A possible unified picture for both scaling and non-scaling plateau-to-plateau transitions in quantum Hall systems

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Abstract – The effect of the inter-Landau-band overlap on electron localization in integer quantum Hall systems is studied. A narrow band of extended electronic states is formed near each Landau band center due to interband mixing. However, these bands are greatly modified by the degree of interband overlap. The width of an extended-state band goes to zero when the interband overlap is below a critical value. This leads to a possible unified explanation to both scaling and non-scaling experimental results on plateau-to-plateau transitions.

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The scaling theory of localization [1] says that all electrons in a disordered two-dimensional (2D) system are localized in the absence of a magnetic field. In the presence of a strong magnetic field, a series of disorder-broadened Landau bands (LBs) will appear, and extended states emerge around the center of each LB while states at other energies are localized. The integrally quantized Hall (QH) plateaus are observed when the Fermi energy lies in localized states, with the value of the Hall conductance, $\sigma_{xy} = ne^2/h$, related to the number of filled LBs (n). As a function of the magnetic field, the Hall conductance jumps from one QH plateau to another when the Fermi energy crosses each LB center. Many experimental studies have been focused on the nature of such a plateau-to-plateau (P-P) transition [2–10]. Experiments on clean samples follow scaling behaviors of longitudinal or Hall resistance slope and longitudinal resistance peak width with temperature or frequency near transition points [2–6]. However, experiments on relatively dirty samples show that the longitudinal resistance slope remains finite [7,8] and the resistance peak width remains non-zero [9,10] when extrapolated to zero temperature. This means a

non-scaling behavior around a P-P transition, in contradiction with the expectation of continuous quantum phase transitions [11,12]. In recent studies [13,14] we provided a possible picture for such non-scaling behavior. We showed that an extended-state band is formed near each LB center because of the mixing of states with *opposite chiralities*, leading to a narrow metallic phase between neighboring QH plateaus. (The term “chirality” in this letter refers to the direction of the drifting motion of the center of the electron cyclotron motion around valleys and peaks of the random potential in the bulk of the sample. Viewing from the opposite direction of the magnetic field, such drifting direction is clockwise around a valley and counter-clockwise around a peak.) Thus a P-P transition is a cross through a narrow metallic phase rather than a single continuous transition, *consistent* with non-scaling behaviors observed in experiments. (A previous theoretical study based on lattice model by Hatsugai *et al.* [15] also shows that with increasing disorder delocalized states in each Landau band mix in an intermediate region with very large localization length before the system finally evolves into an insulator.) However, the narrow metallic phase does not vanish at any non-zero mixing, *inconsistent* with experiments on clean samples [2–6] where scaling behaviors were

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Fig. 1: The modified two-channel network model. A potential peak (valley) is denoted by P (V). Solid squares denote drifting loops around peaks and occupied valley, while dashed squares denote unoccupied valleys. The drifting directions are indicated by the arrows. Dashed lines stand for interband mixing between a peak and a neighboring occupied valley or intraband tunnelling between two neighboring peaks or occupied valleys.

observed down to the lowest possible temperature. Thus, a unified picture for both scaling and non-scaling behaviors is still lacking.

In this letter, we shall resolve this problem by including a missing physics —interband overlap. We show that extended states in each LB evolve from a single point around the center to a narrow band around the center as the interband overlap increases. Thus the nature of a P-P transition changes from a continuous phase transition at a low interband overlap to a cross of a narrow metallic phase at a high interband overlap. This provides a possible unified explanation to both scaling and non-scaling experiments.

According to the semiclassical theory [16], an electronic state in a strong magnetic field and in a smooth random potential can be decomposed into a rapid cyclotron motion and a slow drifting motion of the guiding center. The kinetic energy of the cyclotron motion is quantized by $E_n = (n + 1/2)\hbar\omega_c$, where ω_c is the cyclotron frequency and n is the LB index. The trajectory of the drifting motion of a guiding center is along an equipotential contour of value $V_0 = E - E_n$, where E is the total electron energy. Each closed loop of the equipotential contour corresponds to one quantum state schematically denoted by squares in fig. 1. Those squares of $V_0 > 0$ are localized around peaks (denoted by the letter P inside the squares in fig. 1). And squares of $V_0 < 0$ are around valleys (denoted by the letter V inside the squares). The drifting direction of each loop is *unidirectional*, *i.e.*, a chiral state. If one views the plane from the direction opposite to the

magnetic field, the drifting is clockwise around valleys and counter-clockwise around peaks. According to the percolation theory [17], there is only one extended state at $V_0 = 0$ if the interband mixing is neglected. Intraband quantum tunnelling does not change this result [16].

