Time-of-Arrival Estimation for IR-UWB Systems in Dense Multipath Environment

Yizhi Xu*, Edward K. S. Au#, Albert K. Wong§, and Qin Wang*

*Wireless IC System Design Center, HKUST Fok Ying Tung Graduate School, Nansha, China
#Modulation and Coding Department, Institute for Infocomm Research, A*STAR, Singapore.
§Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology, Hong Kong
School of Information Engineering, University of Science and Technology Beijing, Beijing, China.
Email: allenxu@ust.hk, ksau@i2r.a-star.edu.sg, eealbert@ust.hk, wangqin@ies.ustb.edu.cn

Abstract—In this paper, a novel time-of-arrival estimation algorithm is proposed for impulse radio-based UWB systems in dense multipath environment. In particular, a dedicated ranging sequence with good cross-correlation property is proposed and a threshold-based search-back unit is introduced, with the purpose of supporting high-precision ranging even at low signal-to-noise ratio. An approximate closed-form expression on the optimal threshold is derived based on the Rayleigh statistical analysis, and its accuracy and tightness are verified by simulations. Results show that the proposed ranging scheme achieves an excellent mean absolute error performance in the order of nanoseconds without introducing too much latency.

I. INTRODUCTION

Ultra-Wideband (UWB) is a promising emerging technology for providing both high data rate and low data rate communications with very low power spectral density. A signal is recognized as UWB if its fractional bandwidth is greater than or equal to 0.20 [1], or if its signal bandwidth is at the minimum of 500MHz measured at the upper and lower frequencies of the -10dB emission points.

Time-of-arrival (TOA) algorithms, which estimate the arrival time of the first path of the received signal, are prevalently employed for ranging and location finding. Impulse radio (IR)-based UWB technology has attracted great attention on precise ranging and positioning, due to its extremely short pulse duration and the fine time resolution. In dense multipath environment, however, the first path is often not the strongest path because of multipath, noise and different kinds of interferences, which make the estimation problem challenging.

Parametric and statistical methods for estimating the first path have been proposed to tackle this challenge. One common and well-established category of TOA-based ranging algorithms for IR-UWB is the matched filter (MF)-based coherent algorithms [3-5]. These algorithms provide a very high resolution in ranging accuracy but require a high sampling rate. Another category is the energy detection-based non-coherent algorithms [6-7], which in contrast are usually with low computational and hardware complexities, at the expense of providing a relatively low ranging accuracy. Additionally, some ranging schemes combining both the coherent and the non-coherent algorithms are investigated [8-9]. In [8], a set of ternary sequences with good auto-correlation property is proposed to support low-power ranging in Wireless Personal Area Network (WPAN), and an energy detection-based search-back scheme is employed to locate the leading edge of the received signal based on the output of the matched filter. However, the threshold utilized in the search-back scheme is only numerically determined by the probability of false alarm, while other important information such as the probability of misdetection is not taken into account. In [9], a two-step approach that jointly employs energy detection and matched filtering is proposed, but the simulation results presented therein show that the high-precision ranging can only be achieved at high signal-to-noise ratio (SNR).

In this paper, a novel ranging scheme for the IR-based UWB system is proposed for supporting high-precision ranging. In particular, we design a dedicated ranging sequence with good cross-correlation property and introduce a MF-based search-back decision unit in order to locate the leading edge of the received signal. Simulation results reveal that high-precision ranging performance, in terms of mean absolute error (MAE), can be achieved even at low SNR.

II. CHANNEL MODEL

Our UWB channel model follows the IEEE 802.15.4a channel model, which covers indoor residential, indoor office, industrial, outdoor, and open outdoor environments with a distinction between Line-of-Sight (LOS) and Non-Line-of-Sight (NLOS) properties [10]. This channel model is derived from the Saleh-Valenzuela (S-V) model with two modifications. Namely, the magnitude of the multipath gain follows the Nakagami distribution, and independent fading is assumed for each cluster as well as the rays within the cluster. Mathematically, the multipath model of this modified S-V model is given by [10]
where \( \{a_k\} \) and \( \{\phi_k\} \) are the tap weight and phase, respectively, of the \( k \)-th component in the i-th cluster, and \( t_{\phi_k} \) is the delay of the \( k \)-th multipath component relative to the i-th cluster arrival time \( T_i \).

