Probing Superconducting Phase Fluctuations from the Current Noise Spectrum of Pseudogapped Metal-Superconductor Tunnel Junctions

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We study the current noise spectra of a tunnel junction of a metal with strong pairing phase fluctuation and a superconductor. It is shown that there is a characteristic peak in the noise spectrum at the intrinsic Josephson frequency $\omega_J = 2eV$ when ω_J is smaller than the pairing gap but larger than the pairing scattering rate. In the presence of an ac voltage, the tunneling current noise shows a series of characteristic peaks with increasing dc voltage. Experimental observation of these peaks will give direct evidence of the pair fluctuation in the normal state of high- T_c superconductors and the pair decay rate can be estimated from the half width of the peaks.

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Direct measurements of the pair susceptibility in the normal state of a superconductor are important for understanding the pseudogap phenomenon in high- T_c superconductors. For conventional superconductors, the pairing fluctuation exists only in a very narrow temperature range above T_c . However, in the underdoped high- T_c materials, it is believed that the pair fluctuation is important in a substantial temperature range in the normal state [1]. This fluctuation is also important in superconductor thin films, where long ranged phase coherence is strongly suppressed [2]. The pseudogap effect was often interpreted as a precursor to the high- T_c superconductivity [1,3]. It was observed in many different experiments [4–10]. However, what was really found in experiments is the reduction of the low-lying density of states associated with the opening of an energy gap, which may or may not be a pairing gap, and a direct observation of the pair phase fluctuation is still absent.

The pair susceptibility in the normal state of an underdoped cuprate can be measured from the linear response of the system to an external pair field, like the measurement of the magnetic susceptibility. Such an external pair field can be introduced, for example, by coupling this underdoped cuprate to an optimally doped cuprate with a higher superconducting transition temperature via a tunneling junction. When the temperature lies between the superconducting transition temperatures of these two materials, the tunneling process will undoubtedly be strongly affected by the pair fluctuation in the normal side [11].

The Josephson effect occurs when both sides of the junction are in the superconducting state. A dc voltage across the junction leads to an oscillating Josephson current at frequency $\omega_J = 2eV$, where V is the applied voltage. If one side of the junction is not in the superconducting phase, the Josephson current is zero on average. However, the Josephson effect does not disappear completely. It manifests in the current noise spectrum and, as

will be shown later, the current noise is greatly enhanced at the Josephson frequency ω_J if the pairing fluctuation is strong. The noise spectrum for an ordinary normal metal-superconductor junction without pairing fluctuation in the normal side has been discussed in Ref. [12].

In this paper, we present a theoretical analysis of the tunneling current noise spectrum of the pseudogapped metal-superconductor junction as discussed above. We assume that the pairing on both sides of the junction has $d_{x^2-y^2}$ -wave symmetry. The extension to other pairing symmetry cases is straightforward. The main results we obtained are as follows: (i) With an applied dc voltage, there is a characteristic Josephson peak in the noise spectrum at the intrinsic Josephson frequency ω_J , although the coherent Josephson current is zero. (ii) When an ac voltage $V_s \cos(\omega_s t)$ is applied in addition to the dc voltage, there are a series of characteristic peaks in the zero frequency noise centered at $\omega_J = n\omega_s$ (n is an integer) with increasing ω_J . (iii) The half width of the peak is proportional to the pair decay rate Γ of the system, but the peak intensity drops very quickly with increasing Γ .

