Opportunity-based Topology Control in Wireless Sensor Networks

Yunhuai Liu, Qian Zhang and Lionel Ni
Hong Kong University of Science and Technology
{yunhuai, qianzh, ni}@cs.ust.hk

Abstract

Topology control is an effective method to improve the energy efficiency of wireless sensor networks (WSNs). Traditional approaches are based on the assumption that a pair of nodes is either “connected” or “disconnected”. These approaches are called connectivity-based topology control. In real environments however, there are many intermittently connected wireless links called lossy links. Taking a succeeded lossy link as an advantage, we are able to construct more energy-efficient topologies. Towards this end, we propose a novel opportunity-based topology control. We show that opportunity-based topology control is a problem of NP-hard. To address this problem in a practical way, we design a fully distributed algorithm called CONREAP based on reliability theory. We prove that CONREAP has a guaranteed performance. The worst running time is $O(|E|)$ where E is the link set of the original topology, and the space requirement for individual nodes is $O(d)$ where d is the node degree. To evaluate the performance of CONREAP, we design and implement a prototype system consisting of 50 Berkeley Mica2 motes. We also conducted comprehensive simulations. Experimental results show that compared with the connectivity-based topology control algorithms, CONREAP can improve the energy efficiency of a network up to 6 times.

1 Introduction

Topology control is an effective method to improve the energy efficiency of Wireless Sensor Networks (WSNs) [15,21]. In traditional methods, the network model is based on the assumption that a pair of nodes is either “connected” or “disconnected”. When all nodes are connected to the network, the network is said to have the full connectivity. Traditional approaches aim to seek a topology of minimized energy cost and the full connectivity. We therefore call these approaches *connectivity-based topology control*. Connectivity-based topology control is a well-known NPC

problem [10] and many heuristic algorithms have been proposed (e.g., [6, 12, 20]).

In real application environments, however, this connectivity-based model is not practical due to the *transitional region phenomenon*. Beyond the “connected” region, there is a *transitional region* that has intermittently connected wireless links, the so-called *lossy links* [3,4,26]. Via such lossy links a pair of nodes are reachable but not always connected. As reported in the literature (e.g., [3,26]), lossy links usually account a major part of a wireless network. In a specific setup, lossy links are more than 90%.

An interesting observation is that lossy links, if used properly, provide a great opportunity to construct energy-efficient topologies. Compared with those reliable links, lossy links allow a transmitter to reach more nodes when they succeed. By taking the succeeded lossy links as an advantage, many transmitters can be saved and the transmission cost can be largely reduced.

The employment of lossy links in topology is not straightforward. When they are employed, the sink may not be able to reach all nodes for a given transmission. An important characteristic of WSNs is the large density and high data redundancy. For many applications, to require the sink reach all nodes for every transmission is not a necessity. As long as an expected percentage of the nodes can be reached, individual nodes are often less concern. This feature allows the employment of lossy links during topology control, by which some applications can trade certain part of the network for higher energy efficiency. We use network-wide broadcast to represent the routing protocol, which is the best that a routing protocol can do. No congestion or collision is considered. Under this transmission paradigm, we define the metric *network reachability* to quantify the percentage of the nodes that can be reached by the sink using a sink initialized broadcast. Based on network reachability, we propose a novel *opportunity-based topology control*, targeting at more energy-efficient topologies.

An illustration is depicted in Fig. 1. The x-axis is the energy cost (which is indicated by the number of links in the final topology). And the y-axis is the network reachabil-

Figure 1. Illustration of opportunity-based and connectivity-based topology control

ity. Connectivity-based topology control only uses reliable links. It seeks the optimal topology of the minimized cost and the full connectivity (point A in the figure). As this is a NPC problem, different heuristic algorithms were proposed to approach this point. Different from connectivity-based algorithms, opportunity-based topology control employs lossy links during topology control. It aims at a topology of higher energy efficiency with certain degraded network reachability (constrained by an application-dependent threshold λ_{TH}). In the example of Fig. 1, given the threshold λ_{TH} (0.8 in this case), point B is the target topology of opportunity-based topology control. Point C is of connectivity-based one. The energy gap between B and C is the main motivation of this work. In some extreme applications of $\lambda_{TH} = 1$, opportunity-based topology control is equivalent to the traditional connectivity-based one.

