Grocery Shopping Behaviour in the Presence of a Power Retailer

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Abstract

The "power retailing" phenomenon, which refers to very large retailers that compete primarily on price, has received much attention in the popular and trade press, and more recently, from academics. The academic concern so far has been with relative power in the channel. In contrast, we take a modelling approach to examine the effect on horizontal competition and customer shopping behaviour in the grocery retailing industry. Assuming rational cost-minimizing consumers, who purchase both perishable and nonperishable goods from two stores differentiated by price and location, we derive the consumers' optimal shopping policies. We show how it may be optimal for consumers to have a planned multi-store shopping policy, even when the stores sell identical goods and there is no price uncertainty. As well, the market share exhibits increasing returns to scale to price reductions, but only when price differentials become large enough. This is offered as a possible reason for the unexpected (to the incumbents) success of power retailers in the grocery industry.

1. Introduction

"Clout! More and More, Retail Giants Rule the MarketPlace", claims a recent Business Week cover story. To quote, "In category after category, giant "power retailers" are using sophisticated inventory management, finely tuned selections, and, above all, competitive pricing to crowd out weaker players" (Business Week, December 21, 1992, p 66). Marketing academics, too, have recognized the phenomenon. The academic emphasis, however, has been primarily on the channel relationships. For example, Farris and Ailawadi (1992) discuss the apparent redistribution of channel power to the retailer. The nature of horizontal
competition, and the impact on consumer shopping behaviour has yet to be investigated. The research in this paper addresses these issues in the context of food retailing.

Specifically, a modelling approach is taken to answer two related questions: first, "How does consumer shopping behaviour change when a power retailer enters the grocery market?"; and, second, "What is the effect of the presence of a power retailer on share of market?". Assuming cost-minimizing shoppers, for whom three costs are important (trip costs, inventory costs, and price), and recognizing that some goods are perishable, the model provides several important insights. First, stockpiling behaviour becomes dramatically more important as the price reduction of the power retailer becomes greater, leading to an "increasing returns" effect of price reductions. As a result, the market entry of a power retailer may have a much greater impact than that anticipated by existing retailers. Second, the difference between perishable and nonperishable goods can lead some consumers to have normative strategies of shopping at more than one store, even when there is no uncertainty and the stores offer the same goods. It is also shown that perishable goods are much less susceptible to the influence of a power retailer entering the market, than are nonperishables.

Food retailing is an appealing industry to begin an investigation of power retailing phenomena for at least three reasons. First, total demand for groceries is relatively inelastic compared to many other retail goods (Ghosh and Harche, 1992). This allows modelling to focus on the allocation of the food dollar to the competing retailers. Second, the food and drug category of retailing is relatively well defined. It will be helpful, for subsequent empirical work, to have well defined market boundaries. Third, the effect of power retailing in the food industry has been not only dramatic, but, to some extent, unanticipated.

One case which helps illustrate and motivate this research is the entry of the Real Canadian Superstore (RCS) to the western Canadian grocery retailing industry, during the last
decade. In the major cities of western Canada (Winnipeg, Calgary, Edmonton, and Vancouver), the retail grocery business was dominated by supermarkets, led by Safeway, in the early eighties. The supermarkets were typically 20,000 to 40,000 square feet, and traded in a relatively localized area of a few kilometres in diameter. Price competition appeared to be healthy, with much promotion and advertising. RCS entered various cities in the mid-

![Image of graph showing Edmonton grocery prices, 1992.](image)

**Figure 1:** Weekly prices of a food basket of approximately $100 value, at two supermarkets, relative to the price of the same basket purchased at RCS, the power retailer, in Edmonton, Alberta, in 1992. (Week 1 = Jan 6)

eighties, with an expansion westward, opening in Edmonton in 1984, and in Vancouver in 1989. RCS stores were over 100,000 square feet, of which about 60% to 70% was devoted to food and drugs. Labour and land costs of the RCS outlets were kept dramatically lower than Safeway’s, by employing non-union staff, and locating in industrial areas. As shown in Figure 1, this led to prices at RCS as much as 30% lower at two main established
supermarket chains, Safeway and IGA (Satanove, unpublished). The price reduction, as well as some increase in variety, appeared to make the outlets a destination, so that the unfavourable locations were a relatively minor disadvantage. RCS, like power retailers in other categories, drew substantial market share with very few outlets. DATA HERE

One of the interesting aspects of the entry of RCS to the market, and one which a model should be able to explain, is Safeway's early reaction. According to private communications with consultants and academics in Winnipeg, Edmonton, and Vancouver, Safeway was surprised by the impact of RCS on the market. The modelling approach in this chapter suggests a reason: the market's highly nonlinear response to price differences that arises when consumers are allowed to stockpile. When price differences between stores are small, consideration of consumer stockpiling has negligible effect, and models (formal or intuitive) of trading areas and market share that consider only price and travel costs work well. As price differences increase, however, the dynamic effects of stockpiling become important. Consumers at unusually long distances from the Superstore can make large purchases, in order to make the long trip less frequently, and thus keep long-run average trip costs down. If the price advantage is great enough, the stockpiling costs will be offset, and the power grocery retailer can capture a huge trading area.

The following section relates this work to the previous literature. Section 3 develops the model. Preliminary results are derived in Section 4, and the complete solution for the full model provided in Section 5. Section 6 uses a numerical example to illustrate the implications of the model for trading areas and shopping behaviour, and Section 7 shows the increasing returns effect on market share. Results are summarized and directions for future research indicated in Section 8.
2. Relation to Previous Literature

The model developed in this research draws on literature in three areas: consumer stockpiling in response to promotions, normative inventory control theory, and multipurpose shopping.

Promotion and Stockpiling

A common concern in the retail promotion literature is that consumers may accelerate their purchases in time or quantity as a response to promotions, and that such stockpiling behaviour may not be beneficial to the retailer or manufacturer (e.g., Neslin, Henderson and Quelch, 1985; Gupta, 1988). While the short term focus on opportunistic behaviour is different from the long term, planned behaviour investigated in this article, some of the results in the promotions literature are relevant to our research\(^1\). First, the literature establishes the value of normative inventory models in consumer shopping (e.g., Meyer and Assuncao, 1992). Second, it establishes that consumers make tradeoffs between the costs of the goods purchased ("price costs") and inventory holdings (e.g., Blattberg, Eppen, and Lieberman, 1981). Third, it recognizes the "fill-in trip", a minor trip that occurs between major trips, which has a parallel with the shopping behaviour studied in this research (e.g., Kahn and Schmittlein, 1992). It should be noted that the existing work defines the difference between trip types, whereas in the research reported here, the different trip types arise endogenously as a result of optimizing behaviour by consumers. Finally, this literature is also useful for structuring consumer's holding costs for nonperishables (e.g., Raju, 1992). Although perishability is recognized as a relevant factor in the promotion literature, it has not previously been explicitly treated as an inventory cost.

