Chaos: Implications for New Product Forecasting
and the Research-Practice Interface

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ABSTRACT

The mathematics of chaotic dynamics are now familiar to marketing researchers. Do possible sightings of chaos in marketing data sets have implications for the way new product studies and launches should be performed? Should these practices be affected by the knowledge that chaos is possible in principle?

Although the mathematics of new product diffusion models clearly allow for chaotic bifurcations and fluctuations, these phenomena have not been reliably observed for actual products. In this paper we offer reasons why this has been so. The reasons include measurement and specification error, and aggregation and data collection interval effects. We conclude that marketers have not been looking in the right places to find chaos (or at least, traditional market research reports do not lend themselves to an effective search for chaos), and that brand managers behave in a way that minimizes chances of observing chaos. The exploration of chaos in the context of new product forecasting leads to an analysis of the implications of chaos for the practice of marketing.
The conception of chance enters into the very first steps of scientific activity, in virtue of the fact that no observation is absolutely correct. I think chance is a more fundamental conception than causality; for whether in a concrete case a cause-effect relation holds or not can only be judged by applying the laws of chance to the observations.

Max Born
Natural Philosophy of Cause and Chance

1. Introduction

To date, publications addressing chaos and related phenomena in marketing have been didactic (Hibbert and Wilkinson 1994), or oriented to detecting chaos in a particular marketing data set (Wen 1994), or have offered novel models and data estimation methods for dealing with specific nonlinear phenomena in marketing (Mulhern and Caprara 1994; Krider and Weinberg 1994). Mulhern and Caprara (1994) aptly summarize the thrust of these publications: "Chaos may or may not exist in the behavior of marketing time series."

We agree that chaos has not been reliably identified in marketing time series. In this paper, we use new product diffusion models as a lens to examine why this is so. Discussion of this question leads to insights into the meaning of chaos for marketing research. Our purpose differs from prior papers in that we focus on the actual practices of marketing research and management in order to offer constructive analysis of the meaning and implications of chaos for best practice in marketing science and new product management.

The issue is of importance because of the question of control of marketing programs and processes. Although current research indicates that controlling a very simple (e.g., univariate) deterministic system in chaos may be possible in some cases, there is no indication that such control is practicable in any way for complex, real-world systems once these have entered a chaotic regime. Thus, for marketers accustomed to believing that markets respond
(more or less) to deliberate changes in the marketing mix, the possibility of deterministic chaos in marketing arenas becomes fundamental.

Although Isaac Newton was apparently aware of the possibility of unstable solutions to systems of nonlinear equations, he did not have the computational facilities to explore this unstable behavior. At the turn of this century, Poincaré began to explore these solutions theoretically, and Volterra (1936) presented data and models of the potentially highly irregular patterns of population growth, especially in systems of predator-prey populations. It was only when electronic computing capacity became cheap and plentiful that computational exploration of nonlinear systems became possible and popular. These explorations first led to a better understanding of certain physical phenomena, including planetary orbits and dripping faucets. Later, for example in the work of Prigogine (e.g., Nicolis and Prigogine 1977; Prigogine, Chen and Wen, forthcoming), instabilities were related to disequilibrium and to the emergence of complexity from simpler systems, including chemical and biological systems.

One class of behavior of certain nonlinear systems has been called "chaotic." Chaos results in successive values of a function or system of functions that appear to be random, but are actually deterministic (though irregular) phenomena. According to Thore (1994), the word "chaos" was first used by Yorke and Li (1975). May's (1976) example of the differing behavior under different parameter values of the iteration of a simple quadratic function has been widely repeated in the literature of various disciplines. Indeed, many scientific disciplines have been transformed by the newly widened awareness of the potentially chaotic behavior of the formulas commonly used in these disciplines.

In marketing, Fortt and Woodlock (1960) presented a model of new product penetration growth that used a similar functional iteration. Their paper gave no hint that a cumulative penetration function would or could show anything but a smooth or regular shape. Narasimhan and Sen (1983) reviewed many later but similar nonlinear models of new product growth, none of which suggested the possibility of chaotic behavior. But subsequent to the work of Yorke and Li (1975) and May (1976) and others, and to widespread attention to the new results in chaos in popular scientific magazines and books (e.g., Gleick 1987; Priemeyer 1994), marketers began to reexamine their data. A paper published in *Journal of the Academy of Marketing Science* (Hibbert and Wilkinson 1994), plus several earlier, privately circulated
working papers by various authors argued the following: If the Fourt-Woodlock and similar models used the same families of mathematical functions that described chaotic behavior of physical systems, then chaos might well be seen in marketing phenomena as well. The reality is that it has not been seen.

