Multifrequency Gap Solitons in Nonlinear Photonic Crystals

Ping Xie and Zhao-Qing Zhang

Department of Physics and Institute of Nano Science and Technology (INST), The Hong Kong University of Science and Technology,
Clear Water Bay, Hong Kong, China

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We predict the existence of multifrequency gap solitons (MFGSs) in both one- and two-dimensional nonlinear photonic crystals. A MFGS is a single intrinsic mode possessing multiple frequencies inside the gap. Its existence is a result of synergic nonlinear coupling among solitons or soliton trains at different frequencies. Its formation can either lower the threshold fields of the respective frequency components or stabilize their excitations. These MFGSs form a new class of stable gap solitons.

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Photonic crystals have been under intensive study both theoretically and experimentally over the past decade [1]. Since a photonic crystal can have a spectral gap in which light propagation is forbidden, it offers the possibility of controlling the flow of photons. To employ a high-technology potential of photonic crystals, it has been turned to nonlinear photonic crystals, where the tunability of the photonic band gap can be realized by changing the light intensity. Based on this idea, Scalora *et al.* [2] and Tran [3] numerically studied the optical switching of a weak probe beam by the use of a strong pump beam with a frequency different from that of the probe. An interesting phenomenon in nonlinear photonic crystals is the existence of nonlinearity-induced self-organized localized states with the frequency in the forbidden gaps, which are called gap solitons. The term gap soliton was first introduced by Chen and Mills in their numerical study of a 1D Kerr nonlinear superlattice [4]. Subsequently, the studies of gap solitons in 1D superlattices have attracted a lot of research interest both theoretically and experimentally [4–8]. Specifically, the formation of a coupled gap soliton by using two orthogonally polarized pulses in a 1D superlattice has been studied theoretically [7] and demonstrated experimentally [8]. Meanwhile, gap solitons in 2D and 3D nonlinear photonic crystals have also attracted considerable attention. John and Akozbek [9] studied gap solitons in 2D and 3D photonic crystals by using the coupled-mode theory, which is valid in small dielectric modulations. By using the numerical Green's function, Mingaleev and Kivshar [10] demonstrated the existence of stable gap solitons in 2D photonic crystals with large dielectric contrast. Recently, we demonstrated the existence of stable gap solitons and soliton trains in 2D photonic crystals by studying the multistability in the transmission coefficient [11]. In all of these studies, only single-frequency gap solitons (SFGSs) have been found.

In this Letter, we predict the existence of multifrequency gap solitons (MFGSs) in both 1D and 2D nonlinear photonic crystals by studying the multistability of the transmission coefficients of finite-sized crystals. Similar to SFGSs found previously [4–11], MFGSs are

also intrinsic self-organized localized modes of nonlinear photonic crystals. However, unlike a SFGS, a MFGS possesses multiple frequencies inside the gap. It is formed via the synergic nonlinear coupling of solitons or soliton trains at different frequencies. The existence of MFGSs can have potential applications. As an example, due to the localized nature of MFGSs, the threshold fields to excite a MFGS can be very low for a large sample. This makes the dynamical control (or switching) of the probe beams with a weak pump beam possible, in sharp contrast to the use of a strong pump to control the probe beam previously proposed [2,3]. Below, we demonstrate the existence of MFGS in both 1D and 2D photonic crystals.

1D superlattice.—Consider a 1D superlattice with N unit cells. One layer in each unit cell is linear with the dielectric constant ε_a and the width a , and the other layer is Kerr nonlinear, with the linear part of the dielectric constant denoted by ε_b and the width b . Here we consider M incoherent beams with frequencies $\omega_1, \omega_2, \dots, \omega_M$, each with arbitrary polarization, normally incident on one surface of the superlattice. The Maxwell equation inside the superlattice takes the form

$$\frac{d^2 E_i(z)}{dz^2} + \frac{\omega_i^2}{c^2} \varepsilon(z) E_i(z) = 0 \quad (i = 1, 2, \dots, M), \quad (1)$$

where $E_i(z)$ is the electric-field amplitude of the ω_i beam, $\varepsilon(z) = \varepsilon_a$ in the linear layer, and

$$\varepsilon(z) = \varepsilon_b + \sum_{i=1}^M \lambda |E_i(z)|^2 \quad (2)$$

in the nonlinear layer, where λ is the Kerr coefficient [12].

