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Optimal Energy-efficient Cooperative Sensing Scheduling for Cognitive Radio Networks with Heterogeneous Primary Channel Access

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Abstract—As a promising technology to implement Dynamic Spectrum Access (DSA), Cognitive Radio (CR) is envisioned to evolve to be *Green*. In this paper, the Cooperative Sensing Scheduling (CSS) problem for CR is analyzed by taking both the performance and energy consumption of spectrum sensing into consideration. We model the CSS problem under a practical scenario where primary user (PU) channels are heterogeneous in terms of channel protection criteria, idling probabilities and channel capacities. With the objective to find an optimal and efficient secondary users (SUs) assignment scheme, we formulate the CSS problem into a nonlinear integer programming problem. The intricate two-dimensional trade-off among energy efficiency, sensing performance and spectrum opportunity exploration is presented and properly tackled. We explore the inherent property of this problem by adopting a two-step approach. In the first step, by fixing the number of SUs participating in sensing, the problem is converted to an M-concave problem and an efficient algorithm for optimal SU assignment is proposed. With this scheme, we then investigate the appropriate number of SUs participating in sensing in the second step. Based on these two steps, the final SU assignment scheme is proposed. The optimality of this assignment scheme is proved theoretically and verified numerically.

Index Terms—cognitive radio, cooperative sensing scheduling, nonlinear integer programming, energy-efficient, M-convex

I. INTRODUCTION

The dramatic growth of wireless application leads to tremendous boost in the volume of communication traffic as well as the associated energy consumption. According to [1], the Information and Communication Technologies (ICT) infrastructures constitute three percent of worldwide energy consumption and generate two percent of worldwide CO2 emissions. As an effort to make the ICT sector more environment-friendly, the next generation wireless communications technology is envisioned to evolve to be *Green* [3].

Dynamic spectrum access (DSA) technology is proposed to resolve the conflict between massive spectrum underutilization and the impending spectrum scarcity. One of the promising implementations of this paradigm is Cognitive Radio (CR) [2], which enables Secondary Users (SUs) to opportunistically access frequency bands which are exclusively allocated to Primary Users (PUs) without introducing harmful interference to the communication process of PUs. In this paper we focus on the energy and performance optimization issues in CR.

Spectrum detection plays a crucial role for the successful operation of CR. Geo-location database and spectrum sens-

ing are two major technologies used for spectrum detection. From both technical and commercial viewpoints [4], spectrum sensing continues to demonstrate value in the operation of CR despite the recent raise in regulatory attention on geo-location database techniques [15] [16]. The Cognitive Radio Network (CRN) adopting spectrum sensing needs to meet stringent performance requirements specified by corresponding regulatory organizations in order to protect the communication process of PUs, and cooperative sensing [6] was proposed to meet these requirements by allowing multiple SUs to collaboratively sense one channel, which can improve the sensing performance significantly.

However, the spectrum sensing process is power intensive [3], which will cause energy limited devices, such as laptops, tablet PCs, smart phones, to consume energy more rapidly. Hence, energy efficiency should be considered as one of the design objectives. To achieve this objective, SUs need to adopt certain strategies to conserve energy while preserving the sensing performance. In this paper, we propose a scheme in which SUs participate in sensing only when “necessary”.

CRNs usually have access to multiple PU channels. For such CRN, the following three problems need to be addressed if cooperative spectrum sensing is adopted.

- (1) How many SUs should participate in sensing?
- (2) How many available channels should SUs sense?
- (3) How much effort should the CRN spend on sensing for different channels?

These problems are referred to as Cooperative Sensing Scheduling (CSS) problem, in which lie two essential trade-offs. The first trade-off is between capturing spectrum opportunity and energy consumption: the more SUs participating in sensing, the more spectrum opportunity can the CRN discover, but in the meantime, more energy will be consumed. The second trade-off lies between sensing accuracy and capturing spectrum opportunity. Assigning more SUs to sense one channel cooperatively will lead to a better sensing performance (i.e., higher sensing accuracy and reliability), the CRN, however, cannot fully explore spectrum opportunity since less channels are sensed in this case. A desirable assignment scheme of SUs should take both trade-offs mentioned above into consideration.