As demonstrated in the literature and in the next section of this paper, it is easy to generate chaotic behavior deductively, i.e., from the iteration of a function and specific parameter values. It is much more difficult - and not even certainly possible (Barnett, et al. 1994) - to detect chaos inductively, from an empirical data set generated by an unknown process. The mathematical tests for empirical chaos are sophisticated, and require hundreds of observations as input. Few marketing data sets of interest contain such a volume of observations.

This paper offers possible reasons why chaos has not been spotted in marketing, and particularly in patterns of new product growth. Some of the reasons are supported by numerical experiments. The remainder involve reasoning by analogy and are offered as conjectures. In general, two questions must be asked: Why is chaos not anticipated in the models commonly used for product growth forecasting? And why is chaos not detected when actual new products are monitored? One possibility, that chaos is mistaken for statistical variation when modelers desire to fit smooth curves, would answer both questions. We explore this and other possibilities below.

The exploration leads to some extended comments on the relevance of chaos and nonlinear complexity to new product marketing and to marketing science, and how awareness of the possibility of chaos may change the practice of both. We find that the nature of marketing data sets and their typical uses in research and practice - and, significantly, the equivocal results of a recent large-scale, single-blind experiment in detecting chaos in simulated data sets (Barnett, et al. 1994) - make the chances of firm findings of chaos in marketing small. However, knowledge that a marketing situation may be driven by a complex, nonlinear deterministic process may change marketing research in several ways. These are detailed in a "summary and implications" section at the close of this paper.
2. Conceptual Basis

Chaos theory has made it clear that equations with very few parameters, or even only one parameter, can generate highly irregular curves. Yorke and Li's simplified example is

\[ X_{t+1} = bX_t(1-X_t). \]  

(1)

Hibbert and Wilkinson (1994) state this equation is equivalent to a logistic function.

Figure 1 shows this series for a starting value of \( X_1 = 0.001 \) and \( b = 1.5 \). The result, \( X_t \), interpreted as a market penetration rate for a new product, shows a pre-takeoff introduction period of about ten iterations \( t \), a rapid growth phase occupying about another eight iterations, followed by a slow asymptotic approach to the ceiling penetration, \((b-1)/b\). As the parameter \( b \) is increased to 2.5, a faster takeoff is observed (Figure 2A), and a more abrupt transition to the terminal level of penetration.

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Figures 1 and 2A about here

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Increasing \( b \) to 3, we see in Figure 2B what appears to be a different kind of convergence to the equilibrium value (0.666667). Here a conceptual problem arises when, in the ninth iteration, the curve overshoots the equilibrium level; a market penetration curve must be monotonic. We can sidestep the issue momentarily by reinterpreting the curve as a sales per unit time curve. When \( b \) is increased to 3.95 (Figure 2C), the smooth growth of sales breaks down in period 8, and no further sensible pattern is evident.

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Figures 2B and 2C about here

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The transition of a smooth time series into an irregular regime, and the subsequent fluctuations within the irregular regime, are typical not only of the Yorke-Li equation. They
are found also in more complex differential or finite difference equations, and in systems of such equations. In the remainder of this paper, we will consider the marketing implications of these graphs when they are (a) empirical, i.e., when all points shown are actual observations, and (b) fitted, i.e., when a forecast is made on the basis of a few or many observed points.

3. Top Eleven Reasons Why New Product Diffusion Does Not Appear To Be Chaotic

The context for the following discussion includes the usual practice of forecasting a new product's performance based on sales or penetration figures observed during the first few months following its introduction to the market. Figures 2A-C show rapid sales growth reaching 80% of maximum in eight time periods according to a (until then) smooth curve. Because most market research data for durable products are collected on a monthly or annual basis, and the noted rate of growth is reasonable in eight months for some products and in eight years for others, we unfortunately cannot dismiss the chaotic regime of the new product diffusion curve as representing unrealistic growth rates.

We must entertain the possibility that a curve fit to the first three or four months of product data may exhibit chaos when it is used to produce forecasts for six or seven months in the future. That this has not been reported in literature or practice means that (i) it has not happened, or (ii) it has been observed and dismissed as "obviously" erroneous. Further specific reasons for not seeing chaos in new product diffusion are the following. Our discussion is based on several examples using Bass-type diffusion models which are widely accepted and used for new product forecasting in marketing.

1. Measurement error. There are many reasons why the first few observations of a new product's performance are error-prone (see Phillips 1990). Hibbert and Wilkinson (1994) repeat the often noted observation that high-quality data are needed in order to detect chaos. We do not share their optimism that scanner data will allow chaos to be detected in marketing; many recent news stories confirm that scanner data are also error prone, perhaps especially so for new products. To date, optimism and claimed successes in finding chaos in economic data
have been confined to stock prices (Wen 1994) and to the time series of certain monetary aggregates (Barnett and Chen 1988).

