Many-Firm Dynamic
Spatial Competition in Retailing:
The Emergence of Self-Organized Criticality

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Series No. MKTG 94.004
March 1994
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Conventional wisdom suggests that rapid change is a dominant characteristic of retailing, and that success in this industry depends on management's ability to respond quickly and effectively to these changes. In spite of this intense dynamism, there is also a tradition that suggests there is an underlying order, such as the "wheel of retailing", in this industry. This paper formalizes the conventional wisdom of rapid change in a spatial interaction model, and, via a stochastic steady state known as self-organized criticality (SOC), connects it to the tradition of underlying order. SOC has a number of interesting and appealing characteristics, including the fact that the market does not smooth exogenous shocks as effectively as if the central limit theorem applied, thereby providing a mechanism to explain observed macroeconomic instability. The state is also robust to parameter changes and details of model structure, and because of its appearance in a wide variety of apparently unrelated disciplines, appears to be a very general organizing principle. Its application here suggests how systematic long-run behaviour, that is entirely different from typical economic equilibria, might arise and be detected in complex dynamic spatial competition. This work represents the first known marketing model that demonstrates SOC.

1. Introduction

Theoretical analyses of competition typically rely on some variation of a Nash equilibrium, and assume various levels, often high, of information and foresight. Frequently, however, competition operates in environments of low information and myopia. In such an environment, management behaviour might better be described as reactive or adaptive, rather than planned. Academic, trade and popular literature frequently identify retailing as such an environment. Corstjens and Doyle, for example, introduce a recent (1989) Marketing Science article as follows:

A central facet of modern retailing management is repositioning--adapting the
business to a changing retail environment. A retailer's existing positioning base is continually being eroded by maturing markets and aggressive competitors seeking opportunities for profit and growth. Often the repositioning required is small and gradual...Sometimes, however, the repositioning has to be more radical--a switch into new types of stores, a change into major new merchandise areas or a total re-presentation of the stores.

The popular press also frequently acknowledges this facet of retailing. The Financial Times of Canada (April, 1993) described how one major supermarket chain (Loblaws) responded successfully to the competitive attack of warehouse clubs:

Loblaws' Gilles Potvin...survived the first wave of the warehouse invasion by scrambling astutely [emphasis added] to put his store on a sound footing. He'll survive the next wave because he's discovered the warehouse owners can't be all things to all people.

"Scrambling" is not a behaviour easily captured in equilibrium models. It implies a reaction to unexpected adversity in the environment. At first glance, attempting to model such behaviour might appear uninteresting--the intuition is that one would get either trivial or uninterpretable results. And yet, conventional wisdom has long held that there is some underlying order to the dynamics of retailing. McNair's (1958) "wheel of retailing", and associated notions like "the accordion of retailing" (see Brown, 1990b, for a summary and integration), suggest that retailers cycle through various formats. These notions persist in introductory marketing textbooks as well as academic journals (May, 1989; Brown, 1990a), in spite of a long history of criticism on both theoretical and empirical grounds.

In this article, we suggest an approach that reconciles the intuition that retailers individually "scramble", with the intuition that there is underlying order in the structure of retailing industries. The link is through a stochastic steady state known as self-organized criticality, or SOC. This state has several appealing properties (to be described shortly) including the property that large economies do not smooth small exogenous shocks as
effectively as would be expected if the central limit theorem applied. In fact, a single small exogenous shock can produce no response, or occasionally, an industry-wide response. As will be elaborated on later, SOC is often called a "poised" state: poised on the border between order and disorder, or perhaps more colourfully, "on the edge of chaos".

The analysis we present is based on the well-established spatial interaction model of retail competition (Huff, 1962; Ghosh and McLafferty, 1987). To our knowledge it is the first demonstration of SOC based on a marketing model, although previous work in economics exists on an insightful, but "an extremely simple model of a multi-sector, multi-stage production process" (Bak, Chen, Scheinkman, and Woodford, 1992, p. 3).

The model dynamics are characterized by four main features. First, the decision maker operates in a low information environment. Specifically, behaviour is limited to reaction to environmental adversity and competitive attacks, as indicated by the firm's own revenue erosion. Second, the reaction involves satisficing behaviour. Revenues (or profits) must be kept above a threshold level. Third, the challenged firm has the ability to respond with a successful innovation. Finally, the entire system is driven by exogenous shocks in the form of indirect outside competition hitting randomly chosen firms. These characteristics capture the notion of "scrambling astutely".

Rather than assume what the characteristics of the equilibrium will be like (or even that there is any equilibrium), the modelling approach is to allow the system to evolve according to rule-based heuristics. We allow the managers to make reasonable decisions, and observe the evolution of the system. When (or if) a steady state is reached, and only then, are the characteristics of the state investigated. The results help to understand how retailing as an industry can change relatively rapidly, with waves of innovative formats sweeping across the system, and yet be at (stochastic) steady state, exhibiting a certain orderliness
consistent with the spirit of "wheels" and "accordions" of retailing. Although there are no managerially actionable variables in the model, one consequence is that at steady state, stores are more susceptible to the effects of exogenous shocks to competitors than would be expected if a market-smoothing dynamic obeying some central limit theorem applied. This susceptibility is indicated by power law distributions--the probability of long range effects decreases according to a power law, rather than the more rapid exponential decay associated with the central limit theorem. One strategic implication is support for the necessity of monitoring distant or indirect competition as well as nearby or direct competition (Montgomery and Weinberg, 1979). By extension, the results suggest that an ability to scramble astutely is important to the long run survival of a retailer.

In the following section, the concept of self-organized criticality will be reviewed. In Section 3, a model of spatial competition in retailing, along with a set of rule-based dynamics that govern management behavior, are developed. Section 4 presents the results of computer experiments to monitor the evolution of the system under a variety of conditions. These results establish the existence of a stochastic steady state and examine its robustness. The rather striking "poised-at-the-edge-of-chaos" feature is also demonstrated in Section 4. Section 5 discusses and summarizes the results, and suggests directions for further research.

2. Self-Organized Criticality

The self-organized critical state has been introduced to the economics literature by Bak, Chen, Scheinkman, and Woodford (BCSW) (1992). It has its origins in several independent developments in such diverse fields as theoretical biology (Kauffman and Johnson, 1992), solid state physics (Bak, Tang, and Weisenfeld, 1988) and computer simulations of artificial life (Langton, 1989, 1992).
BCSW (1992) consider a model of production and inventory dynamics for an artificial economy with a large number of firms. The highly stylized model consists of a two dimensional network of producers on a cylinder, each of whom buy supplies from two of their neighbours at a higher level and sell goods to two other neighbours at a lower level. At one end of the cylinder, final goods are demanded randomly, from the last row of producers, by consumers. This demand creates a flow of goods from progressively higher levels in the supply network. The system converges to a stochastic steady state known as self-organized criticality (SOC). SOC will be discussed in more detail shortly, but first consider BCSW's main point: the law of large numbers does not apply in this situation\(^1\). In the limit of a large number of firms, the (appropriately scaled) aggregate response (production) to the independent exogenous shocks (consumer demand) does not converge to a distribution with zero variance, but to a Pareto-Levy distribution with nonzero variance. In other words, for large but finite economies, the probability of large responses decreases according to a power law distribution; that is, much more slowly than the exponential decrease that would be expected if the central limit theorem applied.\(^2\) To quote BCSW (1992).