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\(^1\)For an excellent current review of the promotions literature, see Blatteberg and Neslin, 1993.
Inventory Control

As an inventory control problem, the situation modelled here would fall into the class of joint replenishment problems (JRP), which, according to Atkins (1993), "is among the most studied of inventory problems." See Muckstadt and Roundy (1993) for a survey of this literature.

Joint replenishment refers to the problem of maintaining inventory of more than one good, when there is the possibility of reducing replenishment costs by ordering the goods simultaneously. The problem analyzed here is different from the solved JRP problems in the types of goods (perishable and nonperishable) analyzed, and in the focus of the cost structure on consumers (prices differ between stores), as opposed to firms (variable costs assumed constant). Our model is a member of the subclass of JRP’s with continuous time, an infinite horizon, and deterministic constant parameters. Optimal policies for this subclass are difficult to find, and almost all research has concentrated on finding optimal periodic polices, and then showing that these are "good" policies. To the best of our knowledge, this work represents only the second nontrivial case where the periodic policy is shown to be generally optimal (the first is Andres and Emmons, 1975).

The second link to the operations research inventory control literature is to the extensive work on perishable goods (see Nahmias, 1982 for a review). This literature is concerned with optimal replenishment times when goods deteriorate slowly, or expire suddenly, as assumed in this paper. The form of the perishability costs are taken from this literature.

Multipurpose Shopping

Multipurpose shopping refers to the observation that consumers will make a single trip to a central location to purchase more than one good or service, resulting in retail
agglomeration. Early work, based on Hotelling (1929), focused on tradeoffs between price and travel costs; more recent work includes inventory costs as well (Ingene and Ghosh, 1990). The cost minimization framework has recently been used (McLafferty and Ghosh, 1986; Ingene and Ghosh, 1990) to show explicitly how qualitatively different kinds of retail centres might arise, as suggested by Central Places theory (Christaller, 1933). The model developed in this research is similar in spirit to the multipurpose shopping literature in that consumers are assumed to make a choice on the basis of cost minimization over transportation, inventory, and price costs. It differs in its objectives (choice between stores differentiated by price, rather than between "order", or level of agglomeration, of centre), the detailed cost structures, and types of goods (perishable vs nonperishable) considered. Furthermore, the optimization method in the multipurpose shopping literature involves relaxing the constraint that the number of trips is a nonnegative integer, so that calculus may be applied. In this article, the integer constraint is retained.

3. Model Development

Assumptions, notation, and rationale for the model components follow.

Supply Characteristics: Goods, Stores, and Prices

Two goods, one perishable and one non-perishable, are available to consumers. Subscripts \( *_p \) and \( *_n \) denote perishable and non-perishable goods respectively.

Each of the two goods is available at two stores, which, in keeping with the power retailer theme, are identified as the "large" and the "small" store, with subscripts \( *_l \) and \( *_s \). The two stores are differentiated by location and by price, with the small store the most expensive on both goods. Thus, there are four prices: \( P_{n,l} \), \( P_{n,s} \), \( P_{p,l} \), and \( P_{p,s} \).
**Demand**

The consumer's demand is determined by the constant consumption rates, $D_n$ and $D_p$, for each of the two goods. Also, the consumer never goes hungry, so she must always have some of each good available (i.e., no consumer stockouts are allowed).

**Consumer Cost Structure**

Consumers minimize long-run average costs over an infinite horizon. Total costs consist of trip costs $C_t$, plus storage costs $C_s$, plus the price of the goods $C_p$.

**Trip Costs:** Trip costs consist of a fixed amount $c_s$ or $c_l$ incurred for each trip to the small or large store. Later, trip costs will be assumed proportional to the Euclidean distance between customer and store, but initially, they will remain general. It should be noted that trip costs correspond to order costs or set-up costs in the inventory management literature, but that, unlike the inventory management literature, the costs are *not* tied to the goods. In particular, once the customer is in the store, there is no further cost whether she purchases one or both of the goods.

**Non-perishable good inventory costs:** Consumers have quantity-dependent instantaneous storage costs $s \cdot Q_a(t)$ for the non-perishable good, where $s$ is the cost per unit quantity per unit time, and $Q_a(t)$ is the quantity of the non-perishable good on hand at time $t$. This serves to limit the quantity purchased, which would be unlimited if only trip costs were considered. Note that since price appears explicitly elsewhere in the model, this formulation makes storage costs NOT dependent on price. This is in contrast to the inventory management literature, where it is implicitly assumed that the holding costs include the cost of capital; and in contrast to the multipurpose shopping models (e.g., Ghosh and McLafferty, 1987), where the holding cost is proportional to the monetary value of the stock. Quantity-dependent holding costs are used here for three reasons. First, given that the frequency of
grocery shopping is on the order of weekly, it seems unlikely that the cost of capital would play a significant role in limiting the amount of consumers' purchases compared to storage and transportation capacity limitations (how big is the car and the cupboard?). Second, an objective of this research is to investigate the effects of price differences between stores, and that objective is facilitated by keeping price as a separate component. The third reason is empirical. Litvack, Calantone and Warshaw (1985) and Blattberg, Eppen and Lieberman (1981) both conclude that "bulkiness" lowers the incentive to stockpile in the presence of promotions. More recently, Raju (1992) investigated how the variability in category sales, where the variability is largely due to price discounting, depends on category characteristics. While the relationship between category expensiveness and variability was not statistically significant, bulky categories had significantly lower variability. To the extent that sales variability reflects a stockpiling response to discounts, this supports the modelling of storage costs as quantity-dependent and price-independent.