The key challenge in opportunity-based topology control is the computation of the network reachability. As we will see later (Sec. 2.1), it is a NP-hard problem. We do not yet have efficient algorithms to compute the network reachability accurately, even given the global network knowledge. The key issue then becomes how to guarantee a derived network topology can satisfy the optimization constraint (i.e., the network reachability is no less than the threshold λ_{TH}). To address the problem, we explore the reliability theory during our algorithm design.

The main contributions of this paper are as follows. First, we identify and highlight the opportunity of using lossy links during topology control in WSNs. We formulate the novel opportunity-based topology control by defining the metric “network reachability”. To the best of our knowledge, we are the first one to attempt to employ the lossy links for energy-efficient topology control purpose. Second, we prove that opportunity-based topology control is a problem of NP-hard. In order to address the problem in a practical manner, we design a fully distributed algorithm called *CONREAP* based on the reliability theory. We prove that the derived topology by *CONREAP* has a guaranteed network reachability, and the network energy cost is significantly reduced. The worst running time of *CONREAP* is

$O(|E|)$ where E is the set of links in the original topology. The space requirement of individual nodes is $O(d)$ where d is the node degree. Third, in order to investigate the performance of *CONREAP*, we design and implement a prototype system consisting of 50 Berkeley Mica2 motes. We also conduct simulations to simulate a 200 node network. Our results show that *CONREAP* can improve energy efficiency of a network up to 6 times compared with the traditional connectivity-based algorithms.

The rest of the paper is organized as follows. In Section 2, we use an example to motivate our work. We then give an overview of opportunity-based topology control in Section 3. In Section 4 we present the proposed *CONREAP* algorithm and show some analytical results. Section 5 shows the implementation and simulation experiment results. We give an overview of the related work in Section 6. The last section concludes this paper and indicates possible future research directions.

2 Motivation

In this section, we use an example to motivate opportunity-based topology control. We first describe the network model that we adopt in this paper to simplify the presentation.

2.1 Network model

We model a WSN as a directed graph $G(V, E)$, where V is the set of nodes and E is the set of links¹. For each link $e = \{u, v\} \in E$, we use $\lambda_e \in (0, 1]$ to indicate the probability that a packet can be successfully delivered from the node u to v , called *link reachability*. As links may fail, whether a node can be reached² by the sink via broadcast is a random event. Let V_r denote the set of reached nodes by the sink for a given broadcast. $|V_r|$ is thus a random variable indicating the number of such nodes. With these concepts available, we define:

Definition Network reachability $\lambda(G)$. Given a network $G(V, E)$, $\lambda(G)$ is defined as the ratio between the expected number of reached nodes by the sink using a sink initialized broadcast, and the total number of nodes in V , i.e.,

$$\lambda(G) = \frac{E(|V_r|)}{|V|}$$

To obtain $E(|V_r|)$, we define:

¹In this paper, we use the term link and edge interchangeably, as well as node and vertex.

²The term “reach” here means that there exists a data path from the sink to the node, along which all the links succeed during the broadcast.

Table 1. Notations used in this paper

Notation	Description
$G(V, E)$	A network topology, V is the set of nodes and E is the set of links
V_{Tx}	V_{Tx} is the set of transmitters in G : $V_{Tx} = \{u \exists v \in V, \{u, v\} \in E\}$
$T_i(V, E_i)$	An intermediate tree topology; $E_i \cap E_j = \emptyset \Leftrightarrow i \neq j$;
$G_R(V, E_R)$	The derived topology; By CONREAP algorithm we have: $E_R = \bigcup E_i$
$\lambda_e, e \in E$	Link reachability, also written as $\lambda_{\{u, v\}}$ when $e = \{u, v\}$;
$\lambda_G(v)$	Node reachability for a node $v \in V$
$\tilde{\lambda}_G(v)$	An approximation of node reachability: $\tilde{\lambda}_G(v) = 1 - \prod (1 - \lambda_{T_i}(v))$
$\lambda(G)$	Network reachability of G : $\lambda(G) = \sum_{v \in V} \lambda_G(v) / V $
λ_{TH}	A threshold to define the minimal required network reachability
$\varepsilon(G)$	The energy cost of a network: $\varepsilon(G) = \varepsilon_{Tx}(G) + \varepsilon_{Rx}(G)$
Nb_v	Node v and its neighboring set: $Nb_v = \{u \forall u \in V, \{u, v\} \in E \vee \{v, u\} \in E\} \cup \{v\}$
$avg(\lambda_G(v))$	Local average of $\lambda_G(v)$ in Nb_v : $avg(\lambda_G(v)) = \frac{\sum_{u \in Nb_v} \lambda_G(u)}{ Nb_v }$

