Calibration of models for pile settlement analysis using 64 field load tests

L.M. Zhang, Y. Xu, and W.H. Tang

Abstract: Due to the presence of uncertainties, errors inevitably arise with the estimations of pile settlement. To properly consider serviceability requirements in limit state design, it is necessary to characterize the performance of commonly used settlement prediction models. In this work, information from 64 cases of long driven steel H-piles from field static loading tests in Hong Kong is utilized to evaluate the errors of three settlement prediction models for single piles: two elastic methods and a nonlinear load-transfer method. Commonly adopted soil parameters recommended in two Hong Kong design guidelines are used to reflect the uncertainty arising from evaluation of soil properties. The model error is represented by a bias factor. A conventional statistical analysis was first conducted to study the variability of model bias. A regression analysis method was then proposed as a supplemental analysis of model bias when only limited test data were available or when the measured settlement data distribute in a large range. Both methods result in very similar mean biases. The mean bias of each prediction model tends to vary with the load level and the bearing stratum at the pile toe; while the coefficient of variation of model bias only varies in narrow ranges.

Key words: driven piles, limit state design, model calibration, pile foundations, settlement analysis, serviceability.

Résumé : À cause de la présence d'incertitudes, des erreurs se produisent inévitablement dans les estimations du tassement des pieux. Pour prendre en considération correctement les exigences de résistance dans la conception à l'état limite, il est nécessaire de caractériser la performance des modèles de prédiction de tassement utilisés communément. Dans ce travail, on utilise les informations d'essais de chargement statique faits sur le terrain à Hong Kong sur 64 cas de longs pieux H en acier foncés pour évaluer les erreurs de trois modèles de prédiction de tassement pour des pieux simples : deux méthodes élastiques et une méthode non linéaire de transfert de charge. On utilise les paramètres de sol communément adoptés et recommandés dans deux guides de conception de Hong Kong pour refléter l'incertitude provenant de l'évaluation des propriétés des sols. L'erreur du modèle est représentée par un facteur de différence systématique du modèle. On a réalisé en premier lieu une analyse statistique conventionnelle pour étudier la variabilité de la différence systématique du modèle. On a alors proposé une méthode d'analyse de régression comme analyse supplémentaire de la différence systématique du modèle lorsque seulement un nombre limité de données d'essai étaient disponibles ou lorsque les données de tassement mesuré se répartissaient sur une large plage. Les deux méthodes donnent des différences systématiques moyennes très semblables. La différence systématique moyenne de chaque modèle de prédiction a tendance à varier avec le niveau de charge et la strate portante à la pointe du pieu; alors que le coefficient de variation de la différence systématique du modèle ne varie que dans des plages étroites.

Mots-clés : pieux foncés, conception à l'état limite, calibrage de modèle, fondations sur pieux, analyse de tassement, utilité.

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Introduction

A number of methods have been used to estimate pile settlement. Due to the presence of uncertainties, errors inevitably arise with the estimations. Two primary sources of error can be distinguished: those arising from the evaluation of input design parameters, such as soil properties, and those arising from geotechnical calculation models. Phoon and Kulhawy (1999), Baecher and Christian (2003), Zhang et al. (2004), Fenton and Griffiths (2005), Fenton et al. (2005) and many others have studied the sources of uncertainty from

the evaluation of soil properties. Even if input design parameters have been estimated correctly, the prediction from a geotechnical calculation model is still expected to deviate from reality due to the systematic errors associated with the model itself (Der Kiureghian 1990; Phoon and Kulhawy 2005). The model errors may be from insufficient representation of the constitutive behavior of materials, simplification of boundary conditions, use of empirical assumptions, and numerical errors.

To develop limit-state design codes, models for both ultimate and serviceability limit states need to be calibrated (e.g., Becker 1996a, 1996b). Many models for pile-capacity prediction have been calibrated (e.g., Briaud and Tucker 1988; Paikowsky 2002; Zhang et al. 2001, 2005; Phoon and Kulhawy 2005). Yet the evaluation of models for pile-movement prediction has not received the same level of attention. Several investigators conducted surveys of model errors associated with settlement prediction. Briaud and Tucker (1988) evaluated five load-transfer methods for

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predicting the single pile settlement based on a 98 pile load test database obtained from the Mississippi State Highway Department. The piles included 64 square concrete piles, 27 H-piles, and 7 drilled shafts. The pile lengths were 3–25 m, with an average of 12.2 m. The accuracy of each method was quantified statistically at one half of the ultimate load. The American Society of Civil Engineers (ASCE) held a pile prediction symposium at Northwestern University to compare the predicted and observed axial behaviors of piles including the load–settlement response (Finno et al. 1989). Paikowsky and Lu (2006) discussed the uncertainties of several analysis methods for calculating deep foundation displacements. In general, pile settlement predictions were widely scattered, but few attempts have been made to calibrate commonly used prediction models.

Ideally, errors in input parameters and models should be calibrated separately. In this paper, since a standard procedure is followed to determine input parameters, the “model error” is broadly taken as the lumped error caused by both input parameters and prediction models. The model error is represented by a bias factor, λ , which is the ratio of the measured settlement, S_m , to the predicted settlement, S_p (Shahin et al. 2005)

$$[1] \quad \lambda = \frac{S_m}{S_p}$$

Model errors are often evaluated based on field or model tests. In this study, performance data from 64 cases of very long driven steel H-piles from field static loading tests in Hong Kong are utilized to evaluate model errors. These test piles were constructed in weathered soils that are commonly encountered in Hong Kong, Mainland China, and many southeast Asian countries. Three typical settlement prediction models for single piles are calibrated: two elastic methods and a nonlinear load–transfer method. Commonly adopted soil parameters recommended in two Hong Kong design guidelines (GEO 1993, 2006) are used to reflect the uncertainty arising from the evaluation of soil properties. The mean model bias and the coefficient of variation (COV) under a particular load (the intended design load or twice the design load) are established through a statistical analysis. Furthermore, a regression analysis is proposed to study the model bias at various load levels, particularly for the failure load at which limited test data are available.