Despite of its importance in DSA, the CSS problem has

received insufficient attention. Research on CSS has been carried out in [11]-[13]. In [11], the authors studied optimal strategies of CSS that were called multichannel coordination. By considering heterogeneous PU occupancy probabilities in different channels and heterogeneous detection error probabilities for SUs, the CSS is modeled as an NP-Hard integer programming problem. They proposed heuristic algorithms to obtain suboptimal results. The optimal solution, however, can be found under a set of relaxed assumptions that all SUs have identical sensing capability and all PU channels have homogeneous characteristics. The CSS is then modeled as a combinatorial optimization problem [12], and the optimal solution was found. Yet this solution might not be feasible in real world situations where the assumptions are usually not valid. A comprehensive design which considered both CSS and optimal sensing parameter setting was provided by [13]. They formulated the channel assignment as a Max-Min optimization problem, which was also shown to be NP-Hard, and a suboptimal greedy algorithm was proposed. We also noticed that issues related to energy efficiency were seldom discussed in the papers mentioned above. We conclude that the research on this topic is still in the early stage, and applications in practical situations are lacking.

In this paper, we consider this CSS problem under a more practical scenario where PU channels are heterogeneous in terms of channel protection criteria, channel idling probabilities and channel capacities. The intricate two-dimensional trade-off among energy efficiency, sensing performance and spectrum opportunity exploration is also properly tackled. After formulating this problem into a nonlinear integer programming problem, we explore the inherent property of this problem based on analysis involving the theory of M-convexity [14]. A two-step approach is adopted to solve the problem. In the first step, by fixing the number of SUs participating in sensing, the problem is converted to an M-concave problem and an efficient algorithm is proposed. In the second step, we investigate the appropriate number of SUs participating in sensing to achieve energy efficiency. After these two steps, an optimal and efficient SUs assignment scheme which takes both sensing performance and energy efficiency into consideration is obtained. The optimality of this assignment scheme is proved both theoretically and numerically.

The rest of this paper is organized as follows. System model is given in section II and problem formulation is presented in section III. We then proceed to analyze the inherent structure of this problem in section IV. By dividing this problem into two subproblems, an algorithm is proposed to obtain an optimal solution in section IV. We also provide two numerical examples in section V to illustrate and validate our theoretical analysis. Finally, conclusion is given in section VI.

II. SYSTEM MODEL

We consider a centralized CRN with M SUs and a Base Station (BS). The BS is in charge of assigning SUs to sense different PU channels and collecting individual sensing results from SUs to make final decisions on the occupancy state of

primary channels. SUs can opportunistically access N primary channels, which are denoted as $\mathcal{N} = \{1, 2, \dots, N\}$. Let $s_n = 0$ ($s_n = 1$) denote primary channel n ($n \in \mathcal{N}$) being idle (busy). Also, these PU channels are heterogeneous in terms of channel protection criteria, channel idling probabilities and channel capacities. Throughout this paper, same as many other works [8] [9], we assume that the SUs are equipped with single radio interface, and therefore can only sense or access one primary channel at a time. We assume the CRN works in a slotted frame structure [7] and the frame length is denoted as T . Each frame is divided into three parts, namely, sensing duration τ , scheduling and results fusion duration η , and data transmission duration $T - \tau - \eta$.

