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Dirac cones at $\vec{k} = 0$ in acoustic crystals and zero refractive index acoustic materials

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We show that two-dimensional acoustic crystals (ACs) can be designed to exhibit Dirac cone dispersion at $\vec{k} = 0$. Effective medium theory finds that some of these ACs can have effectively zero reciprocal of bulk modulus $1/\kappa_{\text{eff}}$ and zero mass density ρ_{eff} , and thus zero refractive indices at the Dirac point. Numerical simulations are used to demonstrate various phenomena associated with the zero spatial phase change inside such materials. © 2012 American Institute of Physics. [doi:10.1063/1.3686907]

Acoustic metamaterials are a class of complex composite materials that have created considerable excitement for they can be engineered to exhibit intriguing physical phenomena such as negative refraction, subwavelength imaging, and invisibility cloaking. Past efforts are focused on realizing acoustic metamaterials, exhibiting effectively negative bulk modulus and negative mass density^{1–8} and demonstrating acoustic applications^{9–13} that require such unusual properties. Zero refractive index materials represent another class of metamaterials that have been realized experimentally^{14–18} for electromagnetic (EM) wave, and the natural question to ask is whether one can also realize zero index for acoustic waves. One-dimensional acoustic transmission line metamaterials with negative/zero/positive index has been¹⁹ reported in the literature. The purpose of this paper is to design acoustic material with effectively zero density and zero reciprocal modulus in two dimensions. Recently, it has been shown that Dirac cone dispersion can be realized at $\vec{k} = 0$ in some two-dimensional photonic crystals with effectively zero permittivity and permeability simultaneously.²⁰ It is well known that many intriguing characteristics of electromagnetic crystals can also be realized in acoustic crystals (ACs). Here, we show that we can indeed realize Dirac cones $\vec{k} = 0$ in two-dimensional (2D) ACs, and some of these 2D ACs can be mapped to a zero refractive index acoustic material. We also note that Dirac cone dispersions^{21–24} are interesting and useful in their own right (such as for beam steering²⁵), irrespective of whether they can be mapped to a zero index or not.

The necessary condition to get a Dirac cone at $\vec{k} = 0$ is linear dispersion, but linear dispersion is forbidden by time reversal symmetry for non-degenerate states.²⁵ This can be circumvented by employing accidental degeneracy.²⁰ We give explicit example, which shows that in 2D ACs, Dirac cones at $\vec{k} = 0$ can indeed be obtained by accidental degeneracy and some of these ACs can be mapped to an acoustic zero refractive index material with both effectively zero reciprocal of bulk modulus $1/\kappa_{\text{eff}}$ and zero mass density ρ_{eff} .

Figure 1(a) shows the band structure of a 2D AC consisting of a square lattice of rubber cylinders embedded in water

calculated using the multiple scattering method.²⁶ Here, the lattice constant is denoted by a , and the radius of the cylinders is set at $R = 0.224a$. The density ρ of rubber is taken to be $\rho = 1.3 \times 10^3 \text{ kg/m}^3$ and that of water is $\rho_0 = 1.0 \times 10^3 \text{ kg/m}^3$. The Lamé constant in rubber is $\lambda = 2.08 \times 10^8 \text{ N/m}^2$ and $\mu = 5.2 \times 10^7 \text{ N/m}^2$ and for water $\lambda_0 = \kappa_0 = 2.22 \times 10^9 \text{ N/m}^2$. Two branches with linear dispersion intersect at a triply degenerate point at $\vec{k} = 0$ forming a Dirac point at $\omega_0 = 0.561(2\pi v_0/a)$. A flat branch with a small group velocity also intersects at the Dirac point. This flat band can be interpreted as a consequence of the zero effective density of the system.² If the system is perfectly homogeneous, the band will be dispersionless, and the band will be a “deaf band,” which does not couple with external waves. In any real ACs composing of discrete building blocks, the spatial dispersion gives the band a finite (albeit small) dispersion. In order to understand the physical nature of the eigenstates near the Dirac point, we show the magnitude of displacement field distributions of the eigenstates near the Dirac point with a small k along the ΓX direction ($k_x = 0.001 \times \pi/a$, $k_y = 0$). In Figs. 1(b) and 1(c), we show the imaginary and real parts of the displacement fields of the lowest frequency state ($\omega_0 = 0.56(2\pi v_0/a)$), which show that the eigenmode is a combination of a monopole and a longitudinal dipole excitation. The eigenmode for the highest frequency state ($\omega_0 = 0.562(2\pi v_0/a)$) (not shown here) is similar, also a combination of a monopole and a longitudinal dipole. The real part of the eigenmode of the flat band is plotted in Fig. 1(d), and the imaginary part is almost zero. We can see that it is a pure dipolar mode with displacement essentially perpendicular to the wave vector $\vec{k}$. Thus, it is a transverse mode near the Γ point. These displacement field distributions show that the three branches involve only a mixture of monopolar and dipolar modes. We note that the square lattice system has C_{4v} symmetry. The non-degenerate monopolar mode and the doubly degenerate dipolar modes at the Dirac point can be labeled as A_1 and E , respectively, at the zone center.²⁷ The monopolar and dipolar modes are not required by symmetry to have the same frequency, but the system parameters (such as the diameter of the cylinder) can be chosen so that they are equal, and the accidental degeneracy of A_1 and E states makes possible the existence of the Dirac cones.