The Hamiltonian of the junction we study is defined by $H = H_S + H_N + H_T$ with

$$H_S = \sum_{p\sigma} \varepsilon_p d_{p\sigma}^\dagger d_{p\sigma} + \sum_p (\Delta \varphi_p d_{p\uparrow} d_{-p\downarrow} + \text{H.c.}), \quad (1)$$

$$H_N = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{kq} [\Psi(q, t) c_{-k+q/2, \uparrow} c_{k+q/2, \downarrow} + \text{H.c.}] + \sum_q \chi_{\text{pair}}^{-1}(q, t, t') \Psi^*(q, t) \Psi(-q, t'), \quad (2)$$

$$H_T = \sum_{kp\sigma} V_{kp} e^{i \int_0^t e V_s \cos(\omega_s t') dt' + i \omega_J t} c_{k\sigma}^\dagger d_{p\sigma} + \text{H.c.}, \quad (3)$$

where $\varphi_p = \cos p_x - \cos p_y$ and $\Psi(x, t) = \Delta_n \exp \times [i\phi(x, t)]$ is the pairing field of the normal lead. H_S is

the Hamiltonian for a d -wave superconductor. H_N is the effective Hamiltonian of a metal with pairing fluctuations [11,13] and χ_{pair} is the pair propagator (or pair susceptibility) in the normal state. In H_T , V_{kp} is the tunneling matrix and can be expanded with the crystal harmonics [11,14]. To the second order approximation, it is given by

$$|V_{pk}|^2 = V_0^2 + V_1^2 \varphi_k \varphi_p. \quad (4)$$

The constant term in $|V_{pk}|^2$ contributes to the single particle tunneling current, but not to the pair tunneling current because of the d -wave pairing symmetry. The integration resulted from the applied ac voltage on the exponent in H_T is difficult to handle analytically. In practical calculations, it is often expanded as a sum of a series of oscillating terms with the Bessel function:

$$e^{i \int_0^t eV_s \cos(\omega_s t') dt'} = \sum_n J_n \left(\frac{eV_s}{\omega_s} \right) e^{in\omega_s t},$$

where $J_n(x)$ is the n th order Bessel function [15].

Let us first consider phenomenologically the effect of pairing phase fluctuation on the current noise. We assume that the phase fluctuation follows a diffusion process and the spatial correlation length is very short. In this case, the correlation function of the phase field is given by

$$\langle e^{i\phi(x,t)} e^{-i\phi(x',0)} \rangle \sim e^{-\Gamma|t|/2} \delta(x - x'), \quad (5)$$

where Γ is the pair fluctuation rate. From this equation, it can be shown that

$$\chi_{\text{pair}}^{\text{Ret}}(q, \omega) = 1/(i\omega + \Gamma/2). \quad (6)$$

The current noise spectrum measures the current fluctuation in the junction [16]. It is defined by the symmetrized current-current function [17]:

$$S(\omega) \equiv \int dt e^{i\omega t} \langle \{I(t) - \langle I(t) \rangle, I(0) - \langle I(0) \rangle\} \rangle, \quad (7)$$

where $I(t)$ is the current across the junction and $\{A, B\} \equiv AB + BA$. In the case where only a dc voltage is applied, the above tunneling Hamiltonian can be treated semiclassically and the current-current correlation function contributed from the pair tunneling is simply given by the thermal average of the phase fields:

$$\begin{aligned} \langle I(t)I(0) \rangle &\sim \int dx dx' \langle \sin[\phi(x, t) + \omega_J t] \sin\phi(x', 0) \rangle \\ &\sim \cos(\omega_J t) e^{-\Gamma|t|/2}. \end{aligned} \quad (8)$$

Substituting this equation into (7), we then obtain the noise spectrum contributed from the pair tunneling,

$$S_{\text{pair}}(\omega) \sim \frac{\Gamma}{(\omega - \omega_J)^2 + \Gamma^2/4} + \frac{\Gamma}{(\omega + \omega_J)^2 + \Gamma^2/4}. \quad (9)$$

This result indicates that the Josephson current across the junction is coherent if the tunneling time is much shorter than the fluctuation time $\tau \sim \Gamma^{-1}$ of virtual cooper pairs.

In the limit $\Gamma \rightarrow 0$, $S(\omega)$ becomes two delta functions centered at $\pm\omega_J$. For finite Γ , these two delta functions are broadened and the half width of the broadened peak is proportional to Γ .

