Low-energy phenomenology and neutral-weak-boson masses in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak model

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(Received 10 August 1981)

Within the framework of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak group, a model-independent analysis of the neutral-current phenomenology at low energy is given both for Dirac and Majorana neutrinos. It is found that if the Higgs structure is not fixed, solutions substantially different from the standard model are still allowed by the present neutral-current data. It is shown how the upper limit on the backward-forward asymmetry in $e\bar{e} \rightarrow \mu\bar{\mu}$ constrains not only the neutral-weak-boson masses but also the mass ratio of the two Majorana neutrinos predicted by the "minimal" left-right-symmetric model.

I. INTRODUCTION

It is now widely accepted that low-energy electroweak dynamics is generated by the effective local $SU(2)_L \times U(1)$ symmetry.¹ This expectation is primarily based on the fact that the parameter $\sin^2\theta_W = 0.233 \pm 0.009$ fits all low-energy neutral- and charged-weak-current data extremely well, and that the predicted value of the second parameter $\rho = 1$ is also in accord with the data.² In spite of this, the evidence for the standard model will not be decisive until one observes the (left-handed) Z^0 and $W^\pm$ weak bosons precisely at the predicted mass values. Indeed, the chief purpose of several large machines under construction is to find the (left-handed) weak bosons and study their properties. Because of the small cross section for single- W production below W^+W^- threshold, it is expected that the neutral weak boson will be the first one detected.

Under these circumstances, we propose to reexamine the existing evidence for the $SU(2)_L \times U(1)$ model, especially with regard to its uniqueness. Admittedly, the standard-gauge-model fit to all the neutral- and charged-weak-current data is very economical but the fact remains that we do not have any experimental information concerning the high-energy behavior of the electroweak interaction. Furthermore, it is the implicit assumption of the standard model that the neutrinos are massless. If the neutrinos possess finite masses, they could be Dirac or Majorana particles. The standard electroweak model would have to be modified³ and the one-parameter fit of the low-energy data would

have to be regarded as somewhat accidental. *A fortiori*, finite-mass neutrinos would require changes in the "minimal" $SU(5)$ grand unified theory with far-reaching consequences.⁴

The possible nonuniqueness of the standard electroweak model must therefore be examined very carefully. Since there seems to be a deep connection between the small neutrino mass and the preponderant left-handed (charged) weak current, it seems natural to consider the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak group, with the right-handed weak bosons heavier than the left-handed weak bosons, as the leading contender for an alternative option to the standard $SU(2)_L \times U(1)$ electroweak group. We shall therefore focus on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model which predicts a different high-energy behavior of the electroweak interaction from the standard model and can accommodate Dirac as well as Majorana neutrinos.⁵ We shall demonstrate that some versions of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model give good fits to all low-energy neutral- and charged-weak-current data which differ substantially in their high-energy predictions from the standard model.

The $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak group had its origin in 1974 and 1975 when Mohapatra, Pati, Salam, and Senjanović⁶ developed an alternative model of the electroweak interaction which restored parity to the status of a high-energy symmetry of weak interactions. It was later shown by Marshak and Mohapatra⁷ that, unlike the case of the $SU(2)_L \times U(1)$ group, the vector $U(1)$ generator in the left-right-symmetric group can be identified with local $(B-L)$ symmetry; this is

called the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak model, thereby relating parity breakdown to the breakdown of a local (internal) $B-L$ symmetry. Whether or not this approach can generate a correct low-energy phenomenology and be experimentally distinguished from the standard model is the subject of this paper.

II. LOW-ENERGY FIT OF LEFT-RIGHT-SYMMETRIC ELECTROWEAK MODEL

Several analyses based on the charged- and neutral-weak-current data have been carried out.⁸ But most of the previous analyses are not model-independent; that is, they usually assume the specific Higgs structure from the beginning. Of course, the Higgs structure is important for a spontaneously broken gauge theory. But the gauge group, the fermion assignments, and the Higgs structure are really three different ingredients of a gauge model; it is not necessary to tie them together at all stages of the analysis. In fact, the neutral-current structure is only determined by the gauge group, fermion assignments, and electric charge identification. The use of a specific Higgs structure only leads to well-defined mass relations between the charged- and neutral-weak-boson sectors, which can be tested when we join together the study of the neutral- and charged-weak-current data. Therefore, a more general analysis is called for. In this section, a model-independent analysis of the neutral-current phenomenology at low energy is carried out within the context of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model.