Storage costs over time are given by

\[ \int_{0}^{T} sQ(t)dt \]  

(1)

An amount \( Q_n \) purchased and immediately consumed at the constant rate \( D_n \) will have a storage cost (Figure 2):

\[ \int_{t_0}^{t_0 + \frac{Q_n}{D_n}} s(Q_n - D_n t) dt = \frac{sQ_n^2}{2D_n} = \frac{sQ_n \Delta t}{2} = \frac{sD_n (\Delta t)^2}{2} \]  

(2)

**Perishable Good Inventory Cost:** The perishable good has a lifetime \( \Delta t \) after purchase, and incurs an inventory cost equal to the price paid for any quantities that expire
before they are consumed. If the expired quantities, which must be disposed of, are given by $Q_{p,\text{expired}}$, the cost is $P_{p}Q_{p,\text{expired}}$. Inventory management models consider both sudden expiry and gradual deterioration. This model only considers the former, which, for groceries has face validity, in that many items are marked with an expiry date. It also seems reasonable that even for goods like vegetables that do not have marked expiry dates, the consumer has the binary choice of consuming or disposing of the product. It is assumed that the quantity-dependent storage costs associated with the perishable good are negligible compared to the other costs in the model, but nonzero. This is consistent with the view that the most important cost for perishable goods relates to their perishability. In other words, the consumer has enough room to transport and store (e.g., in the refrigerator) more perishables than she can consume before they expire, so that the expiry time, rather than the quantity-dependent storage costs, impose the limitation on the amount of perishables purchased on a single trip. The nonzero assumption, which is also reasonable since the perishable goods do take up some room, allows the derivation of analytic results.

**No Discounting:** The time value of money is assumed negligible compared to other costs, and not considered. This is consistent with the promotions literature, the multipurpose shopping literature, and much of the JRP literature. Although the model is infinite horizon, grocery shopping trips occur frequently enough that it is difficult to imagine differences in
the time value of money across shopping situations entering into the consumer’s calculations.

**Decision Variables and Solution Method**

The consumer’s problem is to decide when to shop at which store, and how much to buy of each good. This is formalized by identifying *shopping patterns*, or sequences of visits to the stores (for example the sequence "1) large, 2) small, 3) small, 4) small, 5) large") and associated sequences of types of good purchased (for example, "1) perishable and nonperishable, 2) perishable, 3) perishable, 4) perishable and nonperishable, 5) perishable and nonperishable"), and optimizing over purchase quantities $Q_{n,l}$, $Q_{n,s}$, $Q_{p,l}$, $Q_{p,s}$ and trip timing $t_{n}$ and $t_{p}$. Through a series of lemmas and propositions, the infinity of possible shopping patterns are reduced to a manageable small set of possibly optimal patterns, for which analytic expressions for the cost function are derived. These cost functions can then be minimized over $Q_{(\cdot,\cdot)}$, and the smallest selected as the optimal pattern.

**Domain:** All parameters are positive real. Quantities are non-negative real.

Notation is summarized in Table I.

**4. Preliminary Results**

Before addressing the full problem of choice between two goods and two stores, results for some restricted cases are presented. These results, besides being necessary for the main problem, help to clarify the issues involved and highlight the relationships to existing work. Any proofs not supplied here, and in the remainder of the paper, are available in an appendix from the authors.

**4.1 Non-perishable goods and one store only**

For a *single* good available at a single store, we may ignore price, and the well-known Economic Order Quantity (EOQ) result from inventory theory applies (e.g., Carr and Howe,
Subscripts:
- n: non-perishable
- p: perishable
- s: small (expensive) store
- l: large (cheap) store
- i: visit index to large store
- j: visit index to small store

\[ Q_{n,i}, Q_{n,s}, Q_{p,i}, Q_{p,l} \]: Quantities of each good purchased at each store
\hspace{1cm} \text{(time subscript omitted)}
\[ P_{n,i}, P_{n,s}, P_{p,l}, P_{p,s} \]: Prices of each good at each store
\[ t_{i,j}, t_{j,i} \]: time of \(i^{th}\) \((j^{th})\) visit to each store
\[ \Delta t_{i,j}, \Delta t_{j,i} \]: time intervals between purchases
\[ D_n, D_p \]: consumption rate of each good
\[ \Delta t_e \]: time between purchase and expiry of perishable good
\[ s \]: instantaneous storage cost of nonperishable good
\[ c_i, c_s \]: trip cost to each store
\[ C_t \]: long-run average trip costs
\[ C_s \]: long-run average storage costs
\[ C_p \]: long-run average price costs
\[ C \]: total long-run average costs

1962). Equal quantities are ordered at equal time intervals, with the inventory just at zero at each repurchase occasion ("zero inventory rule"). Omitting the price the customer pays for the goods ("price costs") for now, because they only have an impact when shopping from two sources with different prices, the optimal long-run average quantities, times, and costs are:

\[ Q_n^* = \sqrt{\frac{2cD_n}{s}} \]
\[ \Delta t_i^* = \sqrt{\frac{2c}{sD_n}} \] \hspace{1cm} \text{(3)}
\[ C^* = \sqrt{2scD_n} \]
Now suppose there are two non-perishable goods, differing only on storage costs, \( s_1 \) and \( s_2 \), and purchased at a single store. It is now necessary to consider the possibilities that only one or the other good is purchased on a trip, or that both goods are purchased on a trip. Consider first a fixed horizon, during which time \( f_1 \) purchases of good 1, and \( f_2 \) purchases of good 2 are made. Let the number of trips to purchase only good 1 be \( m_1 \); the number of trips to purchase only good 2 be \( m_2 \); and the number of trips to purchase both goods be \( m_{12} \). We note that \( m_1 + m_{12} = f_1 \), \( m_2 + m_{12} = f_2 \), \( m_1 + m_2 + m_{12} = f_1 + f_2 \), and that the zero inventory rule still holds for each good (if any quantity of the good were on hand at repurchase time, storage costs could always be reduced by purchasing less the time before), so that the integrated storage costs may be expressed as the summation of terms like (2). The consumer's problem is then

\[
\text{MIN} \quad C_T
\]

\[
m_1, m_2, m_{12}, f_1, f_2, Q_1, Q_2
\]

where

\[
C_T = c(m_1 + m_2 + m_{12}) + \sum_{i_1=0}^{f_1-1} s_1 \frac{Q_{i_1}}{2} \Delta t_{i_1} + \sum_{i_2=0}^{f_2-1} s_2 \frac{Q_{i_2}}{2} \Delta t_{i_2}
\]  \( \text{(4)} \)

and, as in the usual single good case, the no-stockout and zero-inventory rules imply constraints on the total quantity purchased over the horizon \( T \):

\[
\sum_{i_1=0}^{f_1-1} Q_{i_1} = Q_1 = D_1 T
\]

\[
\sum_{i_2=0}^{f_2-1} Q_{i_2} = Q_2 = D_2 T
\]  \( \text{(5)} \)

With only one store, and two non-perishable goods with different storage costs, the following
proposition states that the customer purchases both goods each time he shops.