Explaining the observed instability of economic aggregates is a long-standing puzzle for economic theory. A number of possible reasons for variation in the pace of production are easily given, such as stochastic variation in the timing

\(^1\)See Judge, Griffiths, Hill, Lutkepohl, and Lee, (1985), page 156 for a discussion of various central limit theorems and assumptions.

\(^2\) For a discussion of Pareto-Levy, or "scaling", or "stable" distributions, and an empirical example of Pareto-Levy fluctuations in the price of cotton see Mandelbrot (1982) pp 338-340. It should be noted that the evidence for stable infinite-variance distributions in many financial and commodity markets ( Mandelbrot, 1963 a and b, 1966, 1967; Fama 1963, 1965, 1970 ) has been challenged. Blattberg and Gonedes (1974) suggest that Fama's 1965 data could be fit by a finite-variance \(t\)-distribution. Others (e.g., Westerfield 1977) have shown that if time is redefined so that the analysis is based on prices per transaction, rather than per calendar time, the distribution is normal.

The generalized central limit theorem and the resulting stable distributions are discussed in Levy, (1925,1954).
of households' desired consumption of produced goods, or stochastic variation in the costs of production. But it is hard to see why there should be large variations in those factors that are synchronized across the entire economy—why most households should want to consume less at exactly the same time, or why most firms should find it an especially opportune moment to produce at the same time. Instead it seems more likely to suppose that variations in demand or in production costs in different parts of the economy should be largely independent. Thus, one might ask, should one not expect these local variations to cancel out, for the most part, in their effects on the aggregate economy, due to the law of large numbers? Fluctuations in activity of macroeconomic significance, it might be thought, should occur only when many independent shocks happen by coincidence to have the same sign, and this should be an extremely unlikely event (with the probability of occurrence decreasing exponentially with the square of the size of the event, by the central limit theorem).

There are a number of ways in which the law of large numbers can be made to fail\(^3\), and more ways still that aggregate instability can arise\(^4\). BCSW offer SOC as one way in which the law of large numbers may fail.

In fields other than economics, SOC has been shown to arise in an amazing variety of very different models and circumstances. While it would be possible to describe SOC in the context of BCSW’s model, we will use the prototype "sandpile" model introduced by Bak, Tang, and Weisenfeld (1988) in statistical physics. This simplest of models displaying SOC is mathematically isomorphic to the subsequent economic model of BCSW, and is easier to describe and understand. For more detail in relatively painless prose, the reader is referred to Bak and Chen’s (1991) *Scientific American* article.

Consider a flat tabletop on which grains of sand are individually dropped. Eventually, the sand will pile up and start falling off the edge. At some point, the pile will reach a

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\(^3\) These include Shleifer’s (1986) innovations; see Jovanic (1987) for more discussion.

\(^4\) For example, periodicities or deterministic chaos arising from low dimensional nonlinear equations involving relations between macro variables; see Frank and Stengos (1988); Boldrin and Woodford (1990). A little known marketing example is the Bass diffusion equation, which is theoretically capable of producing chaotic behaviour.
maximum height, with constant slope in all directions to the edge of the table. At this point in time, the addition of a grain of sand may have no effect, or it may trigger a small avalanche of sand, or, occasionally, it may trigger a large avalanche, on the order of the size of the whole sandpile. The distribution of avalanche sizes follows a power law: the probability of an avalanche involving $N$ grains of sand occurring is proportional to $N^{-\alpha}$, where $\alpha$ is a constant. This state is robust to model details, and is an attractor for the system dynamics. For example, it is arrived at whether the grains of sand are dropped at random locations, or at a single location. It can also be arrived at from the other direction—by putting barriers around the edge of the table, filling the resulting box with sand, and then removing the barriers, allowing the pile to relax to its natural slope. The resulting state of the sandpile will be "critical".

The notion of criticality comes from condensed matter physics, which has described "critical states" in a variety of circumstances, usually associated with phase transitions (e.g. from a solid to a liquid phase). In the subcritical state, local disturbances in the phase have only a weak effect on neighbouring parts of the system, and die out in a finite distance. More specifically, correlations of fluctuations between different parts of the system approach zero exponentially as the distance between the parts increases. By varying a "tuning parameter" such as temperature, however, the fluctuations can be made to propagate further. At a certain critical value of the tuning parameter, a state is reached where disturbances can just barely propagate to infinity. At this point, correlations no longer fall off exponentially, but with a power law. This is the critical state. As the tuning parameter is changed further, a "phase transition" typically occurs, a new structure forms, and correlations again fall off
exponentially. Disturbances, or shocks, can propagate to infinity only at the critical point. The critical state is a region where spontaneous macroscopic instability may occur. BCSW state.

The problem with this as a model of spontaneous macroeconomic instability is that, traditionally, critical states were thought to be associated with certain "critical" parameter values (such as temperature), that would almost certainly not occur in any existing system unless they were "tuned" to be at the critical value in a laboratory experiment. (Jovanovic's (1987) examples of economic models in which independent sectoral shocks produce aggregate fluctuations no matter how large the number of sectors...are special in exactly this sense). But [the sandpile model provides a mechanism whereby] large interactive dynamical systems can "self-organize" into a critical state. That is, the critical state can actually be an attractor for the dynamical system, toward which the system naturally evolves, and to which it returns after being perturbed by some large external shock.

The power law distribution of the sizes of disturbances, or "avalanches", is mixed news for a competitor living in a SOC state. The bad news is that a competitor may be affected by a shock happening anywhere in the system. The good news is that the probability of being affected by the shock decreases as the distance from the shock increases. This "mixed news" has substantial intuitive appeal.