Field static loading tests on driven piles

A database of dynamic and static tests conducted on steel H-piles in Hong Kong, the Hong Kong Driven Pile Database, was established (Zhang et al. 2006). In this study, the information from 64 cases of field static loading tests conducted in the past decade in five sites was selected for model calibration. Table 1 shows the general information from these load tests. The pile toe was driven to slightly (grade II) to completely (grade V) decomposed rocks. The corresponding penetration lengths of the test piles varied from 21.6 to 78 m with an average of 45.7 m. Two sizes of steel H-piles were used in the field tests: 305 mm $\times$ 305 mm $\times$ 180 kg/m piles (15 cases) and 305 mm $\times$ 305 mm $\times$ 223 kg/m piles (49 cases). Detailed properties of the pile sections are summarized in Table 2.

Within the 64 test cases, 33 were conducted on working piles and 31 were conducted on preliminary piles (Table 1). Two and three load cycles were applied on working piles and preliminary piles, respectively, following the load-test procedures outlined by Hong Kong Housing Authority (2000) or Buildings Department (2000). For working piles, the peak loads of the two load cycles corresponded to one times ($1\times$) and twice ($2\times$) the design compression capacity of the pile (intended design load), respectively, which were 2950 kN for 305 mm $\times$ 305 mm $\times$ 180 kg/m piles and 3548 kN for 305 mm $\times$ 305 mm $\times$ 223 kg/m piles. For preliminary piles, in addition to the intended design load and twice the design load, the piles were loaded to failure in terms of either the geotechnical capacity or the structural capacity. In this work, the failure load is cut off at three times the design load if it is larger than three times the design load. A typical load–settlement curve involving three load cycles from a load test and the corresponding simplified virgin load curve are shown in Fig. 1. Figure 2 presents the simplified load–settlement curves for the 64 load tests.

Models for settlement analysis

A number of settlement analysis methods for single piles are available. These methods may be broadly classified into three categories: elastic continuum methods, load–transfer methods, and numerical methods (GEO 2006). Examples of such methods are the elastic methods proposed by Vesic (1977) and Poulos and Davis (1980), the simplified elastic methods proposed by Randolph and Wroth (1978) and Fleming et al. (1992), the nonlinear load–transfer methods proposed by Coyle and Reese (1966) and McVay et al. (1989), and the numerical methods based on advanced constitutive models of soil behavior proposed by Jardine et al. (1986). In this paper, three representative methods are adopted for the calibration exercise: the elastic method proposed by Vesic (1977), the simplified analysis method proposed by Fleming et al. (1992), and a nonlinear load–transfer method (McVay et al. 1989) implemented in program FB-Pier (BSI 2003).

In Vesic’s method, the settlement of a pile under vertical loading, S , includes three components

$$[2] \quad S = S_1 + S_2 + S_3$$

where S_1 is the elastic pile compression; S_2 is the pile settlement caused by the load at the pile toe; and S_3 is the pile settlement caused by the load transmitted along the pile shaft. If the pile material is assumed to be elastic, the elastic pile compression can be calculated by

$$[3] \quad S_1 = \frac{(Q_b + \xi Q_s)L}{A_p E_p}$$

where Q_b and Q_s are the loads carried by the pile toe and pile shaft, respectively; A_p is the pile cross-section area; L is the pile length; E_p is the modulus of elasticity of the pile material; and ξ is a coefficient depending on the nature of unit friction resistance distribution along the pile shaft. In this work, the distribution is assumed to be uniform and hence $\xi = 0.5$. Settlement S_2 may be expressed in a form similar to that for a shallow foundation

Table 1. General information from 64 tests on long driven piles.

No.	Weight per unit length (kg/m)	Length (m)	Material at pile toe	Load cycles	Site No. ^a	No.	Weight per unit length (kg/m)	Length (m)	Material at pile toe	Load cycles	Site No. ^a
1	180	28.4	Grade II-III tuff	3	1	33	223	59.3	Grade IV - V granite	2	4
2	180	40.6	Grade II-III tuff	3	1	34	223	43.0	Grade IV - V granite	2	4
3	180	40.2	Grade II-III tuff	3	1	35	223	51.9	Grade IV - V granite	2	4
4	180	49.3	Grade II-III tuff	3	1	36	223	45.3	Grade IV - V granite	2	4
5	180	21.6	Grade II-III tuff	3	1	37	223	45.4	Grade IV - V granite	2	4
6	180	46.4	Grade II-III tuff	3	1	38	223	42.1	Grade IV - V granite	2	4
7	180	41.4	Grade II-III tuff	3	1	39	223	56.1	Grade IV - V granite	2	4
8	180	46.5	Grade II - III rhyolite	3	2	40	223	54.2	Grade IV - V granite	2	4
9	180	32.5	Grade II - III rhyolite	3	2	41	223	52.5	Grade IV - V granite	2	4
10	180	32.5	Grade II - III rhyolite	3	3	42	223	48.5	Grade IV - V granite	2	4
11	180	55.0	Grade IV - V rhyolite	3	3	43	223	45.2	Grade IV - V granite	2	4
12	180	78.0	Grade IV - V rhyolite	3	3	44	223	47.8	Grade IV - V granite	2	4
13	180	63.5	Grade IV - V rhyolite	3	3	45	223	47.0	Grade IV - V granite	2	4
14	180	56.5	Grade II - III rhyolite	3	3	46	223	58.1	Grade IV - V granite	2	4
15	180	62.4	Grade II - III rhyolite	3	3	47	223	48.1	Grade IV - V granite	2	4
16	223	45.5	Grade II - III granite	2	4	48	223	44.0	Grade IV - V granite	2	4
17	223	36.0	Grade II - III granite	2	4	49	223	52.7	Grade IV - V granite	2	4
18	223	36.5	Grade II - III granite	2	4	50	223	42.1	Grade IV - V granite	2	4
19	223	37.2	Grade II - III granite	2	4	51	223	55.6	Grade IV - V granite	3	4
20	223	35.5	Grade II - III granite	2	4	52	223	45.1	Grade IV - V granite	3	4
21	223	34.6	Grade II - III granite	2	4	53	223	55.4	Grade IV - V granite	3	4
22	223	39.0	Grade II - III granite	2	4	54	223	47.3	Grade IV - V granite	3	4
23	223	35.7	Grade II - III granite	2	4	55	223	58.8	Grade IV - V granite	3	4
24	223	36.4	Grade II - III granite	2	4	56	223	53.1	Grade IV - V granite	3	4
25	223	37.8	Grade II - III granite	2	4	57	223	59.8	Grade IV - V granite	3	4
26	223	38.0	Grade II - III granite	2	4	58	223	58.0	Grade IV - V granite	3	4
27	223	39.1	Grade II - III granite	2	4	59	223	51.0	Grade IV - V granite	3	4
28	223	33.3	Grade II - III granite	2	4	60	223	52.0	Grade IV - V granite	3	4
29	223	34.2	Grade II - III granite	3	4	61	223	42.9	Grade IV - V granite	3	5
30	223	38.6	Grade II - III granite	3	4	62	223	35.6	Grade II - III granite	3	5
31	223	54.4	Grade IV - V granite	2	4	63	223	32.8	Grade II - III granite	3	5
32	223	53.0	Grade IV - V granite	2	4	64	223	32.0	Grade II - III granite	3	5