Equation (9) is similar to the result obtained by Martin and Balatsky very recently [18]. However, since both sides of the junction in the experimental setup they suggested are in the normal state, the total pair decay rate is larger than the case discussed here. Since the intensity of the peak compared with the average noise drops sharply with increasing Γ (see Fig. 2 below), it may be difficult to observe the characteristic Josephson fluctuation in the finite frequency conductance measurement suggested in their paper. However, in our paper, the pair decay rate is determined only by the zero frequency noise which is easier to measure.

In an s -wave pairing system, since no quasiparticle excitations exist within the energy gap, the single particle tunneling can be ignored when the noise frequency is lower than the gap. For the d -wave pairing case, the contribution of the single particle tunneling cannot be ignored because of the existence of gap nodes. To find out the contribution from both the quasiparticle and the pair tunneling to the noise, we perform a microscopic calculation of the current-current correlation function using the closed time Green's function technique [19,20]. The tunneling Hamiltonian H_T is treated as a perturbation. Since the normal Josephson effect is absent in the junction, the second order contribution of H_T contains only the quasiparticle tunneling terms. However, the noise caused by the quasiparticle tunneling varies very slowly with frequency. When the noise frequency is smaller than Δ , it adds only a featureless background to the noise spectra.

The most important contribution of H_T to the noise is from the fourth order perturbation, in particular from the virtual pair tunneling process. When the pairing fluctuation in the normal state is strong, a Cooper pair can tunnel from the superconductor to the normal lead and contributes a significant term to the noise at low frequencies. Figure 1 shows the Feynmann diagram for this pair tunneling term in the current-current correlation function. The solid line is the Green's function of electrons. The dashed line is the propagator of the pair field in the normal state.

FIG. 1. The Feynmann diagram for the pair tunneling term, where the solid line represents the electronic Green's function and the dashed line represents the pairing fluctuation.

To the leading order approximation in both the quasiparticle and the pair tunneling channels, we find that the noise spectra is given by

$$S(\omega) \approx N_q(\omega) + N_p(\omega) + N_q(-\omega) + N_p(-\omega), \quad (10)$$

where N_q and N_p are, respectively, the contributions from the quasiparticle and pair tunneling:

$$N_q(\omega) = V_0^2 \sum_n J_n^2(x) \rho_n \int d\nu \rho_d(\nu) \times [n_f(\nu) - n_f(\nu + \omega_n'')] \coth \frac{\beta \omega_n''}{2}, \quad (11)$$

$$N_p(\omega) = V_1^4 C^2 \sum_{n, n_1, n_2} \frac{J_{n_1}(x) J_{n-n_1}(x) J_{n_2}(x) J_{n-n_2}(x)}{\omega_n'^2 + \Gamma^2/4} \coth \frac{\beta \omega_n'}{2} \times \sum_q \{ \omega_n' [\phi_2^2(\omega_n', q) - \phi_1^2(\omega_n', q)] + \Gamma \phi_2(\omega_n') \phi_1(\omega_n', q) \}. \quad (12)$$

In the above equations, $x = eV_s/\omega_s$, $\omega_n' = \omega + \omega_J + n\omega_s$, $\omega_n'' = \omega + eV + n\omega_s$, $C = -\sum_p \Delta \varphi_p^2 / (\varepsilon_p^2 + \Delta^2 \varphi_p^2)$, ρ_d is the density of states of the d -wave superconducting phase, and ρ_n is the density of states of the normal side. In Eq. (12),

$$\phi_1(\nu, q) = \pi \sum_k \varphi_k^2 (1 - n_k - n_{-k+q}) \delta(\nu - \varepsilon_k - \varepsilon_{-k+q}),$$

$$\phi_2(\nu, q) = - \sum_k \varphi_k^2 \frac{1 - n_k - n_{-k+q}}{\nu - \varepsilon_k - \varepsilon_{-k+q}},$$

and $n_k = 1/(1 + e^{\beta \varepsilon_k})$ is the Fermi-Dirac function.

