Quantum spinon oscillations in a finite one-dimensional transverse Ising model

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The full quantum dynamics of a spinon under external magnetic fields is investigated by using the time-evolving block decimation (TEBD) method within the microcanonical picture of transport. We show that the center of the spinon oscillates back and forth in the absence of dissipation. The quantum many-body behavior can be understood in a single-particle picture of transport and Bloch oscillations (BOs), where quantum fluctuations induce finite lifetimes. Transport, oscillations, and lifetimes can be tuned to some degree separately by external fields. Other nontrivial dynamics, such as resonance and chaos, are also discussed.

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Controlling the time evolution of quantum states and manipulating the dynamics of quantum matter have attracted considerable theoretical and experimental attention in recent years because of their relevance to fundamental physics, as well as potential applications in information storage, encoding, and processing. Up to now, most efforts in this field have been based on solid-state systems. Recent progress in ultracold atomic gases in optical lattices not only provides an ideal platform for simulating quantum many-body models in condensed-matter physics, but also paves the way for quantum manipulation. Because of the unique properties of the ultracold atomic gases in optical lattices, such as extremely low dissipation rates and long coherence times, we can explore the dynamics of quantum many-body systems in a clean and dissipation-free environment. This enables us to manipulate the quantum many-body state with an unprecedented degree of precision, and to study new physical effects. An early breakthrough in the field was the first direct observation of Bloch oscillations (BOs) in a tilted optical lattice. In solid-state systems, scattering by impurities would result in strong damping of BOs, while in an optical lattice the perfect optical crystals are free from any imperfections, and what is more, the long coherence times and high tunability of optical lattices enable us not only to observe BOs directly, but also to manipulate the dynamics of the quantum particles via external fields.

Compared with the dynamics of the quantum particles, it may be more interesting to explore and manipulate the quasiparticles that emerge as elementary excitations in quantum systems consisting of a collection of interacting particles, and that may behave differently compared to the original particles. The most interesting example of this is known as fractionalization: the particles are effectively split into smaller constituent quasiparticles that carry only a fraction of the quantum numbers.

In this paper we study the quantum dynamics of one of the most known quasiparticles: the spinon, which is an excitation separating two degenerate states of opposite magnetization in a quantum spin chain and usually carries a fractionalized quantum number (spin-$\frac{1}{2}$). By using the time-evolving block decimation (TEBD) method within the microcanonical picture of transport, we study the time evolution from a quantum many-body initial state with a spinon and show how to manipulate its dynamics via external fields. In particular, we find that this quantum many-body problem can be interpreted in the picture of a quantum quasiparticle at the single-particle level, which shows controllable linear transport and BO behavior depending on perpendicular external fields, where the field along the magnetization direction controls the BO physics and the transverse field controls the linear velocity. The broadening of the quasiparticle is also tunable by the field strength. We also find a quantum resonance behavior of the spinon under a periodically driven external field. This may pave the way to the controlled manipulation of quasiparticles, as well as potential applications in condensed-matter and ultracold atomic physics.

Our Hamiltonian is a finite one-dimensional transverse Ising model with an additional magnetic field along the $z$ direction:

$$H = \sum_{i} J \sigma_{i}^{z} \sigma_{i+1}^{z} + g \sigma_{i}^{x} + h \sigma_{i}^{z},$$  

(1)

with a ferromagnetic coupling $J = -1$. Pauli matrices $\sigma_{i}^{x,y}$ on sites $i$, and a transverse field along the $x$ direction that breaks magnetization conservation. Without the magnetic field $h \sigma_{i}^{z}$, the transverse Ising model can be solved exactly by a Jordan-Wigner transformation and is known as a classic paradigm of a quantum phase transition.

Recently, the transverse Ising model was realized experimentally in cobalt niobate, and an $E_{8}$ symmetry was observed in the vicinity of the quantum critical point of this model. In the presence of a magnetic field along the $z$ direction $h$, Eq. (1) can no longer be solved exactly by Jordan-Wigner transformation, because it would induce a nonlocal term. In this paper, $h$ is a constant or a time-dependent field, and in both cases it leads to nontrivial dynamics of the spinon.

