Hyperfine splitting of P states in heavy quarkonia

James Pantaleone

Physics Department, Purdue University, West Lafayette, Indiana 47907

S.-H. Henry Tye

Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

(Received 14 December 1987)

The splitting between the 1P_1 and the center of gravity of the 3P_J states of heavy quarkonia is evaluated using the perturbative QCD expression for the hyperfine potential. The theoretical result is about 1% of the absolute fine-structure splitting and disagrees with an experimental value.

Heavy-quark systems are accurately described by a nonrelativistic potential model. The static, spin-independent potential has been determined empirically and by lattice gauge theory with good agreement. The status of the various spin-dependent potentials is less certain. These are small, relativistic corrections to the model and are more difficult to observe. The perturbative QCD expressions for these potentials have been quite successful in describing the data. Fitting these expressions to the data yields values for the QCD scale parameter which are consistent with other determinations of this parameter.^{1,2} The ratio of fine-structure splittings,

$$R \equiv [M(^3P_2) - M(^3P_1)] / [M(^3P_1) - M(^3P_0)]$$

is predicted to be $\frac{4}{3}$ by tree-level perturbative QCD, independent of any parameters, and is measured to be 0.48 ± 0.01 and 0.67 ± 0.06 in the $c\bar{c}$ and $b\bar{b}$ $1P$ states.³ This success is in agreement with recent lattice gauge theory calculations⁴ which do not find any significant

long-range contributions to the spin-dependent potentials, except possibly in the spin-orbit potential.⁵⁻⁷ It thus appears that a large and probably the dominant contribution to the presently observed spin splittings is described by perturbative QCD (especially for $b\bar{b}$).

In this paper we examine the hyperfine splitting of the P states using the perturbative QCD expression for the potential. This splitting is often ignored in analyses of spin-dependent effects. However, two experimental groups have recently reported preliminary results for the masses of the $c\bar{c}$ and $b\bar{b}$ 1^1P_1 states. These provide the first measurements on the hyperfine splitting in $l \neq 0$ heavy-quark states. Our theoretical results agree with the preliminary $c\bar{c}$ data⁸ but disagree in sign and magnitude with the preliminary $b\bar{b}$ data.⁹

The hyperfine correction was first calculated correctly, to one-loop order, in Ref. 10 and has since been verified by many groups.^{1,2,11,12} Using the modified minimal-subtraction ($\overline{\text{MS}}$) scheme, the expression for the hyperfine perturbation is

$$\Delta H_{\text{Spin}} = \mathbf{S}_1 \cdot \mathbf{S}_2 \frac{8\pi}{3} C_F \alpha_{\overline{\text{MS}}}(\mu) \frac{1}{m^2} \left\{ \left[1 + \frac{\alpha_S}{\pi} \left[\frac{5}{12} b_0 - \frac{11}{12} C_A - \frac{1}{2} C_F + \frac{3}{2} (1 - \ln 2) T_F \right] \right] \delta^3(\mathbf{r}) + \frac{\alpha_S}{\pi} \left[-\frac{b_0}{4} \frac{1}{2\pi} \nabla^2 \left[\frac{\ln(\mu r) + \gamma_E}{r} \right] + \frac{7}{8} C_A \frac{1}{2\pi} \nabla^2 \left[\frac{\ln(mr) + \gamma_E}{r} \right] \right] \right\}, \quad (1)$$

where $b_0 = \frac{11}{3} C_A - \frac{4}{3} T_F N_f$, with N_f being the number of light quarks. μ is the renormalization scale, and $\gamma_E = 0.5772$. . . is the Euler constant. T_F , C_F , and C_A are gauge-group factors. In QCD they read $T_F = \frac{1}{2}$, $C_F = \frac{4}{3}$, and $C_A = 3$. Corrections to Eq. (1) are at the order¹ of $(\alpha_s/m^2)(v^2/c^2)$ and (α_s^3/m^2) .

