Distributed Power Allocation Algorithm for Spectrum Sharing Cognitive Radio Networks with QoS Guarantee

Yuan WU, Danny H.K. TSANG
Department of Electronic and Computer Engineering
Hong Kong University of Science and Technology
Email: ecewuy@ust.hk, eetsang@ece.ust.hk

Abstract—In this paper we study the distributed multi-channel power allocation for spectrum sharing cognitive radio networks with QoS guarantee. We formulate this problem as a non-cooperative game $G_{MCPA-C}$ with coupled strategy space to address both the co-channel interference among secondary users and the interference temperature regulation imposed by primary systems. We investigate the properties of Nash equilibrium (N.E.) for our $G_{MCPA-C}$, including the existence and QoS provisioning. Furthermore, we derive a layered structure by applying the Lagrangian dual decomposition to $G_{MCPA-C}$ and design a distributed algorithm to find the N.E. via this structure. Simulation results are presented to show both the validity of our game theoretic model and the performance of our proposed algorithm. Finally, we incorporate the Pigouvian taxation into our algorithm to improve the efficiency of N.E. when social optimality is considered.

I. INTRODUCTION

Cognitive radio with its ability to intelligently learn from real-time environment and flexibly adapt its transmission parameters has been considered as a promising technology to realize the dynamic spectrum usage proposed by Federal Communication Commission.

Power allocation plays an important role in multiuser spectrum sharing cognitive radio networks because of the co-channel interference in radio transmission and the interference temperature regulation [1] imposed by primary systems. Current researches found that power allocation greedily aiming to maximize the system utility [2] or each user’s own transmission rate [3] resulted in unfairness and/or inefficiency in spectrum utilization of the networks. [4] studied the unfairness issue with the repeated game. [5] studied the rate-constrained sum-power minimization problem for multi-cell networks with exogenous mediator. However, most of these works were targeted for conventional wireless networks without considering the interference temperature regulation in cognitive radio networks. [6] considered the interference temperature regulation constraint using the auction game, but it only used a single channel spectrum-spread model. Besides, most of the previous work seldom considered the heterogenous QoS requirements of secondary users. A more practical assumption is that each user should selfishly aim to achieve its QoS requirement using as little power consumption as possible. [7] studied the power allocation with SINR constraint for cellular networks using the S-modular game. However, S-modular model cannot be applied in the multi-channel power allocation case because the coupled strategy space in the multi-channel case is no longer a sublattice and the monotonicity condition doesn’t hold. Motivated by these considerations, we study the multi-channel power allocation with QoS guarantee for spectrum sharing cognitive radio networks. Our contributions include:

- We formulate this optimization problem as a non-cooperative game $G_{MCPA-C}$ in which each user selfishly aims to achieve its target QoS using as little power consumption as possible. We introduce the coupled strategy space in our $G_{MCPA-C}$ to address both the co-channel interference among secondary users and the interference temperature regulation of primary systems.
- We analyze the Nash equilibrium (N.E.) for our $G_{MCPA-C}$, including its existence and QoS provisioning. We apply the Lagrangian dual decomposition [12] to $G_{MCPA-C}$ and derive a layered structure for it.
- We design a distributed algorithm (A1) to find the N.E. of $G_{MCPA-C}$ based on the layered structure. Specifically, the algorithm (A1) has an interpretation as a Stackelberg model [17] where each monitoring station and secondary user separately announce the interference regulation dual price and QoS provisioning dual price respectively, which are then followed by a multi-channel power allocation sub-game $G_{MCPA}$ among all secondary users. We investigate our proposed algorithm in details, including its distributed property, awareness of interference regulation and QoS provisioning, convergence, and robustness to insufficient power iteration.

We organize this paper as follows. In Section II, we present the network model and the game-theoretic formulation. We investigate the properties of N.E. in Section III. Decomposition structure is introduced in Section IV and detailed study on the layered structure is presented. We propose a distributed algorithm to find the N.E. in Section V. Simulation results are presented in Section VI and conclusion is in Section VII.
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2009 proceedings.

Fig. 1. Local monitoring network scenario. $N$ secondary pairs share the same set of $K$ channels with primary system. $M$ monitoring stations are equipped to monitor the interference level over each channel.

II. NETWORK MODEL AND GAME FORMULATION

We consider a multi-channel spectrum sharing cognitive radio network as shown in Figure 1, where a set of secondary users $\mathcal{N} = \{1, 2, \ldots, N\}$ share a common set of $\mathcal{K} = \{1, 2, \ldots, K\}$ channels (each with equal bandwidth $B$) with primary systems. Each secondary user $i \in \mathcal{N}$ consists of a transmitter $SU_{Tx_i}$ and an intended receiver $SU_{Rx_i}$. Based on the local monitoring model proposed by [1], a set of $\mathcal{M} = \{N + 1, N + 2, \ldots, N + M\}$ monitoring stations are established and co-located with the primary systems to observe the interference over all channels. Specifically, monitoring station $m$ has a tolerable interference level over channel $k$ upper bounded by $\kappa T_m^k B$, where $\kappa$ is the Boltzmann’s constant and $T_m^k$ is its interference temperature. We assume that each secondary user $i$ has a target QoS represented by data rate $R_{i}^{tar}$ at link layer. Each user $i$ selfishly aims to achieve its $R_{i}^{tar}$ using as little power consumption as possible. In this paper we focus on the cases that the target rate profile ($R_{1}^{tar}, R_{2}^{tar}, \ldots, R_{N}^{tar}$) of all users are feasible to the network. Issues about determining the feasible region of target rate profile and designing an admission control mechanism with which secondary users requiring unacceptable target rates can be identified are our future work.