Equations (1) and (2) are solved by using the transfer-matrix method in conjunction with an iterative procedure, which is described in Ref. [11]. In our calculations, we use a model superlattice with $N = 40$, $\varepsilon_a = 1$, $\varepsilon_b = 2.4$, $\lambda = -10^{-3}$, and $b = a/2.4$.

In the linear case, i.e., $\lambda = 0$, the transmission spectrum of the model superlattice gives a first band gap from $\omega = 0.2637 \times 2\pi c/a$ to $\omega = 0.3428 \times 2\pi c/a$ that corresponds well to that predicted by the dispersion relation.

FIG. 1. Transmission of a single beam with $\omega_1 = 0.265 \times 2\pi c/a$ versus incident amplitude $|E_0|$. The transmission is calculated by first increasing $|E_0|$ and then decreasing $|E_0|$. The inset shows the intensity distribution at M normalized by $|E_0|^2$.

When nonlinearity is included, we first briefly discuss the excitation of SFGSs at $\omega_1 = 0.265 \times 2\pi c/a$ near the lower band edge by using a beam of frequency ω_1 . The transmission coefficient of the beam versus its incident amplitude $|E_0|$ is shown in Fig. 1. With increasing $|E_0|$, the transmission follows the curve $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow H$. When decreasing $|E_0|$, the transmission follows a different curve, $H \rightarrow I \rightarrow J \rightarrow K \rightarrow L \rightarrow$

$M \rightarrow B \rightarrow A$, showing a multistability behavior. The transmission resonances (TRs) at M ($|E_0| = 0.576$), K ($|E_0| = 2.114$), and I ($|E_0| = 4.331$) correspond to single-soliton (see inset of Fig. 1), two-soliton, and three-soliton states, respectively. The electric-field envelope of a single soliton has the hyperbolic secant form [4]. However, when frequency is moved deeper into the gap, the single-soliton state may become unstable and cannot be excited [13]. For example, at $\omega_2 = 0.27 \times 2\pi c/a$, the single-soliton excitation becomes unstable and the first TR ($|E_0| = 2.228$) corresponds to a two-soliton train and the second TR ($|E_0| = 4.923$) to a three-soliton train.

Now we consider the excitation of a MFGS possessing both frequencies ω_1 and ω_2 . Here, we use two incident beams with frequencies ω_1 and ω_2 , respectively. For reasons that will be explained later, the ratio of the incident amplitude $|E_{01}|$ of the ω_1 beam to $|E_{02}|$ of the ω_2 beam is fixed at a constant number of $|E_{02}| = 0.411|E_{01}|$. The transmission coefficients of ω_1 and ω_2 beams are shown by solid and dotted lines, respectively, in Fig. 2(a). We see that a TR occurs at P_1 for both ω_1 and ω_2 beams. The intensity distributions for ω_1 and ω_2 components at P_1 are shown by solid and dotted lines, respectively, in Fig. 2(b). The distribution for the ω_1 component has two envelopes, and that for the ω_2 component has a single envelope. The presence of a two-envelope intensity distribution of the ω_1 component stabilizes the excitation of a single-envelope distribution

FIG. 2. (a) and (c) show transmissions of beam $\omega_1 = 0.265 \times 2\pi c/a$ (solid line) and beam $\omega_2 = 0.27 \times 2\pi c/a$ (dotted line) versus $|E_{01}|$, where $|E_{02}| = 0.411|E_{01}|$ in (a) and $|E_{02}| = 0.204|E_{01}|$ in (c). The transmissions are calculated by first increasing and then decreasing $|E_{01}|$ and $|E_{02}|$. (b) and (d) show intensity distributions for ω_1 (solid line) and ω_2 (dotted line) components at P_1 of (a) and at P_2 of (c), respectively, where intensity distributions are normalized by $|E_{01}|^2$.