The spin-dependent potentials are relativistic corrections to the standard, spin-independent model. Thus first-order nonrelativistic perturbation theory will give us the spin splittings to leading order in (v^2/c^2) , the same level of accuracy as Eq. (1):

$$\begin{aligned} [M(n^3S_1) - M(n^1S_1)] &= \langle nS | \Delta H | nS \rangle, \\ [\bar{M}(n^3P_j) - M(n^1P_1)] &= \langle nP | \Delta H | nP \rangle. \end{aligned} \quad (2)$$

Here $\bar{M}(n^3P_j) \equiv [\sum (2j+1) M(n^3P_j)] / [\sum (2j+1)]$ is the center of gravity of the triplet P states. The dependence of the triplet P states on the spin-orbit and tensor potentials cancels out in this weighted average^{1,2} and only the effects of the hyperfine splitting remain.

For the P states, the hyperfine splitting vanishes at the tree level in QED (Ref. 13) and QCD. This is because the P -state wave function vanishes at the origin so the terms

in Eq. (1) proportional to $\delta^3(\mathbf{r})$ do not contribute. However at one loop, the terms with $\nabla^2(\ln r/r)$ contribute a $-1/r^3$ to the splitting. In QCD, the expression for the splitting is

$$[\bar{M}(n^3P_j) - M(n^1P_1)] = \frac{8}{9} \left[\frac{1}{4} - \frac{N_f}{3} \right] \frac{\alpha_s^2}{\pi} \frac{1}{m^2} \left\langle \frac{1}{r^3} \right\rangle, \quad (3)$$

$$\langle 1 \rangle \equiv \int_0^\infty dr r^2 R_{nl}^2(r) = 1.$$

$R_{nl}(r)$ denotes the radial wave functions of the spin-independent Hamiltonian. We make the following general observations.

(1) The hyperfine potential contains two physically independent potentials. One contributes to $l=0$ splittings and the other contributes for all l .

(2) Even though Eq. (3) comes from one-loop order the expression does not depend on the renormalization point μ . This is to be expected since the expression is the leading-order term in perturbative QCD, it depends only on the generic "strong coupling constant."

(3) The splitting is sensitive to the number of light-quark flavors. In fact the splitting is positive for $N_f=0$ and negative for $N_f>0$. Thus the predicted sign of the splitting is opposite to that of S -state splitting and also opposite to expectations from a simple softening of the Coulomb potential.

(4) This result is much smaller than a typical spin splitting. It is down by factors of α_s/π and by small numerical factors from typical fine-structure splittings.

To evaluate Eq. (3) we need to know the size of $\langle 1/m^2 \rangle \langle 1/r^3 \rangle$. This expression can be calculated using Schrödinger's equation with the static, spin-independent potential, but we choose an alternative approach. This particular combination of quark mass and P -state position average is identical to that which appears in the leading-order expressions for the fine-structure splittings (see, for example, Ref. 1). As noted previously, these splittings have been observed and perturbative QCD has been successful in describing them. Using these leading-order expressions to relate the fine and hyperfine splittings of the P states yields

$$[\bar{M}(n^3P_j) - M(n^1P_1)] = [\bar{M}(n^3P_2) - M(n^3P_0)] \frac{10}{81} \left[\frac{1}{4} - \frac{N_f}{3} \right] \frac{\alpha_s}{\pi}. \quad (4)$$

The largest fine-structure splitting has been arbitrarily chosen to be used above. This expression is evaluated in Table I. Table I also contains the preliminary experimental results for these splittings.

The evaluation of the perturbative expression for the P -state hyperfine splitting has many sources of uncertainty. Because only the leading-order term is known, the expression does not depend directly on the fundamental pa-

TABLE I. The predicted P -state hyperfine splittings as calculated from perturbative QCD and the absolute fine-structure splitting. The preliminary experimental (Refs. 8 and 9) results for the hyperfine splittings are also shown with their statistical significance. $\bar{M}(n^3P_j) \equiv [\sum (2j+1)M(n^3P_j)] / [\sum (2j+1)]$ is the center of gravity of the triplet P states. Here $N_f=3$ and $\alpha_s \approx \frac{1}{3}$ are used.