Due to the co-channel interference and interference temperature regulation, the achievable data rate of each user is conflicting with those of all the other users. Therefore, we formulate this multiuser power allocation problem as a non-cooperative game with the formal description given as follows:

$$G_{MCPA-C} = \{\mathcal{N}, \{\chi_i\}_{i \in \mathcal{N}}, \{\Psi_i\}_{i \in \mathcal{N}}\}$$

(1)

where $\mathcal{N}$ denotes the set of coexisting secondary users. User $i$’s cost function is its total power allocation over all channels:

$$\Psi_i(p_i) = \alpha_i \sum_{k \in \mathcal{K}} p_i^k$$

where $\alpha_i$ is its weighting factor and $p_i = (p_i^1, p_i^2, \ldots, p_i^K)$ is its power allocation strategy over the $K$ channels. In our $G_{MCPA-C}$, each user $i$’s strategy space $\chi_i$ is coupled with all the other users’ strategies, and it can be expressed as:

$$\chi_i(p_{-i}) = \{p_i \in \Omega_i | \sum_k \Phi_{i}^{k}(p_i^k, p_{-i}^k) \geq R_i^{tar}\}$$

$$p_i^k g_{im} + \sum_{j \neq i} p_j^k g_{jm} \leq \kappa T_m^k B, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}$$

(2)

where $\Omega_i = \{p_i \in \mathbb{R}^K | \sum_{k \in \mathcal{K}} p_i^k \leq P_i^{max}; p_i^k \leq p_i^k, \forall k \in \mathcal{K}\}$ is user $i$’s power allocation range, and $P_{i}^{max}$ denotes user $i$’s power capacity. $p_i^k$ and $p_i^k$ denote the practical lower and upper bounds of user $i$’s power allocation on channel $k$. Given all the other users’ power allocations $p_{-i} = (p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$, the QoS constraint:

$$\sum_{k \in \mathcal{K}} \Phi_{i}^{k}(p_i^k, p_{-i}^k) \geq R_i^{tar}$$

(3)

guarantees that user $i$’s target rate is satisfied. Specifically, $\Phi_{i}^{k}$ has the information theoretic capacity formula as:

$$\Phi_{i}^{k}(p_i^k, p_{-i}^k) = B \log(1 + \frac{p_i^k g_{ik}^k}{\eta_i^k}), \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$$

(4)

where $\eta_i^k = \sum_{j \neq i \in \mathcal{N}} g_{jk}^k + n_i^k$ is the power of interference plus background noise measured by $SU_{Rx_i}$ over channel $k$. $g_{jk}^k$ is the channel power gain from user $SU_{Tx_j}$ to $SU_{Rx_i}$ over channel $k$. $n_i^k$ is the power of background noise of $SU_{Rx_i}$ over channel $k$. Meanwhile, the constraint:

$$p_i^k g_{im} + \sum_{j \neq i} p_j^k g_{jm} \leq \kappa T_m^k B, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}$$

(5)

guarantees that the interference temperature regulation of all monitoring stations is satisfied. Based on (2) the whole strategy profile space of $G_{MCPA-C}$ can be expressed as:

$$\chi = \{(p_1, \ldots, p_N) \in \Omega_1 \times \ldots \times \Omega_N | \sum_k \Phi_{i}^{k}(p_i^k, p_{-i}^k) \geq R_i^{tar}, \forall i \in \mathcal{N}; \sum_{i \in \mathcal{N}} p_i^k g_{im} \leq \kappa T_m^k B, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}\}$$

Given $p_{-i}$, each user $i \in \mathcal{N}$ faces a multi-channel power allocation problem as follows:

$$\min_{p_i \in \chi_i(p_{-i})} \Psi_i(p_i)$$

(6)

According to the definition of Nash equilibrium (N.E.) [17], the equilibrium power allocation profile $(p_1^*, \ldots, p_N^*)$ for $G_{MCPA-C}$ should satisfy the following condition:

$$p_i^* = \arg \min_{p_i \in \chi_i(p_{-i}^*)} \Psi_i(p_i)$$

(7)

which guarantees that no single user has an incentive to deviate from N.E. unilaterally.

III. PROPERTIES OF N.E.

In this section, we analyze the properties of N.E. for $G_{MCPA-C}$, including its QoS provisioning and existence.\footnote{In this work we consider a relatively static network, i.e., the set of channel parameters keeps unchanged during the time interval of interest.}

We first introduce the definition of feasibility for a target rate profile $(R_1^{tar}, \ldots, R_N^{tar})$ as follows:

**Definition 1:** A target rate profile $(R_1^{tar}, \ldots, R_N^{tar})$ is feasible if there exists a power allocation profile $(p_1, \ldots, p_N)$ such that: \(\{(p_1, \ldots, p_N) \in (\Omega_1 \times \ldots \times \Omega_N) \]| \sum_{k \in \mathcal{K}} \Phi_{i}^{k}(p_i^k, \rho_{-i}^k) = R_i^{tar}, \forall i \in \mathcal{N}; \sum_{i \in \mathcal{N}} p_i^k g_{im} \leq \kappa T_m^k B, \forall m \in \mathcal{M}, k \in \mathcal{K}\}

\footnote{In this work we only consider the pure strategy N.E.}
We then state the following properties for each secondary user’s optimal power allocation when each user aggressively aims to maximize its own transmission rate:

**Observation 1:** Given all the other users’ power allocations \( \mathbf{p}^- = (\mathbf{p}_1, ..., \mathbf{p}_{i-1}, \mathbf{p}_{i+1}, ..., \mathbf{p}_N) \), then for the secondary user \( i \):

(a): Its optimal power allocation choice \( \mathbf{p}_{i}^* = \arg \max_{\mathbf{p}_i \in \mathcal{P}_i} \sum_{k \in K} \Phi_i(k, \mathbf{p}_i^-) \) is unique for any given \( P_{i,\max} \) because of the strict convexity of \( \sum_{k \in K} \Phi_i(k, \mathbf{p}_i^-) \) on \( \mathbf{p}_i \).

(b): Its optimal achievable transmission rate \( R_{i,\max}^* = \max_{\mathbf{p}_i \in \mathcal{P}_i} \sum_{k \in K} \Phi_i(k, \mathbf{p}_i^-) \) is a continuous, monotonic increasing function of its power capacity \( P_{i,\max}^* \) if the interference temperature regulation constraints (5) is still satisfied, i.e., if \( \sum_{k \in (1,2)} \mathbf{p}_{i,\max} \gtrless \sum_{k \in \mathcal{K}} \Phi_i(k, \mathbf{p}_i^-) \) is a water-filling structure [14, Chap. 5] with water-filling level \( \mathbf{w}_i \) determined by \( \sum_{k \in (1,2)} \mathbf{p}_{i,\max}^* \).

**Remark 1:** We illustrate Observation 1 using a simple two-channels example as shown in Figure 2. For a target secondary user 1, given \( \mathbf{p}^- \), its interference regulation bounds are determined by \( \Lambda_1^* = \min_{k \in (1,2)} (\sum_{j \neq i} \Phi_i(k, \mathbf{p}_j\mathbf{r}_j^-) B \overset{\text{channel 1}}{\gtrless} \sum_{j \neq i} \mathbf{p}_j\mathbf{r}_j^- y_{jm} B, \forall m \in \mathcal{M}, k \in \mathcal{K} \).

