An Analytical Expression for Drain Saturation Voltage of Polycrystalline Silicon Thin-Film Transistors

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Abstract—A physical-based analytical expression for the drain saturation voltage V_{Dsat} of polycrystalline silicon (poly-Si) thin-film transistors (TFTs) is derived. V_{Dsat} is found to be dominated by the grain boundary potential barrier modulation effect, which can be readily estimated from the device transfer characteristic. Straightforward prediction of V_{Dsat} values at arbitrarily given gate voltages based on the proposed formula is demonstrated for both low temperature and high temperature processed poly-Si TFTs in either n- or p-type. The prediction agrees well with experimentally determined V_{Dsat} value. Derivation of the expression is based on our previously proposed analytical ON-state drain-current model for poly-Si TFTs, with no empirical factors included.

Index Terms—Drain saturation voltage, output characteristic, polycrystalline silicon, thin-film transistors (TFTs).

I. INTRODUCTION

WITH THE rapid development of new polycrystalline silicon thin-film transistor (poly-Si TFT) technologies, TFT device modeling [1]–[6] becomes increasingly important to practical applications of poly-Si TFT-based circuits. Drain saturation voltage V_{Dsat} is one of the key parameters for modeling of device output characteristics. For “ideal” MOSFETs, $V_{Dsat} = V_g - V_t \equiv V_{gt}$, where V_t is the gate bias when channel inversion just occurs. By including some first-order effects in MOSFETs, namely carrier velocity saturation and transverse field (E_{eff}) dependent mobility degradation effect, analytical formulas are derived for V_{Dsat} prediction [7]–[9]. These formulas are equivalent and calculated V_{Dsat} values are close. Sometimes for simplicity, a “saturation factor” α is introduced to define V_{Dsat} by $V_{Dsat} = V_{gt}/\alpha$ [8], [9]. α equals 1 for “ideal” MOSFETs and becomes slightly larger than 1 due to the nonideal first-order effects. Here, the physical basis of the saturation factor is clear.

For poly-Si TFTs, however, there is still no generally accepted analytical formula for V_{Dsat} prediction. Instead, some previous studies [1], [2] empirically introduced the saturation factor α as a fitting parameter for V_{Dsat} modeling. Without

knowing the physical basis of α in TFTs, it can only be obtained by fitting the experimental output curve at some specific V_g . In practice, α is found to be V_g dependent and usually considerably larger than one [2], [6]. Therefore, straightforward prediction of V_{Dsat} before measuring the corresponding output characteristic is not possible in this approach. In other previous works [3]–[5], without introducing α , V_{Dsat} was obtained numerically [4], [5] or through an expression very close to that for MOSFETs [3]. With a number of fitting parameters involved there, prediction of V_{Dsat} was not able to be addressed. Up to now, a physical-based analytical formula enabling the straightforward prediction of V_{Dsat} of poly-Si TFTs is still absent, and is therefore highly desired.

Recently, we developed a physical-based analytical ON current model for poly-Si TFTs [6], where the physical basis of α has been clarified. It is shown that three effects, V_g dependent grain boundary (GB) potential barrier modulation, E_{eff} dependent mobility degradation and carrier velocity saturation contribute to its nonideality. This may provide an approach for straightforward prediction of V_{Dsat} . Unfortunately, the expression for α is a rough estimation that includes four physical parameters. In practice, α still has to be empirically used as a fitting parameter. In this letter, we propose for the first time a physical-based analytical formula enabling the straightforward calculation of poly-Si TFTs’ V_{Dsat} at arbitrarily given V_g . The given expression contains only two parameters, both of which are physical explicitly defined and can be readily obtained before measuring actual output characteristics. It is derived by further developing our previous model to precisely describe the drain saturation region. Its applicability is demonstrated on different poly-Si TFTs made from various processes.