State	Experiment	Theory	
	$\bar{M}(^3P_2) - \bar{M}(^3P_0)$ (MeV)	$\bar{M}(^3P_j) - M(^1P_1)$ (MeV)	
$c\bar{c}1P$	141.4 ± 1.2	0.0 ± 1.0 (2.3 σ)	-1.4
$b\bar{b}1P$	52.7 ± 2.4	5.4 ± 1.7 (2.5 σ)	-0.5
$b\bar{b}2P$	$38.6 \pm 2.4 \pm ?$		-0.4

rameter of QCD, Λ , but on the somewhat arbitrary "strong coupling constant" α_s . Also the expression is extremely sensitive to N_f , the number of light-quark flavors, and there is some uncertainty as to the proper choice of N_f at the scale relevant for heavy-quark P -state binding. In addition, Eq. (4) may introduce some additional uncertainty from possible long-range contributions to the fine-structure splittings. These factors combine to render any evaluation of Eq. (3) or (4) uncertain by about a factor of 2. However, they do not change the basic observation that this prediction for the hyperfine splitting is very small, as is clear from the table.

The theoretical predictions do not agree with the preliminary experimental results. From Table I, there is tentative agreement between theory and experiment for the $c\bar{c}$ system but for the $b\bar{b}$ system the experimental result is an order of magnitude larger and has the opposite sign. This is puzzling because one naively expects the theoretical value, obtained from perturbative QCD, to be more accurate in the larger mass, $b\bar{b}$ system. Two likely possibilities suggest themselves; the preliminary $b\bar{b}$ experimental results could be incorrect or nonperturbative contributions to the splitting could be large. The small size of the perturbative QCD contribution to the P -state hyperfine splitting makes it a good test of the perturbative model for the spin splittings. Contributions from long-range, gluon condensate, relativistic, coupled channel, or other effects could dominate over the small, perturbative value. Definite experimental results would provide an insight into how important these corrections are for the spin splittings of heavy quarks. The original calculations of Igi and Ono disagree with our values; however, in a recent erratum their results are similar to ours.¹⁴

We thank Tom Clark, T. K. Kuo, and Sherwin Love for useful discussions. J.P. was supported by the United States Department of Energy and S.-H.H.T. by the National Science Foundation.

¹J. Pantaleone, S.-H. H. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986); Phys. Rev. Lett. **55**, 916 (1985).

²S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **26**, 3305 (1982); S. N. Gupta and S. F. Radford, *ibid.* **24**, 2309

(1981); **25**, 2690 (1982); **25**, 3430 (1982).

³For a recent review of heavy-quark spectroscopy, see W. Buchmüller and S. Cooper, MIT Report No. MIT-LNS-159, 1987 (unpublished).

- ⁴E. Eichten and F. L. Feinberg, Phys. Rev. Lett. **43**, 1205 (1979); Phys. Rev. D **23**, 2724 (1981).
- ⁵M. Campostrini, K. Moriarty, and C. Rebbi, Phys. Rev. D **36**, 3450 (1987); Phys. Rev. Lett. **57**, 44 (1986).
- ⁶P. de Forcrand and J. D. Stack, Phys. Rev. Lett. **55**, 1254 (1985).
- ⁷C. Michael, Phys. Rev. Lett. **56**, 1219 (1986).
- ⁸R704 Collaboration, C. Baglin *et al.*, Phys. Lett. B **171**, 135 (1986).
- ⁹CLEO Collaboration, T. Bowcock *et al.*, Phys. Rev. Lett. **58**, 307 (1987).
- ¹⁰W. Buchmüller, Y. J. Ng, and S.-H. H. Tye, Phys. Rev. D **24**, 3003 (1981).
- ¹¹M. Klein and O. Streit, Phys. Lett. **146B**, 357 (1984).
- ¹²N. Luo, Cornell Ph. D. thesis, 1987.
- ¹³H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum, New York, 1977).
- ¹⁴K. Igi and S. Ono, Phys. Rev. D **36**, 1550 (1987); **37**, 1338(E) (1988).