A Generalized Probabilistic Topology Control for Wireless Sensor Networks

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Abstract—Topology control is an effective method to improve energy-efficiency and increase the capacity in Wireless Sensor Networks (WSNs). To fully characterize WSNs with lossy links, we propose a novel probabilistic network model. Under this model, we meter the network quality using network reachability defined as the minimal of the upper limit of the end-to-end delivery ratio between any pair of nodes in the network. We attempt to find a minimal transmitting power for each node while the network reachability is above a given application-specified threshold, called probabilistic topology control (PTC). We prove that PTC is NP-hard and propose a fully distributed algorithm called BRASP. We prove that BRASP has the guaranteed performance. Two rules that must be followed by any algorithm have been identified. We conduct both simulations and prototype implementations based an 18-TelosB-node test-bed. The experimental results show that the network energy-efficiency can be improved by up to 250%. The average node degree is reduced by 50% which will lead to a great benefit for the network capacity.

I. INTRODUCTION

Topology control is an effective method to improve network energy-efficiency and increase the capacity in Wireless Sensor Networks (WSNs) [1].

In traditional methods, networks are modeled using the *deterministic network model*.

In real environments, however, the deterministic network model cannot fully characterize the behavior of wireless links. This is mainly due to the *transitional region* phenomenon which has been revealed and by many empirical studies [2], [3]. Beyond the “always connected” region, there is a transitional region which probabilistically connects a pair of nodes. Such pairs of nodes are not fully connected but reachable via the so-called *lossy links*. It has been reported that lossy links are often much more than the fully connected links. Their impact can hardly be neglected.

Lossy links provide a great opportunity for higher energy-efficiency and larger network capacity. Compared with those fully connected links, lossy links allow a transmitter to reach more nodes when they succeed. Motivated by this, a large volume of routing algorithms have been proposed to take lossy links as an advantage, which are the so-called opportunistic routing (e.g., ExOR [4] and MORE [5]).

Lossy link have no guarantee on the connectivity. When they are employed, the topology may have no full network connectivity. In order to well characterize a network topology with lossy links employed in, we propose a new model of

Fig. 1. An illustration of the relation between energy cost and network reachability for DTC and PTC

probabilistic topology control (PTC). The core concept of PTC is the *Quality of Connectivity (QoC)*. Given a topology, QoC is metered using *network reachability* defined as the minimal of the E2E delivery ratio between any pair of nodes using network-wide broadcast (i.e., simple flooding). We use broadcast as the benchmark routing algorithm as it is the best that a routing algorithm can do. By broadcast the derived E2E delivery ratio is the upper limit of all routing algorithms. Another physical meaning of the network reachability is the minimal of the probability that any pair of nodes has at least one data path in the network.

The objective of PTC is to seek a topology of minimal energy cost by reducing the transmitting power of each node, while the network QoC satisfies certain constraint (e.g., 60%). Fig. 1 illustrates the relation between the energy cost and the network reachability for both DTC and PTC. The x-axis represents the energy cost and y-axis is the achieved network reachability. DTC only uses reliable links. Its target is the topology of full connectivity and minimized cost (point P in the figure). Different from this approach, PTC involves lossy links. It aims at a topology of higher energy efficiency subject to a certain constraint of network reachability. In the example of Fig. 1, given a threshold Λ_{th} (0.6 in this case), point Q is the target of PTC and R is that of deterministic one.

A key challenge in PTC is the computation of the network reachability. It is known that given a network topology, the computation of E2E delivery ratio is NP-hard when network broadcast is used. Indeed, the E2E delivery ratio is not practically computable unless the data paths between the source and destinations are *series-parallel* topologies (Sec. IV-A). To deal with this challenge, we explore the reliability theory. Instead of computing the accurate network reachability, we design a fully distributed algorithm to approximate the

network reachability. Using this approximation scheme, we propose a fully distributed algorithm called BRASP. The basic idea is that every node granularly increases its transmitting power until certain local condition is satisfied. We prove that when this local condition is satisfied, the global constraint on network reachability will be guaranteed as well. We also prove that this local condition is a tight condition, meaning that it is necessary and sufficient to guarantee the global constraint.