There is another interesting way in which the state is "critical": in terms of how sensitive the evolution of the system is to small changes in initial conditions (that is, "small" relative to the range of the system variables normally observed in the system over a long period of time). This sensitivity may be quantified in terms of the time evolution of the

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5The classic example is spontaneous magnetization of a ferromagnetic material (such as iron) in the presence of a magnetic field as the temperature drops below the "Curie point". Above the Curie point, thermal disruption keeps the magnetization microscopically disordered, and below, the magnetic structure is frozen. In both cases, an external field has little macroscopic effect. However, at the Curie point, macroscopic fluctuations of the aggregate magnetization are possible, and a weak field can determine the final aggregate magnetization as the temperature drops. This effect was key in establishing plate tectonics, by "freezing" the orientation of the earth's magnetic field, in cooling minerals, at various times in geologic history.
separation of two realizations of the system, which are initially separated by small distance in state space. The two systems may converge, diverge or remain at a constant separation, either absolutely or on the average. Divergent systems are commonly referred to as chaotic, and for the classic nonlinear systems, such as the logistic equation, the divergence is exponential. The SOC state is also divergent, but not so strongly. The model developed here, for example, diverges at a constant rate and is therefore said to be weakly chaotic, or "on the edge of chaos".

In summary, the appeal of SOC is, first that it provides a mechanism for generating aggregate fluctuations from independent random shocks that do not have vanishing variance in the large limit; in particular, these fluctuations follow power-law (or Pareto-Levy) distributions, which have been shown to occur in economic time series (Mandelbrot, 1982). Second, the self-organized state is generally robust to model details. This means that it is not necessary to have very special conditions. Third, it has been shown to arise in very different contexts. It remains unclear what criteria are necessary for SOC to arise, but one common feature seems to be that systems have many degrees of freedom, with dynamically interacting

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6This is the "butterfly effect" due to Lorenz (1969), and the rationale behind empirical methods to detect chaos. See Grassberger and Procaccia(1983); Sugihara and May (1990). The butterfly effect states that the difference between a butterfly fluttering its wings or not in Beijing can make the difference between whether or not a thunderstorm occurs in New York a month later. Erickson (1993) provides a theoretical example of the possibility of chaotic behaviour in a marketing context: closed-loop Nash equilibrium strategies of duopolistic competitors in a Lanchester advertising model may be chaotic. While this is a micro-level effect, Erickson's concluding statement is relevant: "Chaos theory can be useful in marketing, because market responses to marketing activities are dynamic and nonlinear. It is also the case that rarely are markets observed to be in steady state, or in a state of repeating cycles, so that exclusively empirical approaches are not likely to be sufficient in the study of dynamical behavior in marketing settings."

elements: not unlike the many agents in an economy, or more specifically, retailers in a geographic area.

3. Model Development

3.1 Introduction

In Section 3.2, the static (i.e., within each period) model of supply and customer choice will be developed in a spatial interaction context. Section 3.3 describes the market structure, and section 3.4, model dynamics.

Before starting detailed model development, three important points regarding the research strategy will be discussed: the approach to dynamic competition, the generation of successful innovations, and the origin of the resulting steady state.

Rule-based dynamics

The model evolves according to recursive rule-based decision dynamics, rather than assuming a particular equilibrium concept. This is not a common approach, but neither is it highly unusual. As mentioned in the introduction, the myopic behaviour ("scrambling") that is more likely to arise in low-information environments may or may not lead to Nash equilibria. In the case where the environment includes unexpected adversity (modelled here as exogenous shocks), it would seem even more difficult to justify a Nash-type solution approach.

The game-theoretic equilibrium approach has well-known advantages and shortcomings, which have been addressed eloquently by many, including Gould and Sen (1984) and Kreps (1990). Regarding the Nash equilibrium, Gould and Sen state that "the definition is essentially ad hoc in the sense that it is not endogenously motivated by the model itself". Kreps (1990, p405) introduces the Nash equilibrium as
...an [author’s emphasis] answer to the question: If there is an obvious way to play the game, what properties must that "solution" possess? ... But this is a very weak question, and it is clear that having the answer "Nash equilibrium" is pretty thin gruel of what we are after is a way to solve games. All we have is a test of solutions derived by some other means."

The dynamic modelling approach which uses only recursive decision rules, in the form of sensible heuristics, directly deals with this problem of "how do we get where we're going". Examples are Baumol and Quandt (1964); Day (1967); Day and Tinney (1968); and Cohen and Axelrod (1984). These models, however, usually involve some explicit form of adaptation or learning behaviour. An example where learning is via Bayesian updating is Eliashberg (1981), who distinguishes circumstances where the competition may evolve cyclically, or it may converge to the game-theoretic static equilibrium. In contrast, our concern is not with the explicit modelling of the adaptation process; rather, the model simply assumes that firms can adapt to adversity.

Successful Innovations

This research focuses on industries, such as retailing, which are dynamic, intensely competitive, and innovative. The model assumes that successful, innovative responses to adversity will be made (the managers scramble astutely). These may arise from some evolutionary or learning process, as for example Eliashberg's (1981) Bayesian learning; the details of the origin of the successful response, however, is not the issue being addressed here. Nor is the precise nature of the innovation. It is simply assumed that an effective response can be made. Modelling at this level is necessary to capture the long-run behaviour, because the retail industry changes dramatically and qualitatively in the long run. It is similar in spirit to Shleifer (1986), who also examines innovations.

Economic Self-Organized Criticality

The third point relates to the outcome state, self-organized criticality. Marketing has
a long tradition of borrowing from other fields, and this research is no exception. In borrowing concepts, however, there is always an issue of appropriateness, and, in that context, we note that the approach here was not to find marketing labels to place on a borrowed model. Rather, the issue was whether the SOC state could arise in an established marketing context. In particular, the model for the interactions between the elements is the spatial interaction model which has been used in many empirical settings, and which involves substantially more complex computations than the sandpile and related models.

Much of the work on decision rules is designed to show how myopic but reasonable heuristics can arrive at states that are close, or identical to, the game-theoretic equilibria implied by optimal strategies of fully informed managers with perfect foresight. A recent marketing example is Jeck (1991). Our model, in contrast, shows how reasonable decision rules can lead to SOC.

3.2 Spatial Competition

Competition between retail outlets involves a spatial component, in terms of the customers' cost, or disutility for travel. A widely used model for empirical analyses and facility location is the spatial interaction model.