2. Specification error. Franses (1994) notes the difficulty of distinguishing whether empirical data are generated by a logistic function or a Gompertz function. The nearly identical behavior (in the usually interesting intervals) of several functions can mean that forecasts are based on a function that seems to fit the data but does not generate chaos - rather than a similarly shaped function which could, under perturbations, exhibit chaos.

For example, the Bass model (1969) of new product growth specifies the market penetration rate (cumulative adoption rate) as:

\[ X_{t+1} = X_t + (p+qX_t)(1-X_t) \]  

(2)

where \( X_t \) is the cumulative adoption rate, and \( p \) and \( q \) are interpreted as the coefficients of external influence and internal influence, respectively (Mahajan, Muller, and Srivastava 1990). This type of model has been widely accepted for new product forecasting in marketing. However, although the Bass model depicts sales growth pretty well in smooth curve for \( t > 0 \) and is meaningful (showing no chaos) for some values of \( p \) and \( q \), it is easily conceivable that there would be little evidence in the immediate post-introduction period for favoring, say, this model against the more volatile Yorke-Li model, or vice-versa.

3. Long data collection interval. Bass and Leone (1986) note that advertising models using different time intervals may yield "dramatically different conclusions." We now extend this notion to conclusions regarding chaos in the diffusion models. The annual data can easily show monotonically increasing sales even when monthly sales are fluctuating alarmingly. In fact, it can be in the brand manager's interest to display the data in this monotonic fashion even when he/she is aware of the fluctuations. Figures 3A and 3B deal with this problem, utilizing the

\[ \text{footnote}{1} \text{ According to Sultan, Farley, and Lehmann (1990), the range for } p \text{ values is 0.00002 to 0.23 and the range for } q \text{ values 0.00003 to 0.99 for the existing Bass model estimates. However, there was no research to our knowledge on why this range should apply to make the Bass diffusion curve "meaningful."} \]
Bass-type diffusion function of equation (2) where $X_t$ is the cumulative market penetration rate with full penetration (market potential) of unity. The starting value is $X_0 = 0.001$.

Figures 3A and 3B about here

Figure 3A shows the sales fluctuations during 120 months, and Figure 3B is the aggregation of the sales in Figure 3A into annual bases. As can be seen in Figure 3B, there appears to be sales at the first year and no sales after that. This sales pattern is typical for fashion products. Even though there are a lot of fluctuations in the cumulative sales with unreasonable negative sales in Figure 3A, the aggregated sales function is a normal one. This simplistic example shows that temporal aggregation may conceal the chaotic behavior of the sales. When we have a small change in $q$ value from 2.5 to 2.0, we get Figures 4A and 4B. Again, the monthly sales shows a chaotic pattern whereas the annual aggregation gives a normal sales pattern for fashion products.

Figures 4A and 4B about here

Interestingly, we note that change in $q$ value of the Bass-type diffusion function has considerable impact on the monthly sales shape (compare Figures 3A and 4A) but causes a minimal change in the annual sales pattern (Figures 3B and 4B). This fact suggests that small changes in forecasting parameters can cause a dramatic change in the chaotic patterns (changing a chaos into another totally different chaos) but do not have much influence on the normal patterns. Accordingly, temporal aggregation may conceal the chaotic sales patterns and keep the concealment stable. This argument partly explains why we cannot find chaotic sales behavior in the new product diffusion models based on annual sales data.

4. Dynamic changes in marketing plan and actions as diffusion proceeds. Engineers are said to be adept at forestalling chaotic vibrations in mechanical systems. This may be because, not
being random, chaos exhibits some characteristic warning signs. This kind of intercession may be feasible in marketing practice as well, and should be a subject for further field/interview research with brand managers. Figure 5 shows the impact of price dynamics on the chaotic behavior of sales in Figure 3A.

Whereas Figure 3A shows a chaotic sales pattern, Figure 5 shows an apparently normal diffusion function, due to the price impact on the sales in Figure 3A. This demonstrates that a diffusion model which includes price impact (see, e.g., Mahajan, Muller and Kerin 1984) could disguise or eliminate chaotic phenomena in a sales series.