The main contributions of this paper are as follows. First, we identify the limitations of the traditional DTC in practical environments and propose a novel PTC for general sensor-to-sensor communications. To the best of our knowledge, we are the first one to solve PTC in general communication paradigms. Second, we prove that PTC is NP-hard. To solve the problem, we propose a fully distributed algorithm BRASP to approach the solution by exploring the reliability theory. We implement BRASP in a TelosB test-bed to verify its feasibility and conduct a large number of simulations to make the performance evaluation. Experimental results show that BRASP can significantly increase the network energy-efficiency by up to 250% comparing to the traditional DTC. And there is also an obvious benefit for network capacity because the average node degree is largely decreased by 50%.

We organize the remainder of this paper as follows. In Section II we give an overview of the related work. We present the probabilistic network model in Section III as well as some empirical results and an motivation example. Section V gives some preliminary information of the PTC, and the detailed design of BRASP is in Section V which also includes the analytic results of BRASP. Section VI presents the performance evaluation results. The last section will be the conclusion and future research directions.

II. RELATED WORK

DTC has been extensively studied in literature. Depending on different energy-saving schemes, these approaches can be classified as power-adjustment based and MCDS-based (Minimal Connected Dominating Set).

In the category of power-adjustment based approaches, the goal is to assign a minimal transmission range for each node while the global network connectivity is preserved. This is NP-complete problem and many heuristics have been proposed such as [6]. In particular, Wattenhofer et al. [7] proposed Cone-based Topology Control algorithm (CBTC). They proved that if in each cone of no greater than $\frac{5\pi}{6}$ there was at least one neighbor for every node, then the derived network preserved the original connectivity. D'Sauza et al. [8] further showed that the cone is up to π , which was a tight bound for CBTC algorithms. Topology control with irregular wireless footprint, robustness, interference-aware and quality-based were also studied (e.g., [9]). Another energy-saving scheme is to remove un-necessary transmitting and receiving by seeking the Minimal Connected Dominating Set (MCDS). This problem is also NP-complete with many heuristic algorithms [10]. All these existing works are based on the deterministic network model and do not take the lossy links into consideration. They therefore share the same limitation.

III. OVERVIEW OF PTC

In this section, we give an overview of PTC. We first present the assumptions we made, and then describe the network model and energy model. The third subsection is the problem statement. In the last part, we point out the key issues and main challenges we are facing when solving the problem.

A. Assumptions

We assume a static WSN in which sensor nodes have no mobility. The individual link quality (Packet Reception Rate PRR) is available, which can be obtained by periodic "Hello" messages, or using Link Quality Index (LQI) to predict [11], [12]. We further assume that the link quality is fixed. This assumption is reasonable as many empirical studies have shown that link quality is pretty stable in a static environment. Note that a link may have different qualities under different transmitting powers. No node failure is considered as it is equivalent a link failure case. No duty cycle is considered either. We do not consider packet collisions or transmission congestions, which are left to the designers of MAC layer. The degradation of the E2E delivery ratio is thus only due to the failure of wireless links. We assume there are finite number of power levels.

B. Network model

We model a WSN as a directed graph $G(V, E)$ where V is the set of nodes and E is the set of links. The K networks derived by different transmitting powers of nodes are denoted as $G_{\min} = G_1(V, E_1)$, $G_2, \dots, G_{\max} = G_K(V, E_K)$ respectively. The derived topology by BRASP is denoted as $G_R(V, E_R)$. For each link $e = \{u, v\} \in E$, we use $\lambda_G(e) \in (0, 1]$ to indicate the probability that node u can directly deliver a packet to v . Let $N_\chi(u)$ denote the χ -hop neighbor set of a node u in $G_{\max}$, i.e.,

$$\begin{cases} N_1(u) = \{v | \lambda_{G_{\max}}(u, v) > 0 \vee \lambda_{G_{\max}}(v, u) > 0\} \\ N_\chi(u) = N_{\chi-1}(u) \cup \{v | \exists w \in N_{\chi-1}(u), v \in N_1(w)\} \end{cases}$$

Definition Give a source node s and a destination node d in G , the node-pair reachability $\Lambda_G(s, d)$ is defined as the probability that s has at least one valid data path to d in G .

Theorem 3.1: An equivalent definition of node-pair reachability $\Lambda_G(s, d)$ is the probability that s can deliver a packet to d by a network-wide broadcast.

We omit all the proofs in this paper due to the space limitation.