In general, the utility of store \( j \) to customer \( i \) is assumed to take the form

\[
U_{ij} = \prod_{k=1}^{K} [f_k(A_{jk})]^{\alpha_k} \prod_{l=1}^{L} [g_l(D_{yl})]^{\beta_l}
\]

(1)

The utility--often referred to as the attraction in share attraction models (Cooper and Nakanishi, 1988)--is a function of \( K \) characteristics \( A_{jk} \) intrinsic to store \( j \); the \( \alpha_k \) are usually assumed positive, so that utility increases with increasing \( A_{jk} \). Ghosh and McLaugherty (1987)
state, "The [intrinsic] attractiveness of a store results from a number of factors, including its size (which is often a surrogate for breadth and assortment of goods carried), its relative prices, and consumer perceptions of quality of merchandise and service" (p.63). Utility is also a function of L travel cost components, D_{ij}, usually measured as distance or travel time between the i^{th} customer and the j^{th} store; the \beta_i are assumed negative to capture disutility for travel. If the time-honoured assumption is made that either aggregate market shares or individual choice probabilities (Luce's choice axiom) are proportional to the share of utility (or attraction), the j^{th} store's share of i^{th} customer's purchases is given by (Ghosh and McLafferty, 1987)

\[
M_{ij} = \frac{U_{ij}}{\sum_h U_{ih}} 
\]

(2)

In this relation, total demand is inelastic. Elasticity is introduced by including an extra term, K_j, in the denominator, representing a generic choice, or a no-purchase option. (See, for example, Choi, DeSarbo, and Harker, 1990). This operates like indirect competition, in the sense of being outside the market. Further, this term is store specific, so that each store may have a different level of "indirect competition" to deal with. It is through this term that adversity in the environment is modelled. The customer's allocation then takes the form

\[
M_{ij} = \frac{U_{ij}}{(\sum_h U_{ih}) + K_j} 
\]

(3)

In the marketing retail location literature, the functions f_x and g_i are almost always the identity function, giving the multiplicative competitive interaction (MCI) model. Early
versions were limited to $K = L = 1$, with size as the surrogate for intrinsic attractiveness, and Euclidean distance for $D_{ij}$. If the absolute value of beta is much greater than alpha, the model approaches "nearest centre" models, as in Christaller's (1933) Central Places formulation. In Reilly's 1931 "Law of Retail Gravitation", which focuses on intermetropolitan trading area boundaries, alpha is one and beta is -2. Huff (1962) assumes alpha is one, and estimates beta to be 2.1 to 3.7 for various types of outlets in Los Angeles. Later authors consider more attributes, such as sales area, number of checkout counters, and whether credit cards are taken for the $A_{jk}$ (Jain and Mahajan, 1979); and auto travel time, transit travel time, and travel cost per unit income for the $D_{ij}$ (Weisbrod, Parcells, and Kern, 1984). Typical empirical estimates (for $\alpha$ and $\beta$) are between one and two, with extremes of 0.1 to 3.7. Fixing $\beta = 0$ gives a more usual MCI market share model, as, for example, in Hansen and Weinberg's (1979) analysis of retail banking outlets.

The particular realization of the model used here is guided by the objective of capturing only the essential features of spatial competition. The above discussion suggests that the minimal requirement for modelling spatial competition is one "intrinsic attractiveness" parameter and one distance parameter. For expositional intuition, the attractiveness parameter will be referred to as "size", as suggested by Ghosh and McLafferty (1987), although it may be equally well thought of as any number of other quantities (such as product valuation minus price). Because the focus here is on geographic space, distance will be taken as the usual Euclidean distance on geographic coordinates. For this research, $f$ and $g$ are taken to be the identity function, to be consistent with the marketing retail location literature.

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8 Exponential $f_k$ and $g_i$ result in the multinomial logit (MNL) share model. An example in the economics spatial competition literature in the Hotelling tradition is dePalma, Ginsburgh, Papageorgiou, and Thisse (1985).

In general, there is little reason to prefer either the identity or the exponential forms for the functions $f$ and $g$ (Cooper and Nakanishi (1988, page 35). The greater analytic
As modelled so far, each customer considers all possible retail outlets, which is not only unrealistic and ignores the notion of consideration sets, but is computationally cumbersome. In a product space context, Carpenter (1989) introduces a "reservation distance" to limit the customer's consideration set. Similarly, Ghosh and Craig (1991) use a reservation distance in a location model for franchises. Analogous to reservation price, this is a distance beyond which customers will not travel to patronize the firm. In many situations, this has substantial appeal. For example, it would not seem reasonable for a customer to allocate some portion of his dry cleaning, however small, to every dry cleaner in a city. Applied to the retail location problem, from the firm's point of view, the reservation distance determines the outlet's trading area.

In the basic spatial interaction model, then, customer $i$ allocates a proportion $M_{ij}$ of his expenditures to store $j$ in each period:

$$M_{ij} = \begin{cases} \frac{S_j^\alpha D_{ij}^\beta}{\left( \sum_k S_k^\alpha D_{ik}^\beta \right) + K_j}, & D_{ij} \leq R \\ 0, & D_{ij} > R \end{cases}$$  

(4)

where $R$ is the reservation distance, the summation is understood to include all stores within customer $i$'s reservation distance, and

$$\alpha \geq 0,$$

tractability of the exponential form in optimization and equilibrium analysis is not relevant for this research, which uses simulation. The marketing retail location literature commonly uses the identity, and the dynamic geographic literature uses the identity for $f$ and the exponential for $g$, the distance function.
\[ R_j = \sum_i M_{ij} \]  \hfill (5)

3.3 Market Configuration

Customers are uniformly distributed on a bounded plane. For the simulation the plane is

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**Figure 1:** Market Configuration: Small circles represent customer origin points, squares are stores, and the trading area of one store is the area inside the large circle.
rectangular, with customer-origin points in a regular grid. It is slightly more intuitive to think of the origin points as city blocks, rather than individual customers, in this setup. Stores are located in a coarser grid in the market—for example, every third block in the north-south direction and every fourth block in east-west direction. In each period, each customer-origin spends one unit (e.g., dollar), allocating the unit to all the stores within the customer-origin’s reservation distance according to share of attraction.

3.4 Dynamics

As previously noted, the approach in this model is to assume a rule-based decision dynamic, and allow the system to evolve, rather than assume a particular equilibrium concept. The dynamics has four main features. First, the decision maker operates in a low information environment; second, the heuristic involves satisficing behaviour; third, the challenged firm has the ability to react with a successful innovation; and fourth, the entire system is driven by exogenous shocks in the form of indirect outside competition hitting individual firms randomly.

Low Information Environment and Satisficing Behaviour

Corstjens and Doyle (1989) characterize as a "central facet" of retailing the need to respond to "continual erosion" of the retailer’s position. The Financial Times of Canada article, cited earlier, in reference to the retail grocery industry, speaks of "scrambling".

We model this behaviour by assuming that decision makers explicitly observe, and respond to, only their own revenue levels. This may arise not only because information is difficult to obtain, but because managers simply don’t use information that is available. Jeck (1991) states,

For example, The Marketing Workbench Laboratory at Duke University has found that store by store reports of prices, which can be obtained by the
decision makers, have not been used by many firms even thought it is felt that many consumer purchase decisions are based on available stimuli at the point of purchase...