**Lemma 1:** Assume there exists a pure strategy N.E. \( (\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \) for \( G_{MCPA-C} \) on the strategy profile space \( \chi \), then the following property always holds at the N.E. if the target rate profile \( (R_{i,\max}^* \text{ for } i = 2, ..., N) \) is feasible:

\[
\sum_{k \in \mathcal{K}} \Phi_i(k, \mathbf{p}_i^*, \mathbf{p}_i^-) = R_{i,\max}^*, \forall i \in \mathcal{N}
\]

**Proof:** Lemma 1 can be proved by contradiction. Assume there exists a N.E. \( (\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \) for which a particular user \( i' \), its equilibrium power allocation \( \sum_{k \in \mathcal{K}} \Phi_i(k, \mathbf{p}_{i'}^*, \mathbf{p}_i^-) \) is greater than \( R_{i',\max} \). Then according to Observation 1(b), user \( i' \) always has the incentive to reduce its power allocation on some channels such that its total power cost \( \Psi_i(\mathbf{p}_{i'}) \) is reduced while keeping the constraint (3) still satisfied, i.e., user \( i' \) will deviate from this assumed N.E. unilaterally. Therefore, \( (\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \) cannot be the N.E. for \( G_{MCPA-C} \).

**B. Existence and Uniqueness**

Based on Lemma 1, we state the existence of N.E. for \( G_{MCPA-C} \) as follows:

**Lemma 2:** If the target rate profile \( (R_{1,\max}^*, ..., R_{N,\max}^*) \) is feasible, then there always exists a pure strategy N.E. for \( G_{MCPA-C} \) on the strategy space \( \chi \). Meanwhile, let \( (\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \) denote a N.E., then for each secondary user \( i \in \mathcal{N} \), its equilibrium power allocation has the modified water-filling structure as follows:

\[
p_i^* = \left\{ \begin{array}{ll}
[w_i - \frac{k_i \eta_i}{\mathbf{g}_i}]^+ & \text{if } k \in \mathcal{K} \setminus \mathcal{I} \mathcal{S} \mathcal{C}_i \\
\min_{m \in \mathcal{M}} \left( \frac{k_i \eta_i}{(\mathbf{g}_i - \mathbf{y}_{im}) B} \right) & \text{if } k \in \mathcal{I} \mathcal{S} \mathcal{C}_i
\end{array} \right.
\]

where the water-filling level \( w_i \) in our \( G_{MCPA-C} \) is chosen such that:

\[
\sum_{k \in \mathcal{K}} \Phi_i(k, \mathbf{p}_i^*, \mathbf{p}_i^-) = R_{i,\max}^*
\]

\( \mathcal{I} \mathcal{S} \mathcal{C}_i \) denotes the channel set of user \( i \) where interference is saturated, i.e., \( k \in \mathcal{K} \setminus \mathcal{I} \mathcal{S} \mathcal{C}_i \), \( \{w_i - \frac{k_i \eta_i}{\mathbf{g}_i}\}^+ \) is always the mean interference temperature regulation constraint (5) is binding.

**Proof:** (9) guarantees the necessary condition for \( (\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \) to be a N.E. according to Lemma 1. (8) guarantees that for any particular secondary user \( i' \in \mathcal{N} \), if all the other users have already adopted \( \mathbf{p}_i^- \), then \( \mathbf{p}_i^* \) will be user \( i' \)’s unique optimal power allocation choice for (6) according to Observation 1(a)(b). Besides, the feasibility of target rate profile \( (R_{1,\max}^*, ..., R_{N,\max}^*) \) guarantees the existence of pair \((w_i, \mathcal{I} \mathcal{S} \mathcal{C}_i), \forall i \in \mathcal{N} \), such that both \( \sum_{k \in \mathcal{K}} \Phi_i(k, \mathbf{p}_i^-) \leq P_{i,\max}^* \), \( \forall i \in \mathcal{N} \), and the interference regulation constraint (5) is met (otherwise, \( (R_{1,\max}^*, ..., R_{N,\max}^*) \) will become infeasible). Therefore a pure strategy N.E. exists for \( G_{MCPA-C} \).

The uniqueness of N.E. for \( G_{MCPA-C} \), however, is difficult to guarantee, and strongly dependent on the structure of conditions (8),(9). If conditions (8),(9) only admit a unique solution profile \((\mathbf{p}_1^*, ..., \mathbf{p}_N^*) \in \chi \), then the N.E. is unique.

*Footnote: for the sake of clear expression, we don’t consider the impacts of \( k_i \) on \( \mathbf{p}_i^*, \mathbf{p}_i^- \) in the proposed model in Section V.*

---

**Fig. 2. Example of Secondary User 1’s Optimal Power Allocation**

An important property of \( G_{MCPA-C} \) is that if the target rate profile \( (R_{1,\max}^*, ..., R_{N,\max}^*) \) is feasible, then at the N.E. (assuming its existence) no individual user will get any excess of data rate beyond its target and no power will be wasted to achieve the unwanted rate. We describe this property as follows:
IV. DUAL DECOMPOSITION AND LAYERED STRUCTURE

Distributedly approaching to the N.E. \((p^*_i, \ldots, p^*_N)\) for \(G_{MCPA-C}\) is difficult because both the QoS guarantee constraint (3) and the interference regulation constraint (5) explicitly require the coordination among secondary users. Therefore, we use the partial dual decomposition [12] to relax both constraints (3,5) for each secondary user \(i\). The corresponding Lagrangian function can be written as:

\[
L_i(p_i, p_{-i}, z_i, \mu) = \alpha_i \sum_{k \in K} p_i^k + z_i(R_i^{tar} - \sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k)) + \sum_{m \in M} \sum_{k \in K} \mu_m^k(\sum_{i \in N} p_i^k g_{im}^k - \kappa^k T_m^kB), \forall i \in N
\]

where \(z_i\) denotes the dual price for user \(i\)’s QoS provisioning. Define the \(1 \times N\) dual vector \(\mathbf{z} \triangleq (z_1, \ldots, z_N)\). Further define the \(1 \times K\) dual vector \(\mathbf{\mu} \triangleq (\mu_1^1, \ldots, \mu_M^K)\), where each \(1 \times K\) vector \(\mu_m^k \triangleq (\mu_1^m, \ldots, \mu_K^m), \forall m \in M\), denotes the set of dual prices for managing the interference temperature regulation over the \(K\) channels, which are observed by all secondary users simultaneously. For \(\forall i \in N\), let the parameterized cost function:

\[
\Theta_i(p_i, p_{-i}, z_i, \mu) = \sum_{k \in K} (\alpha_i + \sum_{m \in M} \mu_m^k g_{im}^k) p_i^k - z_i \sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k)
\]