II. DRAIN-CURRENT MODEL

At some point y (where the drain voltage drop is V_c) along the inversion channel of poly-Si TFTs, E_{eff} controlled mobility degradation is modeled as [10]

$$\mu_{ig} = \frac{\mu_0}{1 + (E_{eff}/E_{crit})^\nu} \quad (1)$$

where μ_0 is intra-grain carrier mobility free of surface scattering, and E_{crit} the critical field for mobility degradation, and $\nu = 1$ for holes or 1.6 for electrons [10]. E_{eff} is controlled by the channel inversion charge sheet density Q_{inv} [11], [12]

$$E_{eff} = \eta Q_{inv}/\epsilon_s = \eta C_{ox}(V_{gt} - V_c)/\epsilon_s. \quad (2)$$

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η is 1/2 for electrons and 1/3 for holes [9], [12]. ε_s is the Si permittivity. The intra-grain carrier mobility μ_{ig} should be incorporated into a complete effective channel mobility μ_{eff} model [6] by including carrier transport over V_g modulated GB barrier

$$\mu_{eff} = \mu_{ig} \exp(-S/V_{gt}). \quad (3)$$

S is a constant relating to GB trap areal density [13]. It reflects GB barrier modulation effect and can be experimentally extracted.

Following gradual channel approximation (GCA), I_D at a fixed V_D is given by:

$$I_D = WQ_{inv}(y)v_c(y) \quad (4)$$

where v_c is carrier velocity at some point y along the channel. Taking the velocity saturation effect into consideration, v_c is modeled as [9], [14]

$$v_c = \frac{\mu_{eff} dV_c/dy}{1 + (\mu_{eff}/v_{sat}) dV_c/dy} \quad (5)$$

where v_{sat} is the carrier saturation velocity. Combining all the above equations into (4), and integrating it from the source $y = 0$ to the drain $y = L$, one obtains

$$I_D L = WC_{ox} V_{gt}^2 \int_0^{V_D/V_{gt}} \mu_{eff}(1-x) dx - \frac{I_D}{v_{sat}} \int_0^{V_D} \mu_{eff} dV_c$$

$$x \equiv V_c/V_{gt}.$$

Define $\bar{\mu}_{eff} \equiv \int_0^{V_D} \mu_{eff} dV_c/V_D$, and the saturation field $E_{sat} \equiv v_{sat}/\bar{\mu}_{eff}$, one has

$$I_D = \frac{W}{L + V_D/E_{sat}} C_{ox} V_{gt}^2 \int_0^{V_D/V_{gt}} \mu_{eff}(1-x) dx$$

$$= \frac{W}{L + V_D/E_{sat}} C_{ox} V_{gt}^2$$

$$\times \int_0^{V_D/V_{gt}} \frac{\exp[-(S/V_{gt})/(1-x)]}{1 + [\beta(1-x)]^\nu} \mu_0(1-x) dx \quad (6)$$

where $\beta = \eta C_{ox} V_{gt}/\varepsilon_s E_{crit}$. Define $f(x) = (\exp[-(S/V_{gt})/(1-x)]/(1-x))/1 + [\beta(1-x)]^\nu$, according to the mean value theorem for integration, there always exists an $x_0 \in (0, V_D/V_{gt})$ so that $f(x_0)$ can be taken out from the integral. Therefore, I_D is finally derived as

$$I_D = \frac{W}{L + V_D/E_{sat}} \bar{\mu}_{eff} C_{ox} \left(V_{gt} - \frac{1}{2} V_D \right) V_D$$

$$\bar{\mu}_{eff} = \mu_0 f(x_0), \quad x_0 = \delta V_D/V_{gt} \quad 0 < \delta < 1. \quad (7)$$

Parameter δ defines a position in the inversion channel where local mobility equals $\bar{\mu}_{eff}$. Thus, it could be named ‘‘average mobility parameter.’’ This I_D expression is similar to most forms for MOSFETs and TFTs, except for the modified mobility term. Three important mechanisms, namely, velocity

Fig. 1. I_D model fit of output characteristics at various V_g before saturation (solid lines) for an n-type high temperature processed poly-Si TFT. V_{Dsat} calculated by (11) and (12), and V_{gt} are compared to the experimentally extracted V_{Dsat} . Dashed lines are fitting curves based on channel length modulation after saturation.

saturation, mobility degradation and GB barrier modulation are accurately incorporated. The derivation is purely based on GCA without any other approximations made.