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Our many-body initial state is prepared as follows: a spinon with a finite velocity is located at the center of the lattice. To realize this initial state, we can choose its wave function as \( |\phi_i(0)\rangle = N_0 \sum A_i e^{i h_0} |i\rangle \), where \( N_0 \) is the normalization constant and \( |i\rangle \) denotes a perfect static spinon located between site \( i \) and \( i + 1 \): 
\[ |i\rangle = |\uparrow\rangle_1 \cdots |\uparrow\rangle_i |\downarrow\rangle_{i+1} \cdots |\downarrow\rangle_L \] 
where \( L \) is the length of the lattice. If we choose \( A_i = 1 \), the initial state is a perfect Bloch wave function with definite wave vector \( k_0 \), and the spinon is totally delocalized over the length of the lattice. To ensure that the spinon is initially located at the center of the lattice, we then choose a Gaussian \( A_i = e^{-2((i-\frac{L}{2})/L)^2} \). Given the exponentially fast decay of \( A_i \) away from the center of the lattice, we can just keep \( A_{L/2}, A_{L/2+1}, A_{L/2-1} \) and set all other \( A_i = 0 \) for constructing the initial wave function. In our calculation we furthermore choose \( k_0 = \pi/2 \). We also define the average position of the spinon \( P \) as the position where \( S_z = 0 \).

Because certain excitations can be modeled as quasiparticles, i.e., properties similar to those of real particles, we may conjecture that this is the case here and compare the spinon to a real particle, where the field coupling to \( \sigma^z \) leads to the motion of the particle and the field coupling to \( \sigma \) acts as a constant electric field along the axis in the particle picture, because the Zeeman energy induced by \( h\sigma_z \) is proportional to the displacement of the spinon from its original position. To verify this picture quantitatively, we use TEBD within the microcanonical picture of transport to calculate the time evolution of the spinon from our initial state \( |\phi_i(0)\rangle \), which is not an eigenstate of the Hamiltonian. In the course of the real-time evolution, we use the length of our lattice \( L = 30 \) and take the truncation dimension \( \chi = 80 \) and time step \( \Delta t = 0.05 \). The convergence is checked by taking larger \( \chi \). Times are measured in inverse hopping strengths \( |J|^{-1} \) throughout the paper.

First we focus on the case \( h = 0 \); the real-time evolution from our initial state \( |\phi_i(0)\rangle \) for different values of \( g \) is shown in Fig. 1(a) (\( g = 0.1 \)) and Fig. 1(b) (\( g = 0.3 \)). We find that the spinon propagates along the \(-x\) direction with a velocity proportional to \( g \) as shown in Fig. 2(a). The linear relation between the velocity and \( g \) is numerically verified in Fig. 2(b). The quantum fluctuations increase the width of the spinon during the time evolution, limiting the lifetime of the quasiparticle. The quantum fluctuations make the quantum spinon different from its classical counterpart, the magnetic domain wall, which corresponds to a soliton solution of the nonlinear Landau-Lifshitz-Gilbert (LLG) that can preserve its shape during the propagation. Below we will see that all these results are modified when we introduce the magnetic field along the \( z \) direction.

Next, we apply a constant magnetic field \( h \) along the \( z \) direction. Interpreting the dynamics of the spinon in terms of a real particle in a constant external field suggests the observation of BOs. First we set \( g = 0.3 \) and investigate the dependence of the spinon dynamics on \( h \). The time evolution of the spinon under different magnetic fields \( h \) is shown for \( h = 0.1 \) in Fig. 3(a) and for \( h = 0.2 \) in Fig. 3(b). For BOs, the frequency depends only on the strength of the external constant field, which can also be verified via our numerical result, as shown in Fig. 4, where we set \( h = 0.2 \). We find that for different values of \( g \), the period of the BO is the same \( (T_b = 15.5) \), while the amplitude of BO \( A \) is determined by \( g \). If we set \( g = 0.3 \), the time evolution of the spinon for different
values of $h$ is shown in Fig. 5(a). Following the interpretation in terms of a BO, we expect the oscillation frequency to be proportional to the strength of the external field $h$, which can be verified numerically [Fig. 5(b)].