Definition *Node reachability* $\lambda_G(v)$. Given a node v in a network G , $\lambda_G(v)$ is the probability that the sink can reach v by a broadcast.

By reliability theory [1], it is known that the computation of node reachability is a problem of NP-hard.

Theorem 2.1 *Network reachability $\lambda(G)$ is equal to the averaged node reachability of all nodes in the network, i.e.,*

$$\lambda(G) = \frac{\sum_{v \in V} \lambda_G(v)}{|V|}$$

Theorem 2.2 *The computation of the network reachability for a given network is NP-hard.*

Due to the space limitation, we omit the proof of these theorems.

Definition *Transmitter set* V_{Tx} . Given a network G , V_{Tx} is the set of transmission nodes that have outgoing links in G , i.e.,

$$V_{Tx} = \{u | \forall u \in V, \exists v \in V, \{u, v\} \in E\}$$

Definition *Network energy cost* $\varepsilon(G)$. Given a network G , $\varepsilon(G)$ is the sum of the transmitting cost $\varepsilon_{Tx}(G)$ and receiving cost $\varepsilon_{Rx}(G)$, i.e.,

$$\varepsilon(G) = \varepsilon_{Tx}(G) + \varepsilon_{Rx}(G)$$

The network energy cost is comprised of two parts, the transmitting cost $\varepsilon_{Tx}(G)$ and the receiving cost $\varepsilon_{Rx}(G)$. The energy consumption in WSNs is quite different from that in traditional wireless networks. On one hand, receiving a packet has a similar power level as transmitting it [19]. On the other hand, the consumptions of different transmission power levels are not significantly different [7, 8]. The electronic current of the minimal power is about half of the maximum power. To simplify the analysis, we assume that for any sensor node, each transmitting or receiving consumes one unit of energy. More information about the energy consumptions in WSNs can be found in the references [7, 8, 19].

The transmitting cost $\varepsilon_{Tx}(G)$ depends on $V_{Tx} \subseteq V$. It not only relates to the size $|V_{Tx}|$ but also the node reachability $\lambda_G(v), v \in V_{Tx}$. A transmitter of higher reachability is likely to consume more energy cost as it has more chances to forward packets. Therefore we define: $\varepsilon_{Tx}(G) = \sum_{v \in V_{Tx}} \lambda_G(v)$.

The receiving cost $\varepsilon_{Rx}(G)$ depends on the receivers in G that have incoming links. Each transmitter v introduces $d_G(v) \cdot \lambda_G(v)$ receiving cost to the network, where $d_G(v)$ is the number of outgoing links of v . Therefore, $\varepsilon_{Rx}(G) = \sum_{v \in V_{Tx}} d_G(v) \cdot \lambda_G(v)$, where $d_G(v)$ is the node v 's out-degree in G .

Combining $\varepsilon_{Tx}(G)$ and $\varepsilon_{Rx}(G)$, we have the definition of the network energy cost. It is an expected value.

Definition *Reachability-energy ratio* $\eta(G)$. Given a network G , $\eta(G)$ is defined as the ratio of $\lambda(G)$ and $\varepsilon(G)$:

$$\eta(G) = \frac{\lambda(G)}{\varepsilon(G)}$$

Table 1 lists some notations used in this paper.