### III. PROPOSED RANGING SCHEME

In this section, we present a TOA-based ranging algorithm for IR based UWB systems that works with bandwidth of at least 500MHz and achieves excellent accuracy in terms of MAE even at low SNR.

#### A. Ranging Preamble

In [3], Stein and Leeper utilize the MB-OFDM time-domain base sequence [11] for ranging in WPAN. Since this base sequence is designed to achieve a good auto-correlation property with very low side lobes, it is expected that the leading edge of the received signal can be easily identified even in the presence of noise. In a dense multipath environment, however, the strongest MPC (multipath component) may not be the first path. This leads to an overestimate of the actual leading edge and makes the estimation very challenging.

![Packet/Frame Sync. Sequence Ranging Sequence](image)

**Figure 1.** The proposed ranging preamble

In order to tackle this challenge, we propose a ranging preamble in Figure 1 that consists of a time-domain packet/frame synchronization (P/FS) sequence [11] and a PN sequence dedicated for ranging. The P/FS sequence is generated by the time-domain base sequence, a spreading sequence and zero-suffix [11]. Its main purposes are to perform acquisition and timing synchronization, or equivalently as coarse ranging that provides a coarse timing estimate for the subsequent ranging process. For the PN sequence dedicated for ranging, its purpose is to detect the leading edge. It is designed to achieve perfect zero side lobes when cross-correlation is performed with the received signal such that the contamination from the delayed multipath components is eliminated.

![Packet/Frame Sync. Sequence Ranging Sequence](image)

**Figure 2.** The proposed dedicated ranging sequence \( s(t) \).

Figure 2 depicts the structure of the dedicated ranging sequence, which consists of \( N \) symbols transmitted consecutively, with \( T_r \) being the ranging symbol period, \( T_p \) being the time instant estimated by the P/FS sequence, and \( T_i \) being the time instant of the leading edge we would like to estimate.

In order to provide the robustness against multipath and poor channel realizations, \( N \) is an integer greater than 1. For each of the \( N \) symbols \( s_k(t) \) in the sequence, it is made up of a preamble, followed by a PN sequence \( \Theta(t) \) and zero-suffix as shown in Figure 3. The function of both the preamble (which is a copy of the tail part of the PN sequence) and the zero-suffix is to avoid inter-symbol interference, and their lengths are equal to that of the maximum multipath delay. For the PN sequence, it is an m-sequence of length \( L_m = 2^d - 1 \) [12] which is generated by a d-stage linear feedback shift register (LFSR).

![Preamble PN sequence Zero-suffix](image)

**Figure 3.** The structure of a ranging symbol.

#### B. Transmitter Structure

We follow [8] and consider a relatively simple transmitter structure of our proposed ranging algorithm in Figure 4. Each symbol is mapped to an m-sequence of length \( L_m \), followed by inserting the preamble at the beginning of the sequence. After zero padding at the end of the sequence, this extended symbol is repeated with \( N \) times to form our proposed ranging sequence. Then, the whole ranging sequence is pre-appended with the time-domain P/FS sequence to create a ranging preamble. Lastly, this ranging preamble is modulated by the pulse generator for transmission.

![Preamble PN sequence Zero-suffix](image)

