Modelling of piezoelectric micro positioning system using direct mapping and numerical analysis for precision grinding process control

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Abstract: For micro/nano positioning systems using piezoelectric actuators for precision grinding process control, sinusoidal command signals will be used and will give additional problems in comparison with the commonly used step signals due to the hysteresis effect, which require good modelling of the effect. To avoid discontinuity in obtaining the values of the piezoelectric constant, a method of direct mapping with polynomial fit has been proposed. Compared with the single polynomial approach, the model obtained was able to reduce the modelling error by 6.5%. To further reduce the modelling error, a new method using direct mapping and numerical analysis is proposed while discontinuity is avoided. Theoretical and experimental studies are conducted and comparative studies are made. Compared with the single polynomial approach, it is able to reduce the modelling error to 7.3%, which is 45.5% lower. Compared with the direct mapping with polynomial approach, the proposed method is able to reduce the modelling error by 41.7%. For a typically fast piezoelectric micro/nano positioning system, hysteresis distortion is more significant towards modelling error than dynamic distortion. The proposed new approach should be useful in precision grinding process modelling and control.

Keywords: micro/nano positioning; piezoelectric hysteresis; direct mapping; numerical analysis; precision grinding process control.

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1 Introduction

It is known that, in surface grinding, a high speed workpiece table motion is involved. In order to achieve dynamic and active grinding process control for improved machining accuracy, the control commands will include substantial amount of high frequency signals. The signals may not be sufficiently implemented by a single actuating unit, typically the wheel infeed system, since the wheel system has a small frequency bandwidth for dynamic operations. A piezoelectric micro/nano positioning system utilizing a piezoelectric actuator may be used as an additional unit for providing small but fast control actions. The infeed system and the piezoelectric micro positioning system will work together under the composite control scheme (Gao, 1996; Gao, 2001; Gao *et al.*,

2002; Gao *et al.*, 2002).

In order to investigate the characteristics of the piezoelectric micro/nano positioning system that operates over a range of frequencies and to obtain the response of the system for performance analysis, sinusoidal signals are utilized to be the command signals for analysis of the piezoelectric micro/nano positioning system.

For the system under the sinusoidal excitation, the hysteresis of the piezoelectric actuator has a significant effect (Fisher *et al.*, 2004; Yang *et al.*, 2000). Based on our previous studies (Gao *et al.*, 2001-2006; Tse *et al.*, 2005-2006), sinusoidal signals would give additional problems in comparison with the commonly used step signals. This requires further studies of the hysteresis characteristics of the piezoelectric actuator that is typically used in many micro/nano positioning systems.

In our previous investigations (Gao *et al.*, 2004-2006; Tse *et al.*, 2005-2006), models for the hysteresis behaviour and models for the transient response of the system under sinusoidal excitation were established (Gao *et al.*, 2004-2006; Tse *et al.*, 2005-2006). In one case, the hysteresis was modelled based on a single polynomial regression approach (Gao *et al.*, 2004). An exhaustive search method was used to reduce the modelling error (Gao *et al.*, 2004), which was up to approximately 15% (Gao *et al.*, 2004).

Problems of discontinuity in obtaining values for the piezoelectric constant were experienced (Gao *et al.*, 2005-2006; Tse *et al.*, 2005-2006) in our previous studies. In order to avoid the problem of the kind and to reduce modelling error, direct mapping with polynomial fit was used (Tse *et al.*, 2006). It was found that this approach was able to reduce the modelling error by 6.5% when compared with the result of the single polynomial approach and the problems in obtaining the values of the piezoelectric constant were avoided.

In order to further reduce the modelling error and solve the problems in obtaining the piezoelectric constant, a new method of direct mapping with numerical analysis is proposed. Theoretical and experimental studies are conducted and comparative studies are made for assessment of the proposed method.

2 Piezoelectric micro positioning system model

The mechanical model of the piezoelectric micro positioning system is shown in Fig. 1. Under the condition of no grinding force $F_n(t)$, at the time instance t , the force generated by the piezoelectric translator $F_{pzt}(t)$ is

$$F_{pzt}(t) = (k_{pzt} + k_s)y_{pzt}(t), \quad (1)$$

where k_{pzt} is the stiffness of the piezoelectric translator, k_s is the stiffness of the springs for preloading, $y_{pzt}(t)$ is the transient displacement of the piezoelectric translator.