It is found that because only neutral-left-handed-current couplings of the neutrino are measured in the present neutrino reactions,⁹ solutions different from the standard-model limit are still allowed by the present neutral-current data. In par-

ticular, the light (left-handed) neutral-weak-boson mass can be substantially lower than the standard value of 93 GeV (Ref. 10) and the second (right-handed) neutral-gauge-boson mass can be as low as 130 GeV if the neutrinos are Dirac or pure Majorana particles (i.e., neutrinos only with Majorana masses). In general, because of the additional mixing angle between the two Majorana neutrinos, the present neutrino neutral-current and polarized-electron-hadron deep-inelastic scattering experiments are still not sufficient to determine all parameters in the neutral-current sector. The measurements of the backward-forward asymmetry in the reaction $e^+e^- \rightarrow \mu^+\mu^-$ (Ref. 11) will be important for a complete determination of the neutral-current structure of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model. And only the choice of Higgs structure will fix the mass ratio of Majorana neutrinos.

We now summarize the key results of our calculations. We recall that in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the fermions are assigned to left-right-symmetric representations of the group as follows:

$$\psi_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \left(\frac{1}{2}, 0, -1 \right), \quad \psi_R = \begin{bmatrix} \nu_R \\ e_R \end{bmatrix} \left(0, \frac{1}{2}, -1 \right), \quad (1)$$

$$Q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix} \left(\frac{1}{2}, 0, \frac{1}{3} \right), \quad Q_R = \begin{bmatrix} u_R \\ d_R \end{bmatrix} \left(0, \frac{1}{2}, \frac{1}{3} \right),$$

and the electric charge is given by

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}. \quad (2)$$

Then, when the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group spontaneously breaks down to $U(1)^{EM}$, independent of any specific Higgs structure, the effective neutral-current interaction at low energy is given by

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = (J_V(i), j_A^3(i)) \frac{e^2}{2 \sin^2 \omega} \begin{bmatrix} \frac{A}{\cos 2\omega} & -\frac{D}{(\cos 2\omega)^{1/2}} \\ -\frac{D}{(\cos 2\omega)^{1/2}} & B \end{bmatrix} \begin{bmatrix} J_V(j) \\ j_A^3(j) \end{bmatrix}, \quad (3)$$

where

$$J_V(i) = j_V^3(i) - \sin^2 \omega j^{\text{EM}}(i), \quad (4a)$$

with

$$j_V^3 = \frac{1}{2} \bar{\nu} \gamma_\mu \nu - \frac{1}{2} \bar{e} \gamma_\mu e + \frac{1}{2} \bar{u} \gamma_\mu u - \frac{1}{2} \bar{d} \gamma_\mu d + \dots$$

and

$$j_A^3 = \frac{1}{2} \bar{\nu} \gamma_\mu \gamma_5 \nu - \frac{1}{2} \bar{e} \gamma_\mu \gamma_5 e + \frac{1}{2} \bar{u} \gamma_\mu \gamma_5 u - \frac{1}{2} \bar{d} \gamma_\mu \gamma_5 d + \dots, \quad (4b)$$

$$A = \frac{\cos^2 \theta_2}{m_{Z_1}^2} + \frac{\sin^2 \theta_2}{m_{Z_2}^2}, \quad (5a)$$

$$B = \frac{\sin^2 \theta_2}{m_{Z_1}^2} + \frac{\cos^2 \theta_2}{m_{Z_2}^2}, \quad (5b)$$

$$D = \sin \theta_2 \cos \theta_2 \left[\frac{1}{m_{Z_1}^2} - \frac{1}{m_{Z_2}^2} \right], \quad (5c)$$

where m_{Z_1}, m_{Z_2} are the masses of the neutral weak bosons (Z_2 is lighter than Z_1) and ω, θ_2 are mixing angles with