2.2 Example

A simple example is illustrated in Fig. 2. G is the original topology with 4 nodes v_0, v_1, v_2 and v_3 . The node v_0 serves as the sink. Solid lines denote reliable links and dash lines denote lossy links. The link reachability is labeled on top of the corresponding link.

Consider two topologies G_1 and G_2 . Since the original topology G has no full connectivity, the best that connectivity-based algorithm can do is G_1 that connects v_1

Figure 2. An example of opportunity-based topology control; G is the original topology; G_1 is by connectivity-based and G_2 is by opportunity-based topology control

and v_2 using reliable links. Different from G_1 , G_2 connects v_2 and v_3 using lossy links.

In G_1 , both v_1 and v_2 are reliably connected to the sink. The node reachability $\lambda_{G_1}(v_1) = 1$, $\lambda_{G_1}(v_2) = 1$, $\lambda_{G_1}(v_3) = 0$. Network reachability is thus $\lambda(G_1) = \text{mean}\{1, 1, 0\} = 0.67$. There are two transmitters v_0 and v_1 , and two receivers v_1 and v_2 . The energy cost is $\varepsilon(G_1) = \varepsilon_{Tx}(G_1) + \varepsilon_{Rx}(G_1) = 4$. The reachability-energy ratio is $\eta(G_1) = 0.67/4 = 0.167$.

In G_2 , $\lambda(G_2) = \text{mean}\{1, 0.9, 0.9\} = 0.93$. There is only one sender v_0 and thus $\varepsilon_{Tx}(G_2) = 1$. There are three receivers v_1 , v_2 and v_3 . Notice that a receiver cannot determine whether a transmission will succeed or not in advance. For any transmission, the receiver will pay the receiving cost no matter the transmission succeed or not. Therefore, $\varepsilon_{Rx}(G_2) = 3$, and $\varepsilon(G_2) = \varepsilon_{Tx}(G_2) + \varepsilon_{Rx}(G_2) = 3 + 1 = 4$. The reachability-energy ratio is $\eta(G_2) = 0.93/4 = 0.233$.

From this example, we make following observations.

First, lossy links do have the opportunity to produce more energy-efficient topologies. Compared with G_1 of no lossy link, G_2 exhibits higher network reachability with the same energy cost. Second, not all lossy links are energy-efficient. In this example, when the link $\{v_0, v_1\}$ has the reachability $\lambda_{\{v_0, v_1\}} = 0.1$, removing it from G_2 will result in a higher reachability-energy ratio. To find the appropriate set of links with the highest energy efficiency is the main objective of opportunity-based topology control. Third, the computation of network reachability becomes critical.

3 Problem Overview

In this section, we give an overview of opportunity-based topology control. We first present the assumptions and the problem classification. We then make a formal statement of

the problem, showing that this problem is NP-hard. Finally, we introduce the main challenges and research issues.

3.1 Assumptions and classifications

In this work, we assume individual link reachability is available, which can be obtained by sending periodic “Hello” messages, or by measuring the Link Quality Index (LQI). For the latter solution, many recent empirical studies have shown promising results (e.g., [18, 22]). We further assume that the link reachability is fixed. This assumption is reasonable as many empirical studies have shown that link reachability is pretty stable in a stationary environment. In real environments, links may be asymmetric as $\lambda_{\{u, v\}} \neq \lambda_{\{v, u\}}$. In this work we only consider sink-to-sensors communications, which are quite useful for many applications such as query issuing, command dissemination, new network program upload [14], and etc. Optimizations for other transmission patterns are left for future work. We do not consider the node failure as it can be easily transferred to the equivalent link failure. No node sleeping is considered in this work. No congestion or packet collision is considered either.

We assume wireless links can be well controlled so that the un-intended receivers are able to save receiving cost. In the example of Fig. 2, v_3 in G_1 will not pay receiving cost when the sink is transmitting, though v_3 may be in the sink’s transmission range. To do so, a control field is needed in RTS/CTS packet header for CSMA-based MAC, or a scheduling-based MAC such as T-MAC is needed as we did in this paper.

It is worth noting that different applications may have different requirements on opportunity-based topology control, leading to different problems and optimal solutions. In this work we only consider one of them, leaving more to future research work.