Based on the mechanical model (Fig. 1), the dynamic model of the system is given as

$$(m_{pzt} + m_m)\ddot{y}(t) + (c_{pzt} + c_s)\dot{y}(t) + (k_{pzt} + k_s)y(t) = F_{pzt}(t), \quad (2)$$

where m_{pzt} is the equivalent mass of the piezoelectric translator, m_m is the mass of the moving part in the micro positioning system, c_{pzt} is the equivalent damping coefficient of the piezoelectric translator, c_s is the equivalent damping coefficient of the springs, and $y(t)$ is the displacement response of the system (Fig. 1).

From Eq. (1) and Eq. (2),

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 y_{pzt}(t), \quad (3)$$

where

$$2\zeta\omega_n = \frac{c_{pzt} + c_s}{m_{pzt} + m_m}, \quad (4)$$

$$\omega_n^2 = \frac{k_{pzt} + k_s}{m_{pzt} + m_m}, \quad (5)$$

ζ is the damping factor, and ω_n is the undamped natural frequency of the system.

In order to avoid the discontinuity problem in obtaining the values of piezoelectric constant d_{33} (Gao *et al.*, 2005-2006; Tse *et al.*, 2005-2006), which requires special treatments, it was found that the displacement of the translator $y_{pzt}(t)$ can be directly mapped with the input command signal $v_d(t)$.

It is found that there is no need to determine a polynomial fit model for $y_{pzt}(v_d(t))$ (Tse *et al.*, 2006). Through the use of the experimentally obtained hysteresis loop (Fig. 2) and mapping, $y_{pzt}(t)$ values can be obtained for given $v_d(t)$ values.

Furthermore, the displacement response $y(t)$ (Eq. (3)) can be directly obtained by use of numerical analysis to solve the differential equation (Eq. (3)). It can be seen that the proposed new approach is further simplified (Tse *et al.*, 2006).

For case study, two motion profiles for $v_0=5V$, and $v_0=9V$ are shown in Fig. 2. The values of the displacement of the translator $y_{pzt}(t)$ in Eq. (3) can be obtained directly from Fig. 2.

Figure 1 Mechanical model of the micro positioning system

Figure 2 Two motion profiles for $v_0=5V$ and $v_0=9V$, respectively

(a) $v_0=5V$

(b) $v_0=9V$

3 Results and discussion

3.1 Theoretical and experimental results

Theoretical displacement response $y(t)$ to a sinusoidal command signal $v_d(t)$, where $v_d(t)=v_0(1-\cos 2\pi f_c t)/2$, can be determined using numerical analysis and Eq. (3). To obtain $y(t)$ (Eq. (3)), a number of parameters of the piezoelectric micro/nano positioning system

are to be determined. In our previous work (Gao *et al.*, 2001-2002), the damping factor of the micro/nano positioning table ζ was found equal to 0.0312. The other parameters of the system were obtained from the specifications of the piezoelectric actuator PI P-840 and a number of tests. The equivalent mass of the translator $m_{pzt} = 4.3\text{g}$, the mass of the moving part $m_m = 1.28\text{kg}$, the stiffness of the piezoelectric translator $k_{pzt} = 17\text{N}/\mu\text{m}$, the stiffness of the springs for preloading $k_s = 1.2 \times 10^4\text{N/m}$, and as such $\omega_n = 3639.22\text{rad/s}$.

To examine the proposed approach, the data obtained in our experimental tests (Gao *et al.*, 2004-2006; Tse *et al.*, 2005-2006) were used. In the experimental tests, the position (Fig. 1) was measured using a laser interferometer capable of dynamic measurement up to 5kHz in range and up to 1nm in resolution.

Figures 3-6 show the experimental response $y_e(t)$ and the theoretical response $y(t)$ for the excitation frequencies $f_e = 408.9\text{Hz}$, 334.5Hz, 322.3Hz and 247.8Hz, respectively. It can be seen that $v_d(t)$ and $y_e(t)$ are smooth while $y(t)$ is distorted. This is due to the distortion by the dynamics (Fig. 1) and the distortion by the hysteresis (Fig. 2). The dynamic distortion causes a difference in shape between $y_{pzt}(t)$ and $y(t)$ (Eq. (3)). The hysteresis distortion causes a difference in shape between $v_d(t)$ and $y_{pzt}(t)$ or $F_{pzt}(t)$ (Fig. 2 and Eq. (1)). Compared with the dynamic distortion, for a typically fast piezoelectric micro/nano positioning system (Fig. 1), hysteresis distortion (Fig. 2) is more significant towards the modeling error between $y(t)$ and $y_e(t)$.