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Table 2. Detailed information on 305 mm × 305 mm × 180 kg/m and 305 mm × 305 mm × 223 kg/m steel H-piles.

	Pile size	
	305 mm × 305 mm × 180 kg/m	305 mm × 305 mm × 223 kg/m
Young's modulus (MPa)	205 000	205 000
Poisson's ratio	0.3	0.3
Shear modulus (MPa)	78 846	78 846
Cross sectional area (m ²)	0.022 9	0.028 5
Width (m)	0.319 7	0.325 7
Depth (m)	0.326 7	0.337 9
Area of pile base (m ²)	0.104 4	0.110 1
Web thickness (m)	0.024 8	0.030 3
Flange thickness (m)	0.024 8	0.030 4
Inertial 2-axis (m ⁴)	0.000 41	0.000 53
Inertial 3-axis (m ⁴)	0.000 14	0.000 18
Torsional inertial (m ⁴)	0.000 005 55	0.000 010 37

Fig. 1. Typical load–settlement curve involving three loading cycles from a field load test and the corresponding simplified virgin load curve.

Fig. 2. Load–settlement curves from 64 field load tests.

$$[4] \quad S_2 = \frac{q_b D}{E_{sb}} (1 - \nu^2) I_b$$

where D is the pile width or diameter; q_b is the load per unit area at the pile toe, $q_b = Q_b/A_b$; A_b is the pile base area; E_{sb} is the modulus of elasticity of the soil at the pile toe; ν is

Poisson's ratio; and I_b is an influence factor, generally $I_b = 0.85$. Finally, S_3 is given as

$$[5] \quad S_3 = \left(\frac{Q_s}{pL} \right) \frac{D}{E_{ss}} (1 - \nu^2) I_s$$

where p is the pile perimeter; E_{ss} is the modulus of elasticity of the soil along the pile shaft; and I_s is an influence factor. The influence factor I_s can be calculated by an empirical relation (Vesic 1977)

$$[6] \quad I_s = 2 + 0.35 \sqrt{\frac{L}{D}}$$

With Vesic's method, both Q_b and Q_s are required. In this work, Q_b and Q_s are obtained using two methods. In the first method (Vesic's method I), these two loads are determined from a nonlinear load–transfer method, which will be introduced later. In the second method (Vesic's method II), these two loads are determined using empirical ratios of Q_b to the total load applied on pile Q based on field test data. Shek (2005) reported load–transfer in 14 test piles, including 11 piles founded in soil and 3 piles founded on rock. The mean ratios of Q_b/Q for the piles founded in soil and the piles founded on rock are summarized in Table 3 and applied in this calibration exercise. The mean values of Q_b/Q at twice the design load and the failure load are very similar. Hence, the average of the mean values is adopted for calibration at both twice the design load and the failure load.

In the Fleming et al. method, the settlement of a pile is given by the following approximate closed-form solution (Fleming et al. 1992):

$$[7] \quad \frac{Q}{G_L r_0 S} = \frac{\frac{4\eta}{(1-\nu)\xi_G} + \frac{2\pi\rho \tanh(\mu L)}{\zeta} \frac{L}{\mu L} \frac{r_0}{r_0}}{1 + \frac{4\eta}{\pi\lambda_p(1-\nu)\xi_G} \frac{\tanh(\mu L)}{\mu L} \frac{L}{r_0}}$$

where $\eta = r_b/r_0$, r_0 and r_b are the radii of the pile shaft and pile toe, respectively (for H-piles, $\pi r_0^2 = \pi r_b^2 = Dh$, h is the depth of the pile cross-section); $\xi_G = G_L/G_b$, G_L is the shear modulus of the soil at depth L , and G_b is the shear modulus of the soil beneath the pile toe; $\rho = G_{ave}/G_L$, G_{ave} is the average shear modulus of the soil along the pile shaft; λ_p is the pile stiffness ratio, $\lambda_p = E_p/G_L$; $\zeta = \ln\{[0.25 + (2.5\rho(1-\nu) - 0.25)\xi_G] L/r_0\}$; $\mu L = (2/\zeta\lambda_p)^{1/2} (L/r_0)$. If the slenderness ratio L/r_0 is less than $0.5\lambda_p^{1/2}$, the pile may be treated as effectively rigid and eq. [7] then reduces to

$$[8] \quad \frac{Q}{G_L r_0 S} = \frac{4\eta}{(1-\nu)\xi_G} + \frac{2\pi\rho L}{r_0}$$

If the slenderness ratio L/r_0 is larger than $3\lambda_p^{1/2}$, the pile may be treated as infinitely long, and eq. [7] then reduces to

$$[9] \quad \frac{Q}{G_L r_0 S} = \pi\rho \left(\frac{2\lambda_p}{\zeta} \right)^{1/2}$$

In this case, G_L' is the soil shear modulus at the bottom of the active pile length L_{ac} , where $L_{ac} = 3r_0\lambda_p^{1/2}$.

In the nonlinear load–transfer method implemented in FB-Pier, the axial τ – Z curve for modeling the pile–soil interaction along the pile is given as (McVay et al. 1989)

Table 3. Empirical ratios of Q_b to total load Q based on field static loading tests.