Reachability Preserving Problem (RPP): It is mainly for those data-critical applications such as disaster detections. In this problem, the objective is to minimize the energy cost while guaranteeing that the network reachability is no less than a given threshold $\lambda_{TH} \in (0, 1]$.

3.2 Reachability preserving problem (RPP)

The problem can be stated as follows:

Given a directed graph $G(V, E)$ and a threshold $\lambda_{TH} \in (0, 1]$, assuming $\lambda(G) \geq \lambda_{TH}$, the problem is to find a sub-network $G_R(V, E_R) \subseteq G$ such that:

$$\text{minimize} : \varepsilon(G_R) \quad (1)$$

$$\text{subject to} : \lambda(G_R) \geq \lambda_{TH} \quad (2)$$

Theorem 3.1 *RPP is NP-hard.*

Due to the space limitation, we omit the proof.

3.3 Key issue and main challenges

The main objective of RPP is to seek a sub-network $G_R \subseteq G$ of the minimized cost in subject to the constraint of $\lambda(G_R) \geq \lambda_{TH}$. The key issue is to guarantee a derived topology can satisfy the constraint. To do so, it is critical to approximate the network reachability as precise as possible. It mainly involves two challenges.

The first one is to compute the node reachability. This is a known NP-hard problem for a general network [23].

The second one is to compute the network reachability based on the node reachability. By Theorem 2.1, we know that the network reachability is the global average of the node reachability. Therefore it is a challenging job to obtain the network reachability in a fully distributed manner.

4 CONREAP Algorithm Design

In essential, CONREAP is a greedy algorithm. CONREAP starts from an empty topology. Each time it adds the link that contributes the highest network reachability to the topology.

In the next, we first introduce how to compute node reachability in a general network. We then present the two approximation algorithms employed by CONREAP, which are for the two challenges mentioned in Sec. 3.3. In the third part we give a detailed description of CONREAP. In the last, we show some analytical results.

4.1 Node reachability computation and approximations in CONREAP

As we mentioned before, the computation of node reachability for a general network is a known NP-hard problem.

Nevertheless, there are some specific topologies that can be computed in polynomial time [1]. By reliability theory, a general network must be converted into such forms to be practically computable. One is *series* topologies, in which there is a unique path from the source to the destination. Another form is *parallel* topology. Currently, only simple combinations of series and parallel topologies, called *series-parallel networks*, can be computed in polynomial time. More details of series-parallel networks can be found in the reference [25].

The first approximation algorithm is to deal with the challenge of the computation of the node reachability. We compute an approximation of the node reachability instead of the accurate one. We use link-disjoint trees as the building block during CONREAP design. The principle behind is that trees are series topologies. The node reachability in a tree can be easily obtained by computing the

Figure 3. An example of CONREAP; the left-up is the original topology; T_1 and T_2 are disjoint-trees (no common link); they merge to derive the final topology G_R

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CONREAP( $v$ )
1.  $\bar{\lambda}_{G_R}(v) = \lambda_{T_i}(v) = 0, i = 1, \dots; E_R = \emptyset$ 
2. send "Hello" to probe neighbor set  $Nb_v$ 
   measure the link reliability  $\lambda(u, v)$  for  $\forall u \in Nb_v$ 
3. do
4.   receive tree information  $\lambda_{T_i}(u), \forall u \in Nb_v$ 
5.    $u_i = \arg \max \{ \lambda_{T_i}(u) \cdot \lambda_{\{u_i, v\}} \} \wedge M_{\{u_i, v\}} = false.$ 
6.    $\lambda_{T_i}(v) = \lambda_{T_i}(u_i) \cdot \lambda_{\{u_i, v\}}, E_i = E_i \cup \{u_i, v\}$ 
7.    $T_k = \arg \max \{ \lambda_{T_i}(v), i = 1, \dots \}$ 
8.    $E_R = E_R \cup \{u_i, v\}; M_{\{u_i, v\}} = true$ 
9.    $\bar{\lambda}_{G_R}(v) = 1 - (1 - \bar{\lambda}_{G_R}(v))(1 - \lambda_{T_k}(v))$ 
10.  broadcast  $\lambda_{T_i}(v), i = 1 \dots k$  to  $Nb_v$ 
11.  exchange  $\bar{\lambda}_{G_R}(v)$  with neighboring set  $Nb_v$ 
12.   $avg(\bar{\lambda}_{G_R}(v)) = \frac{\sum_{u \in Nb_v} \bar{\lambda}_{G_R}(u)}{|Nb_v|}$ 
13. while ( $avg(\bar{\lambda}_{G_R}(v)) < \lambda_{TH}$ )