Next, we study the time evolution of the width of the spinon. The width of the spinon $\Delta$ can be defined similarly to the width of a wave packet. We introduce $n_i = 0.5 - |S_i|$ and $\Delta = \sqrt{\langle n_i^2 \rangle - \langle n_i \rangle^2}$, where $\langle O \rangle = \sum_i O_i / \sum_i$. The result is shown in Fig. 6. Without the magnetic field along the $z$ direction, $h = 0$ [Fig. 6(a)], we observe that the width oscillates strongly at first, whereas after some relaxation time it grows linearly in time, which means the spinon will become wider and wider as the spinon propagates. The situation is different for $h \neq 0$, as shown in Fig. 6(b), where after some short-time behavior, the width of the spinon oscillates around an average value instead of diverging, which corresponds to a solitonic breather mode: not only is the position of the spinon confined to oscillate around the center of the lattice by the magnetic field, but its width oscillates as well around an average value.

The dynamics of the spinon is even more interesting when the system is driven by a time-dependent magnetic field $h(t)$. In our case, we choose a periodic driving magnetic field: $h(t) = h_0 + \delta \cos(\omega t)$. Classically, it is known that if the frequency of the driving force can match the intrinsic frequency of the system, the system can accumulate vibrational energy, and even a small periodic driving force can produce large-amplitude oscillations; in other words, a resonance occurs. This phenomenon can also be observed in our quantum system, where the intrinsic frequency corresponds to the frequency of BOs of the spinon. As shown in Fig. 7, the amplitude of oscillation of the spinon is drastically enhanced when the frequency of the driving potential is comparable to $\omega_0$, the BO frequency corresponding to $h = h_0$. Highly nontrivial dynamics may emerge when the frequencies of the
external driving force and the BO become incommensurate, which could induce chaotic dynamics of the spinon.

Now we discuss several possible trapped ultracold atomic systems that could realize our Hamiltonian [Eq. (1)] experimentally. Likely candidates are Rydberg atoms, which have been used already in quantum simulations and manipulation and have attracted lot of attention in recent years. Albeit highly excited, the lifetime of Rydberg atoms can reach even hundreds of microseconds, which is much longer than the typical lifetime of the first excited state of alkali metal atoms. Two possible states of a Rydberg atom (excited state and ground state) on each site reduce the problem to that of a pseudospin system, and the strongly dipole-dipole interactions contributing to the interaction between the pseudospins $J\sigma_i^x/\sigma_j^z$ and the external magnetic fields $\sigma_i^x$ and $\sigma_j^z$ can be realized by a laser with single-atom Rabi frequency $\Omega_0$ and detuning $\Delta$, respectively. Notice that the dipole-dipole interaction drops off as $r^{-3}$, suggesting that the next-nearest neighbor interaction (NNI) will still be appreciable, but in fact it is much weaker than the NNIs. Therefore, we do not expect that this changes the physics qualitatively for a ferromagnetic interaction.

An alternative approach would be provided by ultracold dipolar atoms or molecules with two internal states in optical lattices in the hard-core limit, with an externally driven intraspecies transition ($g$ field) and a Zeeman term. The recent experimental progress in observing ultracold atoms at the single-atom level (or even single-spin level) would be helpful to detect the dynamics of the single quasiparticles (spinons). The classic counterpart, a magnetic domain wall, has been realized experimentally as a supercold atomic thermometer. More recently, both our Hamiltonian and the magnetic domain wall were experimentally realized in an optical lattice. Considering the rapid development of the field, we expect the nontrivial spinon dynamics proposed in this paper to be accessible experimentally in various setups soon.

Our results could also be illuminating for condensed-matter physics, especially for the domain-wall motion in magnetic nanowires. Classically, the magnetization dynamics of magnetic systems is governed by the LLG equation, where the spin is considered a classic vector and its dynamics is reduced to a classic nonlinear equation. However, with advances in the miniaturization of magnetic structures, one has to anticipate that the classical and phenomenological LLG equation will eventually be inadequate, necessitating a full microscopic quantum description of the magnetization dynamics. Our result actually paves the way to studying the motion of the domain wall in the quantum situation. While dissipation would be very weak in a quantum optical implementation of our model, it would be more relevant in a solid. For the classic domain wall, dissipation is known to play a key role in determining its velocity. By assuming a memory-free bath, dissipation could be modeled in the framework of a Lindblad quantum master equation or a stochastic Schrödinger equation; the numerical method used here can be extended to simulate this model, in either a matrix product operator or a quantum jump approach. Depending on the detailed nature of the physical realization (and hence the bath), much richer physics could emerge because of the interplay between interactions and dissipation, making it an interesting topic for future study.

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