Decision makers react when their revenues fall below a fixed threshold. This is a "satisficing" (Simon, 1965) type of behaviour, rather than optimizing. An example of decision rules which involve satisficing can be found in Day (1967). In Day's model, however, the decision maker keeps attempting to improve his lot until the incremental improvement (in profit) falls below some satisfactory level. As this "satisficing level" is set smaller, the outcome approaches optimal "marginal costs equals marginal revenues" solution. In our model, the satisficing level is a fixed revenue level, rather than a differential level; in the turbulent environment, the retailer simply scrambles to keep his head above the threshold. The implications of replacing the satisficing rule with an optimizing rule is discussed in the concluding section.

Successful Innovations

The decision maker can make a good decision, once prodded, in that the decision results in improved revenues: the scrambling is astute. In the context of the spatial competition model, this means that the store attractiveness can be increased, drawing in more revenue\(^9\). Whether this increase is the result of simply resetting some marketing mix variable, or a truly innovative qualitative change, does not need to be specified. For a food retailer, this might mean a change in advertising strategy (perhaps to increase consumer’s sensitivity to travel costs), or an introduction of new high-margin delicatessen departments, or the extraction of wage concessions from unions followed by price reductions. It might also

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\(^9\)The implication, of course, is that the revenue more than offsets the cost of implementing the decision, so that profits increase. The model can be implemented with profits, rather than revenues, and an example is provided in the results section. The issue of costs, however, is outside the main focus of this work.
be an imitation of a successful strategy—in some Western Canadian and U.S. cities, Safeway introduced its own low-cost superstore chain, Food-For-Less, in response to the introduction of Superstores.

**Exogenous Shocks**

This leads to the fourth feature: the nature of the shocks. In a share attraction model, the total market demand may be constant, allowing the modeller to focus on the distribution of share among firms\(^\text{10}\). Alternately, the customers may be given a choice that is outside the industry, in the form of an additional term in the denominator of the share expression. This device is used by Choi, DeSarbo, and Harker (1990,1992) to introduce price elasticity into their logit model of spatial competition. Choi et. al. refer to this as a "no purchase option" and use it to preclude the possibility of an equilibrium with infinite prices and infinite profits. In our model, the additional term is used to model the effect of "continual erosion", as described by Corstjens and Doyle, by incrementing the term each period at a randomly chosen store.

**Dynamics Algorithm**

At time \(t = 0\), the system is initialized by assigning store sizes randomly to all the stores in the plane. This ensures that the results do not depend on a uniform distribution of store sizes. Revenues are then calculated for each store. A revenue threshold is initialized at some fraction of this initial revenue. The actual value of the fraction doesn't matter—it is only set different from unity in order to investigate the transient behaviour of the system. Because of the random initial store sizes, each store starts with different revenues, and will have a different threshold. Again, this guards against results arising from uniformity in the model.

\(^{10}\)See Bell, Keeney, and Little (1975) for an axiomatic development of the share attraction model and a discussion of its features and limitations.
Stores are shocked by the addition of an increment \( \delta k \), which remains constant over stores and time, to \( K_j \). Stores innovate, when their revenues drop below their individual revenue thresholds, by the addition of an increment \( \delta S \), fixed over stores and time, to \( S_j \). After initialization, the following algorithm is implemented.

**ALGORITHM: SCRAMBLING ASTUTEELY**

1. Shock a store chosen at random.
2. Calculate revenues of all stores.
3. If all stores have revenues above their threshold, increment time and return to 1.
4. All stores whose revenues have dropped below their threshold innovate.
5. Return to 2.
4. Results of Numerical Experiments

In this section, the characteristics of the model are examined. First, the transient behaviour is examined in terms of total system revenues to demonstrate convergence. Then the size distribution of avalanches in the steady state is examined, and found to follow a power law. The robustness of the power law distribution to model changes is then investigated. Finally, the sensitivity of the system to initial conditions is examined by introducing a small perturbation at a single point in time, and then tracking the subsequent evolution in state space.

In most of this section, the concern is with the behaviour of the system as time approaches infinity, that is, the steady state of the system. We must first, therefore, examine transient behaviour, so that we can be assured that any transients have decayed before we start to count responses.

4.1 Transient Behaviour

The nature of the environment and the decision process ensure that each firms’ revenues will eventually be close to its threshold level\(^{11}\). To demonstrate that the system will actually converge on this general region, the threshold level is set away from the initialization level, and the total revenues in the system monitored over time. In the following, the threshold is set at 80\% of the initial revenues. The parameters used in this run are shown in Table 1.

Figure 2 shows the sum of the revenues of all 64 stores over 1000 iterations. Note that it does indeed converge, and to 80\% of the starting value, as expected by the setting of

\(^{11}\)Close means within a distance determined by the size of the revenue response to shocks and innovations.
Table 1: Parameter values for transient test.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 3</td>
<td>STORE SEPARATION IN x &amp; y. These are the number of customer origin points between each store in the x and y directions.</td>
</tr>
<tr>
<td>23, 23</td>
<td>MARKET GRID SIZE. The number of customer origin points in the market in x an y directions. This gives a store grid of 8 x 8, or 64 stores.</td>
</tr>
<tr>
<td>1, 1</td>
<td>EXPONENTS of DIST &amp; SIZE. Alpha and beta in the choice model.</td>
</tr>
<tr>
<td>30</td>
<td>SEED. Initializes the random number generator.</td>
</tr>
<tr>
<td>3.0, .0001</td>
<td>SIZE INITIALIZATION. Size = a + b*ran; a, b are the parameters and ran a random number between 1 and 32,767.</td>
</tr>
<tr>
<td>4.5</td>
<td>RESERVATION DISTANCE. In units of customer origin points.</td>
</tr>
<tr>
<td>1000</td>
<td># OF ITERATIONS. Total number of exogenous shocks delivered.</td>
</tr>
<tr>
<td>0.8 0.2</td>
<td>MINREVENUE and INNOVATION. The threshold level and the size of a,</td>
</tr>
<tr>
<td>0.05 1</td>
<td>ADVERSITY. Initial value of K; and size of shock.</td>
</tr>
</tbody>
</table>

The break in the slope around period 150 indicates the point where enough firms have been driven below their revenue threshold, and consequently innovate, to noticeably slow the decline of total industry revenues. Up to that point, only the external shocks have any impact on revenues, and so they are continually being driven down. As more and more firms become involved in innovation, the total revenue approaches its steady state value. Note that there always remains some variation—the steady state is stochastic. This highlights another feature of this system that is characteristic of complex extended systems that display SOC. The degrees of freedom remain high (although each dimension is range restricted), even after the self-organized state is reached. As BCSW point out, "This is in contrast to a macroscopic description of economics in terms of a few global variables, where micro economic
fluctuations are assumed to average out in the final analysis”.

We also tested the robustness of system convergence to more intense shocks and more powerful innovations. Results were essentially the same with shock and innovation magnitudes 50 times and 500 times greater than those shown above, except that convergence was more rapid, and the break in slope where firms commence innovation occurs sooner.