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Figure 4. CONREAP algorithm running on node v

probability along the path. When multiple trees are constructed, these paths are independent and parallel if trees are link-disjoint to each other. As such, the inclusion-exclusion principle can be applied.

To deal with the second challenge, we design another approximation algorithm to compute the global average in a fully distributed manner. The idea is that every node computes a local average from its neighbors. Observe that the global average is no less than the minimal of local averages (proved in Sec. 4.3). The global average must be above the threshold if all local averages are above the threshold. As such the constraint $\lambda(G_R) \geq \lambda_{TH}$ can be verified locally.

In the next, we show how we implement these two approximation algorithms in CONREAP.

4.2 Detailed description

Fig. 4 shows CONREAP's pseudocode and Fig. 3 gives an example of running CONREAP on a node v_1 .

Initially, v broadcasts a "Hello" message and initials its neighbor set Nb_v . At line 3-13, v enters the main loop of CONREAP. At line 4, v receives the tree propagation in-

formation $\lambda_{T_i}(u), \forall u \in Nb_v$ from Nb_v (Such information is originated from the sink, i.e., $\lambda_{T_i}(\text{sink}) = 1, i = 1 \dots d_G(\text{sink})$). Line 5 is the key step by which CONREAP implements the Dijkstra's shortest path algorithm. The link reachability is used as the distance function. More specifically, from the known $\lambda_{T_i}(u)$, v selects a node u_i that provides v the highest tree reachability $\lambda_{T_i}(v)$ as its parent node in the tree, i.e., $u_i = \arg \max \{ \lambda_{T_i}(u) \cdot \lambda_{\{u_i, v\}} \}$. One additional condition is that $M_{\{u_i, v\}} = \text{false}$, which means that the link $\{u_i, v\}$ has not yet been used. This additional condition ensures that different trees are link-disjoint. In the example of Fig. 3, v_1 selects u_1 as its parent in T_1 and u_2 in T_2 . The node v then updates $\lambda_{T_i}(v) = \lambda_{T_i}(u_i) \cdot \lambda_{\{u_i, v\}}$ at line 6. The line 5 and line 6 is operated for every tree T_i . When $\lambda_{T_i}(v)$ is available, CONREAP is ready to select and include trees in G_R . Line 7 is the step of the operation. At line 7, node v selects the tree with the highest $\lambda_{T_i}(v)$ to join, denoted as $T_k = \arg \max \{ \lambda_{T_i}(v), i = 1, \dots \}$. Upon determining T_k , at line 8 v unions the so-far G_R with the link $\{u_i, v\}$ by storing this record in the local storage. Line 8 also marks the link $\{u_i, v\}$ as "used" ($M_{\{u_i, v\}} = \text{true}$). Line 9 is the essential part of the first approximation algorithm which implements the node reachability update function. By inclusion-exclusion principle, the node reachability is $\tilde{\lambda}_{G_R}(v) = 1 - \prod_{i=1}^K (1 - \lambda_{T_i}(v))$ when there are K trees. It can also be written in an iterative form (line 9) which is more suitable when K is not known in advance. At line 10-11, v broadcasts all the tree information $\lambda_{T_i}(v)$ to Nb_v . v also exchanges the updated $\tilde{\lambda}_{G_R}(v)$ with Nb_v . Upon receiving the Nb_v node reachability, v updates $\text{avg}(\tilde{\lambda}_{G_R}(v))$ by line 12. CONREAP terminates when $\text{avg}(\tilde{\lambda}(v)) \geq \lambda_{TH}$.

It is possible that a node cannot satisfy the constraint when it joins all the available trees. In that extreme case, the original topology is kept.