The diffusion function in Figure 5 is the well-known Robinson and Lakhani (1975) model which includes price impact on the sales as:

\[ X_{t+1} = X_t + (p+qX_t)(1-X_t)\exp(-rP_t) \]  

(3)

where \( P_t \) is the price at time \( t \) and \( r \) is a coefficient for the price. The \( p \) and \( q \) parameters retain the same values used to generate Figure 3A (\( p \) is the diffusion parameter used in the previous section and is not to be confused with the subscripted \( P_t \) which represent the price path). We use two price schemes to illustrate. Price scheme 1, which is for Figure 5, has the following functional form.
SCHEME 1

\[ P_t = 1.2(1/X_t)^{0.5} + 0.5 \text{ for months 1-50} \]

\[ = 1.0(1/X_t)^{0.5} + 0.1 \text{ for months 51-90} \]

\[ = 0.1(1/X_t)^{0.5} + 0.01 \text{ for months 91-120.} \]

We specified two price cuts, at the 51st and 91st months, to gauge their impact on sales. The price path is constructed as an inverse function of the cumulative market sales to reflect an experience curve effect on price. This type of price function has proved realistic (see Robinson and Lakhani 1975). The coefficient \( r \) in equation (3) was selected to be 0.35, following Robinson and Lakhani (1975). If we compare Figure 3A with Figure 5, we can say that the price impact on the chaotic sales functions may lead to "normal" types of diffusion functions. So if we simply look at the revealed phenomena in Figure 5 we cannot capture the chaos which is originally embedded in the sales.

Figure 6 uses a different price scheme (scheme 2) but is are based on the same original sales graph of Figure 3A. In Figure 6 we use a more realistic price scheme which shows two significant price cuts.

SCHEME 2

\[ P'_t = 10.5(1/X_t)^{0.4} + 10 \text{ for month 1-50} \]

\[ = 8(1/X_t)^{0.4} + 7 \text{ for month 51-90} \]

\[ = 6(1/X_t)^{0.4} + 4 \text{ for months 91-120.} \]
As we see in Figure 6, because of the high price compared to scheme 1, the sales curve is picking up not quickly but gradually - a quite "normal" diffusion curve. Diffusion functions of the type in Figure 6 frequently characterize newly developed technological products such as work-stations, high-definition TV, etc. Again, Figure 6 does not show any chaos after the price effect is included in a sales equation that would otherwise generate chaos. This lengthy example illustrates that increasing the dimensionality of the sales equation (by incorporating strategic variables - in our case, price) changes the nature of the finite difference equation so that, even when maintaining its previous parameters, it does not result in chaos. We may loosely extrapolate this mathematical result to the management situation by noting that to effect such a change in the dimensionality of a decision situation is a legitimate and frequently seen act of management.

5. Aggregation effects over outlets or market segments. In Figure 7 we aggregated (averaged, since the market potential is unity for each market) over 8 market segments which are identified by different q values (q = 0.3, 1.0, 1.1, 1.2, 1.3, 1.5, 2, 2.5). Several of these individual q values result in a chaotic sales series.

The aggregate diffusion curve of Figure 7 looks almost normal except for small fluctuations every other month. These fluctuations, if reflected in market research data, are small enough to be mistaken for statistical variation. Of course, if these monthly data are annualized as described in “Reason #3” above (long data collection interval), the small fluctuations disappear. The trend of cumulative and incremental sales in Figure 7 reflect a common new product
diffusion pattern. The conclusion is that chaotic micro-level symptoms can be concealed by the aggregation of sales over different market segments.

6. Early product failure. The Wall Street Journal (1990) reported that of the 6,960 new brands introduced in 1988 and 1989, only 240, or 3%, reached the $1 million sales mark. Presumably, although the article does not make it explicit, the remaining products did not generate enough revenue to cover expenses, and were discontinued. In Figure 8, sales crash dramatically and totally in period 14, with no recovery for two further periods. Any hopes that the two peaks on the right of Figure 8 represent some kind of seasonal popularity of the product are dashed when the series is computed to 50 iterations. Two more peaks occur, then the function drops to negative infinity and stays there.

--- Figure 8 about here ---

What is a manager to make of such a pattern? If it is empirical data, and in the event that the product has not been discontinued in period 10, it is almost sure to be seen as a loser in period 14, when sales go to zero and there is no recovery for two periods. A manager might speculate that there have been distribution difficulties and stockouts. If the pattern is a forecast, we speculate that it is certain to be disbelieved.

7. Traditional management reports, and how they are generated. New product diffusion is usually measured as cumulative penetration - there is no opportunity to see a jagged curve. For products purchased only once or one at a time, the cumulative sales-time curve equals the cumulative penetration curve. A non-cumulative sales curve is usually used as a management tool for mature products.

An empirical cumulative penetration chart is calculated from consumer panel data as follows. In each time period, the number of households buying the product for the first time is added to the number of households making their first purchase in prior time periods. This total is divided by the number of total households in the sample, or if weighting factors are used, by

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the total number of households in the population of interest. For a single chart, this
denominator is held constant for all time periods; in our experience, practitioners feel it would
be inconsistent to do otherwise. even though it is acknowledged (see, e.g., Mahajan and
Peterson 1978) that the total potential buying population may change over time. This
procedure guarantees a monotonically nondecreasing empirical penetration curve. In
addition, this procedure requires much aggregation and transformation of the observed datum -
that is, whether a given household has bought the item or not - in order to generate the
"empirical" penetration curve. As we have seen, data transformations can obscure chaos in
micro-level data. The deck is thus stacked against non-monotonic models being supported by
historical data.