In the sensing duration, each SU is assigned to sense one PU channel. We assume energy detection method is adopted by all SUs during the sensing process. Spectrum sensing accuracy of each SU is measured by the detection probability p_d and the false alarm probability p_f , which are defined as follows:

$$p_d \triangleq \Pr\{\text{decide } \mathcal{H}_1 | \mathcal{H}_1 \text{ is true}\}, \quad (1)$$

$$p_f \triangleq \Pr\{\text{decide } \mathcal{H}_1 | \mathcal{H}_0 \text{ is true}\}, \quad (2)$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ denote the hypotheses that the PU is present and absent respectively, i.e.

$$\mathcal{H}_0 : s_n = 0, \quad \mathcal{H}_1 : s_n = 1. \quad (3)$$

We assume that the CRN adopts cooperative spectrum sensing. All SUs report their one bit sensing results to the BS after spectrum sensing based on energy detection [6]. The BS combines the collected results using “OR” rule to obtain the final decision on the occupancy state of each PU channel. Suppose channel $n \in \mathcal{N}$ is sensed by m SUs. Then the sensing performance for channel n at the BS can be expressed as

$$P_{d,n}(m) = 1 - [1 - p_{d,n}(m)]^m, \quad (4)$$

$$P_{f,n}(m) = 1 - [1 - p_{f,n}(m)]^m, \quad (5)$$

where $p_d^n(m)$ and $p_f^n(m)$ denote the detection probability and false alarm probability of individual SU that senses channel n , respectively. Same as [7], the relationship between p_d and p_f for each SU is given by

$$p_{f,n} = Q(\sqrt{2\gamma + 1}Q^{-1}(p_{d,n}) + \sqrt{\tau f_s \gamma}), \quad (6)$$

where γ represents the signal-to-noise ratio (SNR) of PU signals received at SU, f_s denotes the sampling rate¹, $Q(\cdot)$ represents Gaussian Q-function which is the tail probability of the standard Gaussian distribution, and $Q^{-1}(\cdot)$ is the inverse function of $Q(\cdot)$.

III. PROBLEM FORMULATION

For a CRN with access to multiple PU channels, two basic trade-offs need to be tackled. The first one is the trade-off between the energy spent on sensing and expected aggregated achievable channel rate discovered by spectrum sensing. The

¹Note that values γ are the same for all SUs in this paper. τ and f_s are decided by the BS, and are the same for all SUs.

more SUs participating in sensing, the more spectrum opportunity can the CRN discover, but the energy consumption will be higher as well. The second trade-off lies between the exploration of the spectrum opportunity and the sensing accuracy. If many SUs are assigned to sense a single channel cooperatively, the sensing accuracy of that channel is higher, but fewer channels are sensed within duration τ consequently. On the other hand, if the SUs are scattered sparsely across the channels, sensing accuracy will deteriorate although the number of channels sensed increases. To tackle this trade-off, we find a proper assignment of SUs $\mathbf{a} = \{a_1, a_2, \dots, a_N\}$ to sense PU channels such that a good balance between spectrum opportunity exploration and sensing accuracy is achieved, where a_n is the number of SUs assigned to PU channel n .

The objective of this paper is to find a proper sensing scheduling scheme that achieves a balance between aggregated channel rate discovered in the sensing process and the energy spent on sensing. During the sensing period, SUs consume energy at two different power levels, namely, sensing power ϕ_s and idling power ϕ_i . We assume that all the SUs stay quiet and listen for PU signals during the sensing duration, i.e. SUs stop packet transmission and listen to the PU channels. We also assume that SUs participating in sensing consume energy at power level ϕ_s and SUs not participating in sensing consume energy at power level ϕ_e . Denote κ as the number of SUs participating in sensing, then the total energy consumption in the sensing duration is given by

$$E(\kappa) = \tau\phi_s\kappa + \tau\phi_e(M - \kappa). \quad (7)$$

Denote $C_{0,n}$ and $C_{1,n}$ as the capacity of channel n when SUs operate in the scenario where PU is absent and present, respectively. Then the expected achievable rate of channel n which is represented by $R'(a_n)$ can be expressed as

$$R''(a_n) = C_{0,n}[1 - P_{f,n}(a_n)]P_n(\mathcal{H}_0) + \quad (8)$$

$$C_{1,n}[1 - P_{d,n}(a_n)]P_n(\mathcal{H}_1), \quad (9)$$

where $P_n(\mathcal{H}_0)$ and $P_n(\mathcal{H}_1)$ denote the stationary probability that channel n is idle and busy, respectively. According to [7], $C_{0,n} \gg C_{1,n}$, and thus the first term on the right hand side of (9) dominates. Also, our objective is to maximize the achievable channel rate in the scenario where CRN operates on a overlay fashion, i.e. CRN only transmits when PU signal is absent. Hence, we can rewrite the achievable rate of channel n as follows