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FIG. 1. (Color online) (a) Band structure of a 2D square lattice ACs consisting of rubber cylinders (radius $R = 0.224a$) in water. Two linear bands intersect at a Dirac point $\omega_0 = 0.561(2\pi\nu_0/a)$, with an additional band with small group velocity. The solid red line is the dispersion curve calculated with the effective medium parameters (shown in panel (e)). We do not include the transverse wave “deaf band” that corresponds to $\rho_{eff} = 0$. (b) The imaginary part of displacement field distributions of the eigenmode at frequency $\omega_0 = 0.56(2\pi\nu_0/a)$. (c) The real part of displacement field distributions of the eigenmode at frequency $\omega_0 = 0.56(2\pi\nu_0/a)$. (d) The real part of displacement field distributions of the eigenmode at frequency $\omega_0 = 0.561(2\pi\nu_0/a)$. (e) The effective mass density ρ_{eff} (red solid line) and reciprocal of bulk modulus $1/\kappa_{eff}$ (olive dashed line) of the ACs as a function of frequency. All the effective acoustic parameters have been normalized to the acoustic parameters of water. The quantity ν_0 is the velocity of water.

As the Dirac cone is located at the zone center in the ACs composed of rubber cylinders, there is a possibility that we can employ an effective medium theory to describe the physics of the system at small values of k . The effective mass density ρ_{eff} and reciprocal of bulk modulus $1/\kappa_{eff}$ can be obtained using standard effective medium theory²⁸ developed for electromagnetic waves, with the mapping of variables as $\varepsilon \leftrightarrow 1/\kappa$, $\mu \leftrightarrow \rho$. Fig. 1(e) shows the effective mass density ρ_{eff} (red solid line) and reciprocal of bulk modulus $1/\kappa_{eff}$ (olive dashed line) as a function of frequency, and ρ_{eff} and $1/\kappa_{eff}$ intersect at zero at the Dirac point frequency ($\omega_0 = 0.561(2\pi\nu_0/a)$). Using the effective parameters ρ_{eff} and $1/\kappa_{eff}$, we can obtain the dispersion curve as $k = \omega \sqrt{\rho_{eff}/\kappa_{eff}}$ (red solid line in Fig. 1(a)). The very good agreement between the analytical and the numerical results indicates that the effective medium theory describes the ACs

FIG. 2. (Color online) (a) Band structure of a 2D square lattice consisting of steel cylinders in water with $R = 0.194a$. There is also a Dirac point at frequency $0.945(2\pi\nu_0/a)$. (b) The effective mass density ρ_{eff} (red solid line) and reciprocal of bulk modulus $1/\kappa_{eff}$ (olive dashed line) of the ACs extracted according to effective medium theory.²⁸ All the effective acoustic parameters have been normalized to the acoustic parameters of water.

FIG. 3. (Color online) The numerically simulated displacement field distributions for a homogeneous $\rho = 1/\kappa = 0$ medium (a), the ACs composed of rubber cylinders (b), and the ACs composed of steel cylinders (c) as a “lens” with boundaries outlined by a purple line. The working frequency is $0.561(2\pi\nu_0/a)$ for panels (a) and (b), while for the panel (c) is $0.945(2\pi\nu_0/a)$, which correspond to the Dirac point frequencies of the two ACs, respectively. Note that the rubber-in-water system behaves as if it is a zero index material, while steel-in-water does not.