**Proposition 1:** For \( s_1, s_2 > 0 \), (4) above is minimized only when \( m_1 = m_2 = 0 \).

Proposition 1 gives \( f_1 = f_2 = f \) and identical interpurchase time intervals for both goods, \( \Delta t_{i1}^* = \Delta t_{i2}^* \) (although not yet necessarily equal for all \( i \)). Since each pair of interpurchase intervals are the same, we also have \( Q_{i1} = D_1 Q_{i2} \), so we may write the cost function as

\[
C_T = cf + \sum_{i=0}^{f-1} \left( s_1 + \frac{D_2}{D_1} s_2 \right) \frac{Q_{i1}}{2} \Delta t_i
\]

which is identical to the single good cost function, with the storage cost replaced by a weighted sum of the different storage costs. Hence the usual EOQ solutions given in (3) apply, with \( s \) replaced by the weighted sum of \( s_1 \) and \( s_2 \), and therefore interpurchase times are equal for all \( i \). Times, quantities, and long-run average costs for the case of two non-perishable goods available at one store are thus:

\[
\Delta t_i^* = \sqrt{\frac{2c}{s_1 D_1 + s_2 D_2}}
\]

\[
Q_{i1}^* = D_1 \sqrt{\frac{2c}{s_1 D_1 + s_2 D_2}}
\]

(7)

\[
Q_{i2}^* = D_2 \sqrt{\frac{2c}{s_1 D_1 + s_2 D_2}}
\]

\[
C^* = \sqrt{2c(s_1 D_1 + s_2 D_2)}
\]

### 4.2 Perishable goods at one store only

The optimal shopping policy for one perishable good at one store only is obviously to shop
at intervals equal to the expiry time of the good, and purchase just enough to last to expiry.

4.3 Perishable and non-perishable goods at one store only

A well-established result in the perishable inventory management literature is that if a good has both holding costs and perishability costs, the optimal order quantity is (Nahmias, 1982):

\[ Q^* = \min\left( \sqrt{\frac{2cD}{s}} , D\Delta t_e \right) \]  

(8)

In other words, the optimum is the amount considering only the storage costs, unless that is more than can be consumed before expiry, in which case the consumer purchases only enough to last to expiry.

The case here is similar, in that two goods, each of which has only one type of cost, are purchased at a single store. A controlling constraint is the "perishability constraint",

\[ \Delta t_i \leq \Delta t_e \]  

(9)

which states that the purchase interval cannot exceed the lifetime of the perishable good.

The results given below parallel Equation (8).

\[ Q_n^* = \min\left( \sqrt{\frac{2cD_n}{s_n}} , D_n\Delta t_e \right) \]

\[ Q_p^* = \min\left( D_p\sqrt{\frac{2c}{s_nD_n}} , D_p\Delta t_e \right) \]  

(10)

\[ \Delta t_i^* = \min\left( \sqrt{\frac{2c}{s_nD_n}} , \Delta t_e \right) . \]

In other words, either the storage cost of the nonperishable good or the expiry time on the
perishable will govern the repurchase cycle. The perishability constraint binds when

\[ \sqrt{\frac{2c}{s_n D_n}} > \Delta t_e \]  

(11)

The cost functions in the binding and nonbinding cases, respectively, are

\[ C_{T,b}^* = \frac{c}{\Delta t_e} + \frac{s_n D_n \Delta t_e}{2} \]  

(12)

\[ C_{T, nb}^* = \sqrt{2s_n D_n c} \]  

(13)

5. Two Goods at Two Stores

With the introduction of two stores, each with its own prices, the problem is complicated by the huge variety of qualitatively different shopping behaviours. Given a sequence of store visits (e.g., small, large, small, small...) and an associated sequence specifying which goods are purchased on each visit, one could in principle minimize the total cost over the quantities purchased, and purchase timing. However, with an infinite horizon, there are an infinity of these sequences of shopping patterns. The same problem is encountered in the infinite horizon JRPP literature, where the analysis is typically simplified by restricting the problem to periodic policies, which allows the analysis of only one period (see Iyogun, 1987). In contrast, as noted above, we will demonstrate that a periodic policy is in fact generally optimal (Andres and Emmons, 1975)

First, consider the three-way classification of shopping only at the small store, only at the large store, and at both (Figure 3). The minimal costs (over quantities) for each of these cases must be compared to see which one obtains. For the single store cases, we simply
modify (12) and (13) above by addition of the appropriate subscripts for the large and small stores, and include price costs\(^2\). For the mixed case, in Section 5.1, a series of results restricting the optimal patterns of shopping, in the case where both stores are visited, will be proven. In Section 5.2, optimal quantities and costs are derived.

5.1 Feasible Normative Behaviour When Both Stores Are Visited

In the previous sections the "zero inventory rule" and the associated requirement that both goods be purchased on each trip greatly facilitated the solution by defining the only possible shopping pattern. When there are two stores, it may well be optimal to visit one store, i.e., the more expensive (small) store, while still holding goods purchased at the cheap (large) store. The zero inventory rule thus requires careful consideration.

The next result states that the "zero inventory rule" still applies to the large store.

**Lemma 1**: Any trip to the large store that is part of an optimal policy will only occur when both goods have just been depleted.

Note that there is no zero inventory rule for the small store. As a concrete example, think of spaghetti as the nonperishable, and tomatoes as the perishable that can be stored for

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\(^2\)These are \(D_{p, n, p} \cdot P_{p, s} + D_{n, n, s} \cdot P_{n, s}\) for the small store, and \(D_{p, p, l} \cdot P_{p, l} + D_{n, n, l} \cdot P_{n, l}\) for the large store. They were not included in (12) and (13) because there was only one store with one price. However, for comparison with the two store case, where prices matter in the optimization, we need to include these price terms.
one week. It may well be optimal to, for example, lay in a month’s supply of spaghetti and a week’s supply of tomatoes in one visit to a distant large store, and then make weekly visits to the small, nearby, and expensive store for tomatoes. There will be spaghetti on hand when the small store is visited for tomatoes.

**Corollary 1-1:** Both goods are purchased on each trip to the large store in any optimal policy.

**Lemma 2:** If the optimal policy has three successive occasions where both goods are at zero inventory, then the time-average cost between the first and second occasion is equal to the time average cost between the second and third.