$$\sin \omega = \frac{g'}{(g^2 + 2g'^2)^{1/2}} = \frac{e}{g}.$$

The angle ω is the Weinberg angle when $SU_L(2) \times U(1)$ is the limiting subgroup of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, otherwise not (see below). Introducing the mixing angle ϕ for the Majorana neutrinos η_1, η_2 , we have

$$\nu_L = \eta_{1L} \cos \phi + \eta_{2L} \sin \phi, \quad (6a)$$

$$\nu_R = -\eta_{1R} \sin \phi + \eta_{2R} \cos \phi \quad (6b)$$

and

$$J_V(\eta_1) = \frac{1}{4}(\bar{\eta}_1 \gamma_\mu \eta_1 - \cos 2\phi \bar{\eta}_1 \gamma_\mu \gamma_5 \eta_1), \quad (7a)$$

$$J_A(\eta_1) = \frac{1}{4}(-\cos 2\phi \bar{\eta}_1 \gamma_\mu \eta_1 + \bar{\eta}_1 \gamma_\mu \gamma_5 \eta_1). \quad (7b)$$

Therefore, within the context of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the neutral weak currents, leptonic and hadronic, can be described by five parameters: $m_{Z_1}, m_{Z_2}, \theta_2, \omega$, and ϕ .

On the other hand, the model-independent and detailed phenomenological analyses of the neutral-weak-current data have been done since 1973, and the neutrino-hadron, neutrino-electron, and electron-hadron neutral-weak-current couplings are now almost uniquely determined.² In this model-independent approach, the interaction of neutral weak currents is

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\nu H} + \mathcal{L}^{\nu e} + \mathcal{L}^{eH} + \dots, \quad (8a)$$

where

$$\mathcal{L}^{\nu H} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 - \gamma_5) \nu J_\lambda^H, \quad J_\lambda^H = \sum_i \bar{q}_i \gamma_\lambda (g_V^i - g_A^i \gamma_5) q_i, \quad (8b)$$

$$\mathcal{L}^{\nu e} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\lambda (1 - \gamma_5) \nu J_\lambda^e, \quad J_\lambda^e = \bar{e} \gamma_\lambda (g_V^e - g_A^e \gamma_5) e, \quad (8c)$$

$$\mathcal{L}^{eH} = -\frac{G_F}{\sqrt{2}} \sum_i [C_{1i} \bar{e} \gamma^\lambda \gamma_5 e \bar{q}_i \gamma_\lambda q_i + C_{2i} \bar{e} \gamma^\lambda e \bar{q}_i \gamma_\lambda \gamma_5 q_i], \quad i = u, d. \quad (8d)$$

In comparing the neutral-weak-current interaction in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with this approach, we obtain a set of equations which relate $m_{Z_1}, m_{Z_2}, \theta_2, \omega$, and ϕ to the phenomenological parameters $g_V^i, g_A^i, g_V^e, g_A^e, C_{1i}$, and C_{2i} .

In making the comparison, the neutral-current couplings of the neutrino should be carefully considered for the following reasons. (a) The neutrino experiments are performed using ν 's from π and K decays so that in fact, only the neutral-left-handed-current couplings of the neutrino are measured.⁹ Thus, we have no knowledge of the neutral right-handed current couplings of the neutrino. (b) The neutrino may be a Dirac or Majorana particle. By considering these two points, we find that the present neutral-weak-current data allow different solutions depending on the character of the neutrino as follows.

Case (1)

First, we consider the case in which the neutrino is a Dirac particle, and there are no constraints on the neutral-right-handed-current couplings of the neutrino. In this case, we obtain

$$g_A^u = -g_A^d = -g_A^e, \quad (9a)$$

$$g_V^u : g_V^d : g_V^e = \left(\frac{1}{2} - \frac{4}{3} \sin^2 \omega\right) : \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \omega\right) : \left(-\frac{1}{2} + 2 \sin^2 \omega\right), \quad (9b)$$

and

$$C_{1u}:C_{2u}:C_{1d}:C_{2d}=(-\frac{1}{2}+\frac{4}{3}\sin^2\omega):(-\frac{1}{2}+2\sin^2\omega):(\frac{1}{2}-\frac{1}{3}\sin^2\omega):(\frac{1}{2}-2\sin^2\omega). \quad (9c)$$