III. DRAIN SATURATION VOLTAGE

The output characteristic is modeled based on the above I_D model. At a fixed V_g , with increasing V_D to V_{Dsat} , pinch-off occurs at the drain end. Drain-current wherein defines the saturation current I_{Dsat} , which still observes (7)

$$I_{Dsat} = \frac{W}{L + V_{Dsat}/E_{sat}} \bar{\mu}_{eff} C_{ox} \left(V_{gt} - \frac{1}{2} V_{Dsat} \right) V_{Dsat}. \quad (8)$$

By fitting the measured output curve at a given V_g before saturation, V_{Dsat} can be experimentally extracted as the V_D position at which the fitted I_D curve reaches maximum [9], as shown in Fig. 1. Apparently, this approach is not straightforward and V_{Dsat} cannot be obtained without measuring and fitting each output curve. The accuracy of the method depends on the curve fitting.

To derive an analytical form for V_{Dsat} , let $\partial I_D/\partial V_D = 0$, a quadratic equation of V_D is obtained

$$\frac{1}{2} \xi V_D^2 - (\kappa + \xi) V_{gt} V_D + V_{gt}^2 = 0$$

$$\kappa = 1 + \frac{V_{gt}/V_D - 1/2}{L E_{sat}/V_D + 1},$$

$$\xi = \left(\delta + \frac{\partial \delta}{\partial V_D} V_D \right) \left[\frac{S}{V_{gt}(1 - \delta V_D/V_{gt})^2} \right. \\ \left. + \frac{\nu \beta^\nu (1 - \delta V_D/V_{gt})^{\nu-1}}{1 + \beta^\nu (1 - \delta V_D/V_{gt})^\nu} \right] \quad (9)$$

V_{Dsat} is the solution of (9)

$$V_{Dsat} = \left(1 + \kappa/\xi - \sqrt{(1 + \kappa/\xi)^2 - 2/\xi} \right) V_{gt}. \quad (10)$$

TABLE I
 V_{Dsat} (IN VOLTS) CALCULATED BASED ON (10)–(12) ARE COMPARED
 TO EXPERIMENTALLY EXTRACTED V_{Dsat} AT VARIOUS V_g ,
 FOR BOTH n- AND p-TYPE POLY-Si TFTs BY EITHER
 LOW OR HIGH TEMPERATURE PROCESS

Poly-Si TFTs	n-type HT		n-type LT		p-type HT		p-type LT	
Device threshold	0.54V		5.24V		-2.09V		-14.0V	
V_g (V)	2	4	8	10	-3	-5	-15	-18
$ V_{Dsat} $ extracted	1.89	3.86	4.66	6.36	1.29	3.02	3.79	6.22
$ V_{Dsat} $ Eq. 10	1.92	3.88	4.68	6.37	1.30	3.02	3.76	6.23
$ V_{Dsat} $ Eq. 10 $S=0$	2.97	4.77	8.05	9.85	1.78	3.57	9.08	11.7
$ V_{Dsat} $ deviation (%)	$\delta=0$	-3.70	-7.25	-2.79	-3.77	-2.33	-4.30	-6.90
	$\beta=0$	2.12	4.25	1.50	2.07	0.78	4.30	0.85
	$\kappa=1$	0.53	1.14	1.50	1.92	1.55	2.32	3.68
$ V_{Dsat} $ Eq. 11	δ fitted	1.86	3.79	4.66	6.36	1.28	3.07	3.69
	δ fixed	1.91	3.68	4.66	6.27	1.23	3.11	4.15
$ V_{Dsat} $ Eq. 12	3.03	4.98	8.18	10.1	1.79	3.66	9.63	12.4
$ V_{gt} $	3.04	5.04	8.33	10.3	1.81	3.81	9.95	13.0

