{it}\beta_i + \lambda_i(y_{i,t-1}^* - y_{t-1}^*) + \sigma_i\epsilon_{it}$ for $i = 1$ and 2 . Set $y_t^* = \min\{y_{1t}^*, y_{2t}^*\}$. With S simulation runs, let $E_t(\theta) = \frac{1}{S} \sum_{s=1}^S y_t^{*(s)}$, $V_t(\theta) = \frac{1}{S} \sum_{s=1}^S (y_t^{*(s)} - E_t(\theta))^2$, and $V_{Gt}(\theta) = \frac{1}{S} \sum_{s=1}^S (y_t^{*(s)} - E_t(\theta), y_{t-1}^{*(s)} - E_{t-1}(\theta))' (y_t^{*(s)} - E_t(\theta), y_{t-1}^{*(s)} - E_{t-1}(\theta))$. The SPML approach uses the simulated pseudo-likelihood $\frac{1}{2T} \sum_{t=1}^T [(y_t - E_t(\theta))^2 / V_t(\theta) + \ln V_t(\theta)]$. The GSPML approach uses the pseudo-likelihood $\frac{1}{2(T-1)} \sum_{t=2}^T [(y_t - E_t(\theta), y_{t-1} - E_{t-1}(\theta)) V_{Gt}^{-1}(\theta) (y_t - E_t(\theta), y_{t-1} - E_{t-1}(\theta)) + \ln \det V_{Gt}(\theta)]$.

6.3 Monte Carlo Results

As this model has a smaller number of parameters, sample sizes T for our experiments vary from 30 to 100. The number of simulation runs S varies from 50 to 200. For each case, the number of replications is 400.

Table 4 reports results for the SLSP approach. For $T = 30$, the optimization subroutine fails to converge with 8 or 9 per 400 replications. For $T = 60$, the number of failure is only 1 when $m = 50$. With $T = 100$, all replications converge. In all these cases, estimates of β_1 are nearly unbiased. There are some downward biases in the estimates of the remaining parameters. The magnitudes of these biases decrease as T increases. For $T = 60$ or 100 , the biases are quite small. The estimates are insensitive to S . The insensitivity shows up also in the reported likelihood values $\ln(lk)$. Simulation runs with $S = 50$ are, in general, good enough. In terms of CPU time cost, it takes about 98.99 seconds on average to complete one replication for the case with $T = 60$ and $S = 200$. Table 5 report results on regime classification.

Classification accuracy is insensitive to S . On average (Mean), about 77% of the regimes are correctly classified. In terms of frequency distributions, 50% of the replications have correctly classified regimes by more than 77% of periods. For $T = 60$ or 100, 90% of the replications have correctly classified regimes by more than 70% of periods. The latter percentage is about 63% for $T = 30$.

Tables 6.1 and 6.2 report SPML estimates. The SPML estimates are also not sensitive to S . Results with thousand simulation runs are similar to results with a few hundred runs. The biases of the SPML estimates are similar to the ones of the SLSP estimates except those of the estimates of σ_1 . The SPML estimates of σ_1 have larger downward biases. The GSPML estimates improve upon the SPML estimates and are comparable with the SLSP estimates in term of biases. The main difference is that both the SPML and GSPML have larger variances than the SLSP estimates. On the other hand, the CPU time cost of the SPML approach for this model is inexpensive. The time cost of the GSPML approach is much more expensive than the SPML approach but is still relatively less than the SLSP approach. This is so because the SLSP approach involves the computation of the normal distribution and its inverse function while both the SPML and GSPML do not. For the disequilibrium model, these pseudo-likelihood approaches are numerically stable except for the case with small sample size. For $T = 30$, slightly more than 5% of replications do not converge. In comparison, the simulated pseudo-likelihood approaches behave much better for estimation of the disequilibrium model than the switching regression model.