**Figure 4.** The proposed transmitter structure.

#### C. Receiver Structure

The proposed receiver structure dedicated for ranging is shown in Figure 5 and is summarized as follows. Given \( T_i \) as the time instant estimated based on the preceding P/FS sequence, we start utilizing the dedicated ranging sequence to estimate the leading edge of the signal \( T_r \). Denote \( r(t) \) as the \( i \)-th symbol of the ranging sequence after performing the timing synchronization. It is expressed as a sum of the desired dedicated ranging sequence component \( a_k \cdot s(t) \), \( L_p \) undesired multipath components, and additive white Gaussian noise \( n(t) \) with zero mean and variance \( \sigma^2 \) [13], i.e.,

\[
r(t) = a_k s(t) + \sum_{p=1}^{L_p} a_p s(t - \tau_p) + n(t)
\]

where \( a_p = \beta_p e^{j\phi} \) and \( \tau_p \) are the complex gain and propagation delay of the \( p \)-th MPC, respectively, \( p = 1, 2, \ldots, L_p \). Given the
coarse timing estimate $T_p$ we search for the possible existence of direct path within the time interval $[T_p - \Omega_k T_c, T_p - T_c]$ using a sliding correlator, which correlates the local sequence $\Phi(t)$ with $r_j(t + jT_c)$ for $j = 1, 2, 3, \ldots, \Omega_k$, where $r_j(t + jT_c)$ is a time-shifted version of a received symbol $r(t)$. In the following sub-sections, the two important functional blocks - the sliding correlator and search-back decision units - are presented in details.

![Diagram](image)

Figure 5. The proposed receiver structure dedicated for ranging.

**C.1. Sliding Correlator**

The function of the sliding correlator is to suppress interference from undesired multipath components by correlating each of the $N$ received ranging symbols $r(t)$ with the local sequence $\Phi(t)$ which is generated from the PN sequence $\Phi(t)$ by mapping $+1/-1$ to $+1/0$. When the received $\Theta(t)$ in the first MPC is exactly matched with $\Phi(t)$, a correlation peak of $R = (P + 1)/2$ is created, where $P$ is the auto-correlation peak value of the PN sequence $\Theta(t)$. Figure 6 illustrates the periodic cross-correlation property between a series of PN sequences and the local sequence in the discrete version, namely $\Theta(n)$ and $\Phi(n)$, with the former being generated by the 8-stage LSFR. It is clearly observed from the figure that there are exactly zero cross-correlation values in-between peaks, which implies that contamination from the delayed multipath components can be completely eliminated.

**C.2. Search-back Decision Unit**

In order to locate the leading edge of the received signal, we introduce a MF-based search-back decision unit that identifies the farthest point from the locked point above a pre-determined threshold within the search-back window. Denote $R$ as a matrix that consists of $N \times \Omega_k$ cross-correlation values:

$$R = \begin{pmatrix}
R_{11} & R_{12} & \cdots & R_{1\Omega_k} \\
R_{21} & R_{22} & \cdots & R_{2\Omega_k} \\
\vdots & \vdots & \ddots & \vdots \\
R_{\Omega_k1} & R_{\Omega_k2} & \cdots & R_{\Omega_k\Omega_k}
\end{pmatrix}$$

The absolute value of each entry of $R$ is compared with a pre-determined threshold $\xi$. For an entry whose value is larger than the threshold, it will be converted to "1"; otherwise it is set to "0". Thus, an indicator matrix is created in the form of

$$I_k = \begin{pmatrix}
0 & 1 & 0 & \cdots & 1 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & \cdots & \cdots & 0
\end{pmatrix}$$

Summing each column of $I_k$ yields a vector $S = [S_1, S_2, S_3, \ldots, S_{\Omega_k}]$ of length $\Omega_k$, where $S_j$ refers to the number of correlation values that are greater than the pre-determined threshold at the time instant $T_p - \Omega_k T_c + (j - 1)T_c$.

Given the vector $S$, a backward search is performed. The farthest point from $T_p$ whose value is greater than or equal to a minimum hit-count (MHC), which is 2, i.e., $\min j | j S_j \geq 2$ is considered as the leading edge. This value is chosen in such a way that the probability of false alarm is reduced without increasing the probability of misdetection too much.

**D. Derivation of a Closed-Form Threshold Value**

In order to determine the threshold utilized in the search-back window, one can resort to time-consuming Monte Carlo simulations but it may only provide limited insight to the problem. Alternatively, one challenging but attractive approach is to derive a closed-form expression for the threshold, and investigate how it is affected by some important system parameters, such as the SNR. In this sub-section, we shall consider this alternative and derive a tractable approximate closed-form expression for $\xi$.