In the above equation, dimensionless coefficient κ is related to velocity saturation (E_{sat}); ξ is a combination of average mobility parameter δ with GB barrier modulation (S) and mobility degradation effect (β). Parameter S is extracted as the slope of $\ln \mu_{eff}$ or $\ln(I_D/V_{gt})$ versus $-1/V_{gt}$ plot; and E_{crit} is obtained by fitting the V_g dependence of μ_{eff} [6]. δ and E_{sat} are extracted by fitting the output curve before saturation. In addition, δ is weakly V_D dependent and its differential δ' is estimated by fitting transfer curves at various V_D . Since both coefficients are V_D dependent, the V_{Dsat} solution (10) is implicit and must be calculated through iteration. The value V_{gt} is ideal to be taken as the initial solution for the iteration. As listed in Table I, calculated V_{Dsat} is within 1% error compared to experimentally extracted value.

Since complicated procedures need to be followed to extract all parameters before V_{Dsat} calculation, it is worth to observe which factor is dominant. In (10), neglect GB barrier modulation ($S = 0$), V_D dependence of δ ($\delta' = 0$), mobility degradation ($\beta = 0$) or velocity saturation ($\kappa = 1$), respectively, the corresponding V_{Dsat} value/deviations are compared in the Table. Significant deviation caused by setting $S = 0$ is in contrast to the small error introduced by the other three approximations. It clearly indicates that GB barrier modulation is the dominant factor deciding the V_{Dsat} . Interestingly, the overall error contributed by the three approximations can largely offset. Therefore, setting $\beta = 0$, $\kappa = 1$ and $\delta' = 0$ simultaneously, a much simplified V_{Dsat} expression with reasonable accuracy is obtained

$$V_{Dsat} = V_{gt}(1 + 1/\xi - \sqrt{1 + 1/\xi^2})$$

$$\xi = \delta S V_{gt} / (V_{gt} - \delta V_D)^2. \quad (11)$$

Although only GB effect is included and two parameters S and δ involved in (11), the calculated V_{Dsat} is within 3% deviation from experimental values. S is a GB related parameter independent of V_g , while δ still weakly depends on V_g . In modeling, δ can be accurately obtained by fitting the output curve at specific V_g . Fortunately, as an average mobility

parameter, δ is nearly a constant. All δ values from 4 different types of TFTs are found to fall within $0.35 \sim 0.45$. For a certain kind of device, an average δ value from such device can be used for V_{Dsat} prediction. As shown in the table, straightforward V_{Dsat} prediction based on fixed δ value can typically have 4%, or in the worst condition, 9% deviation. It should be acceptable for many practical applications. In (10), if only velocity saturation is included, i.e., $\xi = 0$, the V_{Dsat} solution reduces to a standard form for MOSFETs [9]

$$V_{Dsat}|_{\xi=0} = L E_{sat} (\sqrt{1 + 2V_{gt}/L E_{sat}} - 1) \quad (12)$$

which could be further reduced to V_{gt} , i.e., the V_{Dsat} of ideal MOSFETs if $E_{sat} = \infty$. As a comparison, calculated V_{Dsat} using (12) is apparently erroneous for poly-Si TFTs, which actually is close to the value obtained by setting $S = 0$ in (10).

IV. CONCLUSION

For the first time, a physical-based full analytical expression for V_{Dsat} of poly-Si TFTs is derived, which, after being simplified, can be used for straightforward V_{Dsat} prediction with reasonable accuracy without measuring device output curves. The applicability of the formula is demonstrated by four different types of poly-Si TFTs.

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