Hibbert and Wilkinson (1994) point out that real marketing situations are characterized
(or in principle could be characterized) by more complex systems of differential equations.
These more complex systems have larger regions of parameter space in which chaos can arise.
Using more linked models, they say, would lead to chaos being forecasted with greater
frequency. This may be true (though our price example above, and the present authors' on-going research, provide illustrations that increasing the number of variables and/or equations can make a system on the edge of chaos converge again), but it is irrelevant in a world where the single new product growth curve is the chart of choice for management presentations and
decision making. There are very few charts that are traditionally used in marketing
management, and researchers offering new kinds of displays are usually at pains to minimize
their complexity.

We do not mean to imply that researchers do not look at diverse and innovative types
of data displays. Our point is that the patterns of demand for management information provide
little opportunity to observe chaos or other exotic functional behaviors in the practical
management context. Yorke remarked (Gleick 1987, p. 67) that physicists had learned not to
see chaos. Marketers might well examine whether cultural/habitual methods of measurement
and analysis have resulted in similar learning.

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2 In ordinary language, "cumulative" implies "nondecreasing." To display a non-monotonic cumulative
penetration curve would be a recipe for cognitive dissonance, for researchers and for managers. This is
another reason researchers construct the empirical curves in a way that is guaranteed to be monotonic.
8. Sole attention on rate of sales growth. Time series of other variables might suggest chaos, viz. Figure 9 which shows the time path of natural gas spot prices following their deregulation. Deregulated natural gas may be regarded as a new product. Compare Figure 9 to Figure 10, which shows the first 15 periods of Figure 3A/Equation 2.

Figures 9 and 10 about here

The evident similarity of shape raises the possibility that Figure 9 was generated by a chaotic process. This possibility is more plausible given the body of literature suggesting that securities prices exhibit chaos. In fact (Nance 1994), the "fluctuations" occurring in Figure 9 in 1987 and thereafter are artifacts. In 1987, sellers of natural gas first thought of charging seasonal prices. Figure 9 shows two data points per year, summer and winter. After 1987, winter prices are high and summer prices low. Nonetheless, it is possible that marketing-related time series other than cumulative sales reflect chaos. Priesmeyer and Baik (1989), for example, claim to find chaotic attractors in sales-profit phase spaces.

9. Chaos may not be recognized per se, or cause alarm, but is seen as an opportunity. Observing the data of Figure 9, major gas consumers including electricity utilities have hedged their exposure by forming options and futures markets (Nance 1994). In these industries, there was, according to Nance, no hesitation in giving up the quest to predict prices. There was simply an easy glide into a different arena of competition, namely, making a maximum amount of money for the utility by trading derivatives while still satisfying customer demand for electricity. Analogies to brand marketing are not easily forthcoming, but the idea of taking lemons and making lemonade is time-honored.

10. Observations confounded by repeat purchasing or multiple-unit purchasing. Whereas the Bass-type diffusion models focus on the front end of Product Life Cycle (PLC), the later part of the PLC has been analyzed to explain repeat purchase or technological substitution (or both). Repeat purchase models usually estimate and predict purchase phenomena which deviate
from the one-time-purchase assumption of conventional new product diffusion models. They describe repeat purchase phenomena as a function of product attributes, marketing efforts of firms, and word-of-mouth impacts (Kim 1993). Accordingly, as can be seen in Lilien, Rao, and Kalish (1981), the fit and prediction accuracy are very much dependent on which variables and/or how many data points are included in a repeat purchase model. Technological substitution models deal with multi-generation technological product sales and focus on replacement of generations (Olson and Choi 1985; Kamakura and Balasubramanian 1987) or leap-frogging (Norton and Bass 1987; Mahajan and Muller 1990).

These types of models assume the existence of a function which can explain such phenomena as repeat purchase or technological substitution and try to fit the function to the data. However, as these models are developed based on a set of assumptions related to the characteristics of a certain product category - e.g., household appliances for Kamakura and Balasubramanian (1987), ethical drugs for Lilien, Rao, and Kalish (1981) and Rao and Yamada (1988), and IBM mainframe computers for Mahajan and Muller (1990) - the lack of applicability of one model to other products can be a problem. In other words, separate models have been used traditionally to model different phases of the PLC, and the diverse character of repeat buying in different industries has inhibited the development of general total-PLC models. The lack of such models, and of experience with estimating them, may in turn have inhibited marketing scientists' exposure to chaotic transitions in marketing time series.