$$R'(a_n) = C_{0,n}(1 - P_{f,n}(a_n))P_n(\mathcal{H}_0).$$

Then the utility function can be given by

$$U(\mathbf{a}(\kappa), \kappa) = w_r R(\mathbf{a}(\kappa)) + w_e E(\kappa), \quad (10)$$

where $R(\mathbf{a}(\kappa)) = \sum_{a_n \geq 0} R'(a_n)$ is the aggregated achievable channel rate, $w_r > 0$ and $w_e < 0$ are two weighting factors for aggregated achievable channel rate and energy consumption, respectively.

In order to protect the quality of service of PUs, the achieved detection probability of each channel in the CRN should not

be smaller than the required threshold. Denote λ_n as detection probability requirement imposed on channel n . We reorder the channels in such a way that the detection probability requirements are in ascending order, i.e. $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$. Then the CSS problem can be formulated as follows

$$(\text{CSS}) : \quad \max_{\mathbf{a}(\kappa), \kappa} U(\mathbf{a}(\kappa), \kappa), \quad (11)$$

$$\text{s.t.} \quad \sum_{n=1}^N a_n = \kappa \leq M, \quad a_n \in \{0, 1, \dots, \kappa\}, \quad (12)$$

$$P_{d,n}(a_n) \geq \lambda_n, \quad \forall n \in \mathcal{N}. \quad (13)$$

The above formulated problem (CSS) is a nonlinear integer programming problem, and the complexity to solve this kind of problem is usually high. In the next section, we will explore the inherent characteristics of problem (CSS), and we expect to find some useful properties which are conducive to solving this problem efficiently.

IV. PROBLEM ANALYSIS

The total energy consumed by sensing is related to the length of sensing duration in a frame and the number of SUs participating in sensing. Since the sensing duration is fixed and the same for all the SUs, the energy consumption of sensing in the entire CRN is only related to the number of SUs participating in sensing. On the other hand, the sensing performance of each channel is a function of the number of SUs assigned to sense it, hence the sensing performance of the entire CRN is only related to the assignment of SUs participating in sensing. Thus we can decouple problem (CSS) into the following two steps: (1) Assume the number of SUs participating in sensing is fixed, find the optimal assignment of SUs to sense different PU channels. (2) Find an appropriate number of SUs to sense the PU channels so that a good balance between sensing energy consumption and spectrum opportunity discovery is achieved. In essence, we ensure that the performance is optimized in the first step, and then we ensure the energy requirements are met in the second step.

A. Step 1: Find the optimal assignment

In this subsection, we assume the number of SUs participating in sensing, i.e. κ , is fixed. Denote $U(a(\kappa)|\kappa)$ as the utility function (10) when κ is given, then $U(a(\kappa)|\kappa)$ can be expressed as

$$U(\mathbf{a}(\kappa)|\kappa) = \sum_{a_n \geq 0} \alpha_n [1 - P_{f,n}(a_n)] + \beta, \quad (14)$$

where $\alpha_n = w_r P_n(\mathcal{H}_0) C_{0,n}$ and $\beta = w_e E(\kappa)$. Both α and β are constant in this case. Hence, problem (CSS) becomes

$$(\text{P1}) : \quad \max_{\mathbf{a}} U(\mathbf{a}(\kappa)|\kappa), \quad (15)$$

$$\text{s.t.} \quad \sum_{n=1}^N a_n = \kappa, \quad a_n \in \{0, 1, \dots, \kappa\}, \quad (16)$$

$$P_{d,n}(a_n) \geq \lambda_n, \quad \forall n \in \mathcal{N}. \quad (17)$$

Similar to [7], we can easily prove that the optimal solution of problem (P1) is achieved with equality constraint in (17). The following lemma will provide us with further insight to solve this problem.

