Multifrequency Gap Solitons in Nonlinear Photonic Crystals

Ping Xie and Zhao-Qing Zhang

Department of Physics and Institute of Nano Science and Technology (INST), The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China

(Received 23 December 2002; published 20 November 2003)

We predict the existence of multifrequency gap solitons (MFGSs) in both one- and two-dimensional nonlinear photonic crystals. A MFGS is a single intrinsic mode possessing multiple frequencies inside the gap. Its existence is a result of synergic nonlinear coupling among solitons or soliton trains at different frequencies. Its formation can either lower the threshold fields of the respective frequency components or stabilize their excitations. These MFGSs form a new class of stable gap solitons.

DOI: 10.1103/PhysRevLett.91.213904 PACS numbers: 42.70.Qs, 42.65.Tg, 63.20.Pw

Photonic crystals have been under intensive study both theoretically and experimentally over the past decade [1]. Since a photonic crystal can have a spectral gap in which light propagation is forbidden, it offers the possibility of controlling the flow of photons. To employ a high-technology potential of photonic crystals, it has been turned to nonlinear photonic crystals, where the tunability of the photonic band gap can be realized by changing the light intensity. Based on this idea, Scalora et al. [2] and Tran [3] numerically studied the optical switching of a weak probe beam by the use of a strong pump beam with a frequency different from that of the probe. An interesting phenomenon in nonlinear photonic crystals is the existence of nonlinearity-induced self-organized localized states with the frequency in the forbidden gaps, which are called gap solitons. The term gap soliton was first introduced by Chen and Mills in their numerical study of a 1D Kerr nonlinear superlattice [4]. Subsequently, the studies of gap solitons in 1D superlattices have attracted a lot of research interest both theoretically and experimentally [4–8]. Specifically, the formation of a coupled gap soliton by using two orthogonally polarized pulses in a 1D superlattice has been studied theoretically [7] and demonstrated experimentally [8]. Meanwhile, gap solitons in 2D and 3D nonlinear photonic crystals have also attracted considerable attention. John and Akozbek [9] studied gap solitons in 2D and 3D photonic crystals by using the coupled-mode theory, which is valid in small dielectric modulations. By using the numerical Green’s function, Mingaleev and Kivshar [10] demonstrated the existence of stable gap solitons in 2D photonic crystals with large dielectric contrast. Recently, we demonstrated the existence of stable gap solitons and soliton trains in 2D photonic crystals by studying the multistability in the transmission coefficient [11]. In all of these studies, only single-frequency gap solitons (SFGSs) have been found.

In this Letter, we predict the existence of multifrequency gap solitons (MFGSs) in both 1D and 2D nonlinear photonic crystals by studying the multistability of the transmission coefficients of finite-sized crystals. Similar to SFGSs found previously [4–11], MFGSs are also intrinsic self-organized localized modes of nonlinear photonic crystals. Unlike a SFGS, a MFGS possesses multiple frequencies inside the gap. It is formed via the synergic nonlinear coupling of solitons or soliton trains at different frequencies. The existence of MFGSs can have potential applications. As an example, due to the localized nature of MFGSs, the threshold fields to excite a MFGS can be very low for a large sample. This makes the dynamical control (or switching) of the probe beams with a weak pump beam possible, in sharp contrast to the use of a strong pump to control the probe beam previously proposed [2,3]. Below, we demonstrate the existence of MFGS in both 1D and 2D photonic crystals.

1D superlattice.—Consider a 1D superlattice with $N$ unit cells. One layer in each unit cell is linear with the dielectric constant $\varepsilon_b$ and the width $a$, and the other layer is Kerr nonlinear, with the linear part of the dielectric constant denoted by $\varepsilon_a$ and the width $b$. Here we consider $M$ incoherent beams with frequencies $\omega_1, \omega_2, \ldots, \omega_M$, each with arbitrary polarization, normally incident on one surface of the superlattice. The Maxwell equation inside the superlattice takes the form

$$\frac{d^2 E_i(z)}{dz^2} + \frac{\omega_i^2}{c^2} e(z) E_i(z) = 0 \quad (i = 1, 2, \ldots, M),$$

where $E_i(z)$ is the electric-field amplitude of the $\omega_i$ beam, $e(z) = e_a$ in the linear layer, and

$$e(z) = e_b + \sum_{i=1}^{M} \lambda |E_i(z)|^2$$

in the nonlinear layer, where $\lambda$ is the Kerr coefficient [12]. Equations (1) and (2) are solved by using the transfer-matrix method in conjunction with an iterative procedure, which is described in Ref. [11]. In our calculations, we use a model superlattice with $N = 40$, $e_a = 1$, $e_b = 2.4$, $\lambda = -10^{-3}$, and $b = a/2.4$.