	At the intended design load	At twice the design load or the failure load
Piles founded in soil	0.19	0.32
Piles founded on rock	0.53	0.63

$$[10] \quad Z = \frac{\tau_0 r_0}{G_{is}} \left[\ln \frac{(r_m - \beta)}{(r_0 - \beta)} + \frac{\beta(r_m - r_0)}{(r_m - \beta)(r_0 - \beta)} \right]$$

where $\beta = r_0 \tau_0 / \tau_f$, τ_0 is the shear stress being transferred to the soil for a given settlement Z ; τ_f is the ultimate shear stress transferred to the soil; r_m is the radius out from the pile shaft where axial loading effects on soil are negligible, assumed to be the pile length times (one minus the soil's Poisson's ratio) times the ratio of the soil's shear modulus at the pile center to the value at the pile toe; G_{is} is the initial shear modulus of the soil along the pile shaft. The nonlinear Q - Z relationship for the pile toe is given as (McVay et al. 1989)

$$[11] \quad Z = \frac{Q_b(1 - \nu)}{4r_0 G_{ib} \left[1 - \frac{Q_b}{Q_f} \right]^2}$$

where Q_f is the ultimate toe resistance and G_{ib} is the initial shear modulus of the soil at the pile toe.

Soil parameters

Most of the 64 test piles were constructed in reclaimed grounds. Before evaluating input soil parameters, the borehole records for the field tests are introduced briefly using a typical borehole record shown in Fig. 3. The primary information in Fig. 3 includes water level, blow count “ N ” from the standard penetration test (SPT), weathering grade, and detailed soil description. According to the borehole record, the ground profile can be simplified as four successive strata: fill, marine deposits (not present in some cases), alluvium, and finally, decomposed rock. Each layer is assumed to be homogeneous and isotropic sandy soil, because sand or silt consists of the majority of these strata. The pile toe was driven to the decomposed rock. A weathering grade system (GEO 2006) was used to determine whether the decomposed rock material is considered to be soil (grade IV or V) or rock (grade II or III). Ultimately, the ground profile is simplified for analysis as shown on the right side of Fig. 3. Based on the pile toe soil or rock conditions, the 64 pile cases are further divided into two groups: 34 cases of piles founded in soil (20 working piles and 14 preliminary piles) and 30 cases founded on rock (13 working piles and 17 preliminary piles).

The basic soil parameters required for the three prediction models are summarized in Table 4. With Vesic's method, the soil parameters required are Poisson's ratio ν and Young's modulus E_s . Poisson's ratio is assumed to be 0.3, as commonly used in practice, and E_s can be estimated using an empirical correlation with the SPT “ N ” value: $E_s = 2N$ MPa (GEO 2006). According to the borehole records, the SPT “ N ” values for fill, marine deposits, and alluvium are generally less than 50 while those for the completely decomposed rocks are often larger than 50. In this work, the SPT “ N ”

value is cut off at 500 if it is larger than 500. To be consistent with the practice in Hong Kong (GEO 2006), E_s is adjusted as $E_s = 1.7N$ MPa if the SPT “ N ” value is less than 50, and $E_s = 2N$ MPa if the SPT “ N ” value is larger than 50. The Young's modulus of the i th soil layer E_{si} is calculated using the average SPT “ N ” value of this soil layer; and E_{sb} is calculated using the SPT “ N ” value of the soil around the pile toe. Following Poulos and Davis (1980), the average Young's modulus of the soils along the pile shaft, E_{ave} , is calculated by

$$[12] \quad E_{ave} = \left(\frac{1}{L} \right) \sum_i E_{si} L_i$$

where L_i is the thickness of the i th soil layer.

With Fleming et al.'s (1992) method, the following soil parameters are required: G_{ave} , G_L and G_b , and Poisson's ratio ν . The value of G_{ave} can be obtained using ν and E_{ave} , which can be obtained in the same way as in Vesic's method, and $G_L = 2G_{ave}$ if the soil modulus is assumed to increase linearly with depth; G_b can be obtained using ν and E_{sb} .

In the nonlinear load-transfer method implemented in FB-Pier, several inputs are required: water level L_w , total unit weight γ , Poisson's ratio ν , shear modulus G , internal friction angle ϕ , ultimate shear stress transferred to the soil τ_f , and axial ultimate toe resistance Q_f . Firstly, L_w is assumed to be the mean value from the borehole records, $L_w = 2$ m for the convenience of computation. Secondly, as most of the ground is submerged and considered to be fully saturated, γ is set to be the maximum of the typical values shown in Table 5 (GEO 1993), $\gamma = 21$ kN/m³ for all the strata. Thirdly, G can be obtained using ν and E_s , as in Vesic's method. Fourthly, ϕ can be determined using the empirical relationship with the SPT “ N ” value shown in Fig. 4 (Schmertmann 1975; GEO 1993). The maximum ϕ is cut off at 44° according to the ranges given in Table 5 (GEO 1993). Finally, τ_f of a particular soil layer and Q_f are calculated using the following equations (GEO 2006):

$$[13] \quad \tau_f = K_s \sigma'_{ave} \tan \delta_s$$

$$[14] \quad Q_f = N_q \sigma'_{v0} A_b$$

where K_s is the coefficient of horizontal earth pressure on pile for the soil layer; σ'_{ave} is the average effective stress of the soil layer; δ_s is the angle of friction at the pile-soil interface through the soil layer; N_q is a bearing capacity factor; and σ'_{v0} is the effective stress at pile toe. The values of K_s , δ_s , and N_q can be determined by their relationships with ϕ : $K_s = 1.5(1 - \sin \phi)$; $\delta_s = 0.75\phi$; and the Berezantzev et al. (1961) curve for N_q shown in Fig. 5. In this work, N_q is limited to an upper bound of 200 for the sake of safety.

Some test piles penetrated to the grade II or III rock layer, so pertinent parameters for the rocks at the pile toe should be used. The field unconfined compression strength of rock, q_{uf} , is set to be 25 MPa, which is the minimum value for grade II or III rock (GEO 2006) and is similar to the minimum value of 28 MPa obtained from the laboratory unconfined compression strength, q_{ul} , of 140 MPa considering scale effects recommended by Goodman (1980). For common Hong Kong rocks, the ratio of the laboratory modulus of deforma-

Fig. 3. A typical borehole record and the simplified profile.