Table 4
Dynamic Disequilibrium Model
SMLE with Simulated Regime Paths Based on Conditional Prob.(SLSP)

T	S		Mean	SD		Mean	SD	# conv	ln(lk)
30	50	β_1	-.9991	.2851	β_2	.9525	.2869	391	-37.54
		λ_1	.4466	.2777	λ_2	.4236	.2816		
		σ_1	.8724	.2815	σ_2	.9621	.2536		
30	100	β_1	-1.0046	.2918	β_2	.9502	.2843	393	-37.50
		λ_1	.4516	.2826	λ_2	.4276	.2793		
		σ_1	.8581	.2733	σ_2	.9601	.2469		
30	200	β_1	-1.0028	.2929	β_2	.9654	.2931	392	-37.53
		λ_1	.4494	.2711	λ_2	.4249	.2762		
		σ_1	.8557	.2710	σ_2	.9655	.2520		
60	50	β_1	-1.0073	.1850	β_2	.9737	.1807	399	-78.88
		λ_1	.4952	.1758	λ_2	.4727	.1814		
		σ_1	.9439	.1679	σ_2	.9917	.1608		
60	100	β_1	-1.0068	.1836	β_2	.9788	.1777	400	-78.85
		λ_1	.4888	.1754	λ_2	.4780	.1650		
		σ_1	.9434	.1744	σ_2	.9922	.1688		
60	200	β_1	-1.0078	.1812	β_2	.9746	.1752	400	-78.85
		λ_1	.4922	.1685	λ_2	.4820	.1638		
		σ_1	.9405	.1708	σ_2	.9890	.1599		
100	50	β_1	-1.0003	.1365	β_2	.9610	.1331	400	-133.45
		λ_1	.5022	.1177	λ_2	.4781	.1219		
		σ_1	.9736	.1376	σ_2	1.0112	.1387		
100	100	β_1	-1.0037	.1375	β_2	.9577	.1342	400	-133.29
		λ_1	.5066	.1146	λ_2	.4847	.1272		
		σ_1	.9686	.1406	σ_2	1.0083	.1417		
100	200	β_1	-1.0038	.1309	β_2	.9661	.1330	400	-133.28
		λ_1	.5042	.1117	λ_2	.4822	.1209		
		σ_1	.9670	.1364	σ_2	1.0002	.1360		

Notes: CPU time per one replication
SLSP: T=60, S=200, 98.99 seconds.

Table 5
Dynamic Disequilibrium Model
Regime Classification - based on Simulated Regimes with Conditional Prob.

T	S	Mean	SD	10%tile	25%tile	median	75%tile	90%tile
regime 1								
30	50	.7547	.1721	.5333	.6667	.7857	.8824	.9412
30	100	.7516	.1731	.5333	.6667	.7692	.8824	.9474
30	200	.7591	.1697	.5455	.6743	.7857	.8750	.9444
60	50	.7806	.1087	.6452	.7097	.7931	.8611	.9062
60	100	.7773	.1099	.6376	.7143	.7879	.8571	.9091
60	200	.7888	.1104	.6405	.7222	.8065	.8750	.9200
100	50	.7581	.0894	.6373	.7034	.7600	.8207	.8717
100	100	.7641	.0876	.6523	.7111	.7708	.8240	.8794
100	200	.7692	.0865	.6667	.7113	.7708	.8300	.8864
regime 2								
30	50	.7639	.1656	.5385	.6667	.8000	.8889	.9474
30	100	.7652	.1652	.5455	.6875	.7895	.8889	.9444
30	200	.7667	.1601	.5385	.6667	.8000	.8824	.9444
60	50	.7764	.1120	.6429	.7097	.7812	.8571	.9130
60	100	.7768	.1085	.6452	.7165	.7868	.8524	.9105
60	200	.7779	.1092	.6364	.7143	.7879	.8621	.9091
100	50	.7726	.0816	.6643	.7216	.7788	.8304	.8714
100	100	.7750	.0797	.6667	.7176	.7835	.8333	.8692
100	200	.7825	.0795	.6809	.7253	.7864	.8438	.8864
overall classification								
30	50	.7662	.0920	.6333	.7000	.7667	.8333	.8667
30	100	.7659	.0868	.6333	.7000	.7667	.8333	.8667
30	200	.7696	.0872	.6667	.7000	.7667	.8333	.8667
60	50	.7829	.0582	.7000	.7500	.7833	.8333	.8500
60	100	.7802	.0592	.7000	.7500	.7833	.8167	.8500
60	200	.7880	.0576	.7167	.7500	.7833	.8333	.8667
100	50	.7669	.0495	.7000	.7300	.7700	.8000	.8300
100	100	.7709	.0527	.7000	.7400	.7700	.8000	.8400
100	200	.7777	.0521	.7100	.7400	.7800	.8150	.8400