In the linear case, i.e., $\lambda = 0$, the transmission spectrum of the model superlattice gives a first band gap from $\omega = 0.2637 \times 2 \pi c/a$ to $\omega = 0.3428 \times 2 \pi c/a$ that corresponds well to that predicted by the dispersion relation.
When nonlinearity is included, we first briefly discuss the excitation of SFGSs at \( \omega_1 = 0.265 \times 2\pi c/\lambda \) near the lower band edge by using a beam of frequency \( \omega_1 \). The transmission coefficient of the beam versus its incident amplitude \( |E_0| \) is shown in Fig. 1. With increasing \( |E_0| \), the transmission follows the curve \( A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow H \). When decreasing \( |E_0| \), the transmission follows a different curve, \( H \rightarrow I \rightarrow J \rightarrow K \rightarrow L \rightarrow M \rightarrow B \rightarrow A \), showing a multistability behavior. The transmission resonances (TRs) at \( M \) (\( |E_0| = 0.576 \)), \( K \) (\( |E_0| = 2.114 \)), and \( I \) (\( |E_0| = 4.331 \)) correspond to single-soliton (see inset of Fig. 1), two-soliton, and three-soliton states, respectively. The electric-field envelope of a single soliton has the hyperbolic secant form [4]. However, when frequency is moved deeper into the gap, the single-soliton state may become unstable and cannot be excited [13]. For example, at \( \omega_2 = 0.27 \times 2\pi c/\lambda \), the single-soliton excitation becomes unstable and the first TR (\( |E_0| = 2.228 \)) corresponds to a two-soliton train and the second TR (\( |E_0| = 4.923 \)) to a three-soliton train.

Now we consider the excitation of a MFGS possessing both frequencies \( \omega_1 \) and \( \omega_2 \). Here, we use two incident beams with frequencies \( \omega_1 \) and \( \omega_2 \), respectively. For reasons that will be explained later, the ratio of the incident amplitude \( |E_{01}| \) of the \( \omega_1 \) beam to \( |E_{02}| \) of the \( \omega_2 \) beam is fixed at a constant number of \( |E_{01}| = 0.411|E_{02}| \). The transmission coefficients of \( \omega_1 \) and \( \omega_2 \) beams are shown by solid and dotted lines, respectively, in Fig. 2(a). We see that a TR occurs at \( P_1 \) for both \( \omega_1 \) and \( \omega_2 \) beams. The intensity distributions for \( \omega_1 \) and \( \omega_2 \) components at \( P_1 \) are shown by solid and dotted lines, respectively, in Fig. 2(b). The distribution for the \( \omega_1 \) component has two envelopes, and that for the \( \omega_2 \) component has a single envelope. The presence of a two-envelope intensity distribution of the \( \omega_1 \) component stabilizes the excitation of a single-envelope distribution.

**FIG. 1.** Transmission of a single beam with \( \omega_1 = 0.265 \times 2\pi c/\lambda \) versus incident amplitude \( |E_0| \). The transmission is calculated by first increasing \( |E_0| \) and then decreasing \( |E_0| \). The inset shows the intensity distribution at \( M \) normalized by \( |E_0|^2 \).