Interband mixing should be considered when the LB width, related to the disorder strength W , is comparable with the Landau band gap $\hbar\omega_c$. Without losing generality, we shall study two adjacent LBs denoted by E_l and E_u as the center energies, where index l and u stand for the lower and upper bands. Using the semiclassical theory described in the previous paragraph, two sets of loops are obtained for an electronic state with its energy E between E_l and E_u . Loops of the upper band are around valleys whose potential is lower than $V_u = E - E_u < 0$ and loops of the lower band are around peaks whose potential is higher than $V_l = E - E_l > 0$. For simplicity, we assume that peaks and valleys form two interlocked square lattices. In the following study, we focus on electronic states slightly above the lower-band center, *i.e.*, $V_l \sim 0$ and $V_u \sim -\hbar\omega_c$. In the previous studies [13,18], all upper-band and lower-band loops were assumed to be present, which is not quite proper. In reality, almost all loops around peaks should be present since they satisfy $V_{peak} > V_l \sim 0$ while only a fraction of valleys provides upper-band loops because the rest are too shallow to satisfy $V_{valley} < V_u \sim -\hbar\omega_c$. The fraction of occupied valleys p measures the degree of interband overlap which is dominated by the ratio $W/\hbar\omega_c$. A small W , or a clean sample, means not only a weak interband mixing, but also a small interband overlap. Thus, it is legitimate to ask whether extended bands still exist in this case. In order to include the effect of interband overlap, we modify the two-channel network model [13,18] as follows. All peaks are occupied, but only a fraction of valleys are randomly selected to be occupied as illustrated in fig. 1 where three of eight valleys are occupied. It is not a simple randomness in interband mixing [18]. Our previous study [13,14] showed that a weak interband mixing always results in extended-state bands around LB centers when $p = 100\%$. We expect that each narrow band shrinks to a singular point as p is below a critical value.

Before going further, we would like to make clear the distinction between interband mixing and interband overlap. In the case that the LB width is much smaller than the Landau gap, adjacent LBs are separate from each other. In other words, no interband overlap exists in this case. When the LB width is larger than the Landau gap, interband overlap begins to exist for electronic energy around the middle of adjacent LB centers. With further increment of the disorder strength W , the region of interband overlap expands from regions around the middle of adjacent LB centers to LB centers. When the LB width is larger than twice the Landau gap, interband overlap exists for all energies. Thus interband overlap is mainly determined by the ratio of LB width and the Landau gap while it does not have much relationship with the detailed distribution of the random potential in the

real space. While interband mixing refers to the typical tunnelling probability between a clockwise loop around a potential valley and a counter-clockwise loop around the neighboring potential peak. Since such tunnelling cannot exist when interband overlap is absent, interband mixing should also depend on the ratio of the LB width and the Landau gap. However, when interband overlap exists, the value of interband mixing is mainly determined by the ratio of the electron cyclotron radius l_c and the typical distance between a clockwise loop around a potential valley and a counter-clockwise loop around a potential peak. Since such typical distance is determined by the distribution of the random potential in the real space, interband mixing depends not only on the ratio of the LB width and the Landau gap but also on the sharpness of the random potential distribution in real space. A weak-disorder potential with sharp fluctuation in real space may lead to weak interband overlap and strong interband mixing. While a strong-disorder potential with smooth fluctuation in real space may lead to strong interband overlap and weak interband mixing. And a strong-disorder potential with sharp fluctuation can lead to both strong interband overlap and strong interband mixing. This means that weak interband mixing does not always lead to weak interband overlap and vice versa. Thus, interband mixing and interband overlap can be viewed as two separate tunable parameters in our model. In experiments it is usually the magnetic field B that is tuned while the strength and landscape of the random potential in real space can be viewed as fixed. Therefore, for a disorder potential with smooth fluctuation, with decreasing B the system starts from the case of no interband overlap with no interband mixing to the case of weak interband overlap with weak interband mixing (each P-P transition in the two cases still belongs to single-point continuous transitions), and then to the case of strong interband overlap with weak interband mixing (each P-P transition in this case turns into the cross of a narrow metallic phase), and finally enters the Shubnikov-de Haas regime of strong interband overlap with strong interband, where all P-P transitions vanish in this case. While for a disorder potential with sharp fluctuation in real space, with decreasing B the system starts from the case of no interband overlap with no weak interband mixing (each P-P transition in this case still belongs to single-point continuous transitions) to the case of weak interband overlap with strong interband mixing (some P-P transitions in this case may turn into the cross of a narrow metallic phase), and then directly enters the Shubnikov-de Haas regime of strong interband overlap with strong interband mixing where all P-P transitions vanish. Thus, it is easier to observe the transition from scaling to non-scaling behaviors in samples of the smooth random potential.