In situ borehole records						Simplified profile				
Water level (m)/Time	SPT "N"/Depth (m)	Depth (m)	Legend	Weathering grade	Description	Description	Layer	Depth (m)	Average SPT "N"	Average water level (m)
1.60/18:00	7/1.92; 7/3.42	7.5			Loose, dark grey and pinkish brown, silty fine to coarse SAND with some angular fine to coarse gravel sized moderately strong rock and concrete fragments (FILL)	Fill		7.5	6	2.15
1.80/08:00	8/4.94; 2/6.41									
6.30/18:00										
1.55/08:00	7/7.94; 5/9.41	10.5			Soft, light grey and light pink, locally light brown, sandy silty CLAY with occasional subangular fine gravel sized quartz fragments (ALLUVIUM)	Alluvium		13.5	14	
2.50/18:00										
1.65/08:00										
0.90/18:00	12/11; 30/12.41	13.5		V	Medium dense, dark brown, slightly silty fine to coarse SAND with subangular fine to medium gravel sized quartz fragments (ALLUVIUM)	CDG (soil)		40	>500	
1.50/08:30										
1.60/14:30										
27/13.93; 28/15.48	1200*/30.12; 1714*/31.61	30.0		V/IV	Extremely weak, light pink and brown spotted white, completely decomposed medium grained GRANITE (Silty fine to coarse SAND with some fine to medium gravel sized quartz fragments)			45.5	>500	
31/16.92; 33/18.44										
44/19.95; 63/21.45										
49/22.96; 59/24.46	1000*/33.45; 2000*/34.58	40.0		III/II	Extremely weak to very weak, pink and light brownish pink, completely to highly decomposed medium grained GRANITE			45.5	>500	
59/25.93; 1091*/27.43										
1200*/30.12; 1714*/31.61										
1000*/33.45; 2000*/34.58	1200*/36.13; 2000*/37.56	45.5			Moderately strong, white and green, moderately decomposed medium grained GRANITE	MDG (rock)		45.5	>500	
1200*/36.13; 2000*/37.56										

*Note: The SPT test is terminated when the blow count reaches 200 if the penetration is smaller than 300 mm. For instance at depth 27.43 m, the penetration is only 55 mm after 200 blows; the calculated blow count is then $N = 200/55 \times 300 = 1091$.

Table 4. Basic soil parameter requirements for the three settlement prediction models.

	Water level, L_w	Total unit weight, γ	Young's modulus, E_s	Poisson's ratio, ν	Shear modulus, G	Internal friction angle, ϕ
Vesic's (1977) method	—	—	✓	✓	—	—
Fleming et al.'s (1992) method	—	—	✓	✓	✓	—
Load-transfer method (McVay et al. 1989)	✓	✓	✓	✓	✓	✓

Table 5. Typical values of geotechnical parameters for selected Hong Kong soils (after GEO 1993).

	Bulk unit weight, γ (kN/m ³)	Dry unit weight, γ_d (kN/m ³)	Shear strength parameters		
			Friction angle, ϕ' (°)	Cohesion, c' (kPa)	Mass permeability, k (m/s)
Compacted fill					
Completely decomposed granite	19–21	15–19	38–42	0–5	10^{-6} – 10^{-7}
Completely decomposed volcanic	18–21	15–19	35–38	0–5	10^{-6} – 10^{-8}
Crushed rock fill	18–21	18–21	45–>50	0	$>10^{-2}$ – 10^{-4}
In situ soil					
Completely decomposed granite	16–21	14–19	35–44	5–15	10^{-5} – 10^{-7}
Completely decomposed volcanic	16–21	14–19	32–38	5–10	10^{-5} – 10^{-7}
Colluvium	15–21	13–19	26–40	0–10	10^{-4} – 10^{-7}

tion of rock material, E_i , to q_{ul} generally lies between 200 and 500 (GEO 1993), and the mean value, 350, is adopted in this work. A mass factor, set to be 0.8 in this work (GEO 1993), is used to relate the field modulus of deformation of rock, E_f , to E_i . Then E_f is calculated as 39.2 GPa, similar to the minimum value of 38.6 GPa proposed by Peck (1969). In addition, ν is assumed to be 0.26, the mean of the typical

values proposed by Peck (1969). The ultimate unit toe resistance on rock can be expressed as (Goodman 1980)

$$[15] \quad q_p = q_{uf}(N_\phi + 1)$$

where $N_\phi = \tan^2(45^\circ + \phi/2)$. A safe ϕ of 44° , which is the same as the cut-off value adopted for soil, is assumed, re-

Fig. 4. Empirical method for estimating ϕ of sand from SPT “N” values (after Schmertmann 1975).

Fig. 5. Relationship between N_q and ϕ (after Berezantzev et al. 1961).

Fig. 6. Frequency diagrams of the model bias for piles founded in soil: (a) with Vesic’s method I at the intended design load; (b) with Vesic’s method I at twice the design load; (c) with Vesic’s method II at the intended design load; (d) with Vesic’s method II at twice the design load.

sulting in $N_{\varphi} = 5.55$. The corresponding q_p from eq. [15] is 139 MPa.

Statistical analysis of model uncertainty

By comparing the predicted settlements using the three performance models against the respective measured settlements, the model biases can be calculated with eq. [1] and

used to estimate the probability density function (PDF) of λ . Due to the limited number of preliminary pile cases, the available data at the third load cycle are limited and hence no statistical analysis is conducted for the model bias at the failure load.

Figures 6–11 show the histograms of the model bias for all the samples for different prediction models (Vesic’s method, Fleming et al.’s method, and the nonlinear load–

Fig. 7. Frequency diagrams of the model bias for piles founded on rock: (a) with Vesic's method I at the intended design load; (b) with Vesic's method I at twice the design load; (c) with Vesic's method II at the intended design load; (d) with Vesic's method II at twice the design load.

transfer method), load levels (the intended design load and twice the design load), and ground conditions (piles founded in soil and on rock). For compatibility with reliability analysis, it is convenient to assume that λ follows a lognormal distribution. The validity of this distribution has been confirmed using the chi-square goodness-of-fit test (e.g., Ang and Tang 2007) on the data in Figs. 6–11 at a significance level of 5%. The respective statistical parameters are estimated by the sample mean μ and the sample standard deviation σ , which are noted in Figs. 6–11. Based on the average biases and standard deviations, the lognormal PDFs are superimposed on the corresponding figures for comparison.