Table 6.1

SPMLE of Dynamic Disequilibrium Model

True parameters: $\beta_d = -1$, $\lambda_d = 0.5$, $\sigma_d = 1$ and $\beta_s = 1$, $\lambda_s = 0.5$, $\sigma_s = 1$

T	S		Mean	SD		Mean	SD	SPML	#conv
30	30	β_1	-.9836	.3666	β_2	.9811	.3112	-10.69	372
		λ_1	.4171	.3579	λ_2	.4180	.3143		
		σ_1	.8830	.3906	σ_2	.9453	.3407		
30	50	β_1	-.9985	.4152	β_2	.9648	.3210	-10.71	373
		λ_1	.4175	.3333	λ_2	.4346	.2691		
		σ_1	.8550	.3634	σ_2	.9637	.3340		
30	100	β_1	-1.0046	.3285	β_2	.9856	.3226	-10.33	368
		λ_1	.4313	.3282	λ_2	.4357	.2865		
		σ_1	.8394	.3877	σ_2	.9521	.3292		
30	500	β_1	-1.0098	.3703	β_2	.9752	.3082	-10.43	382
		λ_1	.4487	.3072	λ_2	.4293	.2897		
		σ_1	.8117	.3543	σ_2	.9372	.3339		
30	1000	β_1	-1.0256	.3646	β_2	.9805	.3044	-10.23	379
		λ_1	.4384	.3206	λ_2	.4357	.2885		
		σ_1	.8011	.3776	σ_2	.9321	.3337		
30	3000	β_1	-1.0198	.3456	β_2	.9846	.3212	-10.30	372
		λ_1	.4343	.3169	λ_2	.4364	.2720		
		σ_1	.8083	.3551	σ_2	.9315	.3399		
100	30	β_1	-1.0112	.1436	β_2	.9774	.1507	-46.15	399
		λ_1	.5032	.1467	λ_2	.4831	.1751		
		σ_1	.9697	.1631	σ_2	1.0393	.1701		
100	50	β_1	-1.0077	.1390	β_2	.9891	.1441	-45.11	400
		λ_1	.4994	.1289	λ_2	.4864	.1298		
		σ_1	.9589	.1547	σ_2	1.0291	.1672		
100	100	β_1	-1.0108	.1374	β_2	.9860	.1408	-44.06	400
		λ_1	.4972	.1266	λ_2	.4809	.1428		
		σ_1	.9496	.1495	σ_2	1.0148	.1709		
100	500	β_1	-1.0113	.1355	β_2	.9883	.1375	-43.42	400
		λ_1	.4973	.1224	λ_2	.4884	.1250		
		σ_1	.9419	.1461	σ_2	1.0067	.1597		
100	1000	β_1	-1.0098	.1359	β_2	.9877	.1396	-43.33	400
		λ_1	.4981	.1238	λ_2	.4871	.1272		
		σ_1	.9417	.1494	σ_2	1.0042	.1677		
100	3000	β_1	-1.0100	.1354	β_2	.9887	.1385	-43.34	400
		λ_1	.4966	.1221	λ_2	.4874	.1239		
		σ_1	.9412	.1459	σ_2	1.0076	.1578		

Notes: CPU time per one replication

SPML: T=30, S=100, 3.32 seconds; T=30, S=500, 16.68 seconds; T=100, S=100, 9.35 seconds; T=100, S=500, 45.28 seconds.

Table 6.2

SPMLE of Dynamic Disequilibrium Model

True parameters: $\beta_d = -1$, $\lambda_d = 0.5$, $\sigma_d = 1$ and $\beta_s = 1$, $\lambda_s = 0.5$, $\sigma_s = 1$