Our coupled strategy game \(G_{MCPA-C}\) can be vertically separated into three subproblems as follows:

(a) The multi-channel power allocation sub-game among all secondary users, given the dual vectors \(\mathbf{z}\) and \(\mathbf{\mu}\):

\[
G_{MCPA}(\mathbf{z}, \mathbf{\mu}) = \{N, \{\Omega_i^P\}_{i \in N}, \{\Theta_i(p_i, p_{-i}, z_i, \mu)\}_{i \in N}\}
\]

(b) The sub-gradient updating of each user \(i\)’s dual price for QoS provisioning, given the equilibrium power allocation profile \((p_1^*(z, \mu), \ldots, p_N^*(z, \mu))\) for the sub-game \(G_{MCPA}(\mathbf{z}, \mathbf{\mu})\):

\[
z_i = z_i + \zeta_1(R_i^{tar} - \sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k)), \forall i \in N
\]

(c) The sub-gradient updating of each monitoring station \(m\)’s dual prices for interference temperature regulation, given the equilibrium power allocation profile \((p_1^*(z, \mu), \ldots, p_N^*(z, \mu))\) for the sub-game \(G_{MCPA}(\mathbf{z}, \mathbf{\mu})\):

\[
\mu_m^k = \mu_m^k + \zeta_2(\sum_{i \in N} p_i^k g_{im}^k - \kappa^k T_m^kB), \forall m \in M, k \in K
\]

where \(\zeta_1, \zeta_2\) denote the updating step-sizes.

Sub-problems (b),(c) actually maximize the dual function:

\[
D(\mathbf{z}, \mathbf{\mu}) \triangleq \sum_{i \in N} L_i(p_i^*(z, \mu), p_{-i}^*(z, \mu), z_i, \mu)
\]

with the sub-gradient method. It can be shown that the sub-gradient of \(D(\mathbf{z}, \mathbf{\mu})\) at \(z_i, \forall i \in N\), and \(\mu_m^k, \forall m \in M, \forall k \in K\), are determined by:

\[
f_i = R_i^{tar} - \sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k), \forall i \in N
\]

\[
\bar{f}_m^k = \kappa^k T_m^B \sum_{i \in N} p_i^k g_{im}^k, \forall m \in M, \forall k \in K
\]

respectively. Note that the constant \(N\) is embedded in the step-size \(\zeta_2\) in (11) for simplicity.

Our layered structure can be considered as a Stackelberg model where each secondary user \(i \in N\) announces its QoS provisioning dual price \(z_i\) and each monitoring station \(m \in M\) announces its interference regulation dual prices \(\mu_m^k\), which are then followed by the multi-channel power allocation sub-game \(G_{MCPA}(\mathbf{z}, \mathbf{\mu})\) among all the users. Figure 3 shows this layered structure. Different from \(G_{MCPA-C}\), in sub-game \(G_{MCPA}\) each secondary user has an independent strategy space only determined by its power allocation range \(\Omega_i^P\), and its power optimization problem can be expressed as:

\[
\min_{p_i \in \Omega_i^P} \Theta_i(p_i, p_{-i}, z_i, \mu)
\]

(12)

For any given \(p_{-i}\), according to the Slater’s conditions, each user \(i\)’s primal convex problem (6) can always be solved by solving (12), (10) and (11) iteratively without any performance loss.\(^5\)

We note that in \(G_{MCPA}\), each user \(i\)’s achievable transmission rate \(\sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k)\) is weighted by the dual price \(z_i\) to indicate its QoS requirement. Meanwhile, a linear pricing mechanism \(\sum_{k \in K} (\alpha_i + \sum_{m \in M} \mu_m^k g_{im}^k)\) is incorporated to address both user \(i\)’s consideration on power saving and the set of interference temperature regulation of monitoring stations. In [3] the authors investigated the transmission rate maximization game \(G_{TRM}\) as follows:

\[
G_{TRM} = \{N, \{\Omega_i^P\}_{i \in N}, \{\sum_{k \in K} \Phi_i^k(p_i^k, p_{-i}^k)\}_{i \in N}\}
\]

(13)

Our sub-game \(G_{MCPA}\) generalizes \(G_{TRM}\) by including both the QoS weighting factor and the pricing factor (for both the interference temperature regulation and the power saving) in each user’s cost function. Since \(G_{MCPA}\) plays an important role in our layered structure and differs from many other studies, we investigate the properties of N.E. for \(G_{MCPA}\) in detail and describe them in the following lemma.

**Lemma 3:** (a) There always exists a pure strategy N.E. for \(G_{MCPA}(\mathbf{z}, \mathbf{\mu})\) on the strategy profile space \(\Omega^P = \Omega_1^P \times \Omega_2^P \times \ldots \times \Omega_N^P\).

\(^5\)Although single user optimization (6) can be solved with zero dual gap, inefficiency still exists in our \(G_{MCPA-C}\) when social performance is concerned, which we will describe in detail later.
... \times \Omega^p_N$ if the QoS weighting factor $z_i > 0, \forall i \in \mathcal{N}$, and the pricing factor $\alpha_i + \sum_{m \in \mathcal{M}} \rho^m g^{k}_{im} \geq 0, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$.

(b) Furthermore, if the set of channel gains satisfies that:

$$\|G^k\|_2 < 1, \forall k \in \mathcal{K}$$

(14)

where $\|\cdot\|_2$ denotes the matrix’s spectrum norm and for each $k \in \mathcal{K}$, the $N \times N$ matrix $G^k$ is defined as:

$$[G^k]_{ij} = \begin{cases} \frac{g^k_{ij}}{\sum_{i} g^k_{ij}}, & \text{if } i \neq j \\ 0, & \text{otherwise} \end{cases}$$

(15)

then there exists a unique pure strategy N.E. on $\Omega^p$ for $G_{MCPA}(z, \mu)$.