Definition Given a network $G(V, E)$, its network reachability Λ_G is the minimal of the node-pair reachability between any pair of nodes, i.e., $\Lambda_G = \min\{\Lambda_G(s, d), \forall s, d \in V\}$.

Theorem 3.2: Given a topology G and a threshold Λ_{th} , it is NP-hard to verify whether $\Lambda_G \geq \Lambda_{th}$.

C. Energy model

The power consumption $\varepsilon(G)$ to support a network topology G mainly consists of two parts, the transmitting cost $\varepsilon_{Tx}(G)$ and the receiving cost $\varepsilon_{Rx}(G)$. We define:

TABLE I
NOTATIONS USED IN THIS PAPER

Notation	Description
$G(V, E)$	A network topology, V is the set of nodes and E is the set of links
V_{Tx}	The set of transmitters in G : $V_{Tx} = \{u \exists v \in V, \{u, v\} \in E\}$
$N_\chi(u)$	The sub-network of u 's χ -hop neighborhood
λ_e	Link reachability
$\Lambda_G(u, v)$	Node-pair reachability from node u to v
$\Lambda(G)$	Network reachability of G : $\Lambda_G = \min\{\Lambda_G(s, d), \forall s, d \in V\}$.
Λ_{th}	A threshold to define the minimal required network reachability
$\varepsilon(G)$	The energy cost of a network: $\varepsilon(G) = \varepsilon_{Tx}(G) + \varepsilon_{Rx}(G)$

Definition The energy cost $\varepsilon(G)$ of a given network G is the sum of the transmitting cost $\varepsilon_{Tx}(G)$ and receiving cost $\varepsilon_{Rx}(G)$, i.e., $\varepsilon(G) = \varepsilon_{Tx}(G) + \varepsilon_{Rx}(G)$

Definition *Reachability-energy ratio* $\eta(G)$. Given a network G , $\eta(G)$ is defined as the ratio of $\Lambda(G)$ and $\varepsilon(G)$:

$$\eta(G) = \frac{\Lambda(G)}{\varepsilon(G)}$$

We use reachability-energy ratio to meter the network energy-efficiency.

D. Problem statement

reachability (e.g., 60%), and the network energy cost should be

Given a directed graph $G(V, E)$ and a threshold $\Lambda_{th} \in (0, 1]$, assume there are K power levels available and the corresponding transmitting powers are $T_1, T_2, \dots, T_K$. The problem is to find a power $k(v) \in [1, K]$ so that the derived topology $G_{BRASP}(V, E_{BRASP})$ satisfies,

$$\begin{aligned} \text{minimize} & : \varepsilon(G_{BRASP}) \\ \text{subject to} & : \Lambda(G_{BRASP}) > \Lambda_{th} \end{aligned}$$

Theorem 3.3: PTC is NP-hard.

The central task in PTC is to determine the power of each node. It mainly has following challenges.

The first challenge is the verification of the constraint $\Lambda(G_{BRASP}) > \Lambda_{th}$. It is a NP-hard problem by Theorem 3.3. In order to guarantee that a derived network satisfies the constraint, we need an efficient algorithm to compute the node-pair reachability.

The second challenge is the fully distributed computing environment. WSNs are usually characterized as high density, large scale with extremely constrained resources. Centralized algorithms are not suitable due to the poor scalability and high energy cost. Instead, fully distributed algorithms are appropriate for many cases.

IV. PRELIMINARY

In this section we first introduce how to compute node-pair reachability. We then give some rules which are the foundation that must be followed during the design.

A. Node-pair reachability computation

By reliability theory we know that there are some specific topologies which can compute node-pair reachability in polynomial time. One is called *series* topologies that nodes are deployed along a chain. Another form is *parallel* that the source and destination has a number of link-disjoint paths. Currently only simple combinations of series and parallel topologies, called series-parallel topologies, can be computed in polynomial time. More recent results about reliability theory can be found in the reference [13].

There are two fundamental techniques to make a series-parallel conversion. One is called *pivotal decomposition* or *factoring*. Since the running time is exponential in the worst case, this technique is more appropriate for small-scale networks. Instead of computing the accurate node-pair reachability, another approach is by approximation algorithms. This second approach attempts to find a series-parallel sub-network instead of the original network. In this paper, we take the second approach.