**D.1. Statistical Modeling**

As discussed in [13], probabilistic analysis of different kinds of range estimation error provides an efficient means of determining the thresholds used in TOA-based ranging algorithms. In general, there are two major sources of error, namely the estimation error of the algorithm and the unknown propagation delay due to any LOS blockage [13]. Since the latter is highly dependent on the channel environment, we follow [13] and consider only the first one throughout the analysis. In the following, we derive the probability density

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Note that this cross-correlation property has also been proved analytically and is presented in the extended journal version [14].
function (PDF) of the sliding correlator output, which serves as
a basis for evaluating the TOA estimation error and
determining the threshold value.

Denote \( r_c \) as the output of the sliding correlator. Due to
the good cross-correlation property of the ranging sequence,
the multipath components in (2) are completely eliminated.
Therefore, the output signal becomes
\[
\begin{align*}
r_c = & \beta e^{\theta} \cdot P_c + n(t) \otimes \Phi(t) \\
& \text{where } \beta \text{ is the channel gain of the first MPC and } n(t) \otimes \Phi(t) \text{ is a noise-like item whose variance is equal to}
\end{align*}
\]
\[
\sigma^2_r = E[(n(t) \otimes \Phi(t))^2] = \sigma^2 \cdot P_r .
\] (4)

Then, the conditional probability density function of \( r_c \) is
expressed as
\[
p(r_c | \beta, \theta) = \frac{1}{2 \sigma^2_r} \exp\left[-\frac{r_c^2}{2 \sigma^2_r} \right] (6)
\]

In order to obtain the unconditional probability density
function of \( r_c \), the PDFs of the phase \( \theta \) and the path gain \( \beta \) are needed. While the former is uniform distributed in the interval
\([0, 2\pi]\), the latter follows Nakagami distribution [10] for the
modified S-V model. However, its expression is very
cumbersome to provide any useful insight and therefore, we
approximate it as the Rayleigh distribution [15]. Thus, the
unconditional PDF of \( r_c \) becomes
\[
P(r_c) = \frac{1}{\pi (2\sigma^2_r + \beta^2 \cdot P_r)} \exp\left( -\frac{r_c^2}{2 \sigma^2_r + \beta^2 \cdot P_r} \right) .
\] (5)

where \( \beta^2 \) is the average power gain of the first path. In
Section IV, we will justify by simulations the accuracy and
tightness of such an approximation based on Rayleigh model.

D.2. Threshold Derivation

There are two types of TOA estimation error, namely the
probability of misdetection \( P_M \) and the probability of false
alarm \( P_{FA} \). While the former refers to the probability that
the actual leading edge is missed and a delayed multipath
component is mis-detected as the solution, the latter means that
the noise-only portion of the received signal is falsely
misdetected and the probability of false alarm could be
consequently derived and given as follows [14]:
\[
P_M = \int \frac{2}{M^2} \frac{1}{\pi (2\sigma^2_r + \beta^2 \cdot P_r)} \exp\left( -\frac{r_c^2}{2 \sigma^2_r + \beta^2 \cdot P_r} \right) d|r_c| = \frac{1 - \exp\left( -\frac{\beta^2}{M^2} \right) }{\frac{\beta^2}{M^2} } .
\] (7)
\[
P_{FA} = \int \frac{2}{\sigma^2_r} \frac{n'^2}{\sigma^2_r} \exp\left( -\frac{n'^2}{\sigma^2_r} \right) d|n'| = \exp\left( -\frac{\beta^2}{\sigma^2_r} \right) .
\] (8)

where \( M = \sqrt{2\sigma^2_r + \beta^2 \cdot P_r \cdot \frac{2}{\sigma^2_r}} \), and \( |n'| \) is the amplitude
of \( n(t) \otimes \Phi(t) \).