**Proof:** Suppose not. Then replacing the higher cost interval with a copy of the lower cost interval will produce an overall lower long-run cost policy, and the original policy cannot be optimal □.

The equality of costs required by lemma 2 can always be achieved, for any arbitrary set of parameters, by having identical shopping patterns, or policies, in the two intervals. It is possible for some special sets of parameters to allow different policies with the same minimal costs in the two intervals. Such parameters could be found by first finding the cost functions $C_1^∗(*)$ and $C_2^∗(*)$ defined over the n-dimensional parameter space, where the subscripts correspond to two different shopping patterns, and the optimization is over the quantities purchased. Imposing the constraint $C_1^∗ = C_2^∗$ defines a lower-dimensional subspace of the parameter space where the two cost functions are equal. (This, in fact, is the procedure that will be used later on to find the transition, or crossover points, in trip cost parameter space, between different optimal policies). Even for this knife-edge, or rare (i.e., for randomly chosen parameters, the probability is arbitrarily close to zero) case, one of the policies could be repeated, so that we may state:

**Corollary 2-1:** If an optimal policy has three successive occasions where both goods are at zero inventory, then either the policy between the first and second occasion is identical to the policy between the second and third, or there exists an equally good policy with such
identical components.

**Proposition 2:** The optimal policy for the two good, two store scenario is either periodic, or no better than periodic, with period defined by the interval between trips to the large store.

**Proof:** Follows immediately from lemma 1, corollary 1-1, and successive application of corollary 2-1.

For verbal economy, from here on we will ignore the boundary case, and refer to the optimal policy as the periodic policy, recognizing that there may, (with vanishingly small probability) be an equally good (but never better) non-periodic policy.

Proposition 2 is a major restriction on the variety of shopping patterns that need to be examined. It restricts the possibilities not only to periodic policies, but to a specific subset of periodic policies, and allows application of finite horizon results (although not fixed horizon, so some care must still be taken). From here on, it is only necessary to consider the interval between two trips to the large store, with the inventory level of both goods at zero at the beginning and end of the interval, and both goods purchased at the large store at the beginning of the interval.

Since we are examining the mixed shopping case, we also know there is at least one trip to the small store. Note, however, that nothing has been said about the purchase patterns at the small store. One cycle, with notation, for m visits to the small store, is depicted below:

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<tbody>
<tr>
<td>&lt;------ ( \Delta t_0 ) ------&gt;&lt;------ ( \Delta t_{l,1} ) ------&gt;&lt;------ ... &lt;------ ( \Delta t_{m} ) ------&gt;</td>
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<td>( ... )</td>
<td>( t_{m,m} )</td>
<td>( t_{m+1} )</td>
</tr>
<tr>
<td>&lt;----------------------------- ( \Delta t_f ) --------------------------------&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now we will examine possible optimal purchases associated with the above sequence.
of store visits.

**Proposition 3:** If both goods are purchased at the small store in an optimal policy that includes shopping at both stores, then the second good will only be purchased once per period, and on the last trip to the small store in the period.

One of the two goods purchased at the large store, either the perishable or the nonperishable, must last the longest. The following results are designated with and "n" or a "p" depending on which good lasts longest. Explicitly, the "n" case is defined by:

\[
\frac{Q_{n,i}^*}{D_n} \geq \frac{Q_{p,i}^*}{D_p}
\]  

(14)

and the "p" case as the complement of (14).

The "p" case means that shopping still occurs at both stores, but the depletion time of the perishable purchased at the large store is longer than the time for the non-perishable good purchased at the large store. This case has questionable face validity—the implication is that a trip is made to the large, inexpensive store for tomatoes (perishable) and spaghetti (nonperishable), but the spaghetti runs out before the tomatoes, and is replenished, by itself, at the small expensive store. Given that the tomatoes expire (rather quickly), spaghetti trips must be quite frequent. The parameters necessary for such behaviour to be optimal (e.g., a large price cost differential on tomatoes, a small differential on spaghetti, large storage costs, and/or small trip costs to the small store) are possible within the model structure, but somewhat pathological. While acknowledging that this is highly unlikely behaviour, the authors have analyzed the "p" case and the results are available from them. Here, we report the results for the much more plausible "n" case.

First, we further restrict the possible optimal policies.

**Proposition 4n:** The amount of perishable good purchased at the large store, in the "n" mixed trip shopping case, is just enough for the lifetime \( \Delta t \) of the good, which must be less than the consumption time \( Q_{n,i}/D_n \) of the non-perishable good purchased at the large store:
\[ \frac{Q_{p,l}^*}{D_p} = \Delta t_e < \frac{Q_{n,l}^*}{D_n} \]  \hspace{1cm} (15)

**Proposition 5n:** The amount of perishable goods purchased on each trip to the small store, and hence the interpurchase intervals, is identical, except possibly for the last trip and interval.

**Proof:** For any number of trips to the small store, only one good is being purchased on all but possibly the last trip. Since the perishable good has a small but nonzero storage cost associated with it, the standard result from perishable inventory theory applies (e.g., Nahmias, 1982): equally spaced trips, with equal purchase quantities on each trip, minimizes costs \( \square \).

**Corollary 5n:** If the non-perishable is never purchased at the small store, proposition 5n applies to all trips in the interval.

**Proposition 6n:** For an optimal policy in the "n" case, if any non-perishable is purchased at the small store, it is purchased only once, and by itself, on the last trip to the small store in the period, at the time when the non-perishable purchased at the large store is just depleted.

The "n" case, then, has only two shopping pattern possibilities, that vary by whether or not any non-perishable is ever purchased at the small store. Even if some is purchased, the shopping pattern differs only by an extra fill-in trip to the small store to top up on a small amount of nonperishables only, just before the major trip to the large store.

We have now established that consumers follow a periodic policy involving shopping only at the small store, only at the large store, or highly restricted mixed-store shopping patterns (Figure 4). In the preliminary results of Section 4, the optimal purchase quantities and costs for the single store cases were given. We must now solve the mixed-store cases. We have argued that the "p" cases lack face validity, and so focus on the "n" cases. In the next section, we derive the optimal quantities and costs for the simpler mixed-store shopping
Figure 4: The complete set of shopping possibilities.

pattern, the "n-p at large store only" case.

5.2 Optimal Policy When Both Stores Are Visited

For the periodic policy with period defined by the time between trips to the large store, and where non-perishables are purchased only at the large store, the problem is to minimize total long run average costs, denoted by \( C \) (we drop the bar and the subscript \( T \) from here on).