It is noted that n_c is the cycle count and the values of the excitation frequencies f_e were estimated by using a signal processing technique (Gao *et al.*, 2005-2006; Tse *et al.*, 2005-2006). The experimental and theoretical results are also shown in Table 1.

Figure 3 Result of $y_e(t)$ and $y(t)$ for $n_c=1$

Figure 4 Result of $y_e(t)$ and $y(t)$ for $n_c=2$

Figure 5 Result of $y_e(t)$ and $y(t)$ for $v_0=5\text{V}$

Figure 6 Result of $y_e(t)$ and $y(t)$ for $v_0=9V$

3.2 Discussion

To assess the proposed approach (Eq. (1)-(3)), the modelling error e_r will be compared. The peak values of the experimental response $y(t)_{\max}$ and that of the theoretical response $y(t)_{\max}$ were examined (Figures 3-6 and Table 1).

Compared with the single polynomial approach (Gao *et al.*, 2004-2005), noted as C1, the proposed new direct mapping with numerical analysis approach is able to reduce the modelling error e_r by 45.5% on average (Table 1), in addition to the advantage of possessing no discontinuity problem which was experienced (Gao *et al.*, 2005-2006; Tse *et al.*, 2005-2006).

Compared with the direct mapping with polynomial fit approach (Tse *et al.*, 2006), noted as C2, the proposed new approach is able to reduce the modelling error e_r by 41.7% on average (Table 1).

For the proposed approach (C3), $e_r=7.3\%$ on average. The modelling error is smaller than the one using the model based on single polynomial and the model based on direct mapping with polynomial fit for a number of typical input voltages and frequencies (Gao *et al.*, 2004; Tse *et al.*, 2006). The results demonstrate the effectiveness of the proposed new method.

It is noted that the relative error e_r for C1-3 with $v_0=9V$ and $n_c=5$ was large (Table 1). This was due to a detachment problem similar to the one by a large step input command signal (Gao *et al.*, 2001-2002).

4 Conclusions

Sinusoidal input signals would give additional problems in comparison with the commonly used step signals due to the hysteresis effect. A good modelling approach is needed. Using the proposed new method of direct mapping with numerical analysis, a modelling error of 7.3% was obtained. The proposed approach is advantageous since it is able to reduce the modelling error by 45.5% when compared with the single polynomial approach and by 41.7% when compared with the approach of direct mapping with

polynomial fit. In addition, the need to obtain the values of the piezoelectric constant is removed. Compared with dynamic distortion, for a typically fast piezoelectric micro/nano positioning system, hysteresis distortion is typically more significant towards modelling error.

Table 1 Results of modeling error e and relative error e_r

Case ^a	v_0 (V)	n_c	f_c (Hz)	$y(t)_{\text{emax}}$ (μm)	$y(t)_{\text{max}}$ (μm)	e (μm)	e_r (%)
C1	5	1	408.9	31.2607	34.6706	3.41	10.91
C2	5	1	408.9	31.2607	34.6431	3.38	10.82
C3	5	1	408.9	31.2607	36.2902	5.03	16.09
C1	5	2	334.5	27.4849	31.9400	4.46	16.21
C2	5	2	334.5	27.4849	31.3803	3.90	14.17
C3	5	2	334.5	27.4849	28.7921	1.31	4.76
C1	5	5	322.3	27.5795	31.1582	3.58	12.98
C2	5	5	322.3	27.5795	31.0229	3.44	12.49
C3	5	5	322.3	27.5795	27.3007	0.28	1.01
C1	9	5	247.8	163.1977	37.9030	125	76.77
C2	9	5	247.8	163.1977	47.3760	116	70.97
C3	9	5	247.8	163.1977	42.8637	120	73.74

^a C1 - Single polynomial regression [$P_i(v_d(t))$, $i=1$] (Gao *et al.*, 2004-2005)

C2 - Direct mapping with polynomial fit

C3 - Direct mapping with numerical analysis

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