Lemma 1: Let m ($m \in [1, +\infty)$) be a continuous variable representing the number of SUs assigned to sense a channel. Then $P_f(m)$ is decreasing and convex if the following condition holds

$$\left[\ln(1 - p_f(m)) - \frac{m}{1 - p_f(m)} \nabla p_f(m) \right]^2 < \frac{2\nabla p_f(m) - m\nabla^2 p_f(m)}{1 - p_f(m)} - \left[\frac{\sqrt{m}\nabla p_f(m)}{1 - p_f(m)} \right]^2. \quad (18)$$

By extensive simulation, we find that condition (18) holds for most practical systems. Without loss of generality, we assume condition (18) holds throughout this paper.

According to lemma 1, $P_f(m)$ is decreasing and convex in practical cases if the variable m is continuous, however, in problem **(P1)**, the variables are integer numbers. Also, the concept of convex function becomes complicated in discrete case. Therefore, we need to analyze problem **(P1)** in the framework of M-convexity which is different from traditional continuous convex analysis. To facilitate further analysis, we should first introduce the following definitions. Interested readers can refer to [14] for more details.

Definition 1: For a vector $\mathbf{a} = (a_v | v \in V)$, the positive and negative support of this vector is defined as

$$\text{supp}^+(\mathbf{a}) = \{v | a_v > 0\}, \quad \text{supp}^-(\mathbf{a}) = \{v | a_v < 0\}. \quad (19)$$

Definition 2: A function $f : Z^V \rightarrow R \cup \{+\infty\}$ ² with $\text{dom} f \neq \emptyset$ is said to be an *M-convex function* if it satisfies the following *exchange axiom*:

(M-EXC[Z]) For $\mathbf{a}, \mathbf{b} \in \text{dom} f$ and $u \in \text{supp}^+(\mathbf{a} - \mathbf{b})$, there exists $v \in \text{supp}^-(\mathbf{a} - \mathbf{b})$ such that

$$f(\mathbf{a}) + f(\mathbf{b}) \geq f(\mathbf{a} - \chi_u + \chi_v) + f(\mathbf{b} + \chi_u - \chi_v), \quad (20)$$

where χ_u and χ_v are unit vectors.

Definition 3: A function g is said to be *M-concave* if $-g$ is M-convex.

One possible interpretation of **Definition 2** is as follows: two points stay “far away” from each other have a larger total value than two points which are relatively “close” to each other. The exchange axiom (M-EXC[Z]) gives a specific fashion for two points approaching each other. An illustration of **Definition 2** and the exchange axiom (M-EXC[Z]) for a two dimensional case is given in Fig. 1. Based on lemma 1 and **Definition 2**, we have the following lemma.

Lemma 2: Let $a_n \in \{0, 1, \dots, \kappa\}$ be the number of SUs assigned to channel n , then $P_{f,n}(a_n)$ is an M-convex function.

Proof: Without loss of generality, we let $\chi_u = 1$ and $\chi_v = 0$, since a_n is integer, i.e. one dimensional. According to lemma 1, $P_f(m)$ is decreasing and convex for any $m \in [1, +\infty)$. Hence, for any $a_m \leq a_n$ ($a_m, a_n \in \{0, 1, \dots, \kappa\}$), we have

$$P_{f,n}(a_m) - P_{f,n}(a_m + 1) \geq P_{f,n}(a_n - 1) - P_{f,n}(a_n). \quad (21)$$

²The notation Z^V means the integer vector space with coordinates indexed by the elements of V

Fig. 1. Illustration for M-convex

Since $\chi_u = 1$ and $\chi_v = 0$, equation (21) equals to

$$P_{f,n}(a_n) + P_{f,n}(a_m) \geq P_{f,n}(a_n - \chi_u + \chi_v) + P_{f,n}(a_m + \chi_u - \chi_v). \quad (22)$$

According to **Definition 2**, equation (22) reveals that $P_{f,n}^n(a_n)$ is an M-convex function. This completes the proof. ■

According to [14], we have the following lemma.