B. Rules in distributed algorithm design

Existing approximation algorithms for node-pair reachability are centralized based on given global network information. They are not applicable in distributed WSNs. Moreover, the coherent system feature prohibits these algorithms from being simply extended to fit distributed environments. The inclusion-exclusion principle, the basic tool when we compute node-pair reachability, cannot be applied directly due to the dependence between different components. For these we have to develop new principles.

Rule Serialism : let $\Lambda(u, w)$ be the node-pair reachability from a node u to w in G , that from w to v is $\Lambda(w, v)$. Then the node-pair reachability from u to v is guaranteed to be at least,

$$\Lambda(u, v) \geq \Lambda(u, w) \cdot \Lambda(w, v) \quad (1)$$

if and only if $\Lambda(u, w)$ and $\Lambda(w, v)$ are independent.

Rule Parallelism : let w and t be two different intermediate nodes. The node-pair reachability from a source node u to a destination node v is guaranteed to have ,

$$\Lambda_G(u, v) \geq \Lambda(u, w) \cdot \Lambda(w, v) + \Lambda(u, t) \cdot \Lambda(t, v) \quad (2)$$

$$- \Lambda(u, w) \cdot \Lambda(w, v) \cdot \Lambda(u, t) \cdot \Lambda(t, v) \quad (3)$$

if and only $\Lambda(u, w)$, $\Lambda(u, t)$, $\Lambda(w, v)$ and $\Lambda(t, v)$ are mutually independent.

These two rules must be strictly followed when we compute node-pair reachability. Later we will see how to apply these two rules.

Fig. 2. Flowchart of BRASP for a node u 's perspective; there are four main components labeled by a circled number

V. BRASP DESIGN

In this section we describe our BRASP algorithm. We first introduce the general architecture, and then describe the components of BRASP. With these components available, we give a detailed description of BRASP. In the last subsection we show some analytic results, including the key properties of BRASP, the proof of BRASP's correctness, and the space and time complexity.

A. General architecture

BRASP is based on a simple power-adjustments scheme. Since there are only a limited number of power levels, we merely need to determine the lowest power level that a node should take so that the global constraint can be guaranteed. In BRASP, nodes start from the minimal power. They then granularly increase their powers until certain local condition is satisfied. Later we will show that this local condition is a tight condition in order to guarantee the global constraint being satisfied.

Fig. 2 illustrates the workflow of BRASP from a node u 's perspective. The pseudocode and detailed steps can be found in the full version of the paper [14]. Initially, u measures the link qualities under different transmitting powers $\lambda_{G_k}(u, v)$, $k \in [1, K]$ for any node in its neighborhood $N_x(u)$ in the maximal power network $G_{T_{max}}$. Node u sets an initial power setting. The sink will then estimate the network diameter L (i.e., the number of hops in the shortest path between furthest pair of nodes). In the meanwhile, nodes exchange their link qualities with the neighboring nodes. As long as this information is available, u is able to compute and test the local condition. It increases its power until this local condition is satisfied. It then reports to other nodes its new power together with the corresponding link qualities.

G_{BRASP} is derived by applying the power settings to the original network G . Initially it is when every node uses an initial power setting. As nodes execute BRASP and change their power level settings, G_{BRASP} evolves too.

The detailed design of BRASP involves several critical problems. We give an introduction to our proposals and more details are in the next subsection.

The first problem is that under which local condition a node can guarantee a satisfied global constraint. It directly relates to the first major challenge in Sec. IV. Recall that there are no optimal solutions. To address this problem, we

design an approximator algorithm to approximate the node-pair reachability in a fully distributed and localized manner. This approximator intelligently leverages the two rules we exploited in Sec. IV.

The second problem is the computation sequence of BRASP in different nodes. BRASP is a fully distributed algorithm. Different nodes run BRASP in an asynchronized manner. Therefore some nodes have to execute BRASP first, and others wait for the results of these pioneer nodes. Here we employ a heuristic algorithm based on the node degree. A node with a higher node degree will have a higher priority to run BRASP.

The third question is of the initial power setting. On one hand, the initial power setting is the lower bound of the setting. An incorrect lower bound above the real value reduces the effectiveness of topology control. On the other hand, the greatest lower bound is desired as the pioneer nodes use the initial power setting as the input of their neighbor's decision. A higher power setting often results in better link qualities. Therefore pioneer nodes will have more high quality links by which the local condition is easier to satisfy.