Moreover, since the points where sales experience second and third take-offs in the later part of PLC are explained by various reasons according to the product category (repeat purchase, purchase of the new product generation, revival of the old vogue, etc.) we also contend that chaotic sales phenomena could lead to such perturbations at the later part of PLC. Of course, these chaotic phenomena can be intermixed with random error. This logic can be supported in part by Figure 3, which may be regarded as renewed take-offs of sales in PLCs.

Figure 11 displays historical data on black and white television sales in the United States.

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Figure 11 about here

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14
The initial rapid takeoff reflected the volume of new household formations in the aftermath of World War II, and the relative affluence of postwar America. Some of the later cycles can also be attributed to demographic waves. The takeoff was extended by a buying panic preceding the Korean War, which cannibalized sales that would have occurred in 1950. Over the more than 25 years shown in the Figure, replacement of worn-out sets was surely a factor in the second, third and fourth up-cycles. Color sets, first sold in the mid-1950s, took off slowly due to high cost and few color broadcasts. By the mid 1960s, B/W sales declined as American households bought color sets - but took off again in the late 60s and 70s as households first came to own two sets, the second set usually a black and white. This example shows the effects on a sales cycle of external events, obsolescence and replacement, and second units. The irregularity of the sales series is surely a result of these effects more than of the effects of any chaos that may have been present. That is to say, when marketers can obtain long time series of marketing data, these are exactly the databases that can most easily be confounded by external events.

Although we do not display it here, the data of Figure 11 can also illustrate the effects of a changing denominator in the penetration ratio. We have noted that for most "new product" management displays, the denominator (total potential buying population) is held constant, reflecting both the short time frame usually encompassed by such displays, and the desire for simplicity of understanding. Over longer time periods - especially like that of Figure 11 which encompasses waves of household formation and dissolution due to three wars, the baby boom and its later echo, a shift from market research based on families to research based on households, and a sudden increase in divorce rates - the denominator changes, and "cumulative" penetration curves might well be non-monotonic if care were not taken to force them into monotonicity.

11. Estimation method. Hibbert and Wilkinson point out that the initial development of the Bass diffusion model is similarly built on a recursive definition involving innovation and imitation parameters. (Bass, thinking probabilistically, referred to this formulation as the hazard function of a distribution law.) Although we have seen that such iterative arrangements can lead to chaos, marketing models are often estimated under different parameterizations. For
example, a researcher wishing to estimate a logistic curve using the logistic distribution function

$$F(x) = 1 - \{1 + \exp \left[ \frac{(x-\mu)}{k} \right] \}^{-1}$$ (4)

directly would note that its parameters are $\mu$, the mean, and $k$, which is equal to $\sigma \sqrt{3}/\pi$. Thus, any estimate of a finite $\mu$, and a finite, positive standard deviation $\sigma$ results in a smooth, s-shaped curve. Gleick (1987, p. 220) discusses the chaotic behavior of Newton's method of computing square roots. The square root function is, of course, smooth for nonnegative arguments. It is clear that in the context of curve fitting, it is not the estimated function itself that determines whether chaos may emerge, but the estimation procedure.

4. Summary and Implications

Several of the papers cited above contributed to the marketing literature by bringing chaos concepts from physics to the attention to marketers. They have suggested that chaotic dynamics that have been demonstrated theoretically or seen in physical systems may soon be found in marketing phenomena as well. In this paper, by looking in detail at the interplay of these concepts with actual practices of data collection, modeling and reporting, we have begun to re-examine both the detection and the relevance of chaotic dynamics in marketing. We have noted, and illustrated by example or simulation, the following points:

- Marketing data are usually aggregated over time periods, sales regions or outlets, and this can obscure the identification of a time series as stochastic or deterministic. More generally,
data transformations of many sorts that are common in marketing - including summing, averaging, logarithmic substitutions, and moving back and forth between iterative and analytic forms of a function - can do the same.

- Marketing events are affected by environmental and economic forces as well as by the changing tactics of each producer and its competitors. These forces and tactics can both mimic chaotic fluctuations (thus leading to "false sightings" of chaos), and obscure the presence of true chaos.

- Data sources and reporting formats in market research are largely dictated by the needs of business. These are motivated by a desire for simplicity rather than complexity. Moreover, marketing is a field that straddles theory and practice, thus mixing mathematical formalism with vernacular language. The need to bridge between the two modes of expression constrains what can be expressed in either. Both the current theory and practice are results of a cultural paradigm that has been decades in the making. Such firmly entrenched cultures can act as blinders, preventing the recognition of new phenomena such as chaos in marketing.