The problem is

\[
\text{MIN} \quad C_m \\
Q_{n,p}Q_{p,T}Q_{p,m}
\]

(where the "m" subscript on the cost refers to the mixed case described above and in Figure

\footnote{The procedure and results for the more complex "n-p at both stores" case are similar, and are available from the authors.}
4. and minimization is over quantities, and \( m \), the number of trips per period to the small store) subject to the perishability constraints,

\[
\Delta t_{p,s} \leq \Delta t_e \quad (16) \\
\Delta t_{p,l} \leq \Delta t_e \quad (17)
\]

Note (Figure 4) that Proposition 4n requires the quantity of the perishable good purchased at the large store to be exactly enough to last to expiry, so that the perishability constraint (17) is always binding on \( Q_{p,l} \), so it is unnecessary to minimize over that quantity. Corollary 5n also fixes the total amount purchased at the small store during the period as the number of trips, \( m \), to the small store, times \( Q_{p,s} \), which can be expressed in terms of \( Q_{n,l} \):

\[
mQ_{p,s} = (\Delta t_{n,l} - \Delta t_e)D_p
\]

\[
Q_{p,s} = \left( \frac{Q_{n,l}}{D_n} - \Delta t_e \right) \frac{D_p}{m}
\]

Consequently, the minimization need only be done over \( Q_{n,l} \) and \( m \).
The cost function is

\[ C = \frac{D_n}{Q_{n,l}} \left( c_l + mc_s + \frac{sQ_{n,l}^2}{2D_n} + Q_{n,l}P_{n,l} + Q_{p,l}P_{p,l} + mQ_{p,s}P_{p,s} \right) \]  \hspace{1cm} (19)

Substituting for \( Q_{p,l} \) and \( Q_{p,s} \) from above, and letting \( P_{p,l} - P_{p,s} = \Delta P_p \),

\[ C = \frac{D_n}{Q_{n,l}} \left( c_l + mc_s + \frac{sQ_{n,l}^2}{2D_n} + Q_{n,l}P_{n,l} - D_p \Delta t_e \Delta P_p + \frac{D_p P_{p,s} Q_{n,l}}{D_n} \right) \]  \hspace{1cm} (20)

The effect of the binding perishability constraint on \( Q_{p,l} \) has already been incorporated into equation (20), but constraint (16) is not necessarily binding on \( Q_{p,s} \). Since \( Q_{p,s} \) has been eliminated from the cost function, the constraint is rewritten as a minimum on the number of trips that must be taken to the small store in the period\(^4\):

\[ m \geq \frac{\Delta t_{n,l} - \Delta t_e}{\Delta t_e} \]  \hspace{1cm} (21)

or, in terms of \( Q_{n,l} \):

\[ m + 1 - \frac{Q_{n,l}}{\Delta t_e D_n} \geq 0 \]  \hspace{1cm} (22)

In the following, the subscripts \( n,l \) will be dropped from \( Q_{n,l} \), as that is the only quantity being dealt with in the optimization.

The cost function (20) is convex in \( Q \) and the constraint (22) is linear. Therefore the

\(^4\)A discussion of the effect of the perishability constraint on optimal shopping patterns at the small store, and the relation to optimal period length, appears in an appendix available from the authors.
Kuhn-Tucker first-order conditions are sufficient for minimization with respect to Q. The Lagrangian is

\[
L = C - \mu \left( m + 1 - \frac{Q}{\Delta t_e D_n} \right) \\
= \frac{D_n}{Q} \left( c_l + mc_s + \frac{sQ^2}{2D_n} + QP_{n,l} - D_p \Delta t_e \Delta P_p + \frac{D_p P_{ps} Q}{D_n} \right) \\
- \mu \left( m + 1 - \frac{Q}{\Delta t_e D_n} \right) \tag{23}
\]

where \( \mu \) is the Kuhn-Tucker multiplier. The first order condition with respect to Q is

\[
\frac{\partial L}{\partial Q} = \frac{\partial C}{\partial Q} + \frac{\mu}{\Delta t_e D_n} = 0 \tag{24}
\]

The other FOC is the perishability constraint (22).

If the constraint is binding, (22) gives

\[
Q_{b,m}^* = D_n \Delta t_e \left( m + 1 \right) \tag{25}
\]

where the subscripts on Q indicate "binding" and dependence on the trip parameter "m". The binding case implies \( \mu > 0 \); differentiating the cost and substituting in (24),

\[
sQ_{b,m}^2 + 2D_n \left( \Delta P_p \Delta t_e D_p - c_l - c_s m \right) < 0 \tag{26}
\]

Substituting (22) into (26) gives the parameter range where the constraint binds:

\[
c_l + mc_s \geq \frac{\Delta t_e^2 D_n \left( 1 + m \right)^2 s}{2} + \Delta P_p \Delta t_e D_p \tag{27}
\]

Finally, the minimal (binding) cost is
\[ C_{b,m}^* = \frac{c_l + mc_s}{\Delta t_e(1 + m)} - \frac{\Delta P_p D_p}{1 + m} + D_n P_{n,l} + D_p P_{p,s} + \frac{s \Delta t_e D_n(1 + m)}{2} \]  

(28)

When the constraint is not binding, \( \mu = 0 \), and

\[ Q < (m + 1)D_n \Delta t_e \]  

(29)

The first-order condition (24) becomes, after differentiation and simplifying,

\[ Q_{nb,m}^* = \sqrt{\frac{2D_n}{s}(c_l + mc_s - \Delta P_p \Delta t_e D_p)} \]  

(30)

Substituting into the constraint gives

\[ \frac{2D_n}{s}(c_l + mc_s - \Delta P_p \Delta t_e D_p) < (m + 1)^2 D_n^2 \Delta t_e^2 \]  

(31)

or

\[ c_l + mc_s < \frac{(m + 1)^2 D_n \Delta t_e^2 s}{2} + \Delta P_p \Delta t_e D_p \]  

(32)

which is the complement of the set defined in (27). The (not binding) cost is

\[ C_{nb,m}^* = D_n P_{n,l} + D_p P_{p,s} + \sqrt{2D_n s(c_l + mc_s - \Delta P_p \Delta t_e D_p)} \]  

(33)

It will be helpful to consider how the minimum cost varies with the travel cost per period, \( c_l + mc_s \). From equations (28) and (33), the minimum cost is linear with travel cost in the binding case, and varies with the square root of travel cost in the nonbinding case. At the point where the constraint just binds, i.e., equality in (27), the two cost functions (substituting (27) at equality) and their first derivatives are equal:
\[ C_b^{*} = D_n P_{n,l} + D_p P_{p,s} + \Delta t_e D_n s(1 + m) \] (34)

\[ \left. \frac{\partial C_b^{*}}{\partial (c_l + mc_s)} \right|_{b=nb} = \frac{1}{\Delta t_e(1 + m)} \] (35)

where the notation is intended to indicate that the derivatives are evaluated at the point where equality obtains in (27). As can be readily seen in Figure 5, the binding of the \( m \)th perishability constraint occurs, \textit{ceterus paribus}, at higher trip costs; and that \( C_b^{*} > C_{nb}^{*} \).