Proof: We only describe the key points in the proof of Lemma 3 due to limited space. Detailed proof can be referred to [10]. It is noted that the conditions in part (a) can be easily satisfied because the dual variables always satisfy that $z_i \geq 0, \forall i \in \mathcal{N}$, and $\mu^m_k \geq 0, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}$ (we assume $z_i > 0, \forall i \in \mathcal{N}$, here and will explain the special cases of $z_i = 0$ for some $i$ later). Besides, we choose $\alpha_i > 0, \forall i \in \mathcal{N}$. Thus to prove the existence, we show that: (i) for each user $i \in \mathcal{N}$, its strategy space $\Omega^p_i$ is a compact and convex set; (ii) for each user $i \in \mathcal{N}$, its cost function $\Theta_i(p, \mu, z, \mu)$ is continuous on the whole strategy profile space $\Omega^p_i$ and strictly convex on $p_i$, i.e., its Hessian matrix satisfies:

$$\nabla^2_{p,p_i} \Theta_i(p, \mu, z, \mu) = diag \{ \frac{z_i (g^{(k)}_{ii} - p^{k}_{i})^2}{(g^{k}_{ii} + \rho^k_i g^{k}_{ii})^2}, \forall k \in \mathcal{K} \} > 0$$

Therefore, according to Theorem 4.3 [17], there exists a pure strategy N.E. on $G_{MCPA}(z, \mu)$ over $\Omega^p$. With respect to the proof of uniqueness, we provide the sufficient condition (14) by extending Theorem 2 in [3]. Lemma 3(b) states the sufficient condition for the uniqueness of N.E. is channel power gains related and particularly independent of the adopted linear contractive condition for the uniqueness of N.E. is channel power gains $\frac{g^{k}_{ii}}{g^{k}_{ii} + \rho^k_i g^{k}_{ii}}$, the uniqueness condition on the whole strategy profile space $\Omega^p_i$ and strictly convex on $p_i$, i.e., its Hessian matrix satisfies:

$$\nabla^2_{p,p_i} \Theta_i(p, \mu, z, \mu) = diag \{ \frac{z_i (g^{(k)}_{ii} - p^{k}_{i})^2}{(g^{k}_{ii} + \rho^k_i g^{k}_{ii})^2}, \forall k \in \mathcal{K} \} > 0$$

Remark 2: The uniqueness condition on the spectrum norm of matrix $\|G^k\|_2 < 1, \forall k \in \mathcal{K}$, actually requires that the cross path power gain $g^{k}_{ij}$ should be small enough compared to the value of direct path gain $g^{k}_{ii}$, and the uniqueness of N.E. only resides in the relatively low interference (i.e., high SINR) region.

V. DISTRIBUTED POWER ALLOCATION ALGORITHM

We design a distributed algorithm to find the N.E. for $G_{MCPA-C}$ based on the layered structure. To facilitate the proposed algorithm, we assume autonomous time synchronization among users and monitoring stations [13], and design two different time scales for the power and dual prices updatings. Specifically, let $\Delta t^l$ denote the long updating interval for both dual prices updatings, i.e., QoS provisioning and interference temperature regulation. Meanwhile, let $\Delta t = \frac{\Delta t^l}{W}$ denote the short updating interval for power updating, where $W$ is large enough such that for any given dual prices, $(p^1(z, \mu), ..., p^N(z, \mu))$ for sub-game $G_{MCPA}(z, \mu)$ can be reached before the end of $\Delta t$. Figure 4 shows this process in a single $\Delta T$. With the proposed two time scales for updating, we design a distributed power allocation algorithm as in (A1).

We remark our proposed algorithm (A1) as follows:

Remark 3: Algorithm (A1) actually solves (8), (9) and determines the pair $(w_1, ISC_1), \forall i \in \mathcal{N}$, using the dual decomposition method. Specifically, (a) if the optimal dual price $\mu^m_{im} = 0, \forall m \in \mathcal{M}$, then $k \notin ISC_1, \forall i \in \mathcal{N}$; (b) the optimal dual prices $z^*, \mu^*$ jointly determine $w_i, \forall i \in \mathcal{N}$, according to the first term in (16), which also takes the impact of user $i$’s power capacity constraint into consideration.

Remark 4: The distributed property of algorithm (A1) is described as follows: (a) given $z_i$ and $\mu$ (16) shows that each user $i$ requires the channel gains $g^{k}_{ii}, g^{k}_{im}$ and the power of interference plus background noise $\eta^k_i$ to implement the Jacobian iteration. $\eta^k_i$ can be measured by $SU_Rx_i$ over channel $k$, and $g^{k}_{ii}, g^{k}_{im}$ can be obtained via channel estimation; (b) given $p_i$, (17) shows that each user $i$ can locally update $z_i$ with the measured values of $\eta^k_i, \forall k \in \mathcal{K}$; (c) (18) shows that each monitoring station $m$ can locally update $\mu^m_m$ with the measured aggregate interference $\sum_{k \in \mathcal{K}} \varphi^k_{im}$ on each channel $k$. Besides both $z_i$ and $p_i$ are user $i$’s internal information, and only the broadcast of $\mu$ from monitoring stations requires message exchange. Therefore, algorithm (A1) relies mostly on local measurements and requires few message exchange.

Remark 5: The convergence of algorithm (A1) can be described as follows. First, we provide a convergence lemma for the Jacobian power iteration (16) as follows:

Lemma 4: The Jacobian power iteration (16) during each long updating interval $\Delta T$ always geometrically converges to the equilibrium power allocation profile for sub-game $G_{MCPA}(z, \mu)$ if (a) $z_i > 0, \forall i \in \mathcal{N}$, $\alpha_i + \sum_{m \in \mathcal{M}} \rho^m g^{k}_{im} \geq 0, \forall k \in \mathcal{K}, \forall i \in \mathcal{N}$, and (b) the channel condition (14) is satisfied. Meanwhile, this convergence has the decay modulus $\varrho$ upper bounded by $\varrho = \max_{k \in \mathcal{K}} \|G^k\|_2$.

Proof: We only describe the key points in the proof of
Lemma 4 due to limited space. Detailed procedures can be referred to [10]. Sufficient convergence condition for iterative water-filling fashion power allocation algorithm was provided in [11], which however, required the strict binding of each user’s power capacity constraint. We prove Lemma 4 by extending Theorem 7 [11] to both cases of active and inactive power capacity constraints. Specifically, we analyze the cases in which some users’ power capacities are not binding during their power iterations (16), and find that as long as condition (14) is met, the Jacobian power iteration (16) can still be expressed as a piece-wise affine mapping and on each piece of restriction region the corresponding mapping is contractive. Similar to Lemma 3 the convergence of (16) requires relatively small values of the cross path channel gains (i.e., low interference among users), which is attributed to the fact that the lower the interference level, the smaller the coupling effect among users, thus the more easily (16) can converge.