Lemma 3: A separable convex (concave) function

$$f(\mathbf{a}) = \sum_{n=1}^N f_n(a_n) \quad (\mathbf{a} \in \text{dom} f) \quad (23)$$

with univariate discrete convex (concave) functions $f_n(a_n)$ is M-convex (M-concave).

Then we have the following proposition.

Proposition 1: The objective function in problem **(P1)**, i.e. $U(\mathbf{a}(\kappa)|\kappa)$, is M-concave.

Proof: Let $f_n(a_n) = \alpha_n[1 - P_{f,n}(a_n)]$. Since α_n is a positive scalar for all channels, according to lemma 2 and **Definition 3**, $f_n(a_n)$ is M-concave. By referring to equation (14), we have $U(\mathbf{a}(\kappa)|\kappa) = \sum f_n(a_n) + \beta$. Function $U(\mathbf{a}(\kappa)|\kappa) - \beta$ consists of separable M-concave functions f_n , according to lemma 3 $U(\mathbf{a}(\kappa)|\kappa) - \beta$ is M-concave. Only a vertical translation is needed to transform function $U(\mathbf{a}(\kappa)|\kappa) - \beta$ to $U(\mathbf{a}(\kappa)|\kappa)$, because β is a constant. Since the shape and inherent property of function $U(\mathbf{a}(\kappa)|\kappa) - \beta$ is not changed, $U(\mathbf{a}(\kappa)|\kappa)$ is also M-concave. This completes the proof. ■

Linear constraint (16) is both M-convex and M-concave [14]. Therefore, we can conclude that problem **(P1)** is an M-concave problem and the local optimality guarantees the global optimality. Hence, the following greedy algorithm **(A1)** can be used to find the optimal assignment scheme for fixed number of SUs participating in sensing.

Algorithm **(A1)** assigns the given κ SUs to PU channels one at a time. In each iteration, algorithm **(A1)** searches for the channel that gives the largest improvement to the objective function of problem **(P1)** and assigns the SU to that channel. The physical meaning of steps 4 and 5 is as follows: when one additional SU becomes available, the CRN will compare the improvements of false alarm probability of

Algorithm (A1)

-
- 1: **Input:** The number of SUs participating in sensing κ , the PU channel number N , the detection probability requirements $\lambda_1, \lambda_2, \dots, \lambda_N$ and $\alpha_n = w_r P_n(\mathcal{H}_0) C_{0,n}$.
 - 2: **Initialization:** Set $a_n = 0$ for all $n \in \mathcal{N}$, $k = 0$.
 - 3: **while** ($a_1 + a_2 + \dots + a_N < \kappa$) **do**
 - 4: Find the channel number $i(k) = \arg \max \{\alpha_1 \Delta P_{f,1(k)}, \alpha_2 \Delta P_{f,2(k)}, \dots, \alpha_N \Delta P_{f,N(k)}\}$, where $\Delta P_{f,n(k)} = P_{f,n}(a_{n(k)}) - P_{f,n}(a_{n(k)} + 1)$.
 - 5: $a_{i(k)} = a_{i(k)} + 1$, $k = k + 1$.
 - 6: **end while**
 - 7: **Output:** a_n ($n \in \mathcal{N}$).
-

each channel if this SU is assigned to sense that channel. This SU will then be assigned to the channel which gives the largest improvement. One advantage of this algorithm is its low computational complexity ($O(M)$). Another advantage of this algorithm is that it is incremental, which will facilitate its practical implementation greatly. There is no need to execute this algorithm from the beginning to the end when there is a new arrival or departure of SU. The CRN can simply compare the increment or decrement in terms of false alarm probability and make appropriate adjustments to the present assignment.