4.3 Analytical results

To show the correctness of CONREAP, in this subsection we prove $\lambda(G_R) \geq \lambda_{TH}$ when CONREAP terminates. Based on CONREAP, we can state $\forall v \in V, \text{avg}(\tilde{\lambda}_{G_R}(v)) \geq \lambda_{TH}$. Therefore, it is sufficient to prove that $\lambda(G_R) \geq \min\{\text{avg}(\tilde{\lambda}_{G_R}(v))\}$.

To simplify the presentation, we define *hub graph* as follows. Given an arbitrary graph $G(V, E)$, if G contains a node v that has links to every other node, v would have $\text{avg}(\lambda_G(v)) = \lambda(G)$. We call such a graph a hub graph.

Lemma 4.1 $\lambda(G) \geq \min(\text{avg}(\lambda_G(v))), \forall v \in G$ when G is a hub graph.

Lemma 4.2 $\lambda(G) \geq \min(\text{avg}(\lambda_G(v)))$ for any graph G .

Lemma 4.3 $\forall v \in V, \lambda_{G_R}(v) \geq \tilde{\lambda}_{G_R}(v)$

Theorem 4.4 $\lambda(G_R) \geq \min\{\text{avg}(\tilde{\lambda}_{G_R}(v)), v \in V\}$

Due to the space limitation, we omit the proof of these lemmas and theorem.

For the worst running time of CONREAP, since different trees are disjoint to each other, a link can be added in at most one tree. In the worst case, all the links are added. Therefore, the worst running time is $O(|E|)$ where $|E|$ is the link set of the original network. The space requirement for individual nodes depends on the number of trees constructed. For each node, the maximum number of trees it can join is its degree d in the original network. Therefore, the space requirement is $O(d)$.

5 Performance Evaluation

In this section we evaluate the performance of CONREAP. We design and implement a prototype system which consists of one sink node and 50 Berkeley Mica2 nodes [7]. These nodes are uniformly deployed in our laboratory at random (indoor). The transmission power is set to be -10dbm so that the maximal distance is about 5 hops. Link reachability λ_e is measured using 1000 "Hello" messages for each link. To measure the node reachability, the sink nodes broadcasts 1000 packets and other nodes count the number of successfully received packets. Each measurement is averaged from 5 independent runs. In order to save the cost of un-intended receivers, we adopt a scheduling-based MAC protocol T-MAC.

Our experiments apply following evaluation metrics:

- The network reachability $\lambda(G)$;
- The energy cost $\epsilon(G)$;
- The reachability-energy ratio $\eta(G)$.

We compared CONREAP with a recent Cone-Based Topology Control (CBTC) [11] algorithm which represents the connectivity-based topology control algorithm. The derived topology by CBTC is denoted as G_{CBTC} . And the topology of CONREAP is G_R .

5.1 Prototype system results

Fig. 5 depicts the network reachability of CONREAP and CBTC against the energy cost. As only reliable links are taken into account, CBTC has a steady performance regardless of the target network reachability. $\lambda(G_{CBTC})$ is almost linearly proportional to its cost. The upper limit of network reachability is about 90% since in a real setting wireless links are hardly 100% reliable. In our experiments we also implement CBTC using lossy links as reliable ones. In such settings the network reachability can never exceed

Figure 5. $\lambda(G_{CBTC})$ and $\lambda(G_R)$ against network energy cost in the prototype system

Figure 6. $\lambda(G_{CBTC})$ and $\lambda(G_R)$ against the threshold λ_{TH} in the prototype system

Figure 7. $\lambda(G_{CBTC})$ and $\lambda(G_R)$ against the threshold λ_{TH} in the prototype system

0.2 (not shown) even when the whole network is connected. This is nature as lossy links have no guarantee of delivery. When they are used as reliable ones, the network will suffer from extremely low delivery rate. Since such low network reachability can hardly satisfy the constraint, we do not consider this approach in latter studies.