4.2 Size Distribution of Avalanches of Innovation in the Steady State

A shock to an individual firm’s market share through the attraction function reduces that firm’s revenues. This reduction may or may not drive the firm to react, depending on whether or not its minimum revenue threshold is crossed. If it doesn’t react, another shock is delivered to another random site. If it does react, however, it increases its intrinsic
attractiveness, and captures share (and hence revenues) from not only the extrinsic sources (K), but from any competitors with whom it shares customers—that is, any stores that have overlapping trading areas with its own trading area. This causes a reduction in revenues of those stores, some of which may be also be driven to respond. In this way, it is possible for innovative responses to cascade across the market.

Once the system is in steady state, we would like to examine the distribution of avalanche sizes. This involves counting the number of stores that innovate after each shock and plotting a frequency distribution\textsuperscript{12}.

\textbf{Avalanche Size Distribution---Base Case}

Figure 3 is a log-log\textsuperscript{13} plot of the size distribution of the avalanches at steady state for the parameters given in Table 1, except that 1300 shocks are delivered and the innovation size is 1.0; this gives avalanches for 776 of the 1300 shocks. The striking feature is the apparent power law behaviour. The probability of large responses does not fall off exponentially, but rather with an exponent of about -1.6 (the slope of the regression line, $\gamma$

---
\textsuperscript{12}A technical issue is the relative sizes of the shocks and innovations. Since each store is only allowed to innovate once for each shock, the shocks must not be too large relative to the innovation, or the stores will never be able to fully recover. Customer allocations will eventually all go to the external competition, and revenues in the system will approach zero. If the shocks are relatively small, on the other hand, it will take many shocks to drive revenues down after an innovation. Since we are only interested in counting innovation avalanches, small shocks with no response place extra demands on computer resources. In practice, the relative sizes of the shocks are set so that about half of them produce at least one innovation. This is a compromise between keeping computer runs from being expensive, and allowing the system to recover between shocks. We have experimented with different relative sizes to ensure that results are not sensitive to this choice, and results are available from the authors.

\textsuperscript{13}Logarithms are to base $e$. 

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Figure 3: Size Distribution of avalanches in the steady state, for the base case, with approximately 60% of shocks producing avalanches superimposed on the plot\textsuperscript{14}). This is the footprint of self-organized criticality (Bak, Tang and Weisenfeld, 1988; Bak and Chen, 1991; Chau and Cheng, 1992).

Many of the numerical experiments conducted with this model show "finite size effects" (Bak, Tang, and Weisenfeld 1988) to varying degrees. Propagating cascades of innovations may be prematurely truncated when they encounter a system boundary. The effect appears as a deviation from power law behaviour at large avalanche sizes, i.e., there are fewer observations in which a large number of stores innovate in a time period. The problem is a generic one of trying to infer system behaviour in the large limit, using only a finite system. Krider (1993) shows that the deviation here from power law behaviour at large

\textsuperscript{14}The regression line and reported slope on this, and subsequent plots, are intended only as reference points to help with interpretation of the data.
avalanche sizes is consistent with the finite size effect.

4.3 Robustness to model parameters

One of the characteristics of the self-organized critical state is that the power law distribution is relatively insensitive to model details. We next investigated the effect of various values of the size and distance exponents, and of the reservation distance, on steady state behaviour. If the steady state we observe here is the same as the steady states labelled "SOC" that appear in the physical and life sciences, we should expect that the power law distribution holds over a range of parameter values. And, if it does hold over a range of parameter values, the generality (and consequent likelihood of empirical observation) of the state is increased.

Empirical estimates of the exponents of the spatial interaction model have been made in a variety of contexts. In Huff's (1962) original work, the size exponent $\alpha$ was assumed to be unity, and the distance exponent estimated. In suburban Los Angeles, a beta value of 2.6 to 3.7 was found for clothing stores, and 2.1 to 3.2 for furniture stores. Haines, Simon, and Alexis (1972) estimated beta, again assuming alpha fixed, for grocery stores in various suburban and inner city neighbourhoods in a U.S. city. They found values between 0.5 and 1.8. Jain and Mahajan (1979) estimated a multiattribute model for supermarkets in a "large northeastern metropolitan area", and found alpha values between .02 and .56 for their four intrinsic attractiveness attributes (sales area, number of checkout counters, credit cards accepted, and intersection location), and a beta value of 0.3, for the distance exponent.

In summary, the range of exponents estimated empirically is from near zero to three, which defines the range of values of interest for sensitivity tests.

The reservation distance is a limit on how far the customer is willing to travel. It
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Range</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>size exponent</td>
<td>0.5 - 3.0</td>
<td>Determines customer sensitivity to &quot;size&quot; or &quot;intrinsic attractiveness&quot; of store. Empirical estimates of $\alpha$ are in this range.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>distance exponent</td>
<td>0.5 - 3.0</td>
<td>Determines customer sensitivity to distance. A larger value means store attractiveness drops more rapidly with distance, implying weaker competitive interactions between stores. Range is representative of published estimates.</td>
</tr>
<tr>
<td>$R$</td>
<td>reservation distance</td>
<td>2.9 - 6.5</td>
<td>Maximum distance customers will travel to a store. Units are related to &quot;customer origin points&quot;, which are separated by 1 unit of distance, and store locations, which are separated by 3 units. $R$ also is the radius of a store's trading area. Like $\beta$, $R$ affects the strength of competitive interaction. At the low value (2.9), no store is within any other's trading area, and each has a monopoly in a small area (of radius 0.1). If $R$ dropped to 1.5, the system would be entirely decoupled, and each store would be a monopolist. When $R$ is 6.5, the trade area is 5 stores in diameter, which approaches the size of the 8 x 8 system.</td>
</tr>
</tbody>
</table>

determines the number of stores to which the customer allocates his expenditures. From the stores' point of view, it represents the radius of the trading area. The larger the reservation distance, the more stores compete directly with each other. Conversely, the smaller the reservation distance, the more monopolistic each store can become. As the reservation distance decreases, the whole system will eventually decouple.

The base case used a store separation of 3 (measured in terms of customer origin units) and a reservation distance of 4.5. (This puts eight stores in the trading area of every store not on the boundary of the system). Reservation distances of 6.5 (with 20 competitor stores)
stores in a trade area), and 2.9 were also tested. The latter is small enough that, even though stores share customers, no store is actually in any other's trade area.

The parameter meanings and ranges investigated are summarized in Table 2.

![DISTRIBUTION OF AVALANCHE SIZES](image)

**Figure 4:** Avalanche size distribution when size exponent is 0.5.

Figures 4, 5 and 6 show selected results for the three parameters investigated. The ranges of parameters investigated and a compact summary of resulting size distributions is presented in Table 3. With two exceptions, to be discussed next, the power law distribution of the number of innovations was observed over these parameter ranges.