B. Step 2: Find the optimal number of SUs

Let $U(\mathbf{a}^*(\kappa), \kappa)$ denote the utility function (10) when SUs are assigned according to the optimal assigning mechanism given above. And define $\Delta U(\mathbf{a}^*(\kappa), \kappa) \triangleq U(\mathbf{a}^*(\kappa), \kappa) - U(\mathbf{a}^*(\kappa + 1), \kappa + 1)$ as the first order difference of function $U(\mathbf{a}^*(\kappa), \kappa)$. Then we have

$$\Delta U(\mathbf{a}^*(\kappa), \kappa) = [U(\mathbf{a}(\kappa)|\kappa) - U(\mathbf{a}(\kappa + 1)|\kappa + 1)] - \beta', \quad (24)$$

where $\beta' = \tau w_e(\phi_s - \phi_e)$. Then we have the following proposition which presents the property of optimal κ .

Proposition 2: The optimal value of $U(\mathbf{a}^*(\kappa), \kappa)$ is achieved when κ satisfies the following condition

$$\alpha_{i(\kappa)} \Delta P_{f,i(\kappa)} \geq \beta' \text{ and } \alpha_{i(\kappa+1)} \Delta P_{f,i(\kappa+1)} < \beta', \quad (25)$$

where the channel number can be expressed as $i(\kappa) = \arg \max \{\alpha_1 \Delta P_{f,1(\kappa)}, \alpha_2 \Delta P_{f,2(\kappa)}, \dots, \alpha_N \Delta P_{f,N(\kappa)}\}$.

Proof: According to algorithm (A1), the improvement on objective function is compared whenever the CRN wants to add one more SU to sense PU channels. Therefore, the improvement on objective function by increasing the number of SUs participating in sensing from κ to $\kappa + 1$ is given by

$$U(\mathbf{a}(\kappa)|\kappa) - U(\mathbf{a}(\kappa + 1)|\kappa + 1) = \alpha_{i(\kappa)} \Delta P_{f,i(\kappa)}, \quad (26)$$

where the channel number can be expressed by $i(\kappa) = \arg \max \{\alpha_1 \Delta P_{f,1(\kappa)}, \alpha_2 \Delta P_{f,2(\kappa)}, \dots, \alpha_N \Delta P_{f,N(\kappa)}\}$. The optimal value of $U(\mathbf{a}^*(\kappa), \kappa)$ is achieved when

$$\begin{aligned} \Delta U(\mathbf{a}^*(\kappa), \kappa) &= [U(\mathbf{a}(\kappa)|\kappa) - U(\mathbf{a}(\kappa + 1)|\kappa + 1)] - \beta' \\ &= \alpha_{i(\kappa)} \Delta P_{f,i(\kappa)} - \beta' \end{aligned}$$

The optimal value is achieved at the turning point where $\Delta U(\mathbf{a}^*(\kappa), \kappa)$ changes from positive to negative (i.e., the point where condition (25) holds). This completes the proof. ■

Based on proposition 1 and proposition 2, we propose the following algorithm that can solve problem (CSS) efficiently and optimally.

Algorithm (A2)

-
- 1: **Input:** The number of SUs M , the PU channel number N , the detection probability requirements $\lambda_1, \lambda_2, \dots, \lambda_N$, $\alpha_n = w_r P_n(\mathcal{H}_0) C_{0,n}$, and $\beta' = \tau w_e(\phi_s - \phi_e)$.
 - 2: **Initialization:** Set $a_n = 0$ for all $n \in \mathcal{N}$, $k = 0$.
 - 3: **while** ($a_1 + a_2 + \dots + a_N < M$) **do**
 - 4: Find the channel number which generates the largest improvement on objective function $i(k) = \arg \max \{\alpha_1 \Delta P_{f,1(k)}, \alpha_2 \Delta P_{f,2(k)}, \dots, \alpha_N \Delta P_{f,N(k)}\}$, where $\Delta P_{f,n(k)} = P_{f,n}(a_{n(k)}) - P_{f,n}(a_{n(k)} + 1)$.
 - 5: **if** $\alpha_{i(k)} \Delta P_{f,i(k)} > \beta'$ **then**
 - 6: $a_{i(k)} = a_{i(k)} + 1$, $k = k + 1$.
 - 7: **else**
 - 8: Stop and the optimal solution is found.
 - 9: **end if**
 - 10: **end while**
 - 11: **Output:** $\kappa^* = k$, $a_n^* = a_n$ ($n \in \mathcal{N}$).
-

In algorithm (A2), steps 4 and 5 search for the optimal SUs assignment scheme for fixed number of SUs participating in sensing according to proposition 1 and algorithm (A1). In steps 5 and 6, algorithm (A2) looks for the optimal number of SUs participating in sensing based on proposition 2.