These equations are the same as those in the standard model. But the relations between g_V^u , g_A^u , and C_{1u} are replaced by the following equations:

$$\frac{A}{\cos 2\omega} + \frac{D}{(\cos 2\omega)^{1/2}} = \left[\frac{\sqrt{2}G_F}{\pi\alpha} \right] \sin^2\omega \frac{6g_V^u}{3-8\sin^2\omega}, \quad (10a)$$

$$\frac{D}{(\cos 2\omega)^{1/2}} + B = \frac{\sqrt{2}G_F}{\pi\alpha} \sin^2\omega 2g_A^u, \quad (10b)$$

$$\frac{D}{(\cos 2\omega)^{1/2}} = \frac{\sqrt{2}G_F}{\pi\alpha} \sin^2\omega \frac{(-3C_{1u})}{3-8\sin^2\omega}. \quad (10c)$$

Solving these equations, we get

$$0.21 \leq \sin^2\omega \leq 0.29, \quad (11a)$$

$$\tan 2\theta_2 = \frac{-C_{1u}(\cos 2\omega)^{1/2}}{g_V^u \cos 2\omega - g_A^u(1 - \frac{8}{3}\sin^2\omega) - C_{1u}\sin^2\omega}, \quad (11b)$$

$$\frac{1}{m_{Z_1}^2} = \frac{\sqrt{2}G_F}{\pi\alpha} \frac{\sin^2\omega}{1 - \frac{8}{3}\sin^2\omega} (K - \sqrt{L}), \quad (11c)$$

$$\frac{1}{m_{Z_2}^2} = \frac{\sqrt{2}G_F}{\pi\alpha} \frac{\sin^2\omega}{1 - \frac{8}{3}\sin^2\omega} (K + \sqrt{L}), \quad (11d)$$

where

$$K = g_V^u \cos 2\omega + g_A^u(1 - \frac{8}{3}\sin^2\omega) + C_{1u} \cos^2\omega, \quad (12a)$$

$$L = [g_V^u \cos 2\omega - g_A^u(1 - \frac{8}{3}\sin^2\omega) - C_{1u}\sin^2\omega]^2 + C_{1u}^2 \cos 2\omega. \quad (12b)$$

Some typical results are shown in Table I. It is found that the neutrino neutral-current and the polarized-electron-nucleon deep-inelastic scattering experiments are not too restrictive on the masses of the neutral weak bosons. They allow the masses of the neutral weak bosons to vary over a very wide range. However, more stringent limits are obtained when the backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ is considered. The predicted backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ is also shown in Table I. Phenomenologically, the effective Lagrangian for the processes $e^+ + e^- \rightarrow \mu^+\mu^-$ has been defined as (see last work in Ref. 8)

$$\begin{aligned} \mathcal{L}^{e\mu} = \frac{G_F}{\sqrt{2}} [& h_{VV}(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu)(\bar{e}\gamma^\lambda e + \bar{\mu}\gamma^\lambda \mu) - 2h_{VA}(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu)(\bar{e}\gamma^\lambda \gamma_5 e + \bar{\mu}\gamma^\lambda \gamma_5 \mu) \\ & + h_{AA}(\bar{e}\gamma_\lambda \gamma_5 e + \bar{\mu}\gamma_\lambda \gamma_5 \mu)(\bar{e}\gamma^\lambda \gamma_5 e + \bar{\mu}\gamma^\lambda \gamma_5 \mu)] \end{aligned} \quad (13)$$

and the backward-forward asymmetry as

$$A_{\mu\mu} = \frac{\int d\Omega[(d\sigma/d\Omega)(\theta) - (d\sigma/d\Omega)(\pi - \theta)]}{\int d\Omega[(d\sigma/d\Omega)(\theta) + (d\sigma/d\Omega)(\pi - \theta)]} \sim \frac{3}{4} \left[\frac{G_F}{\sqrt{2}\pi\alpha} \right] h_{AA}^S, \quad (14)$$

where $\sqrt{s}$ is the c.m. energy and in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model

$$\frac{G_F}{\sqrt{2}} 2h_{AA} = \frac{e^2}{8\sin^2\omega} \left[\frac{\sin^2\theta_2}{s - m_{Z_1}^2} + \frac{\cos^2\theta_2}{s - m_{Z_2}^2} \right]. \quad (15)$$

TABLE I. Neutral-weak-boson masses in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model where neutrino is Dirac particle or “unmixed” Majorana particle).