Figure 2 shows $S(\omega)$ normalized by $S(0)$ as a function of ω for the case $\omega_J = 0.4\Delta$ and $\omega_s = 0$. The parameters used are $\Delta_n = \Delta$, $T = 0.2\Delta$, $V_0 = 2V_1 = 2\Delta$, and $\rho_s = 1/10\Delta$. The bandwidth of the normal lead is assumed to be 10Δ . From the figure, it is clear that if the fluctuation rate Γ is sufficiently small, the noise peak centered at $\omega = \pm\omega_J$ is very sharp. The half width of the peak is determined by Γ . When $\omega_J \gg \Gamma$, the thermally excited pair tunneling current will oscillate a few times before it loses the phase coherence. But with increasing Γ the noise peak at ω_J broadens and vanishes eventually.

In the presence of both dc and ac voltages, the pair tunneling is enhanced at the frequency $\omega = \omega_J + n\omega_s$ and the noise spectrum shows a series of local maxima at these frequencies. If the applied microwave frequency and the Josephson frequency satisfy the condition $\omega_J + n\omega_s = 0$, a local maximum will appear at the zero frequency. Figure 3 shows the zero frequency normalized current noise spectrum as a function of ω_J for the case $\omega_s = 0.4\Delta$ (the other parameters are the same as in Fig. 2). There are a series of peaks for $\Gamma = 0.2\Delta$. When Γ is increased, these peaks become broader and weaker.

The above results indicate that the pair fluctuation in the normal lead has strong effects on the noise spectrum of the tunneling current. The broadened peaks at the characteristic frequencies in the noise spectrum shown in Figs. 2 and 3 are unique to the normal-superconductor junction with strong pair fluctuations. Experimentally, it is not easy to measure the finite frequency noise, but when the ac voltage is applied we need only to measure the zero frequency

noise which is quite easy to do. Thus experimental measurements of the current noise allow us to judge how strong the pairing fluctuation is in the pseudogap phase. This is apparently important for a direct test of the superconducting precursor scenario.

To detect the peaks in the noise spectrum, the scattering rate of the virtual Cooper pairs must be less than the interval between these peaks which is the frequency of the electromagnetic wave applied to the junction. However, if the scattering rate is too large when compared with the pairing energy gap, the quasiparticle noise dominates the total noise spectrum and it is difficult to probe the pair tunneling contribution. This indicates that the pairing tunneling noise peaks can be resolved only when the scattering rate of the virtual Cooper pairs in the normal side is less than the pairing energy gap. Since the lifetime of the virtual Cooper pairs becomes divergent at the superconducting transition temperature T_c of the underdoped side, the noise peak can therefore be observed provided that the measurement temperature is close enough to T_c .

FIG. 2. The normalized current noise spectrum as a function of the noise frequency for three different Γ . $\omega_J = 0.4\Delta$.

FIG. 3. The normalized current noise spectrum at zero frequency as a function of the intrinsic Josephson frequency $\omega_J = 2 \text{ eV}$ for three different Γ , where V is the dc voltage applied to the junction. $\omega_s = 0.4\Delta$.

In summary, we have shown that the noise spectrum of the tunneling current between an underdoped cuprate in the normal state and an optimally doped cuprate in the superconducting state is a useful probe of the pairing fluctuation in the pseudogap phase. We have calculated the noise spectrum using the closed path Green's functions and found that the current noise is strongly enhanced at a series of characteristic frequencies determined by the Josephson pair tunneling processes. Notice that the system is out of equilibrium when dc voltage is applied, which is why the closed time Green's function is required. Experimental observations of the noise peaks at these characteristic frequencies will help us to establish the correct picture of the pair fluctuation in the pseudogap phase of high- T_c cuprates.

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