- Marketing data sets may be biased by virtue of excluding cases (e.g., products that failed early after introduction) that might have been indicators of chaos. Indeed, zero sales - or an erratic sales pattern which is deemed unmanageable because inventory and cash flow cannot easily be smoothed - means both product distribution and data collection will be discontinued. If rational responses to chaos, like options, futures and derivatives markets, are deemed interesting from a research point of view, then research will focus on these responses rather than on the chaos that may have spurred them.

Not all these points are original, but by considering them in ensemble we have tried to establish an initial framework for assessing the meaning of chaos and related dynamics in marketing. In light of this framework, the new developments in chaos theory present a number of exciting challenges and new points of view for best practice in new product development and marketing science. Some of these are noted below.

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is control of a marketing program that has material consequences. Given e.g., the controversy over top executive pay in the U.S., and linkage of pay to performance, it is relevant to ask whether and when a CEO's actions can have an impact on the firm's (possibly chaotic) stock price.
Tension between statistical and deterministic methods

Although Hibbert and Wilkinson (1994) note that deterministic chaos cannot be detected by statistical techniques, they do not pursue the implication that this presents a very fundamental conflict for marketers, one that may be overcome perhaps only by a paradigm shift. Marketers are accustomed to efficient estimators, unbiased estimators, etc. But chaotic estimators not only are not part of the statistical toolbox, they cannot be part of it. Nor is the conflict then only between statistical and deterministic research methods. Users of statistical tools are accustomed to proving that an estimator is optimal, or "best linear unbiased," and so on, and then to using that estimator and that estimator only. The possibility that chaotic data are impervious to BLUEs means that more than one method of data analysis (e.g., maximum likelihood and the Lyapunov exponent) may have to be applied to the same data set, and the final inferences made heuristically (see Barnett, et al. 1994).

Sensitivity analysis

Another component of management tradition speaks against the complex models that could yield chaotic results, and that is sensitivity analysis. In linked nonlinear equation models, sensitivity analysis is next to impossible. This is because tiny changes in initial conditions can lead to enormous and unpredictable divergence in later portions of the trajectory. But sensitivity calculations are crucial in business and social analysis, not least for answering basic questions like "How accurate do our data have to be?" or "What if we have to cut the budget?" This partially explains the slow acceptance of "systems dynamics" models of the Club of Rome type (see Phillips 1972), and why the early models of this type used piecewise linear functions rather than curvilinear ones. As computing expense continues to decrease (and speed to increase), sensitivity analysis by repeated simulations will become more common.

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4 Certainly any inference based on observed data may be called statistical. Here, however, we use the word statistical to mean data-based inferences that explicitly admit random variation of observed values around a "true" value, and/or random errors of observation (measurement).
Predictability

If a marketing data set is chaotic, then a deterministic model (if one can be devised) will provide superior short-term predictability. This short-term predictability is desirable for management purposes, and also constitutes a test for chaos. See Hibbert and Wilkinson (1994) and Sugihara and May (1990).

A new view of marketing managers’ skills

In “Reason #4” above, we suggested very tentatively that marketing managers may, like engineers, be able to perceive early signs of chaos in the systems they are assigned to manage, and perhaps even take measures that forestall it. Could new product managers be exercising skills that are not yet articulated or described, even by the managers themselves? Now that we are aware of the mathematics of chaos, we are able to ask this and related questions, and devise research to attempt to answer them.

Data quality

Knowing that a data set may have been generated by a deterministic phenomenon will focus marketers’ attention on issues of data quality. Data quality issues should, and now perhaps will, receive attention equal to parameter estimation and the other phases of marketing modeling. Efforts will be made to obtain much larger high-quality data sets.

Insights into the buying mechanism; parsimony in modeling

Stocks are purchased because of the buyer’s expectations about the future performance of the firm. But sometimes stocks are bought because a buyer expects that other buyers have increased their expectations about the firm’s performance. In other words, the market is about expectations about expectations about expectations. To what extent should models incorporate this seemingly infinite regress? Wen’s (1994) work shows that very low-order differential equations can generate time series that are spectrally identical to the Standard & Poors stock price series. It remains to be explored how such simple equations can capture such a seemingly complex phenomenon as the stock market, and one may expect similar challenges to arise in other marketing arenas.
**Implications for gravity-type models**

In this paper we have explored product diffusion models. There is another important class of nonlinear models in marketing. This is the gravity-type models that include individual choice models (for brands, transportation modes and routes, etc.), retail location models, and brand shifting models - Haynes and Phillips (1982) and Phillips (1994) survey many of these. Krider and Weinberg (1994) explore implications of a nonlinear dynamic phenomenon of self-organizing criticality for some retailing situations. We may expect much more attention to these areas to appear in the marketing literature.