$\sin^2\omega$	Input		C_{1u}	Neutral-weak-boson masses		Backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$
	$g_V(u)$	$g_A(u)$		m_{Z_1} (GeV)	m_{Z_2} (GeV)	
0.21	0.157	0.477	-0.17	477.8	95.4	-0.076
0.21	0.157	0.477	-0.18	1013.1	95.7	-0.074
0.23	0.171	0.531	-0.17	329.1	86.7	-0.085
0.23	0.134	0.534	-0.16	8783.1	87.3	-0.089
0.25	0.161	0.541	-0.17	3489.1	83.5	-0.080
0.29	0.225	0.545	0.095	73.5	54.3	-0.26
0.29	0.185	0.585	0.095	78.9	53.5	-0.28
0.29	0.280	0.528	-0.1	123.7	73.6	-0.087
0.29	0.198	0.538	-0.1	126.0	73.9	-0.090
0.29	0.225	0.545	-0.12	132.8	71.8	-0.089
0.29	0.215	0.555	-0.12	134.8	72.2	-0.086

It is clear from Eq. (15) that the closeness of m_{Z_1}, m_{Z_3} to the c.m. energy $\sqrt{s}$ predicts a large backward-forward asymmetry, so the measurement of the backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ places very stringent limits on the masses of the neutral weak bosons. Recently, the various PETRA groups have obtained the asymmetry data.¹¹ The data from the various groups can be combined and one gets at the 95% confidence level

$$|A_{\mu\mu}| < 9\% \text{ for } 27 < \sqrt{s} < 35 \text{ GeV}. \quad (16)$$

The interesting point is that even within this limit the (left-handed) neutral-weak-boson mass could still be substantially lower than the standard value of 93 GeV and the heavier (right-handed) neutral weak boson could be as light as 130 GeV (see Table I).

Case (2)

Next, we consider the case in which the neutrino is a Dirac particle but the vanishing of the neutral-right-handed-current couplings of the neutrino is required. In this case, the additional constraints are

$$\frac{A}{\cos 2\omega} = \frac{D}{(\cos 2\omega)^{1/2}} = B. \quad (17a)$$

The solution is

$$\sin^2\omega = \frac{3}{8} \left[1 - \frac{g_V^u}{g_A^u} \right], \quad (17b)$$

$$\tan 2\theta_2 = \frac{(\cos 2\omega)^{1/2}}{\sin^2\omega}, \quad (17c)$$

$$\frac{1}{m_{Z_1}^2} = 0, \quad (17d)$$

$$\frac{1}{m_{Z_2}^2} = \frac{\sqrt{2}G_F}{\pi\alpha} \sin^2\omega \cos^2\omega 2g_A^u. \quad (17e)$$

The above exactly correspond to the standard-model limit, because the neutral-current couplings are the same as those in the standard model, namely,

$$\frac{e}{\sin\omega \cos\omega} (j_L^3 - \sin^2\omega j^{EM})_{Z_2}. \quad (18)$$

Case (3)

Now we consider the case in which there are two Majorana neutrinos, and there are no constraints on the neutral-right-handed-current couplings of these neutrinos. Clearly, this is a more realistic case for the left-right-symmetric model. In this model, it is expected that the Dirac and Majorana mass terms both will be included in the Lagrangian of the electroweak interaction and, in general, lead to two Majorana neutrinos with a nonvanishing mixing angle. The result is that one gets the same equations as Eqs. (9a)–(10c), with g_V^u and g_A^u replaced by $g_V^u/\cos^2\phi$ and $g_A^u/\cos^2\phi$. Because of the additional mixing angle ϕ between the two Majorana neutrinos, the present neutrino neutral-current and polarized-electron–hadron deep-inelastic