Both Vesic's method I and the nonlinear load-transfer method underestimate the pile settlement at the intended design load. The similar results with these two methods are due to the fact that the two required loads (i.e., Q_b and Q_s) in Vesic's method I are obtained using FB-Pier. Compared to the field load test results, the value of Q_b appears to be underestimated in FB-Pier while that of Q_s is overestimated, especially at the intended design load. It is noted that the settlement reduction due to a decrease in Q_b is larger than the settlement increment due to the same increment in Q_s with either Vesic's method or the load-transfer method. Therefore, the error in the distribution of Q_b and Q_s can lead to underestimation of pile settlement with these two methods. The values of Q_b and Q_s in Vesic's method II are obtained based on mean ratios of Q_b/Q (Table 3) from field tests; therefore the mean λ values from Vesic's method II are closer to unity than those from Vesic's method I. In ad-

dition, Fleming et al.'s method overestimates the pile settlement because of the assumption that an active part of pile exists and no load reaches the lower end, resulting in a conservative pile capacity. However, the mean λ at twice the design load with either Fleming et al.'s method or the nonlinear load-transfer method is closer to unity than that at the intended load. This can be explained by comparing the load-settlement curves from a pile-load test and from the calculation models, shown in Fig. 12. Fleming et al.'s method predicts a linear relation, thus the bias would increase with the applied load monotonically. The predicted curve from the nonlinear load-transfer method shows stronger nonlinearity than the measured curve, so the bias would decrease with the applied load.

To compare the prediction models more conveniently, the lognormal distributions of the model bias for piles founded in soil and on rock are summarized in Fig. 13. In general, the model biases are widely scattered. One possible reason is that the same relationship between the Young's modulus and the SPT "N" value was used for different prediction models and load levels. This is a problem for the elastic methods in particular because the soil modulus must decrease with the strain level. The measurement errors associated with the measured settlement S_m also contribute to the discrepancy.

Table 6 summarizes the model biases (mean μ and coefficient of variation COV) for the three prediction models from the statistical analysis. The mean bias of each prediction model tends to vary with the load level and the bearing str-

Fig. 8. Frequency diagrams of the model bias with Fleming et al.'s method for piles founded in soil: (a) at the intended design load; (b) at twice the design load.

tum at the pile toe. In contrast to the mean model bias, the COV of λ appears to fall within a narrow range of 0.19–0.28, despite prediction models, load levels, and ground conditions. The COV values are smaller than those found by Briaud and Tucker (1988); namely, COV is between 0.35 and 1.30 for driven pile samples larger than 10 cases. One reason may be that the test piles in this study were installed in five similar reclaimed grounds following similar construction procedures, thus leading to less uncertainty in the evaluation of soil parameters and less human errors from construction workmanship. Another reason is that the average pile length in this study is much larger (45.7 m versus 12.2 m), so that the variance in soil properties is reduced by the spatial averaging effect (e.g., Baecher and Christian 2003).

Regression analysis of model uncertainty

In the aforementioned statistical analysis, the model bias is studied under the intended design load or twice the design load. No analysis has been conducted for the failure load because only a few piles were loaded to failure. However, these limited available data under the failure load can be utilized for model calibration with a regression analysis. Based on the statistical results obtained, the model bias varies with

Fig. 9. Frequency diagrams of the model bias with Fleming et al.'s method for piles founded on rock: (a) at the intended design load; (b) at twice the design load.

the magnitude of load or settlement. To take into account this feature, eq. [1] is rewritten as

$$[16] \quad S_m = \lambda' S_p^{\beta'}$$

where λ' is a general model bias, different from the definition of λ and β' is a coefficient. Equation [16] can be converted to a linear regression equation

$$[17] \quad \ln \hat{S}_m = \beta'' \ln S_p + \ln \lambda''$$

where $\ln \hat{S}_m$ is the estimate of $\ln(S_m)$; and β'' and $\ln(\lambda'')$ are estimated regression coefficients. The scatter about the regression line, represented by a calibration error, $\sigma_{\ln(S_m) | \ln(S_p)}$, is assumed to be constant over the whole range of $\ln(S_p)$. The least squares estimates of β'' and $\ln(\lambda'')$ have been described in textbooks (e.g., Ang and Tang 2007). It is convenient to assume that S_m and S_p are both lognormally distributed. This is consistent with the previous assumption that λ , the ratio of S_m to S_p , follows a lognormal distribution. Hence, $\ln(S_m)$ and $\ln(S_p)$ all follow the normal distribution. Figures 14 and 15 show the relationships between $\ln \hat{S}_m$ and $\ln(S_p)$ from eight samples for the three prediction models and the two pile-toe ground

Fig. 10. Frequency diagrams of the model bias with the load-transfer method for piles founded in soil: (a) at the intended design load; (b) at twice the design load.

conditions. An unbiased estimate of the calibration error about the regression line, $\sigma_{\ln(S_m) | \ln(S_p)}$, is (e.g., Ronald 1982)

$$[18] \quad \sigma_{\ln(S_m) | \ln(S_p)}^2 = \frac{n-1}{n-2} \left(s_{\ln(S_m)}^2 - \beta'^2 s_{\ln(S_p)}^2 \right)$$

where $s_{\ln(S_m)}$ and $s_{\ln(S_p)}$ are the sample standard deviations of $\ln(S_m)$ and $\ln(S_p)$, respectively; and n is the sample size. In addition, due to a limited number of data points available in the regression analysis, the values of β'' and $\ln(\lambda'')$ may be subject to estimation errors. Therefore, the estimated mean value, $\overline{\ln \hat{S}_m}$, of $\ln \hat{S}_m$ is also a random variable. Assuming $\ln \hat{S}_m$ is normally distributed, its standard deviation $\sigma_{\ln \hat{S}_m}$ at a specified site $\ln(S_p)$ is (e.g., Ronald 1982)

$$[19] \quad \sigma_{\ln \hat{S}_m | \ln(S_p)}^2 = \sigma_{\ln(S_m) | \ln(S_p)}^2 \left[\frac{1}{n} + \frac{(\ln S_p - \overline{\ln S_p})^2}{(n-1)s_{\ln(S_p)}^2} \right]$$

where $\overline{\ln S_p}$ is the sample mean of $\ln(S_p)$.