**Equilibrium models in marketing**

Nonlinearity and chaos may be associated with disequilibrium and contrasted with the linear models that characterize equilibrium processes. Recent work such as that of Arthur (1989), Thore and Phillips (1993), Krider and Weinberg (1994), and Mulhern and Caprara (1994) emphasize emergent properties and disequilibrium. They contrast sharply with e.g., Goodhardt, et al. (1984) which developed an equilibrium context for the negative binomial model in marketing. Wider awareness of the mathematics of chaos should bring more intense research attention to issues of equilibrium in marketing, and perhaps more published work on nonequilibrium models.

**Local vs. global estimation**

Awareness of the possibly strange behavior of functions outside the range of parameters considered in their models might constructively lead researchers to reemphasize noting the range of parameters tested and the range within which their models might be expected to be valid.

**Construction of estimators**

Mathematically oriented researchers in marketing have traditionally been quite careful about parameter estimation methods. But the considerations raised in this paper, particularly "Reason #11" above, indicate an additional element of importance concerning the construction of estimators. Because the estimator can affect whether the forecasts are smooth or chaotic,
the construction of the estimator becomes equally as important as the representational model (the model whose parameters are being estimated) in determining the truth about the marketing phenomena under study.

5. Concluding Remarks

According to Prigogine, Chen and Wen (forthcoming), "Sciences dealing with human behavior have always been influenced by the dominating paradigms in physical sciences. Now these paradigms are shifting, and that will likely have a lasting effect on economic sciences. Moreover, a condition associated with chaos - sensitivity to initial conditions - is obviously satisfied in most human activities." The existence of chaos and other emergent properties of complexity makes marketing science a more difficult pursuit, or at least makes the goalposts of understanding and control seem farther away. We have no doubt marketing scientists will rise to the challenge.

Meanwhile, much chaos research seems irrelevant or misdirected, from the point of view of a manager -- for example, research on ways of locating all the bifurcations (points at which a transition may occur, perhaps to a chaotic state) of a nonlinear system. Businesses, thankfully, do not seem to bounce from cusp to cusp very rapidly. A competent management won't experience more than one or two purely internal crises before the market changes or the competition changes. In chaos jargon, but speaking informally, we might say a business occupies a small region of a nonlinear space, defined, in principle, by a system of differential equations. The time it takes to leave that region is less than the time it takes for the descriptive equations to become invalid.

It is more important to monitor a business's "path through space" than to attempt a complete description of that space. That is why market research companies are now focusing

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5 In Akira Kurosawa's film Ran (Ran translates as "chaos"), chaos outside the daimyo's household could not be forestalled because of the chaos within it. In the martial arts practice known as randori (defense against free-form multiple person attack, literally translated as "taking chaos") success depends on precise and rapid repositioning, exquisite timing, and a rhythmic sense of when to assert one's position and when to allow the flow of events to unfold. But success in randori is never assured, even for master practitioners. Its lesson is that it is preferable to forestall ran before it can begin.
on improved real-time information delivery. The additional lessons we can learn now from chaos theory are qualitative and strategic: that random forces have relatively greater weight when a system is far from equilibrium; that outliers can gather strength and become new norms; and that healthy systems grow by tolerating both conservative and disruptive forces.
Figure 1.

YORKE AND LI MODEL (b=1.5)

Figure 2A.

YORKE AND LI MODEL (b=2.5)
Figure 2B.

YORKE AND LI MODEL  (b=3.0)

Figure 2C.

YORKE AND LI MODEL  (b=3.95)
Figure 3A.

New Product Diffusion with Chaos (Monthly)

(p=0.002, q=2.5)

Market Penetration Rate
With Full Penetration=1

Cumulative Sales (Xt)
Sales (Gt)
NEW PRODUCT DIFFUSION WITH CHAOS

Figure 3b.

MARKET PENETRATION RATE
WITH FULL PENETRATION=1

(\( p=0.002, q=2.5 \) AGGREGATED BY YEAR

CUMULATIVE SALES (x)- SALES (dx/dt)
Figure 4b.

(p=0.002, q=2) ACGREGATED BY YEAR
NEW PRODUCT DIFFUSION WITH CHAOS

MARKET PENETRATION RATE
WITH FULL PENETRATION=1

YEAR

0- CUMULATIVE SALES (XII) - - - - - - - - SALES (XIVII)
NEW PRODUCT DIFFUSION WITH CHAOS (p=0.002, g=2.5) AND PRICE IMPACT (SCHEME 2)
NEW PRODUCT DIFFUSION WITH CHAOS AGGREGATED ACROSS 8 MARKET SEGMENTS

Figure 7
Figure 8.

YORKE AND LI MODEL (b=4.00925)
Figure 9.
Natural Gas Spot Price at Wellhead
(Source: Nance 1994)

Figure 10.
Curve Generated by Equation (2)
Figure 11.

Sales of Black and White TV Sets
(Source: Electronics Industries Association - Wasson 1972)
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