![Figure 6: Perishability constraint binds as trip costs increase (fixed m).](image)

Also note that if trip costs are some positive monotonically increasing function of distance, and the stores are \textit{separated}, so that there is a minimum total trip cost, that it is quite possible for the constraint to be binding for all customers that are homogeneous on other parameters. The period \( \Delta t_{u,l} \) therefore takes values equal to integer multiples of the expiry time \( \Delta t_n \) for all these customers.

Figure 5 is for a fixed value of \( m \). How does it change with different values of \( m \)? Note that in (27) or (32) that the RHS increases as \( m^2 \), while the LHS as \( m \), implying that for larger values of \( m \), there is greater chance of the constraint being nonbinding. The point of
tangency in Figure 12, defined by equality in (27), moves to the right. So, as trip costs increase for fixed \( m \), the optimal cost function will switch from nonbinding to binding (if nonbinding is ever feasible); but for larger values of \( m \), the switch will come at higher trip costs. One useful application is the following: suppose one were working with a particular set of parameter values, and distance related trip costs, and found that, because of fixed store separation, the constraint was always binding for the \( m = 3 \) case. Then one would know that the constraint was always binding for \( m = 1 \) and \( m = 2 \).

The final step is to determine the optimal number of trips, \( m \), to the small store, where \( m \) is restricted to integers greater than or equal to one. One approach is to assume \( m \) is continuous, and solve the first order conditions. Then for any particular numerical values of the parameters, take the value of \( m \) rounded up or down as appropriate.

If one is interested in a particular parameter (or parameters), an alternate approach is to take the cost functions (28) and (33), and determine at what parameter value the cost with \( m \) trips equals the cost with \( m+1 \) trips. For a sequence of values for \( m \), this will give the points where the shopping behaviour changes by addition of an extra trip to the small store, as a function of the parameter. Because the next section addresses the change in shopping behaviour as a function of household location (which translates to trip costs), the latter approach will be taken here. The method can easily be adapted for other parameters.

We want to find when \( C^*_m = C^*_{m+1} \). Because there are different cost functions for the binding and non-binding cases, there are four possibilities for the transition region:

1) \( C^*_b,m = C^*_b,m+1 \),
2) \( C^*_b,m = C^*_nb,m+1 \),
3) \( C^*_nb,m = C^*_nb,m+1 \),
4) \( C^*_nb,m = C^*_b,m+1 \)

**Proposition 9:** For optimal mixed-store shopping as parameters vary, direct transition from the non-binding case, to a case with one extra trip to the small store, will not occur.
Proposition 9 eliminates the last two of the four possibilities above. For the numerical optimization to be carried out below, it now remains to find the relation between the parameters that defines the region of parameter space where the remaining two allowed transitions occur. Because the examples later in this chapter focus on the effects of trip costs, the relation between the parameters are solved for $c_i$, the trip cost to the large store.

The first case, binding $m$ trips, to binding $m+1$ trips is:

$$ c_i = c_s + \Delta t_e [\Delta P_P D_P + \frac{s}{2} D_n \Delta t_e (1 + m)(2 + m)] $$

(36)

The last case, binding $m$ trips to non-binding $m+1$ trips, is

$$ c_i = \Delta P_P D_P \Delta t_e - c_s m + \frac{s}{2} \Delta t_e^2 D_n (m + 1)^2 $$

$$ \pm \frac{\Delta t_e (1 + m)}{2} \frac{\sqrt{8c_s D_n s}}{s} $$

(37)

The above transition equations (36) and (37) are bounds of the areas where various values of $m$ are optimal in the mixed shopping cases. With the constraint equations (27) and (32), which give the regions in parameter space where, for each value of $m$, the constraint is binding or not binding, the transition equations are helpful in the task of comparing the optimal cost surfaces for the mixed and single store shopping behaviours.

6. An Application: Two-Dimensional Trading Areas and Market Shares

The model assumes that customers shop deterministically, with shopping behaviour that minimizes long-run average costs. It is natural to then ask what this sort of shopping
behaviour implies for the trade areas and market shares of the grocery stores. This, of course, will depend on the specific parameters describing each customer in the competitive market space. The two parameters which are immediately related to customer location, and hence market area, are the trip costs to the large and small store. In the absence of information on other customer-related parameter values, trip costs based on distance can provide a first approximation of market areas, assuming customers are homogeneous on other parameters. In this section, we show a numerical example of the consequences of the model for the shopping behaviour of spatially heterogeneous customers, and the resulting market areas of the two stores.

In particular, assume trips costs are directly proportional to the Euclidean distance between the customer’s household and the store. This is consistent with many theoretical models (Hotelling 1929; Ingene and Ghosh 1990) as well as being a special case of the many empirical Huff-type spatial interaction models (Huff, 1962). For the small store:

$$c_s = \tau \sqrt{(x - x_s)^2 + (y - y_s)^2}$$

(38)

where $\tau$ is the sensitivity to travel in dollars per kilometre, the customer is located at $(x,y)$, and the small store is located at $(x_s,y_s)$. The expression for the large store trip cost is similar. Table 2 shows the arbitrary, but reasonable, parameter values used in the calculations.

The households represented by these parameters would spend $100.00 per week if they shopped exclusively at the small store, and $80.00 per week if they shopped exclusively at the large store. Travel costs can be compared to this savings: a difference in 1 km. to the stores means a savings of $4.00. Storage costs of $2 per unit per week appear relatively small by comparison, but since they are quadratic in quantity purchased, if (for example) the customer stocks up on 4 weeks supply, the storage cost will be $16.00 for that purchase. A lifetime of 1 week seems reasonable for many vegetables and dairy products. The
Table 2: Parameter values used in numerical calculations.