In order to derive the threshold, one may consider the
differentiation of the total probability of error \( P_M + P_{FA} \) with
respect to the threshold \( \xi \) [13]. However, due to the fact that
there are \( N \) ranging symbols in our proposed decision scheme
and the condition \( S_j \geq 2 \) for the leading-edge verification,
the threshold is derived by considering minimizing \( P_M + P_{FA} \), i.e.,
\[
d(P_M + P_{FA}) / d\xi = 0
\] (9)

\[
P_{FA} = N \cdot (1 - P_M)^N \cdot P_{FA} = N \cdot (1 - P_M)^{N-1} \cdot P_{FA} .
\] (10)

IV. NUMERICAL RESULTS AND DISCUSSION

A. Simulation Configurations

System simulations are provided to evaluate the
performance of the proposed algorithm, and assess the
accuracy and tightness of the approximate closed-form
expression on the threshold utilized in the search-back decision
unit. For the channel models, we consider CM1 and CM2 of
IEEE 802.15.4a [10], which refer to residential LOS and
NLOS scenarios, respectively. Unless otherwise stated, we
consider no sampling frequency offset and define SNR as the
averaged transmitted signal power of one sample of a single
symbol without the zero-suffix divided by the noise power.
Additionally, the length of the PN sequence and the zero-suffix
is 255 and 60, respectively. Without loss of generality, the
system bandwidth is set to 500MHz.

Figure 7. Comparison of the derived threshold (11) and the actual optimal
threshold in CM1 and CM2.

B. Results and Discussion

Recall in Section III.D that our analysis is based on
approximating the path gain as Rayleigh distributed. In order
to verify its accuracy, we consider the comparison between the
derived threshold (11) and the actual optimal threshold by
considering $N=5$ ranging symbols, the searching range as
$\Omega_a=10$ and assuming that the distance between the
located time instant detected by the preceding timing synchro-

ization block and the actual time instant of the first path is uni-

formly distributed within $[1,10]$. The methodology employed to
find the actual optimal threshold is to set the searching range
around the derived threshold at different SNRs, and regard the
value within this range that achieves the minimum MAE as the
optimal threshold. Referring to Figure 7, we see that this
derived threshold based on the Rayleigh statistical model is
close to the actual optimal threshold under different channel
models in which the path gain follows the Nakagami
distribution.

![Figure 8. Comparison of the MAE performance of the proposed algorithm
with Stein and Leeper’s algorithm [3] that employs matched filtering
based on the PFS sequence, and the optimal MF-based threshold
approach](image)

We then compare the performance of our proposed ranging
scheme with Stein and Leeper’s algorithm [3] which employs
matched filtering based on the PFS sequence, and the optimal
factor MF-based threshold approach in [9]. Figure 8 compares
the MAE of these three algorithms in CM1 and CM2, with the
simulation configurations of $N=10$, $P=128$, and $\Omega_a=15$.
Referring to the figure, we observe that the proposed algorithm
outperforms Stein and Leeper’s algorithm for both channel
models, in which they do not utilize the fact that the strongest
multipath component may not be the first path. Additionally,
the performance of our proposed algorithm is significantly
better than the optimal MF-based threshold approach for the
NLOS scenario in CM2, because the optimal MF-based
threshold approach does not take into account any knowledge
of the channel.

V. CONCLUSIONS

In this paper, a novel threshold-based ranging scheme for
impulse-based UWB systems in dense multipath environment
is proposed. In particular, a dedicated ranging preamble is
proposed, which consists of a MB-OFDM Packet/Frame
synchronization sequence and a PN sequence. While the
former is used for timing synchronization that provides a
coarse timing estimate for the subsequent ranging process
estimation, the latter is designed with good cross-correlation
property such that the multipath components can be
completely eliminated at the receiver. Additionally, a
threshold-based search-back unit is also introduced, with the
aim at estimating the leading edge of the received signal with
high precision. An approximate closed-form expression on the
optimal threshold is also derived and its accuracy and tightness
are demonstrated by simulations. Numerical results reveal the
effectiveness of the proposed ranging algorithm by comparing
its performance with two conventional ranging schemes. As a
future work, the SOP interference will be analyzed, and the
robustness of our proposed scheme against the SOP
interference will be investigated.

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