of the ω_2 component through the modification of the spatial distribution of the dielectric constant, i.e., $\delta\epsilon_1(z) = \lambda|E_1(z)|^2$. Similarly, the excitation of the ω_2 component significantly lowers the threshold field of the two-envelope excitation of the ω_1 component from $|E_0| = 2.114$ (K point in Fig. 1) to $|E_{01}| = 0.992$ at P_1 through the modification of $\delta\epsilon_2(z) = \lambda|E_2(z)|^2$. Furthermore, the shape of each envelope now deviates from the hyperbolic secant form due to large spatial variations in both $\omega_2^2\delta\epsilon_1(z)$ and $\omega_1^2\delta\epsilon_2(z)$. Thus, the formation of a MFGS is a result of synergic nonlinear interaction between solitons and soliton trains at different frequencies, which, in turn, leads to a lower threshold field and more stable excitation for each component. We have checked that the width and shape of these distributions are insensitive to the number N of the unit cells, when N is sufficiently large. The values of $|E_{02}|$ and $|E_{01}|$ at which the TR occurs (P_1) decrease near exponentially with N , and they are equal to the fields of the localized wave function at the surface of the sample. When $N = 40$, their ratio is 0.411, which is the ratio used in our calculation. Thus, the localized excitation at P_1 is viewed as intrinsic to the superlattice of infinite length, and it is only weakly perturbed by the surface of a superlattice of finite length and can be considered as a MFGS. Here it should be mentioned that another TR at B for the ω_1 component only in Fig. 2(a) corresponds to neither a MFGS nor a SFGS, whereas the TR at A corresponds to a SFGS shown in the inset of Fig. 1 because the ω_2 beam is completely reflected and has no effects on the dielectric constant inside the superlattice.

The MFGS shown in Fig. 2(b) represents only one of the new class of gap solitons predicted in this work. By using the same set of source beams at frequencies ω_1 and ω_2 , we are able to excite other MFGSs with more structures in the intensity distributions. For example, if we fix the ratio at $|E_{02}| = 0.204|E_{01}|$, we obtain another MFGS at P_2 in Fig. 2(c) with higher threshold fields. The intensity distributions of this MFGS are shown by solid and dotted lines in Fig. 2(d) for ω_1 and ω_2 components, respectively. Now the intensity distribution for ω_1 has three envelopes and that for ω_2 still has a single envelope but its form is different from that in Fig. 2(b). It is seen that, for the ω_1 component, the peak intensity in the middle envelope is smaller than that of the other two. The lowering of intensity in the middle envelope also stabilizes the excitation of the ω_2 component.

In the following, we use $\{n, m\}$ to denote a MFGS with n envelopes in the ω_1 component and m envelopes in the ω_2 component. In addition to $\{2,1\}$ and $\{3,1\}$ shown in Figs. 2(b) and 2(d), we have also found other MFGSs such as $\{3,2\}$, $\{3,3\}$, $\{4,1\}$, and $\{4,2\}$. However, we do not find $\{1,1\}$ and $\{2,2\}$. For these configurations, the peak positions of the intensity distributions are the same for both components. The addition of two intensities at the same location enhances the spatial intensity variations and makes the formation of a MFGS less favorable.

In addition to two-frequency gap solitons shown above, there also exist gap solitons with more frequency components. For example, we consider the excitation of a MFGS with three frequencies at $\omega_1 = 0.265 \times 2\pi c/a$, $\omega_2 = 0.2685 \times 2\pi c/a$, and $\omega_3 = 0.2715 \times 2\pi c/a$. The transmission coefficients of ω_1 , ω_2 , and ω_3 beams as a function of $|E_{01}|$ are shown by solid, dashed, and dotted lines, respectively, in Fig. 3(a), where $|E_{02}| = 0.746|E_{01}|$ and $|E_{03}| = 0.37|E_{01}|$. A TR occurs at P , which corresponds to a three-frequency gap soliton. The intensity distributions for ω_1 , ω_2 , and ω_3 are shown by solid, dashed, and dotted lines, respectively, in Fig. 3(b). The distribution for ω_1 has three envelopes, that for ω_2 has two envelopes, and that for ω_3 has a single envelope. The threshold fields for ω_1 and ω_2 components are $|E_{01}| = 2.03$ and $|E_{02}| = 1.51$, which are smaller than the threshold fields of 4.33 for the three-soliton train of the ω_1 component (point I in Fig. 1) and 2.22 for the two-soliton train of the ω_2 component, respectively. Here we have also checked that these distributions are independent of sample size and they represent an intrinsic mode of the system.