We note that large values of beta and small values of the reservation distance imply weak competitive coupling between the stores. At some point, for example, the reservation distance will become small enough that the system will become entirely decoupled. We would expect the SOC state to become progressively more difficult to attain as the competitive coupling decreases, and in fact (see Table 3 and Figure 7), the power law starts to break down in these limits. An interesting technical issue (beyond the scope of this
Figure 5: Size distribution when distance exponent = 2.0. The departure from linearity at large avalanche sizes is the finite size effect.

Figure 6: Distribution of avalanche sizes with reservation distance = 6.5.

research) is the system behaviour as the stores become independent monopolists. Does the SOC state hold as long as there is at least one customer with divided loyalties? It would
### Table 3: Summary of parameter sensitivity tests.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>β</th>
<th>R.D.</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>1</td>
<td>1</td>
<td>4.5</td>
<td>power law size distribution</td>
</tr>
<tr>
<td>Alpha tests</td>
<td>0.5</td>
<td>1</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>1</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td>Beta tests</td>
<td>1</td>
<td>0.5</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.0</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.0</td>
<td>4.5</td>
<td>deviation from power law at large sizes</td>
</tr>
<tr>
<td>R.D. tests</td>
<td>1</td>
<td>1</td>
<td>2.9</td>
<td>steeper slope, deviation from power law</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>6.5</td>
<td>similar to base case</td>
</tr>
</tbody>
</table>

**Figure 7:** Distribution with a small reservation distance. Note curvature of data points compared to "power law" line.

Undoubtedly require very long computer runs to answer this question; and in any case, the rationale for monopolists "scrambling astutely" is unclear.

#### 4.4 Robustness to Model Structure
Is the steady state dependent on the particular dynamics and model structure used in the preceding? To investigate this question, a second model was implemented. Rather than incrementing the size, or intrinsic attraction only, the increment was added to the complete attraction function. This represents an innovation that uniformly increases the store's attractiveness to all its customers. The customer now allocates expenditures according to

\[
M_{ij} = \begin{cases} 
\frac{a_j + S_j^\alpha D_{ij}^\beta}{\left( \sum_k (a_k + S_k^\alpha D_{kj}^\beta) \right) + K_j}, & D_{ij} \leq R \\
0, & D_{ij} > R
\end{cases}
\]  

(6)

where the increment is now added to \( a_j \).

The model structure was also changed to introduce a cost linear with \( S_j \), and the decision to innovate based on profits rather than revenues. Since the store sizes are all initialized at different values, the costs, profits, and profit thresholds will all be different across the system.

Despite the fact that stores are now making decisions on profits, and that the innovations affect the total attractiveness (rather than just intrinsic) of the store, the power law distribution of the number of stores innovating again emerges. This may be seen in Figure 8, which uses the base case parameters (\( \alpha, \beta = 1; \) reservation distance = 4.5). The model was also run with a variety of other parameters. The distribution only departs substantially from the power law when the reservation distance becomes small, or the distance exponent large; that is, when the stores are only weakly coupled through competition in the market. In all other cases, the distribution of avalanche sizes followed a reasonable power
Figure 8: Changing the form of the innovation dynamic and the threshold from profits to revenues does not affect the avalanche size distribution.

law. These results parallel the original model. The steady state is robust to these changes in system structure and dynamics, which, once again, is consistent with other SOC models, and highlights the generality of the state.

4.5 Sensitivity to Initial Conditions

We would like to know what happens at steady state in this market when the "butterfly flaps its wings". When two realizations of the SOC system start out very close to each other in state space, do they converge or diverge? To test this, the system was initialized and run for 200 iterations (shocks) to reach steady state. At this point, a copy was made of the system and perturbed by reducing $K_j$ by for each store by a small amount. The two systems were then allowed to run for another 300 iterations. Each system receives the identical sequence of random shocks. The state space examined was the 64 dimensional space of store revenues. The Euclidean distance between perturbed system and the original system, given by
\[ D = \sqrt{\sum_{k=1}^{64} (R_k - R'_k)^2} \]  

was calculated each period, where the prime indicates the perturbed system. This results in a 300-period trace measuring the separation of the two systems over time.

Any single realization of the separation trace fluctuates wildly, with large spikes, because a store in one system often will innovate before its counterpart in the other system, causing a temporary, maximal separation along that coordinate, and a consequent large separation in the distance measure. To reduce the effect of these large spikes, an ensemble average of 100 different realizations\(^{15}\) of the separation trace was taken. The average trace, shown in Figure 9, indicates that the systems diverge and are hence chaotic; however, the rate is constant (or power law with exponent one) rather than exponential, and so is only weakly chaotic. Consistent with SOC in other fields, we may say this system, in steady state, is on the edge of chaos.

The reader will recall that we have stated previously that the system converges to the critical state, and that we showed this by examining the transient behaviour of total revenues. The divergence analyzed above is for two systems at the critical state. In other words, the critical state is an attractor for the entire dynamics, but the state itself is weakly chaotic. In steady state, each store’s revenues will be above its threshold, but below that threshold plus an increment determined by the innovation increment. This defines (in the case above) a 64 dimension hypercube in state space where the system will be located at steady state. If the system is outside this hypercube in the negative direction on any dimension (a store is below

\(^{15}\)Each trace was developed from pairs of systems which had a different seed for the random number generator across pairs, but the same seed within pairs.
Figure 9: Divergence of two systems at steady state with small initial separation. Ensemble average of 100 realizations shows linear divergence rate in revenues state space. The threshold, it will spontaneously track back into the hypercube (the store will innovate until above the threshold). If the system is outside the hypercube in the positive direction on any dimension (a store is much above its innovation threshold), the shocks will eventually drive it back into the hypercube. In this sense, the hypercube is an attractor for the dynamics. Once at steady state, the system bounces around inside the hypercube. The trajectory of the systems here are the ones which are now weakly chaotic.
5. Concluding Remarks

The primary contributions of this research include the modelling of a facet of retail dynamics often discussed, but not modelled, and the demonstration of steady state model behaviour known as self-organized criticality in a marketing context.

The conventional wisdom that much of competitive retailing involves repositioning in response to unexpected adversity, or scrambling astutely, is formalized by a shock-and-innovation dynamic embedded in a spatial competition model. To the best of our knowledge, this is the first attempt to formally capture this type of behaviour. The resulting stochastic steady state is characterized by a continually innovating industry, with firms innovating in response to the shocks, in waves, or avalanches, of various sizes.