**FIG. 2.** (a) and (c) show transmissions of beam \( \omega_1 = 0.265 \times 2\pi c/\lambda \) (solid line) and beam \( \omega_2 = 0.27 \times 2\pi c/\lambda \) (dotted line) versus \( |E_{01}| \), where \( |E_{02}| = 0.411|E_{01}| \) in (a) and \( |E_{02}| = 0.204|E_{01}| \) in (c). The transmissions are calculated by first increasing and then decreasing \( |E_{01}| \) and \( |E_{02}| \). (b) and (d) show intensity distributions for \( \omega_1 \) (solid line) and \( \omega_2 \) (dotted line) components at \( P_1 \) of (a) and at \( P_2 \) of (c), respectively, where intensity distributions are normalized by \( |E_{01}|^2 \).
of the $\omega_2$ component through the modification of the spatial distribution of the dielectric constant, i.e., 
\[ \delta \varepsilon_2(z) = \lambda |E_2(z)|^2. \] Similarly, the excitation of the $\omega_2$ component significantly lowers the threshold field of the two-envelope excitation of the $\omega_2$ component from $|E_{01}| = 2.114$ (point in Fig. 1) to $|E_{01}| = 0.992$ at $P_1$ through the modification of $\delta \varepsilon_2(z) = \lambda |E_2(z)|^2$. Furthermore, the shape of each envelope now deviates from the hyperbolic secant form due to large spatial variations in both $\omega_2^2 \delta \varepsilon_1(z)$ and $\omega_2^2 \delta \varepsilon_2(z)$. Thus, the formation of a MFGS is a result of synergic nonlinear interaction between solitons and soliton trains at different frequencies, which, in turn, leads to a lower threshold field and more stable excitation for each component. We have checked that the width and shape of these distributions are insensitive to the number $N$ of the unit cells, when $N$ is sufficiently large. The values of $|E_{02}|$ and $|E_{01}|$ at which the TR occurs ($P_1$) decrease near exponentially with $N$, and they are equal to the fields of the localized wave function at the surface of the sample. When $N = 40$, their ratio is 0.411, which is the ratio used in our calculation. Thus, the localized excitation at $P_1$ is viewed as intrinsic to the superlattice of infinite length, and it is only weakly perturbed by the surface of a superlattice of finite length and can be considered as a MFGS. Here it should be mentioned that another TR at $B$ for the $\omega_2$ component only in Fig. 2(a) corresponds to neither a MFGS nor a SFGS, whereas the TR at $A$ corresponds to a SFGS shown in the inset of Fig. 1 because the $\omega_2$ beam is completely reflected and has no effects on the dielectric constant inside the superlattice.

The MFGS shown in Fig. 2(b) represents only one of the new class of gap solitons predicted in this work. By using the same set of source beams at frequencies $\omega_1$ and $\omega_2$, we are able to excite other MFGSs with more structures in the intensity distributions. For example, if we fix the ratio at $|E_{02}| = 0.204|E_{01}|$, we obtain another MFGS at $P_2$ in Fig. 2(c) with higher threshold fields. The intensity distributions of this MFGS are shown by solid and dotted lines in Fig. 2(d) for $\omega_1$ and $\omega_2$ components, respectively. Now the intensity distribution for $\omega_1$ has three envelopes and that for $\omega_2$ still has a single envelope but its form is different from that in Fig. 2(b). It is seen that, for the $\omega_1$ component, the peak intensity in the middle envelope is smaller than that of the other two. The lowering of intensity in the middle envelope also stabilizes the excitation of the $\omega_2$ component.

In the following, we use $\{n, m\}$ to denote a MFGS with $n$ envelopes in the $\omega_1$ component and $m$ envelopes in the $\omega_2$ component. In addition to [2,1] and [3,1] shown in Figs. 2(b) and 2(d), we have also found other MFGSs such as [3,2], [3,3], [4,1], and [4,2]. However, we do not find [1,1] and [2,2]. For these configurations, the peak positions of the intensity distributions are the same for both components. The addition of two intensities at the same location enhances the spatial intensity variations and makes the formation of a MFGS less favorable.

In addition to two-frequency gap solitons shown above, there also exist gap solitons with more frequency components. For example, we consider the excitation of a MFGS with three frequencies at $\omega_1 = 0.265 \times 2\pi c/a$, $\omega_2 = 0.2685 \times 2\pi c/a$, and $\omega_3 = 0.2715 \times 2\pi c/a$. The transmission coefficients of $\omega_1$, $\omega_2$, and $\omega_3$ beams as a function of $|E_{01}|$ are shown by solid, dashed, and dotted lines, respectively, in Fig. 3(a), where $|E_{02}| = 0.746|E_{01}|$ and $|E_{03}| = 0.370|E_{01}|$. ATR occurs at $P$, which corresponds to a three-frequency gap soliton. The intensity distributions for $\omega_1$, $\omega_2$, and $\omega_3$ are shown by solid, dashed, and dotted lines, respectively, in Fig. 3(b). The distribution for $\omega_1$ has three envelopes, that for $\omega_2$ has two envelopes, and that for $\omega_3$ has a single envelope. The threshold fields for $\omega_1$ and $\omega_2$ components are $|E_{01}| = 2.03$ and $|E_{02}| = 1.51$, which are smaller than the threshold fields of 4.33 for the three-soliton train of the $\omega_1$ component (point $I$ in Fig. 1) and 2.22 for the two-soliton train of the $\omega_2$ component, respectively. Here we have also checked that these distributions are independent of sample size and they represent an intrinsic mode of the system.