scattering data do not suffice to determine the neutral-current structure of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model. Nothing can be said about the neutral-weak-boson masses in this case. In order to complete the analysis of the neutral-weak-current data, additional independent experiments are needed. It is easy to show that the measurements of the backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ (Ref. 11) is one such independent experiment. With a nonvanishing mixing angle between the two Majorana neutrinos, case (3) would generally yield lower neutral-weak-boson masses and therefore a larger backward-forward asymmetry than the Dirac-neutrino case [case (1)] for the same neutrino neutral-current and polarized-electron-hadron scattering data. Results are shown in Figs. 1 and 2 for a choice of parameters that meet the requirements of the minimal Higgs structure, namely, $g_V^u = 0.177$, $g_A^u = 0.500$, $C_{1u} = -0.127$, and $\sin^2 \omega = 0.28$; sets of parameters consistent with the "nonminimal" Higgs structure yield values of the weak-boson masses that are represented in Table I. With the minimal set of parameters and using the 95% upper confidence limit for the backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ ($|A_{\mu\mu}| \leq 9\%$), we

FIG. 1. Light-neutral-weak-boson mass as function of mixing angle between two Majorana neutrinos for a choice of phenomenological parameters (see text) consistent with "minimal" $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

FIG. 2. Heavy-neutral-boson mass as function of mixing angle between two Majorana neutrinos for same choice of parameters as in Fig. 1.

can set an upper bound to the mixing angle between the two Majorana neutrinos of approximately 20° (see Fig. 3) and a lower bound on the mass of the neutral weak boson of ~ 74 GeV (see Fig. 1).

What can we say about the masses of the two Majorana neutrinos? As is well known, the neutrino-mass effects can be neglected in the neutrino reactions studied so far. Thus, no direct experimental information about the masses of the two Majorana neutrinos can be obtained in the above analysis. However, in the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak model, in which the Higgs scalars only consist of Dirac and Majorana condensates,⁵ the mass ratio of the two Majorana neutrinos is related to their mixing angle (ϕ), the neutral-weak-boson masses, and the mixing angle (θ_2) in the neutral-weak-current sector. Therefore, it is possible to say something about the mass ratio of the two Majorana neutrinos within the framework of the minimal left-right-symmetric model. In fact, the mass ratio as a function of mixing angle in this model can be written as

$$\frac{m_1}{m_2}(\phi) = \frac{\chi(\phi)\cos^2\phi - \sin^2\phi}{\cos^2\phi - \chi(\phi)\sin^2\phi}, \quad (19)$$

FIG. 3. Backward-forward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ as function of mixing angle between two Majorana neutrinos for same choice of parameters as in Fig. 1.

where m_1, m_2 are, respectively, the masses of the light and heavy Majorana neutrinos, and

$$\chi(\phi) \equiv \frac{V_L}{V_R}(\phi) \equiv \left[1 + \frac{C_{1u}}{g_A^u(1 - \frac{8}{3}\sin^2\omega)} \cos^2\phi \right]^{1/2}, \quad (19a)$$

with V_L, V_R the vacuum expectation values of the two Majorana condensates Δ_L, Δ_R (8, last reference). Choosing the same set of values of $\sin^2\omega$, g_A^u , and C_{1u} as were used to obtain Fig. 3, we plotted $m_1/m_2(\phi)$ as a function of ϕ in Fig. 4. It is seen that the minimum value of m_1/m_2 corresponds to $\phi=0$ (no mixing) as long as we accept the upper limit of 9% on the backward-forward asymmetry in the process $e^+e^- \rightarrow \mu^+\mu^-$. Without this condition, a large mixing angle (of the order of 40°) would be compatible with a very small value of m_1/m_2 , as required by the experimental limits of neutrinoless double- β decay, etc.¹²

Another way to make the same point is to note that the mass ratio is very small when the mixing angle satisfies the following equation:

FIG. 4. Mass ratio of light to heavy Majorana neutrinos in "minimal" $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as function of mixing angle for same choice of parameters as in Fig. 1.