CONREAP constructs more than four trees to achieve $\lambda(G_R) \geq 0.98$. Each tree has the cost about 70 units of energy. The first tree provides $\lambda(G_R) = 0.82$, compared with $\lambda(G_{CBTC}) = 0.32$ under the same cost. To further increase the network reachability, the cost becomes high. The second tree improves $\lambda(G_R)$ from 0.82 to 0.91. And the third tree only has 0.03 improvement. These results are mainly because the first tree has used up most high reachability links. To further increase the reachability, those low reachability links have to be employed. It can also be seen from Fig. 6 that plots reachability-energy ratio against the network reachability. For $\lambda(G_R)$ from 0 to 0.8, $\eta(G_R)$ is steadily 1.2×10^{-3} , three times of $\eta(G_{CBTC}) = 0.4 \times 10^{-3}$. From 0.82 to 0.91, $\eta(G_R)$ decreases to 0.6×10^{-3} and when more than three trees are constructed, CONREAP is similar to CBTC.

Fig. 7 depicts network reachability $\lambda(G_{CBTC})$ and $\lambda(G_R)$ against λ_{TH} . From this figure we can investigate the performance of the two approximation algorithms in CONREAP. We see that since CBTC use only reliable links, the network reachability is nearly the same as the threshold. For CONREAP algorithm, first we see that it can guarantee a satisfied constraint when it terminates. However, there is a big gap between $\lambda(G_R)$ and λ_{TH} . When more trees are constructed, the gap becomes larger. It implies that many links in G_R are not necessary. With better approximation algorithms, there is a further improvement space.

6 Related Work

In this section we give an overview of related works. As energy is the most concern in WSNs, many valuable efforts

have been made in literature.

One category is the traditional connectivity-based topology control [21]. While the network connectivity is preserved, various energy-saving schemes have been adopted. One such scheme is by reducing the transmission power of individual nodes. As this problem is NPC, a lot of heuristics have been proposed such as MST-based [17, 20] and Cone-Based algorithms (CBTC) [11, 24]. Notice that [11] does not need location information such as [16]. Very interesting, a recent work [13] also noticed the impact of lossy links to the topology control. The authors studied the network reachability (but a different term) using the probabilistic model instead of the traditional deterministic model. The results are however, still preliminary as only a specific topology is studied. The result cannot be applied to a general network. Further more, no energy issue is studied.

Another energy-saving scheme is to reduce the transmitter set by seeking the Minimal Connected Dominating Set (MCDS) of the network. It is also a known NPC problem [9]. Many heuristic algorithms were proposed such as [25]. We prove reachability preserving problem is NP-hard based on MCDS problem. However, MCDS is also a connectivity-based topology control algorithm and thus no lossy link advantage is taken.

Designers of routing algorithms also noticed the transitional region phenomenon in WSNs. In particular, ExOR [2] and MORE [5] are proposed to explore packet overhearing along lossy links in routing. These opportunistic routing schemes are however, based on the network topology that provided by topology control algorithm. Without appropriate support from the beneath topology control, routing algorithms can hardly find those energy-efficient routing schemes. For the example in Fig. 2, no opportunistic routing can be carried out when the beneath topology is G_2 . In that sense, opportunity-based topology control aims at providing energy-efficient topology service for upper routing-layer protocols.

7 Conclusion and Future Work

In this paper, we identify and highlight the opportunity of lossy links during energy-efficient topology control. To seize this opportunity, we propose a novel opportunity-based topology control. We focus on the reachability preserving problem and show that this problem is NP-hard. In order to address this problem in a practical way, we propose CONREAP algorithm by exploring reliability theory. We prove that CONREAP has the guaranteed network reachability and the energy cost can be significantly reduced. The worst running time is $O(|E|)$ and the space requirement is $O(d)$. Experimental results show that CONREAP is more appropriate for low requirement, large transitional region environments. Compared with connectivity-based topology control, CONREAP can improve the energy efficiency up to 6 times.

Future work can be carried out along following directions. First, we study the reachability preserving problem and leave other two problems open. Second, joint consideration with adjustable transmission power is also interesting. Third, we consider communications from sink to sensors. Optimizing topologies for other patterns are also important. Finally, some practical considerations are in concern. For example, we assume fixed link reachability which is a function of time in practice. All these issues are calling for further investigations.

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