2D photonic crystal.—Consider a square periodic crystal depicted in the inset of Fig. 4. The sample is made of

FIG. 3. (a) Transmission of beam $\omega_1 = 0.265 \times 2\pi c/a$ (solid line), beam $\omega_2 = 0.2685 \times 2\pi c/a$ (dashed line), and beam $\omega_3 = 0.2715 \times 2\pi c/a$ (dotted line) versus $|E_{01}|$, where $|E_{02}| = 0.746|E_{01}|$ and $|E_{03}| = 0.37|E_{01}|$. The transmission is calculated by first increasing and then decreasing $|E_{01}|$, $|E_{02}|$, and $|E_{03}|$. (b) Intensity distributions for ω_1 (solid line), ω_2 (dashed line), and ω_3 (dotted line) components at P of (a), where the intensity is normalized by $|E_{01}|^2$.

FIG. 4. Transmission of beam $\omega_1 = 0.407 \times 2\pi c/a$ (dotted line) and beam $\omega_2 = 0.41 \times 2\pi c/a$ (solid line) versus $|E_{01}|$ in 2D photonic crystal depicted in the inset, where $|E_{02}| = 1.379|E_{01}|$ and the beam width $d = 6a$.

231 infinitely long parallel cylinders along the z axis possessing Kerr nonlinearity. The period of the lattice is a , and the radius of the cylinders is $R = 0.1a$. Now consider M incoherent E -polarized beams, in the form of either a slit or a plane wave, with frequencies $\omega_1, \omega_2, \dots, \omega_M$ propagating in the (x, y) plane. The electric field with frequency ω_i obeys the Maxwell equation $\nabla^2 E_i(x, y) + (\omega_i^2/c^2)\varepsilon(x, y)E_i(x, y) = 0$, where $\varepsilon(x, y) = \varepsilon_a$ in the background medium and $\varepsilon(x, y) = \varepsilon_b + \sum_{i=1}^M \lambda |E_i(x, y)|^2$ in the Kerr nonlinear cylinder. The above equations are solved using the multiple-scattering method [14] in conjunction with the iterative procedure. Here, we take $\varepsilon_a = 1$, $\varepsilon_b = 13$, and $\lambda = -10^{-3}$.

In the linear case, the calculated band structure gives a first full gap from $\omega = 0.405 \times 2\pi c/a$ to $\omega = 0.494 \times 2\pi c/a$ and a partial gap from $\omega = 0.405 \times 2\pi c/a$ to $\omega = 0.569 \times 2\pi c/a$ along the $\Gamma - M$ direction. The calculated transmission spectrum along the $\Gamma - M$ direction shows excellent agreement with the partial gap. To excite a MFGS, we use two beams with frequencies at, for example, $\omega_1 = 0.407 \times 2\pi c/a$ and $\omega_2 = 0.41 \times 2\pi c/a$ incident upon the crystal from the left face. The transmission coefficients for the ω_1 and ω_2 beams as a function of the incident amplitude $|E_{01}|$ are shown by dotted and solid lines, respectively, in Fig. 4. Here, we have chosen $|E_{02}| = 1.379|E_{01}|$. At P , a TR occurs for both frequencies. This corresponds to the excitation of a MFGS. The intensity distributions of the MFGS for ω_1 and ω_2 components are shown in Figs. 5(a) and 5(b), respectively, which indicates a $\{3,2\}$ MFGS. Here it should be pointed out that the excitation of this intrinsic MFGS is independent of the width of the incident beams, and the ratio between the incident amplitudes of the two

FIG. 5. Intensity distributions of the MFGS for ω_1 (a) and ω_2 (b) components in the 2D photonic crystal.

beams is nearly the same for all widths. It should be mentioned that the solutions found are stable MFGSs. In fact, if a solution is not stable in time, i.e., the solution is not a fixed attractor, the iterative procedure will never be convergent to a limiting result.

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