Second, the sub-gradient updateings (17) and (18) can converge to the optimal \( z^* \) and \( \mu^* \) if the step-sizes \( \zeta_1, \zeta_2 \) are set according to the diminishing rule [16]. Furthermore, according to the Slater’s condition, each user’s optimal power allocation for problem (6) can be found via solving (12), (10) and (11) iteratively. Therefore, algorithm (A1) can converge to the N.E.

**Remark 6:** Although the QoS guarantee constraint (3) is strictly binding at N.E. for each user, i.e., \( z_i^* \neq 0, \forall i \in N \), some user \( i \) may encounter \( z_i = 0 \) during iteration. However, as long as \( \alpha_i + \sum_{m \in M} h_{im}^k g_{im}^k > 0, \forall i \in N, \forall k \in K \) (which always holds in \( G_{MCPA} \) because we choose \( \alpha_i > 0, \forall i \in N \)), the best response of user \( i \) is just to set its power allocation over all channels as \( p_i^k = p_{i,0}^k, \forall k \in K \). Thus the validity of Lemma 3 still holds if \( z_i \geq 0 \) and \( \alpha_i + \sum_{m \in M} h_{im}^k g_{im}^k > 0, \forall i \in N, \forall k \in K \). (16) can still converge under these special cases, i.e., the occurrence of \( z_i = 0 \) during iteration doesn’t influence our previous analysis.

**VI. SIMULATION RESULTS**

**A. Convergence and Robustness**

To test the performance of our proposed algorithm (A1), we build a network scenario where \( N = 6 \) users share \( K = 10 \) channels with bandwidth \( B = 1 \) per channel. \( M = 4 \) monitoring stations are installed and the set of monitoring stations is represented by \( M = \{ a, b, c, d \} \). For the sake of easy representation, let \( \Gamma^k = kT^k_B, \forall m \in M, k \in K \), denote the upper bound of the tolerable interference level set by monitoring station \( m \) over channel \( k \). Users are located at different places and \( g_{ij}^k, \forall i, j \in N, \forall m \in M, \forall k \in K, \) are generated according to the path-loss model\(^8\). Same background noise power \( n_{i}^k = 0.01 \) is assumed and we only consider the channel realization with condition (14) satisfied.

Figure 5 show the convergence of algorithm (A1) under a random channel realization. We make the parameter setting as

\( g_{ij}^k = d_{ij}^{-\gamma} |c_{ij}^k|^\gamma, \) where \( d_{ij}^{-\gamma} \) is the path-loss between \( SU_{TX_j} \) and \( SU_{RX_i} \) with exponent \( \gamma \) (we set \( \gamma = 2 \)) and \( c_{ij}^k \) is a complex Gaussian random variable modeling the frequency selective fading across different channel with distribution \( CN(0, 1) \).

8Specifically, \( g_{ij}^k = d_{ij}^{-\gamma} |c_{ij}^k|^\gamma \), where \( d_{ij}^{-\gamma} \) is the path-loss between \( SU_{TX_j} \) and \( SU_{RX_i} \) with exponent \( \gamma \) (we set \( \gamma = 2 \)) and \( c_{ij}^k \) is a complex Gaussian random variable modeling the frequency selective fading across different channel with distribution \( CN(0, 1) \).

**Distributed Power Allocation (A1)**

**Initialization Step:**

Each user \( i \in N \) initializes \( p_i(t_{0,0}) \in \Omega_i^P \) and \( z_i(s_0) > 0 \). Each monitoring station \( m \in M \) initializes \( \mu_m^k(s_0) > 0, \forall k \in K \), where we set \( t_{0,0} = s_0 \) and let \( v = 0 \).

**Iteration Process:**

(a) **Jacobian Iteration for Power Allocation:**

After receiving the dual prices \( z_i(s_v) \) and \( (\mu_1(s_v), ..., \mu_M(s_v)) \), each user \( i \) participates in the sub-game \( G_{MCPA}(z(s_v), \mu(s_v)) \) during the \( v \)th long updating interval. Specifically, during the \( v \)th \( \Delta T \), each user \( i \in N \) updates \( p_i \) at the beginning of the \( w \)th short updating time \( t_{v,w} \) as follows: (Initialize \( p_i^{k}(t_{v,0}) = p_i^{k}(t_{v,1, w}), \forall k \in K \))

\[
p_i^{k}(t_{v,w}) = \left[ \frac{z_i(s_v)}{\alpha_i + \sum_{m \in M} h_{im}^k g_{im}^k + \lambda_i(t_{v,w})} - \frac{\eta_i^{k}(t_{v,w-1}) \mu_i^{k}}{g_{i1}^k} \right] p_{i,0}^{k}
\]

where \( \lambda_i(t_{v,w}) \) is the optimal dual price for user \( i \)’s power capacity constraint (i.e., \( \sum_{k \in K} p_i^k \leq P_{i, max} \) at \( t_{v,w} \).

(b) **Sub-gradient Updating for Dual Prices:**

Each user \( i \in N \) updates \( z_i \) at the end of the \( v \)th long updating time as follows:

\[
z_{i}(s_{v+1}) = [z_i(s_v) + \zeta_1 (R_{i, tar}^{k} - \sum_{k \in K} \Phi_i^{k}(p_i^{k}(t_{v,W}), p_{-i}^{k}(t_{v,W})))^+]
\]

Each monitoring station \( m \in M \) also updates \( \mu_m \) at the end of the \( v \)th long updating time as follows:

\[
\mu_m^{k}(s_{v+1}) = [\mu_m^k(s_v) + \zeta_2 \left( \sum_{i \in N} p_i^{k}(t_{v,W}) g_{im}^k - \kappa T_m^k B \right)]^+, \forall k \in K
\]

(c) Set \( v = v + 1 \) and Repeat Steps (a), (b), (c) until Convergence
some channels to protect the primary systems. To test these cases, we make the parameter setting as follows (Setting 2): $\Gamma_a = \Gamma_b^5 = \Gamma_c^5 = \Gamma_d^5 = 0.01$, all the other parameters are the same as (Setting 1). Figure 6 shows the convergence of each monitoring station’s interference regulation dual price under a random channel realization. It is shown in Figure 6 that $\mu_a^\star, \mu_b^\star, \mu_c^\star, \mu_d^\star$ all converge to nonzero values, meaning that these corresponding interference temperature regulation constraints become strictly binding, while the values of all the other interference regulation dual prices are going to zeros. Figure 7 further shows the convergence of received interference level experienced by monitoring stations $a, b, c, d$ over all the channels. It is shown in Figure 7 (a) that the aggregated interference level experienced by monitoring station $a$ over channel 1 is almost upper-bounded by $\Gamma_a^5$, which verifies the result in Figure 6. Similar upper-bounded results are also observed for monitoring station $b$ over channel 3, monitoring station $c$ over channel 5, and monitoring station $d$ over channel 7 in Figures 7 (b), (c) and (d) respectively.