In this letter, we shall consider a classical version of the two-channel network model in fig. 1. Let us put a classical particle in one link of a peak at the up edge of the network and let it go along the arrows on links.

It can choose with given probabilities to stay in a peak (valley) or to hop to the neighboring peak (valley) by tunnelling at nodes or to the neighboring valley (peak) by mixing on links. For simplicity, the choice of whether to stay or to hop at each node and on each link is quenched for each realization. Thus, the propagation of the particle is *deterministic* in each realization of the network. Periodic boundary condition is taken in the left and right edges, *i.e.*, the left and right edges are connected. If the particle can arrive at the down edge, then it is recorded as percolated through the network. If the particle follows a closed path, then it is recorded as not percolated. Take many realizations of the network, we can obtain the percolation probability from up edge to down edge. The size dependence of the percolation probability can reveal the localization property of the particle in the network. If percolation probability increases with the lattice size L and tends to one, the particle should be delocalized. If percolation probability decreases with L and tends to zero, it should be localized. It should be noted that in the original quantum model the particle can choose whether to hop or to stay each time it passes a node or a link while in this classical model the choice at each node and link is quenched. Thus, the effect of tunnelling at nodes and mixing on links is underestimated in the classical model. However, our purpose is to see whether interband mixing can induce bands of extended states and whether they will shrink into single points when interband overlap is reduced below a critical value. This simplified classical model, although not quantitatively faithful for the original quantum model, includes all necessary parameters and can serve for such a purpose.

In the following simulation, we assume that tunnelling at each node occurs for the same band. Let us denote $s_{u(l)}^L$ and $s_{u(l)}^R$ as the probability of scattering a particle of the upper (lower) band into its left-hand-side and its right-hand-side outgoing channels, respectively. Since each node scatters the particle into either the left-hand-side channel or the right-hand-side channel, we have $s_{u(l)}^R + (s_{u(l)}^L) = 1$. The potential around a saddle point is assumed to be $V(x, y) = -Ux^2 + Uy^2 + V_c$ [18], where U measures disorder strength and V_c is the random potential at saddle points. V_c proved to be irrelevant [18] and will be set to zero. $s_{u(l)}^L$ is then given by [19]

$$s_{u(l)}^L = [1 + \exp(-\pi\epsilon_{u(l)})]^{-1}, \quad (1)$$

where $\epsilon_{u(l)} = [E + V_c - (n_{u(l)} + 1/2)E_2]/E_1$, $E_1 = \frac{\hbar\omega_c}{2\sqrt{2}}\sqrt{K-1}$, $E_2 = \frac{\hbar\omega_c}{\sqrt{2}}\sqrt{K+1}$, $K = \sqrt{\frac{64U^2}{m^2\omega_c^4} + 1}$, E the electron energy, and $n_{u(l)}$ the band indices with $\Delta n = n_u - n_l = 1$. The dimensionless ratio $E_2/E_1 = 2\sqrt{1 + \frac{2}{K-1}}$ is set to be 2.2. For convenience, we set E_2 as energy unit and the cyclotron energy of the lower band as the reference point. The energy range between two band centers is thus $E \in [0, 1]$. We restrict to $E \in [0, 0.5]$ because the model is identical for E and $1 - E$ [18]. Interband mixing

Fig. 2: Percolation probability *vs.* E for $J = 0.05$ with (a) $L = 100, 200, 400, 500$ at $p = 10\%$ and (b) $L = 100, 200, 400, 500, 600, 800$ at $p = 100\%$.

on each link is represented by the hopping probability J between a lower-band loop and its adjacent occupied upper-band loops. J will take the same value for all links in our calculations.