Given a specified $\ln(S_p)$, the expectation of $\ln \hat{S}_m$ is (e.g., Ronald 1982)

$$[20] \quad \ln \hat{S}_m = \overline{\ln S_m} = \beta'' \ln S_p + \ln \lambda''$$

Fig. 11. Frequency diagrams of the model bias with the load-transfer method for piles founded on rock: (a) at the intended design load; (b) at twice the design load.

The variance of $\ln \hat{S}_m$, denoted by $\sigma_{\ln(S_m)}$, is the sum of the variance of the mean and the variance about the mean,

$$[21] \quad \sigma_{\ln S_m}^2 = \sigma_{\ln S_m | \ln S_p}^2 + \sigma_{\ln S_m}^2$$

Finally, eq. [1], which defines the model bias factor, can be expressed as

$$[22] \quad \ln \lambda = \ln S_m - \ln S_p$$

Then the mean and variance of $\ln(\lambda)$, $\overline{\ln \lambda}$ and $\sigma_{\ln \lambda}^2$, are, respectively, calculated as

$$[23] \quad \overline{\ln \lambda} = \overline{\ln S_m} - \overline{\ln S_p}$$

$$[24] \quad \sigma_{\ln \lambda}^2 = \sigma_{\ln S_m}^2 + \sigma_{\ln S_p}^2 - 2\rho_{\ln(S_m), \ln(S_p)} \sigma_{\ln S_m} \sigma_{\ln S_p}$$

where $\rho_{\ln(S_m), \ln(S_p)}$ is the correlation coefficient between $\ln(S_m)$ and $\ln(S_p)$. After the mean and standard deviation of $\ln(\lambda)$ at a load level have been obtained, the mean and COV of λ can also be obtained.

Take the settlement predictions for the piles founded in soil with Vesic's method I as an example. The values of $\overline{\ln S_p}$ at the intended design load, twice the design load, and the failure load are 2.82, 3.74, and 4.24, respectively; β'' and

Fig. 12. Comparison of load–settlement curves from a pile load test and two calculation models.

Fig. 13. Lognormal distributions of the model bias: (a) for piles founded in soil; (b) for piles founded on rock. P_d , design load.

$\ln(\lambda'')$ are 0.87 and 0.61, respectively, as shown in Fig. 14a. Using eq. [20], the corresponding values of $\overline{\ln S_m}$ at the three loads are calculated as 3.06, 3.87, and 4.30, respectively. Then using eq. [23], the values of $\overline{\ln \lambda}$ at the three loads are calculated as 0.24, 0.13, and 0.06, respectively. The values of $\sigma_{\ln(S_m) | \ln(S_p)}$ at the three loads are all 0.22;

and the values of $\sigma_{\ln(S_p)}$ at the three loads are 0.04, 0.03, and 0.04, respectively. Using eq. [21], the values of $\sigma_{\ln(S_m)}$ at the three loads are calculated and the values are all 0.23. The values of $\sigma_{\ln(S_p)}$ at the three loads are 0.11, 0.09, and 0.15, respectively. In addition, $\rho_{\ln(S_m), \ln(S_p)}$ can be approximated by the square root of the coefficient of determination R^2 , and in this case $\rho_{\ln(S_m), \ln(S_p)} = 0.91$. Therefore, using eq. [24], $\sigma_{\ln(\lambda)}$ at the three loads are calculated as 0.14, 0.15, and 0.11, respectively. Given the values of $\overline{\ln \lambda}$ and $\sigma_{\ln(\lambda)}$, the values of the mean of λ at the intended design load, twice the design load, and the failure load are calculated as 1.29, 1.14, and 1.07, respectively, and their respective COV values are 0.14, 0.15, and 0.11. The model biases of all the samples calculated from the regression analysis are summarized in Table 6.

Similar to the results of the statistical analysis, the mean bias of each prediction model tends to vary with the load level and the bearing stratum at the pile toe. It is also found that the mean biases from the statistical and regression analyses are very similar (see Table 6). This indicates that the mean of λ can be obtained with a reasonable accuracy through a statistical analysis without the need for many test data. This lends convenience to calibrating settlement analysis models for regional practices. When limited displacement measurements vary in a large range due to, for example, large variations in pile length and design load, the regression method will be more effective.

The values of the COV of λ from the regression analysis again fall within a narrow range of 0.10–0.21, which is somewhat smaller than the values of 0.19–0.28 obtained from the statistical analysis. This is caused by the larger samples used for the regression analysis. As found in the previous section, each statistical analysis is conducted under a particular load for 34 cases of piles founded in soils or 30 cases of piles founded on rocks. The regression analyses, on the other hand, are carried out utilizing all data at the three load levels, 82 data points for the piles founded in soil, and 77 data points for the piles founded on rock. Indeed, more information can result in less uncertainty. This is somewhat consistent with the observation by Phoon and Kulhawy (2005) that fewer data are required to estimate the mean model bias than the COV, for a given level of statistical precision. To further show the effect of the number of data, 34 data points for the piles founded in soil (8 working piles and 6 preliminary piles) and 30 data points for the piles founded on rock (6 working piles and 6 preliminary piles) are selected randomly and used to calibrate the non-linear load–transfer model with the regression analysis. For the piles founded in soil, the COV values at the intended design load, twice the design load, and the failure load are calculated as 0.22, 0.23, and 0.16, respectively, and those for the piles founded on rock are 0.21, 0.19, and 0.19, respectively. These COV values are indeed similar to those shown in Table 6 obtained using the same amount of data through the conventional statistical analysis.

Conclusions

The information from 64 field load tests on long driven piles has been used to evaluate the performance of three settlement prediction models for single piles (i.e., Vesic's

Table 6. Summary of model bias factors from statistical and regression analyses.