[9] proposed the “margin-adaptive” algorithm (MA) to minimize each user’s total transmission power subject to its rate requirement. Essentially, algorithm (MA) consists of an inner loop for the iterative water-filling allocation and a outer loop for the power capacity adaptation. However, algorithm (MA) is interference-unaware and incapable of guaranteeing the interference regulation imposed by monitoring stations. Tables I, II show the comparison between our proposed algorithm (A1) and algorithm (MA) under parameter (Setting 2) (same channel realization as the results in Figures 6 and 7). We set the tolerable level $\epsilon = 5\%$ and 15\% of the target rate and power iteration step-size $\delta = 0.02$ in algorithm (MA). It is shown in Table I that with algorithm (MA) each user can achieve an oscillating transmission rate around its target rate after convergence. The oscillation ranges can be reduced by using a larger $\epsilon$, which, however, introduces larger deviation from the target rate. In comparison each user can almost achieve its target rate with our algorithm (A1). It is further shown in Table II that algorithm (MA) is interference-unaware, i.e. the interference regulations imposed by monitoring stations are all violated under both $\epsilon = 5\%$ and 15\%, while these violations can be avoided by using algorithm (A1). In addition, it is required in algorithm (MA) that each user’s initial power allocation should be set as the minimum value (i.e., zero in our simulation). While our algorithm (A1) can be randomly initialized. Note that algorithm (MA) requires the same condition (14) to guarantee the convergence of inner loop iterative water-filling allocation.

We also compare the effect of different values of $W$ in our proposed algorithm (A1) to show its robustness to the insufficient power iteration for sub-game $G_{MCPA}$. Figure 8 shows the relative error of power allocation and dual prices $z^g$ with $W = 2, 4$ and 20 compared to the results that $W$ is sufficiently large (i.e., 50) such that the equilibrium power $(p_1^\star(z, \mu), ..., p_N^\star(z, \mu))$ for sub-game $G_{MCPA}(z, \mu)$ is reached during each $\Delta T$. Every point in Figure 8 is averaged over 100 random channel realizations. Figure 8 shows that a large value of $W$ is not strictly required for the convergence of the whole algorithm (A1). Specifically, when $W = 4$, i.e., 4 power iterations are conducted per $\Delta T$, the relative errors are below $10^{-2}$ for both power allocation and dual price after around 100 long interval iterations. However, Figure 8 also shows that large $W$ can speed up the convergence process and improve the relative accuracy.

**B. Social Optimality and Pigouvian Taxation**

Due to the selfish nature of each secondary user induced by its individual optimization problem (6), $G_{MCPA-C}$ may produce an inefficient N.E. when the social optimization (19) is considered.

$$\min_{(p_1, ..., p_N) \in \chi} \sum_{i \in N} \alpha_i \sum_{k \in K} p_k^i$$  \hspace{1cm} (19)

One method to restore optimality is the Pigouvian taxation [18]. The main idea of Pigouvian taxation is that each user individually carries out a cost-optimization problem which

---

**TABLE I**

<table>
<thead>
<tr>
<th></th>
<th>user1</th>
<th>user2</th>
<th>user3</th>
<th>user4</th>
<th>user5</th>
<th>user6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>2.99</td>
<td>4.98</td>
<td>6.99</td>
<td>8.98</td>
<td>10.98</td>
<td>12.98</td>
</tr>
</tbody>
</table>

**TABLE II**

<table>
<thead>
<tr>
<th></th>
<th>Station a channel 1</th>
<th>Station b channel 3</th>
<th>Station c channel 5</th>
<th>Station d channel 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.0107</td>
<td>0.0101</td>
<td>0.0102</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

---

Relative error of power allocation is defined as $\log_{10}(\frac{p_i - p_i^\star}{p_i})$ and relative error of dual price is defined as $\log_{10}(\frac{(z - z^\star)}{z^\star})$. Both values are averaged over all the users.

---

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE INFOCOM 2009 proceedings.
considers the negative externality\(^\text{10}\) that it imposes on other users so that the competitive equilibrium state can coincide with the social optimum. Meanwhile, the optimality-restoring tax exactly equals to the marginal externality at the social optimum. We apply the Karush-Kuhn-Tucker necessary condition to the social optimization problem (19)\(^\text{11}\), which can be expressed as (assuming an interior solution within each user’s power capacity range, \(\Omega_i^p, \forall i \in \mathcal{N}\)):

\[
\alpha_i + \sum_{m \in \mathcal{M}} \mu_{im}^* \delta_{i,m} - \frac{\partial}{\partial p_{ik}} \pi_i^k(p_{ik}^*, p_{ik}^*)|_{p_{ik}^*} = 0, \forall i \in \mathcal{N}, \forall k \in \mathcal{K} \quad (20)
\]

Comparing with the first order condition for each user \(i\)’s cost function \(\Theta_i(p_i, p_{-i}, z_i, \mu)\) in sub-game \(G_{MCPA}\), we add a taxation component \(\pi_i\) into \(\Theta_i\) as follows:

\[
\tilde{\Theta}_i(p_i, p_{-i}, \pi_i, z_i, \mu) = \sum_{k \in \mathcal{K}} (\alpha_i + \sum_{m \in \mathcal{M}} \mu_{mik}^* |_{p_{i,k}^*}) p_{i,k}^* + \sum_{k \in \mathcal{K}} \pi_i^k p_{i,k} - z_i \supseteq_k \pi_i^k(p_{i,k}^*, p_{-i}) \quad (21)
\]

where user \(i\)’s tax vector \(\pi_i = (\pi_1^i, \ldots, \pi_K^i)\), and each element \(\pi_i^k = -\sum_{j \neq i} \frac{\partial}{\partial p_{i,j}} z_j \Phi_j^k(p_{i,k}^*, p_{i,k}^*)\) denotes the value that user \(i\) has to pay for its negative externality to all the other users over channel \(k\). Assume the sub-gradient updateings (10) and (11) have converged to the optimal dual prices \(z_i^*\) and \(\mu_{mik}^*\) respectively, then the first order optimality condition for (21) (which also equivalently serves as the equilibrium condition for \(G_{MCPA-C}\)) is exactly the Karush-Kuhn-Tucker necessary condition (20) for the social optimization (19). Similar methods appeared in [2] [8] to achieve the sum-utility (or sum-rate) maximization in wireless ad-hoc networks.