In the last, it is still possible that a node fails the local condition when it maximizes its power. Under such scenarios, we employ a recovery mechanism to solve the problem.

We address these problems by three major components as in Fig. 1 where the component numbers are circled. Now we give a detailed description of these components.

VI. PERFORMANCE EVALUATION

In this section we study the impact of the key control parameters and evaluate the performance of BRASP. We design and implement a prototype system consisting of one sink node and 18 TelosB nodes. We also conduct comprehensive simulations to study BRASP under large-scale deployments. The evaluation metrics include: 1) network reachability $\Lambda(G)$; 2) energy cost $\varepsilon(G)$; 3) reachability-energy ratio $\eta(G)$; and 4) node degree $d(G)$. We apply Monte-Carlo method to measure the measure the node-pair reachability. Each measurement is averaged from 5 independent runs.

We compare BRASP with a recent Cone-Based Topology Control (CBTC) [8] algorithm which represents the deterministic topology control. The derived topology by CBTC is G_{CBTC} and that of BRASP is G_{BRASP} . We do not compare BRASP with our previous work CONREAP [15] because they were designed for different applications. CONREAP is for sink-to-sensor communications only and BRASP is for the general sensor-to-sensor communications. CONREAP and BRASP can hardly be put in the same investigation context.

Fig. 3 presents $\Lambda(G_{CBTC})$ and $\Lambda(G_{BRASP})$ against the energy cost in our prototype system. There is a base cost below which the network cannot provide any reachability. For BRASP this base cost is 48 units of energy. For CBTC, this cost increases to 151, more than three times of that of BRASP. As the requirement of the network reachability is increased, the cost is increased too. To provide a reachability of 85%, the cost is 120. Notice that this cost is still less than the base cost of CBTC. As expected, the energy-efficiency

Fig. 3. $\Lambda(G_{CBTC})$ and $\Lambda(G_{BRASP})$ against energy cost in the prototype system

Fig. 4. $\eta(G_{CBTC})$ and $\eta(G_{BRASP})$ against network reachability in the prototype system

Fig. 5. $d(G_{CBTC})$ and $d(G_{BRASP})$ against network reachability in the prototype system

can be improved from 80% when $\Lambda_{th} = 0.8$ to 140% when $\Lambda_{th} = 0.2$ (Fig. 4). These results also indicate that BRASP is more appropriate for environments of low threshold Λ_{th} . When Λ_{th} is increased, the improvement of BRASP against CBTC decreases quickly. Fig. 5 shows the degree distribution of nodes in G_{CBTC} and G_{BRASP} , respectively. Clearly, the network density is dramatically reduced. The average, the minimal, and the maximal node degree are reduced by 50%, 80% and 40% respectively. These prototype implementations are consistent with our simulations, which verify the feasibility of BRASP in real environments.

VII. CONCLUSION AND FUTURE WORK

In this paper, we have studied the topology control using a new model of PTC. PTC is based on the empirical studies that most wireless links are lossy links that only probabilistically connect pairs of nodes. Different from the traditional DTC which assumes that links are either connected or disconnected, the new probabilistic network model enable the employment of lossy links in a smart manner. The key concept in PTC is the QoC metered by network reachability. In this paper we focus our major attention on finding a minimized transmitting power for each node while the network reachability satisfies certain threshold-based constraint. We proved that this is an NP-hard problem and proposed BRASP a fully distributed algorithm to address the problem. BRASP has the provable performance in terms of the network reachability and the energy cost can be significantly decreased. Comprehensive simulations as well as a prototype implementation based on a TelosB test-bed have shown that the energy-efficiency can be improved by up to 250%. The average network node degree is decreased by 50%, which implies a great benefit on network transmission capacity. BRASP has the worst running time of $O(|E|)$, the communication overhead of $O(|E|+|V|)$, and a constant space requirement.

Future work can be carried out along following directions. First, we assume that the quality of wireless links are available and fixed. In practice, link qualities are functions of time and are quite difficult to be accurately measured. We need more understanding of the behaviors of wireless links. Second, besides the E2E delivery ratio, there are some other important metrics for the quality of a network such as fairness, stability, and robustness. Third, we do not consider the impact of multi-

channel capability of sensor nodes, which may present further opportunities for topology control in WSNs.

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