FIG. 4. (Color online) (a) The numerically simulated displacement field distributions for incident waves going through a 180° bending waveguide filled with the ACs composed of rubber cylinders. (b) A comparative simulation with the ACs composed of rubber cylinders removed. The boundary walls are hard walls. The working frequency is at the Dirac point frequency $\omega_0 = 0.561(2\pi v_0/a)$.

composed of rubber cylinders near the Dirac point frequency very well for this particular configuration.

However, not all 2D ACs with Dirac cone dispersion at $\vec{k} = 0$ can be mapped to an acoustic zero refractive index material. Figure 2(a) shows the band structure of a 2D square lattice consisting of steel ($\rho_{\text{steel}} = 7.9 \times 10^3 \text{ kg/m}^3$, $\lambda_{\text{steel}} = 1.15 \times 10^{11} \text{ N/m}^2$, and $\mu_{\text{steel}} = 8.28 \times 10^{10} \text{ N/m}^2$) cylinders in water. Note that there is also a Dirac point at frequency $0.945(2\pi v_0/a)$, and the formation of Dirac point is again due to the accidental degeneracy of a double degenerate E mode and a non-degenerate A_1 mode. But, as the wave velocity of steel is higher than that of water, the displacement fields mainly localize in the water, and quite unlike the case of rubber in water, there are no noticeable excitation in the steel cylinders. The physics is quite different from the case of rubber cylinders in water. In addition, the Dirac point frequency here is very high, and we do not expect effective medium theory can describe the physics correctly. If one insists, one can still use the standard formulas²⁸ to extract the effective parameters, which are shown in Fig. 2(b). These effective parameters obviously cannot produce a dispersion that mimics the dispersion shown in Fig. 2(a). This shows that the Dirac point in this system cannot be mapped accurately to a zero refractive index system.

As ρ_{eff} and $1/\kappa_{\text{eff}}$ go through zero simultaneously and linearly, the effective refractive index also goes through zero but the group velocity remains finite. Waves do not experience any spatial phase change as they propagate inside a zero refractive index material. Such peculiar properties have been exploited to achieve interesting wave manipulation effects such as wave tunneling in bending waveguides^{17,18} and tailoring the radiation phase pattern¹⁶ in electromagnetic waves. As we have realized an effectively zero refractive index in the ACs composed of rubber cylinders, we show below that can achieve similar wave manipulation for acoustic waves.

Fig. 3(a) shows a numerical simulation (using COMSOL MULTIPHYSICS), which demonstrates the acoustic wave manipulation properties of a zero refractive index material. A plane wave is incident from the bottom and impinges normally on the flat bottom of a “lens” made of homogeneous $\rho = 1/\kappa = 0$ medium, and as there is no phase change across the medium, the phase of the transmitted wave is the same on every point of the upper interface of the “lens.” As the left part of the “lens” is concave, a focal point appears on the upper side of the “lens.” In Fig. 3(b), the homogeneous medium is replaced by the ACs composed of rubber cylinders, and at its Dirac point frequency $0.561(2\pi v_0/a)$, the field pat-

tern is similar to the homogeneous case. In contrast, if the homogeneous medium is replaced by the ACs composed of steel cylinders, the field pattern (Fig. 3(c)) is totally different at its Dirac point frequency $0.945(2\pi v_0/a)$. Therefore, the ACs composed of rubber cylinders can be mapped to an acoustic zero refractive index material while the steel cylinder system cannot.

Fig. 4(a) shows another numerical simulation demonstrating the peculiar property of the effectively zero refractive index system. At the Dirac point frequency, acoustic wave incident from the left channel go around a 180° bending waveguide filled with the ACs composed of rubber cylinders and emerge through the right exit channel with little distortion. This is what a zero refractive material is expected to behave. Fig. 4(b) shows a control calculation in which the ACs composed of rubber cylinders is removed, and the results show that most of the incidence waves reflect back into the entrance channel. Comparing Fig. 4(a) with Fig. 4(b) shows the ACs composed of rubber cylinders at its Dirac point frequency can guide acoustic waves through waveguide with large bending angle.

In summary, we showed that we can obtain Dirac cone dispersion at the zone center in a 2D square lattice ACs. Some of these Dirac cone systems (such as those consisting of rubber cylinders in water) behave as if they have effectively zero reciprocal of bulk modulus $1/\kappa_{\text{eff}}$ and zero mass density. Numerical simulations are performed to demonstrate various applications associated with the acoustic zero refractive index material. In addition to the realization of zero refractive index, these ACs with accidental degeneracy should be interesting platforms to explore Dirac cone physics in acoustic wave systems. We note in passing that Dirac cones can also be found in elastic wave systems but the physics there is quite different.²⁹

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