T	S		Mean	SD		Mean	SD	SPML	#conv
Generalized Simulated Pseudo MLE (GSPML)									
60	30	β_1	-1.0144	.2077	β_2	.9886	.2075	-26.41	396
		λ_1	.4807	.2167	λ_2	.4589	.2219		
		σ_1	.9246	.2271	σ_2	1.0328	.2317		
60	50	β_1	-1.0152	.2014	β_2	.9930	.2091	-25.68	400
		λ_1	.4933	.1939	λ_2	.4694	.2020		
		σ_1	.9241	.2351	σ_2	1.0125	.2179		
60	100	β_1	-1.0146	.1896	β_2	.9975	.2010	-25.14	399
		λ_1	.4837	.1829	λ_2	.4719	.1796		
		σ_1	.9065	.2199	σ_2	1.0089	.2130		
60	500	β_1	-1.0130	.1952	β_2	1.0003	.1949	-24.87	399
		λ_1	.4810	.1799	λ_2	.4837	.1717		
		σ_1	.9158	.2162	σ_2	.9884	.2248		
60	1000	β_1	-1.0128	.1935	β_2	1.0014	.1935	-24.77	397
		λ_1	.4844	.1784	λ_2	.4821	.1756		
		σ_1	.9097	.2273	σ_2	.9840	.2220		
60	3000	β_1	-1.0128	.1926	β_2	1.0028	.1935	-24.74	399
		λ_1	.4826	.1805	λ_2	.4849	.1720		
		σ_1	.9094	.2263	σ_2	.9853	.2184		
Generalized Simulated Pseudo MLE (GSPML)									
60	50	β_1	-1.0164	.1985	β_2	.9963	.2075	-50.41	397
		λ_1	.4956	.1874	λ_2	.4808	.1819		
		σ_1	.9440	.2018	σ_2	1.0191	.2176		
60	100	β_1	-1.0135	.1896	β_2	.9986	.1941	-49.11	398
		λ_1	.4834	.1837	λ_2	.4796	.1692		
		σ_1	.9234	.1934	σ_2	1.0082	.1988		
60	500	β_1	-1.0122	.1891	β_2	1.0009	.1913	-48.17	399
		λ_1	.4834	.1749	λ_2	.4874	.1646		
		σ_1	.9225	.2029	σ_2	.9899	.2109		

Notes: CPU time per one replication

SPML: T=60, S=500, 28.11 seconds.

GSPML: T=60, S=500, 177.52 seconds;

7. Concluding Remarks

Evidence from the Monte Carlo experiment reveals that how discrete states are simulated is important for likelihood simulation. The approach by simulating regime paths based on regime probabilities conditional on current and past sample observations (Hendry and Richard 1992) is superior to other approaches which do not use current sample information. The other approaches suffer from the difficulty of selecting or simulating the proper regime paths in the very large space of regime paths. Comparing the simulated likelihood approach and the simulated pseudo-likelihood approaches, the simulated maximum likelihood estimates are numerically stable and are relatively efficient even in sample with moderate size. For the dynamic switching regression model, the simulated pseudo-likelihood approaches are numerically unstable and their estimates are poor. They can, however, be valuable for the estimation of dynamic disequilibrium models.

This article is in search of computationally tractable and reasonably good simulation approaches for the estimation of general dynamic switching models. While the simulated likelihood approach can be applied to the estimation of such general models, there are statistical issues that have neither been addressed in the article by Hendry and Richard nor in this article. One of the difficulties that has yet to be overcome is to provide a satisfactory procedure for the estimation of standard errors of parameter estimates. As the simulated likelihood approach here has to simulate latent discrete state variables in addition to continuous latent variables, the simulated likelihood function is not smooth. For the simulated pseudo-likelihood approach, Laroque and Salanie (1993) reported the difficulty of constructing adequate estimates of standard errors of their parameter estimators since simulated pseudo-likelihood functions are not differentiable. For the simulated likelihood approach, even though in principle, the information matrix can be used to derive an estimate of the variance of the maximum likelihood estimate, it does not take into account the additional finite sample error due to simulation.⁹ Direct derivatives of the simulated likelihood do not exist as the simulated likelihood function is not differentiable.¹⁰ Future research on this and related issues on statistical inference is important and remains open.

⁹We have tried to simulate the hessian matrix of the exact likelihood function but the resulted variance estimates are quite inadequate. They can underestimate the correct (empirical) standard deviations by more than fifty percent. A similar magnitude of underestimation has been reported for the SPMLE in Laroque and Salanie (1993).

¹⁰For simulation estimation of discrete choice models, one gets around this problem by simulating probabilities even though choice events are discrete. This is possible, because in a discrete choice model simulation of latent continuous variables by conditioning on observed discrete choices does not require the simulation of latent states. The simulated likelihood approach in Lee (1994) provides a smooth simulated likelihood because it simulates only continuous latent variables. For that approach, the hessian matrix of the simulated likelihood is available and is useful for the construction of estimates for the variance of that simulated likelihood estimator (see Lee 1994 for those results).

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