Before providing numerical results, we would like to consider the relation between the lattice size of the network model and the sample size in experiments. As the network model is based on the semiclassical picture, the distance between two neighboring nodes should be at least one order of magnitude larger than the electron cyclotron radius $l_c = \sqrt{\hbar/(eB)} \sim 10^{-2} \mu\text{m}$ since the typical order of magnitude of the magnetic field B around P-P transitions in experiments is $\sim 10^0$ tesla. Thus a network model with lattice size $L = 500$ corresponds to a sample of size $5000l_c \sim 50 \mu\text{m}$ which is comparable with typical sample sizes used in experiments. Therefore, we expect that finite-size effects in our numerical results may not be much worse than data obtained in experiments.

Curves in fig. 2 are percolation probability *vs.* E with lattice size $L = 100, 200, 400, 500$ for $J = 0.05$ at (a) $p = 10\%$ and $L = 100, 200, 400, 500, 600, 800$ for $J = 0.05$ at (b) $p = 100\%$. In fig. 2(a), there is a single point around $E = E_c = 0.0002$ where curves of $L = 400$ and $L = 500$ meet with each other. According to our criteria, there is only one extended state in this case. While there is a region between two points in fig. 2(b) where curves of $L \geq 400$ merge together. This means that there is a narrow band of extended states between $E_{c1} \approx 0.002$ and $E_{c2} \approx 0.0027$. Thus one can conclude that for interband mixing $J = 0.05$ there should be a non-zero critical value of interband overlap, p_c . For $p > p_c$ the extended states form a narrow energy band while at and below p_c they shrink into a single point in energy. A similar behavior is obtained for other values of J . Thus, for each J there should be a critical $p_c(J)$ below which the extended states shrink into a single point in energy. Detailed results and discussions for p_c *vs.* J will be presented elsewhere.

By plotting the above results schematically in the plane of disorder W and magnetic field B , we obtain a modified topological QH phase diagram in fig. 3(a) which should be compared with the one in our recent letter [13] as shown in fig. 3(b). The crucial difference is that fig. 3(a) is separated into two regions by the straight line c which corresponds to

Fig. 3: Comparison of the topological QH phase diagram in the W - B plane with and without considering interband overlap. W stands for the disorder strength, and B for the magnetic field. The shadowed regime is for the metallic phase. The area indicated by the symbol n is the n -plateau IQHE phase. The rest area is the insulating phase. (a) Present result with interband overlap: line a corresponds to a process in scaling experiments; line b corresponds to a process in non-scaling experiments; line c represents the curve $p = p_c(E \rightarrow 0)$ which separates the scaling and non-scaling regions; B_0 is the magnetic field when half of the lowest LB is filled. (b) Previous result [13]: line a corresponds to a process in non-scaling experiments.

$p = p_c$ at small J . (Since p is dominated by $W/\hbar\omega_c \propto W/B$, $p = p_c(J \rightarrow 0)$ in the plane of W *vs.* B should be a straight line.) Below line c , both p and J are small, and only a single extended state exists in each LB. With increasing W or decreasing B , p increases drastically while J remains small. Thus a narrow extended band emerges around each LB center above line c . When W is further increased, J begins to increase considerably, and extended bands begin to float up in energy and merge [20]. Experiments on clean samples are represented by a process along line a in fig. 3(a) and scaling behaviors are expected for this case. While experiments on relatively dirty samples correspond to a process along line b in fig. 3(a) where non-scaling behaviors are expected [13]. Therefore, the modified phase diagram in fig. 3(a) gives a possible unified explanation to both scaling and non-scaling behaviors in experiments.

In summary, our numerical results show that there should exist a critical interband overlap which separates scaling and non-scaling behaviors. Above the value, extended states form bands around each LB center and a P-P transition is a cross through a narrow metallic phase, leading to a non-scaling behavior. Below the value, there is only a singular extended state in each LB and a P-P transition is a true continuous quantum phase transition with a scaling behavior. Correspondingly, a modified phase diagram is proposed which provides a possible unified explanation to both scaling and non-scaling P-P transitions in QH systems.

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