The model's steady state has some interesting characteristics. The exogenous shocks driving the system are delivered at random to firms in the model industry, so it is natural to look at distributions of responses. The probability of a large response, where the size is measured as the number of firms responding to a single shock, declines according to a power law. The model identifies and describes a mechanism where the law of large numbers does not apply, as one would expect an exponential decline in that case. In other words, the likelihood of a system-wide wave of innovation in response to a small shock is relatively high.\textsuperscript{16} From the viewpoint of an individual firm, this means that distant small events may

\textsuperscript{16} The possibility of waves of innovations have been addressed by Judd (1985) and Shleifer (1986). In both articles, "innovation cycles" occur. However, the cycles are driven by almost opposite mechanisms. In Judd's model, the introduction of too many new products within a short time causes competition for consumer resources, and reduces profits for each one. After a period of innovation, entrepreneurs delay introducing new products until introduction is again profitable. Shleifer's model, on the other hand, is driven by expectations of large profits. If firms share beliefs about the timing of a boom, they can make the boom a reality by releasing innovations. Both models differ from ours in the driving mechanisms and the nature of the response. A consequence that our model shares with both is that the results, as Shleifer puts it, "shed doubt on a frequently articulated view that a market economy smooths exogenous shocks."
ultimately impact the firm with a relatively high probability, highlighting the importance of
monitoring distant events, and the importance of being able to "scramble astutely".

The high probability of industry-wide waves of innovation parallels the observation
that retailing as a whole tends to undergo dramatic shifts, such as described in the December
21, 1992 Business Week cover article, "Clout! How Giant Retailers are Revolutionizing the
Way Consumer Products Are Bought and Sold". The fact that a micro-level dynamic,
motivated by the conventional wisdom that retailers must "scramble astutely," leads to a
stochastic steady state involving sweeping changes at the macro-level, which also parallels
conventional wisdom, is intuitively appealing.

The steady state has the characteristics of self-organized criticality, and the research
therefore contributes to the SOC literature (and, more generally, the "complexity" literature)
by providing, to the best of our knowledge, the first example of SOC in marketing, and the
second in economics. Furthermore, SOC arises here from a purely marketing model, whereas
the previous economic example (BCSW, 1992) is mathematically isomorphic to the prototype
sandpile model of statistical physics.

SOC is of interest not only because of its rather unusual characteristics, but because
it is quite robust to parameter values and model details. Furthermore, it appears to be a
general organizing principle that has been applied to models in many different areas of the
physical and life sciences. While the models vary widely, two common features are the
dynamic interaction of entities, and many degrees of freedom; these are characteristic of many
marketing situations. This, plus the robustness and generality of SOC, increase the probability
of it occurring in marketing situations.

Extensions to the model, such as positive shocks, ineffective response and entry and
exit are relatively easy to simulate. The extra layers of complexity, however, increase the
difficulty of exploring the model and generalizing results. Nevertheless, insightful ways of exploring these issues would be interesting.

A more important extension, however, is to investigate optimizing, rather than satisficing, behaviour. Furthermore, although the model assumes the ability of management to make good decisions, it captures the decision through its impact on the attraction function. This approach allows for any type of change, for example, an adjustment of a specific marketing mix variable, or a qualitative and innovative change in strategic direction. Optimizing on explicit marketing mix variables is an important next step for this line of research to lead to normative results. The models of theoretical biology, which incorporate short-term optimizing, and relate SOC to Nash states, will be helpful (e.g., Kauffman and Johnson 1992). The likely approach is to consider managers who operate on the basis of short-term profit maximizing heuristics, and to compare the long run industry behaviour (such as static Nash equilibrium, cyclic, SOC, or no orderly steady state) on the basis of system-wide profits. On the face of it, we might expect that neither the rigid, unchanging economic system of a static Nash equilibrium, nor a completely disordered, strongly chaotic system would be optimal. In fact, the hypothesis one would infer from the biological literature is that the critical state, poised on the edge of chaos, provides the highest system profits\(^7\). This would have obvious policy implications: how much and what kind of control does the economy need to achieve the critical state? Or, if left alone, would it self-organize to that state, by virtue of system profits being highest?

The numerical method of analysis, which allows the investigation of complex models, lacks the generality of analytic methods. Analytic approaches to SOC is an active area of

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\(^7\) Routledge (1993), in a finance context, has shown behaviour similar to Kauffman and Johnson's in a model of many agents playing a repeated prisoner's dilemma game.
research in statistical physics, and BCSW utilize some of this work to demonstrate the Pareto-Levy distribution of avalanche sizes in their producer-supplier network model. The analytic work to date is in the context of relatively simple discrete cellular-automaton models. It is not at all clear that it will be applicable to the more complex model described here, and it is therefore likely that numerical approaches will dominate for the foreseeable future.

The final issue is empirical. A crucial problem in detecting the kind of behaviour implied by this research, whether in the context of retailing or elsewhere, is that of counting avalanches. In this, and other SOC models, each shock triggers a single avalanche, which comes completely to rest, before another avalanche is triggered. This is unlikely to occur in any real setting. If shocks are occurring more rapidly, avalanches may run into each other. Even if they don’t physically interfere with each other, there is the likelihood that several avalanches in any large system are occurring simultaneously and econometric data sets will only record the aggregate response. The general question of aggregation of responses is also an active area of research in statistical physics. It is well known that when events, or pulses, of various shapes and sizes in time, are superimposed at random starting times, to produce a time series, the resulting series is highly correlated. The correlation is most often expressed in the frequency domain in terms of the power spectra, which can be shown to be a power law (A. van der Ziel, 1950). The power law behaviour of power spectra arising from aggregates of the pulses produced by systems in the SOC state has been an important part of SOC research from the beginning. Exponents between one and two were suggested originally by Bak, Tang, and Weisenfeld (1988), depending on the dimensionality of the system. Chau and Cheng (1992) suggest that whether the exponent is one or two depends on whether the
model is continuous or discrete.\textsuperscript{18}

Relating power law power spectra in economic time series to SOC models seems to be the best hope for empirical support for the model. The difficulties inherent in sorting out the values of the exponents are compounded by the possibilities of other explanations. Random walks, for example have power spectra that obey power laws, with the simple Gaussian random walk having an exponent of two (Mandelbrot, 1982). This implies that the SOC explanation for power law behaviour will, at least, have to be contrasted with existing explanations of the origin of random walks.

\textsuperscript{18}Some initial work on aggregate traces produced by superposition of the pulses generated by the model in this chapter gives a power spectra of $1/f$, that is an exponent of 1. By comparison, a time series of monthly Canadian bankruptcy data for retail businesses for 12 years has a power spectra at low frequencies that goes as $1/f^2$. The hypothesized connection is that if some fixed proportion of the firms involved in each avalanche are unable to effectively innovate, and exit the system, they will show up in bankruptcy data.
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