V. NUMERICAL RESULTS

In this section, we provide several numerical examples to illustrate and validate the analysis. The parameters used in the simulation are as follows: the total number of SUs $M = 50$, the PU channel number $N = 10$, the detection probability requirements $\{\lambda_n | n = 1, 2, \dots, 10\}$ for these ten channels are $\{0.55, 0.6, 0.65, 0.7, 0.78, 0.85, 0.9, 0.92, 0.95, 0.99\}$ and the coefficients $\{\alpha_n | n = 1, 2, \dots, 10\}$ for these channels are $\{2, 2.1, 2.14, 2.21, 2.53, 2.66, 2.8, 2.88, 3.15, 3.3\}$.

In Fig. 2, the sensing scheduling results are shown for the following system parameter setting: the sampling rate $f_s = 1\text{MHz}$, the sensing duration $\tau = 50\text{ms}$ and the SNR at each SU $\gamma = -20\text{dB}$. As shown in Fig. 2, all SUs participate in sensing when $\beta' = 0$. The physical interpretation for $\beta' = 0$ is that energy consumption is not taken into consideration. In this case, the more SUs joining in sensing, the better performance can the CRN achieve, since energy consumption is not properly penalized. When β' is set to 0.05, 28 SUs participate in sensing in total and these SUs are assigned to these ten channels unevenly to achieve maximum performance. With energy taken into consideration, the number of SUs participating in sensing will also be optimized in our analysis. When β' becomes larger, i.e. the CRN concerns more about energy consumption, the SUs participating in sensing will

Fig. 2. Sensing scheduling results for parameters setting 1

Fig. 3. Sensing scheduling results for parameters setting 2

further decrease to conserve energy. As shown in Fig. 2, when β' increases to 0.2, there are only 18 SUs assigned to sense PU channels. In order to verify the correctness of the results above, we also solve the problem by using exhaustive search which guarantees to give an global optimal solution. And we find that the results are exactly the same to the solution obtained in Fig. 2. This validates the correctness of our algorithm.

In Fig. 3, the system parameters are set as follows: the sampling rate f_s equals 1kHz, sensing duration is set to 2ms and the SNR at each SU is -3dB . This example illustrates the intricate two-dimensional tradeoff among energy consumption, sensing accuracy and exploration of spectrum opportunity. As shown in Fig. 3, when $\beta' = 0$, all the 50 SUs join in sensing and 9 channels out of the 10 channels are sensed. When β' increases to 0.18, the number of SUs participating in sensing decreases to 27 and 8 channels are sensed. When β' increases further to 0.30, only 14 SUs are assigned to sense 7 PU channels. This example demonstrates that when a CRN concerns relatively more about energy consumption, less SUs will be assigned to sense PU channels. This will result in less spectrum opportunity being explored or less SUs being assigned to PU channels (i.e., the sensing performance deteriorates).

VI. CONCLUSION

In this paper, we study the CSS problem in the context of CR. The CSS problem is analyzed under a more practical scenario where PU channels are heterogeneous in terms of channel protection criteria, idling probabilities and channel capacities. We consider both the performance and energy consumption of sensing in the analysis. The intricate two-dimensional trade-off among energy efficiency, sensing performance and spectrum opportunity exploration is presented and properly tackled. An optimal and efficient SUs assignment scheme which takes both sensing performance and energy efficiency into consideration is obtained. The optimality of this assignment scheme is proved both theoretically and numerically. As for future work, we anticipate devising algorithms considering heterogeneous characteristics in both PUs channels and SUs.

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