	Piles founded in soil						Piles founded on rock					
	At the intended design load		At twice the design load		At the failure load		At the intended design load		At twice the design load		At the failure load	
	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV	Mean	COV
Statistical analysis												
Vesic's method I ^a	1.30	0.22	1.22	0.21	—	—	1.20	0.26	1.05	0.21	—	—
Vesic's method II ^b	1.02	0.23	1.05	0.20	—	—	0.96	0.27	0.98	0.25	—	—
Fleming et al.'s method	0.66	0.22	0.78	0.19	—	—	0.81	0.28	0.87	0.26	—	—
Load-transfer method	1.34	0.22	1.10	0.21	—	—	1.16	0.24	1.01	0.21	—	—
Regression analysis												
Vesic's method I	1.29	0.14	1.14	0.15	1.07	0.11	1.19	0.15	1.03	0.17	0.96	0.14
Vesic's method II	1.07	0.16	1.02	0.16	1.00	0.17	1.04	0.17	0.92	0.17	0.86	0.17
Fleming et al.'s method	0.70	0.16	0.73	0.16	0.75	0.17	0.75	0.19	0.76	0.19	0.77	0.21
Load-transfer method	1.35	0.17	1.03	0.14	0.88	0.10	1.15	0.19	0.97	0.17	0.89	0.13

^aIn Vesic's method I, the values of Q_b and Q_s are determined using a nonlinear load-transfer method.

^bIn Vesic's method II, the values of Q_b and Q_s are based on mean ratios of Q_b/Q in Table 3 from field tests.

Fig. 14. Relationships between the mean $\ln(S_m)$ and $\ln(S_p)$ for piles founded in soil with: (a) Vesic's method I; (b) Vesic's method II; (c) Fleming et al.'s method; (d) the load-transfer method.

method, Fleming et al.'s method, and a nonlinear load-transfer method), with a conventional statistical analysis and a regression analysis. Statistics of the model bias for the three prediction models at three load levels have been obtained separately for piles founded in soils and piles founded on rocks. Based on these two analyses, several conclusions can be drawn:

- (1) The mean bias of each prediction model tends to vary with the load level (or magnitude of settlement) and the bearing stratum at the pile toe. The variation of the mean model bias with the load level depends on the nonlinearity of predicted load-settlement curves.
- (2) The mean model bias can be obtained accurately through the conventional statistical analysis without the need for many test data. However, when limited displacement measurements vary in a large range due to large variations in pile length and design load, it would be more effective to use the regression method to calibrate settlement analysis models.
- (3) The COV of model bias depends on the amount of data available, 0.19–0.28 from the conventional statistical analysis (using about 30 data points) and 0.10–0.21 from the regression analysis (using about 80 data points). The regression analysis proposed in this paper utilizes

Fig. 15 Relationships between the mean $\ln(S_m)$ and $\ln(S_p)$ for piles founded on rock with: (a) Vesic's method I; (b) Vesic's method II; (c) Fleming et al.'s method; (d) the load-transfer method.

more data points; hence the COV values obtained from such analysis are smaller than those from the conventional statistical analysis.

- (4) The COV of model bias appears to vary in narrow ranges. This may be caused by the fact that the test piles in this study were installed in five similar reclaimed grounds following similar construction procedures, thus involving less uncertainty in the evaluation of soil parameters and less human errors from construction workmanship.

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List of symbols

A_b	area of pile base
A_p	area of pile cross section
D	pile width or diameter
E_{ave}	average Young's modulus of the soil along the pile shaft
E_f	field modulus of deformation of rock
E_l	laboratory modulus of deformation of rock
E_p	modulus of elasticity of pile material
E_{sb}	modulus of elasticity of the soil near the pile toe
E_{si}	Young's modulus of the i th soil layer
E_{ss}	modulus of elasticity of the soil along the pile shaft
G_{ave}	average shear modulus of the soil along the pile shaft
G_b	shear modulus of the soil beneath the pile toe
G_{is}	initial shear modulus of the soil along the pile shaft
G_{ib}	initial shear modulus of the soil near the pile toe
G_L	shear modulus of the soil at depth L
G'_L	soil shear modulus at the bottom of the active pile length
I_b, I_s	influence factors
H	depth of the pile cross-section
K_s	coefficient of horizontal earth pressure on pile
$\ln \hat{S}_m$	estimate of $\ln(S_m)$
$\frac{\ln \hat{S}_m}{\ln \hat{S}_p}$	estimated mean value of $\ln \hat{S}_m$
$\frac{\ln \hat{S}_p}{\ln \lambda}$	sample mean of $\ln(S_p)$
$\overline{\ln \lambda}, \sigma_{\ln \lambda}^2$	mean and variance of $\ln(\lambda)$
L	pile length
L_{ac}	active pile length
L_i	thickness of the i th soil layer
L_w	water level
N	sample size
N_q	bearing capacity factor
N_φ	a factor related to the friction angle of rock
P_d	design load
P	pile perimeter
q_b	load per unit area at the pile toe
q_p	ultimate unit toe resistance
q_{uf}	field unconfined compression strength of rock
q_{ul}	laboratory unconfined compression strength
Q	total load applied on pile
Q_b	load carried at the pile toe
Q_f	ultimate toe resistance
Q_s	load carried by frictional resistance
r_0	radius of the pile
r_b	radius of the pile toe
r_m	radius out from the pile shaft where axial loading effects on soil are negligible
R^2	coefficient of determination for the regression line
$s_{\ln(S_m)}, s_{\ln(S_p)}$	sample standard deviations of $\ln(S_m)$ and $\ln(S_p)$
S	total pile settlement
S_1	elastic compression of pile
S_2	settlement of pile caused by the load at the pile toe
S_3	settlement of pile caused by the load transmitted along the pile shaft
S_m	measured settlement
S_p	predicted settlement

$\beta'', \ln(\lambda'')$	estimated regression coefficients	$\rho_{\ln(S_m), \ln(S_p)}$	correlation coefficient between $\ln(S_m)$ and $\ln(S_p)$
γ	total unit weight	σ_{ave}	average effective stress of a stratum
δ_s	angle of friction at pile–soil interface	σ_{v0}	effective stress at pile toe
λ	model bias	$\sigma_{\ln(S_m)}$	variance of $\ln \hat{S}_m$
λ_p	pile stiffness ratio	$\sigma_{\ln(S_m) \ln(S_p)}$	calibration error about the regression line
λ'	general model bias	$\sigma_{ml \ln(S_p)}$	standard deviation of $\ln \hat{S}_m$ at a specified site $\ln(S_p)$
ϕ	internal friction angle	τ_0	shear stress being transferred to the soil for a given settlement
ν	Poisson's ratio	τ_f	ultimate shear stress transferred to the soil
ξ	coefficient for elastic pile compression calculation		