2D photonic crystal.—Consider a square periodic crystal depicted in the inset of Fig. 4. The sample is made of

FIG. 3. (a) Transmission of beam $\omega_1 = 0.265 \times 2\pi c/a$ (solid line), beam $\omega_2 = 0.2685 \times 2\pi c/a$ (dashed line), and beam $\omega_3 = 0.2715 \times 2\pi c/a$ (dotted line) versus $|E_{01}|$, where $|E_{02}| = 0.746|E_{01}|$ and $|E_{03}| = 0.370|E_{01}|$. The transmission is calculated by first increasing and then decreasing $|E_{01}|$, $|E_{02}|$, and $|E_{03}|$. (b) Intensity distributions for $\omega_1$ (solid line), $\omega_2$ (dashed line), and $\omega_3$ (dotted line) components at $P$ of (a), where the intensity is normalized by $|E_{01}|^2$. 

213904-3
example, shows excellent agreement with the partial gap. To excite the calculated transmission spectrum along the $a$-index axis for the incident upon the crystal from the left face. The transverse integrated field with frequency $\omega$ obeys the Maxwell equation $\nabla^2 E_i(x, y) + (\omega^2 / c^2) \varepsilon(x, y) E_i(x, y) = 0$, where $\varepsilon(x, y) = \varepsilon_a$ in the background medium and $\varepsilon(x, y) = \varepsilon_b + \sum_{j=1}^{M} \lambda |E_j(x, y)|^2$ in the Kerr nonlinear cylinder. The above equations are solved using the multiple-scattering method [14] in conjunction with the iterative procedure. Here, we take $\varepsilon_a = 1$, $\varepsilon_b = 13$, and $\lambda = -10^{-3}$.

In the linear case, the calculated band structure gives a first full gap from $\omega = 0.405 \times 2 \pi c / a$ to $\omega = 0.494 \times 2 \pi c / a$ and a partial gap from $\omega = 0.405 \times 2 \pi c / a$ to $\omega = 0.569 \times 2 \pi c / a$ along the $\Gamma - M$ direction. The calculated transmission spectrum along the $\Gamma - M$ direction shows excellent agreement with the partial gap. To excite a MFGS, we use two beams with frequencies at, for example, $\omega_1 = 0.407 \times 2 \pi c / a$ and $\omega_2 = 0.41 \times 2 \pi c / a$ incident upon the crystal from the left face. The transmission coefficients for the $\omega_1$ and $\omega_2$ beams as a function of the incident amplitude $|E_{01}|$ are shown by dotted and solid lines, respectively, in Fig. 4. Here, we have chosen $|E_{02}| = 1.379 |E_{01}|$. At $P$, a TR occurs for both frequencies. This corresponds to the excitation of a MFGS. The intensity distributions of the MFGS for $\omega_1$ and $\omega_2$ components are shown in Figs. 5(a) and 5(b), respectively, which indicates a $\{3, 2\}$ MFGS. Here it should be pointed out that the excitation of this intrinsic MFGS is independent of the width of the incident beams, and the ratio between the incident amplitudes of the two beams is nearly the same for all widths. It should be mentioned that the solutions found are stable MFGSs. In fact, if a solution is not stable in time, i.e., the solution is not a fixed attractor, the iterative procedure will never be convergent to a limiting result.

2D photonic crystal depicted in the inset, where $|E_{01}| = 1.379 |E_{02}|$ and the beam width $d = 6a$.

FIG. 4. Transmission of beam $\omega_1 = 0.407 \times 2 \pi c / a$ (dotted line) and beam $\omega_2 = 0.41 \times 2 \pi c / a$ (solid line) versus $|E_{01}|$ in 2D photonic crystal depicted in the inset, where $|E_{02}| = 1.379 |E_{01}|$ and the beam width $d = 6a$.

FIG. 5. Intensity distributions of the MFGS for $\omega_1$ (a) and $\omega_2$ (b) components in the 2D photonic crystal.

[12] Equation (2) is derived from $\varepsilon(z) = \varepsilon_b + \frac{1}{\beta} (\sum_{j=1}^{M} E_j(z) e^{-i(\omega_j + c z)})^2$, where the angular brackets represent a time average. For the values of $\omega_1, \omega_2, \ldots, \omega_M$ taken in this paper, the value such as $1/|\omega_1 - \omega_2|$ ($\sim 10^{-4}$ s for the visible light) is several orders smaller than the response time of the Kerr effect (usually $\sim 10^{-9}$ s). Therefore, the contributions of the terms such as $e^{i(\omega_1 - \omega_2)t}$ are averaged to zero. See, for example, R. W. Boyd, Nonlinear Optics (Academic, New York, 1992), Chap. 4.