To facilitate the distributed implementation, the calculation of tax vector can be simplified as follows:

\[
\pi_i^k = -\sum_{j \neq i} \frac{\partial}{\partial p_{i,j}} z_j \Phi_j^k(p_{i,k}^*, p_{i,k}^*) |_{p_{i,k}^*} = \sum_{j \neq i} \omega_j^k g_{ij}^k \quad (22)
\]

\[
\omega_j^k = -\frac{\partial}{\partial z_j} z_j \Phi_j^k(p_{i,k}^*, p_{i,k}^*) = z_j B (n_j^k + p_{j,k}^* g_{jj}^k) \quad (23)
\]

where \(\omega_j^k\) denotes the tax that user \(j\) collects for its suffering of interference on channel \(k\). Our algorithm (A1) can be modified to incorporate the Pigouvian taxation without any difficulty. Specifically, in Iteration Process (a) of algorithm (A1), each user announces \(\omega_j^k, \forall k \in \mathcal{K}\), by (23) and updates its power allocation by taking into consideration of \(\omega_j^k, \forall k \in \mathcal{K}\) (22). We name the modified version algorithm (M-A1) and omit...
TABLE III
COMPARISON OF SOCIAL PERFORMANCE METRIC AMONG ALGORITHM (A1), (M-A1) AND LINGO (SETTING 3)

<table>
<thead>
<tr>
<th>$R_{\text{S1}}$</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>1.126</td>
<td>0.2029</td>
<td>0.3672</td>
<td>0.797</td>
<td>1.766</td>
<td>2(+)</td>
<td>2(+)</td>
</tr>
<tr>
<td>M-A1</td>
<td>0.1178</td>
<td>0.1860</td>
<td>0.2978</td>
<td>0.4880</td>
<td>0.8342</td>
<td>1.4252</td>
<td>2(+)</td>
</tr>
<tr>
<td>LINGO</td>
<td>1.1174</td>
<td>0.1860</td>
<td>0.2978</td>
<td>0.4871</td>
<td>0.8210</td>
<td>1.4060</td>
<td>Infeasible</td>
</tr>
</tbody>
</table>

TABLE IV
RELATIVE IMPROVEMENT OF ALGORITHM (M-A1) OVER (A1)

<table>
<thead>
<tr>
<th>Rel. Impr.</th>
<th>5</th>
<th>5.5</th>
<th>6</th>
<th>6.5</th>
<th>7</th>
<th>7.5</th>
<th>8</th>
<th>8.5</th>
<th>9</th>
<th>9.5</th>
<th>10</th>
<th>10.5</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.125</td>
<td>0.149</td>
<td>0.184</td>
<td>0.249</td>
<td>0.300</td>
<td>0.335</td>
<td>0.440</td>
<td>0.500</td>
<td>0.441</td>
<td>0.348</td>
<td>0.287</td>
<td>0.130</td>
<td>0.080</td>
<td>0.019</td>
</tr>
</tbody>
</table>

the detailed description here due to limited space. Algorithm (M-A1), however, requires coordination among users and involves some difficulties in practical implementation. For example, designing a self-incentive mechanism so that each user broadcasts its values of $w_k, \forall k \in K$, truthfully is a challenging issue and this is considered as our future work.

To test algorithm (M-A1), we build a small network with 2 users sharing 4 channels. 1 monitoring station $a$ is used. We make the parameter setting as (Setting 3): $P_{\text{max}}^1 = P_{\text{max}}^2 = 1$, $\Gamma_k^a = 0.6, k = 1, 2, 3, 4$ (we set $\Gamma_k^a$ high enough so that the worst case efficiency loss can be found), and $\alpha_i = 1, \forall i \in N$. The social performance metric is $\sum_{i \in N} \alpha_i \sum_{k \in K} p_k^i$. Table III shows the comparison of algorithms (A1), (M-A1) and LINGO [20] (an optimization software) under one random channel realization by changing $R_{\text{tar}}^1 = R_{\text{tar}}^2$ from 6 to 12. Because of the direct control of externality generation, algorithm (M-A1) always outperforms algorithm (A1) in terms of the social optimality at the equilibrium. Besides, when algorithm (A1) cannot find a feasible N.E. for $R_{\text{tar}}^1 = R_{\text{tar}}^2 = 11$ (In Table III, 2(+) denotes the result that both users’ power capacities are exhausted, but some user’s target rate still cannot be met), algorithm (M-A1) can still find a feasible equilibrium solution. Table III also shows that the equilibrium solutions found by algorithm (M-A1) always approach to the local minima found by LINGO. Table IV further shows the relative improvement of algorithm (M-A1)\(^{12}\). Every entry in Table IV is averaged over 100 channel realizations. Table IV shows that the relative improvement of algorithm (M-A1) begins to decrease when target rate goes beyond some threshold. It is because when target rate increases, the possibility that algorithm (M-A1) cannot find a feasible equilibrium solution under a random channel realization increases. These statistical results also show that the efficiency loss in algorithm (A1) is bounded.

**VII. CONCLUSIONS**

In this paper we study the distributed power allocation for spectrum sharing cognitive radio network with QoS guarantee. We formulate this optimization problem as a non-cooperative game $G_{\text{MCPA-C}}$ with the coupled strategy space to address both the co-channel interference among secondary users and the interference regulation imposed by primary systems. We study the N.E. of $G_{\text{MCPA-C}}$ and design a distributed algorithm (A1) to find it. We further adopt the Pigouvian taxation in our algorithm when the social optimality is considered.

**REFERENCES**


\(^{12}\)Let $v_1$ and $v_2$ denote the social performance metric of the solutions found by algorithms (A1) and (M-A1) respectively. The relative improvement of algorithm (M-A1) over algorithm (A1) is defined as $\frac{v_1 - v_2}{v_2}$. 

989