$$\frac{\tan^4\phi - 1}{\cos^2\phi} = \frac{C_{1u}}{g_A^u(1 - \frac{8}{3}\sin^2\omega)} \equiv \xi. \quad (20)$$

Using the same value of ξ as before, namely, $\xi \simeq -1$, we see from Fig. 5 that the two-solutions are $\phi \sim 0^\circ$ and $\phi \sim 38^\circ$. But the second solution is ruled out by the condition $|A_{\mu\mu}| \leq 9\%$ (Fig. 3) and we are left with the "zero" mixing solution in the minimal left-right-symmetric model. The mass ratio then takes on a simple form in terms of V_L/V_R , to wit:

$$\frac{m_1}{m_2}(\phi=0) = \frac{V_L}{V_R} + \left[1 - \frac{V_L^2}{V_R^2} \right] \left[\frac{V_R}{2V_L} - 1 \right] \phi^2. \quad (21)$$

Equation (21) has a minimum for $\phi=0$ and consequently $V_L/V_R \rightarrow 0$ is favored to secure compatibility with a reasonable mass ratio of the two Majorana neutrinos.

Thus, in order to explain a very small mass ratio of the two Majorana neutrinos and the observed limit for the backward-forward asymmetry $|A_{\mu\mu}| \leq 9\%$, not only must the mixing angle be very small but also V_L/V_R must be very close to

FIG. 5. Behavior of ξ [defined by Eq. (20)] as function of mixing angle between two Majorana neutrinos for very small neutrino mass ratio.

zero within the framework of the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model. These conclusions are in essential agreement with those of Rizzo and Senjanović⁸ who studied the neutral-current phenomenology of the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model although they did not explicitly consider the constraints on the parameters of the model imposed by the mass ratio of the two Majorana neutrinos. As has been shown above, some regions of the parameters allowed by Rizzo and Senjanović [e.g., all those with $\eta_L \neq 0$ (Ref. 8)] should, in fact, be ruled out. The situation would be quite different if $|A_{\mu\mu}|$ turned out to be 15–18 % (departing from the presently accepted limit by 2 standard deviations) and the large-mixing-angle solution was valid.^{12a}

If a nonminimal Higgs structure is fixed in the left-right-symmetric electroweak model, the above analysis relating the mixing angle, the mass ratio of the two Majorana neutrinos, and the masses of the neutral weak bosons would then lead to different results.

Case (4)

We finally consider the case in which there are two Majorana neutrinos and the vanishing of the

neutral-right-handed-current coupling of these neutrinos is also required. The solutions are as follows.

Case (4a) $\phi = 0$; that is, while the neutrinos have nonvanishing Majorana masses, there is no mixing between the light and heavy Majorana neutrinos. In this case, we recapture case (1) and the mass of the extra neutral gauge boson Z_1 could again be as low as 130 GeV.

Case (4b) $\phi = 0$ but

$$A / \cos 2\omega = D / (\cos 2\omega)^{1/2} = B. \quad (22)$$

We then get

$$\sin^2 \omega = \frac{3}{8} \left[1 - \frac{g_V^u}{g_A^u} \right], \quad (23a)$$

$$\cos^2 \phi = -\frac{g_V^u}{C_{1u}}, \quad (23b)$$

$$\tan 2\theta_2 = -\frac{(\cos 2\omega)^{1/2}}{\cos^2 \omega}, \quad (23c)$$

$$\frac{1}{m_{Z_1}^2} = 0, \quad (24a)$$

$$\frac{1}{m_{Z_2}^2} = \frac{\sqrt{2}G_F}{\pi\alpha} \sin^2 \omega \cos^2 \omega 2g_A^u. \quad (24b)$$

This case corresponds to an extended version of the standard model with two Majorana neutrinos.¹³

Summary

The present neutral-weak-current data allow solutions different from the standard-model limit. The observation of the light-neutral boson at a mass level substantially lower than the standard value of 93 GeV would lend support to the left-right-symmetric electroweak model.

ACKNOWLEDGMENTS

We thank Professor L. N. Chang, Professor K. C. Chou, and Professor Riazuddin for useful discussions. We also thank Mr. Ray Ng for computer calculations. This work was supported in part by the Department of Energy.

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