<table>
<thead>
<tr>
<th>Locations:</th>
<th>(x,y) = (-5,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large store:</td>
<td></td>
</tr>
<tr>
<td>Small store:</td>
<td>(x,y) = (5,0)</td>
</tr>
<tr>
<td>(store separation = 10 km.)</td>
<td></td>
</tr>
<tr>
<td>Prices:</td>
<td></td>
</tr>
<tr>
<td>Perishable, large store:</td>
<td>$P_{pl} = $40.00/unit</td>
</tr>
<tr>
<td>Perishable, small store:</td>
<td>$P_{ps} = $50.00</td>
</tr>
<tr>
<td>Perishable difference:</td>
<td>$\Delta P_p = $10.00</td>
</tr>
<tr>
<td>Non-perishable, large store:</td>
<td>$P_{nl} = $40.00</td>
</tr>
<tr>
<td>Non-perishable, small store:</td>
<td>$P_{ns} = $50.00</td>
</tr>
</tbody>
</table>

| Demand rates:                  |               |
| Perishable:                    | $D_p = 1$ unit/week |
| Non-perishable:                | $D_n = 1$      |
| Travel cost:                   | $\tau = $4.00/km |
| Storage cost:                  | $s = $2.00/unit/week |
| Expiry time:                   | $\Delta t_e = 1$ week |

Consumption rates are fixed at one unit per week, and are essentially scaling parameters. Alternatively, one could think of the above parameters as representing some average of all perishables and all nonperishables purchased by the household. The point is to illustrate the nature of shopping behaviour and trading areas that emerge from the model.

The parameter values are used to evaluate the optimal (over quantity) cost functions derived in the last section. The smallest of these functions at each point in the plane determines the shopping pattern at that point. These calculations (which are straightforward, but tedious, and therefore not reproduced here) allow us to describe the shopping behaviour in the plane in some detail (see Figure 7). There is a tiny region immediately around the small store that is its own market area exclusively. At about one-quarter of a kilometre away, shopping behaviour switches to a mixed. $m = 4$, or $m = 5$ type of behaviour. Moving strictly to the right along the x axis, this behaviour doesn’t change. In any other direction, however,
Figure 7: Market areas of the large and small store, located at (-5,0) and (5,0), for the parameters given in the text.

It changes to the mixed, m = 4 behaviour. In directions towards the large store, the behaviour goes through progressively fewer fill-in trips to the small store, until eventually shopping is done only at the large store.

From these market areas, market shares can be calculated from a given population density distribution. For purposes of illustration, assume that the city is the twenty by twenty kilometer square area that has been plotted, and that the population is uniformly distributed up to the boundaries of the city, and zero outside the boundaries. Everyone except for those living inside the small store's exclusive region purchases all of their nonperishables at the
large store. This area is a circle of radius 0.2 km, so the small store’s share of nonperishable good is 0.03%, and the large store’s share is 99.97%.

Share of the perishables varies with the different regions.\(^6\) For the entire region, the large store captures 71.86% of the perishables, and the small store has a 28.14% share.

**Figure 8:** Market areas with a 10% price difference between stores. From left to right, the regions are exclusively the large store’s; mixed with \(m = 1\) and \(m = 2\); and exclusively the small store’s.

**Smaller Price Differences:** The model has two parameters under the control of management: price and store location. Price, of course, is a much shorter term strategic variable, and it is interesting to see what happens to the market areas if the price difference is less than the 20% postulated in the previous numerical example. A price difference of 10% gives the trading areas shown in Figure 8. Unlike the case with 20% price difference, the small store now has a large market area exclusively its own. The mixed behaviour is confined to a narrow region between the two exclusive areas. The market shares that this shopping pattern corresponds to are 36.6% of the nonperishable for the small store and 63.4% for the large store; and 40.5% of the perishables for the small store and 59.5% of perishables for the large store. Changing the price difference between stores from 20% to 10% affects the share of non-perishables much more than perishables. As price differences increase, it becomes worthwhile to travel further, but less frequently, for goods that can be stored.

**Increased Consumption Rate:** One would expect that larger households would be more

---

\(^6\) The detailed calculations are available from the authors.
sensitive to price differences, if only from income effects. However, even if all sensitivity parameters are kept the same, the increased consumption rate will make it worthwhile for a larger household to stockpile. Figure 9 shows the trading areas for the same prices (10% difference between stores), and all other parameters the same, except for doubling the consumption rates: $D_a = D_p = 2$. The regions of mixed shopping are increased again, although not to the same extent as in Figure 19. The small store still has a substantial region of exclusivity. The market areas are, not surprisingly, sensitive to the absolute differences in consumer expenditure rates at the two stores. Another way of saying this is that people with large families will go further for a deal, even if they have the same price, travel, and stockpiling sensitivities as a single person.

The small store's market shares for the three preceding sets of parameters are summarized in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>20% price difference</th>
<th>10% price difference</th>
<th>10% price difference; 2x consumption rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>nonperishable shares</td>
<td>0.03%</td>
<td>36.6%</td>
<td>18.5%</td>
</tr>
<tr>
<td>perishable shares</td>
<td>28.14%</td>
<td>40.5%</td>
<td>29.1%</td>
</tr>
</tbody>
</table>
7. Increasing Returns and Power Retailer Entry

We have described a case where a 20% price difference produced almost complete domination of the nonperishables market by the large store, due to the consumers ability to reduce long-run average trip costs by stockpiling and shopping less frequently at the large store, and thus being able to travel longer distances to take advantage of low prices.

Figure 10: Share response of the large store to its own price reductions, using the numerical example of the last section. At small price differences, no mixed shopping occurs, and the two goods have the same share.

Figure 10 shows this effect, using the shares calculated in the preceding section, as and intermediate points at 5% and 15% price differences. Nonperishables exhibit pronounced increasing returns of share to price reductions. Note that the perishable goods are much less sensitive to price differences than nonperishables: and that the shares of the two goods are the same when price differences are small, corresponding to optimal shopping patterns of
shopping exclusively at one store or the other for everyone in the region, that is, no mixed-trip cases anywhere.

One of the questions posed in the introduction was "Why was Safeway taken off-guard by the entry of the Real Canadian Superstore?" One possibility lies in the highly nonlinear response to the price differences that arises when stockpiling is considered. The retailing literature has many examples of location models that do not consider stockpiling. These usually trade off store attractiveness (which may be a function of several different variables such as price or image) against distance, and can be traced from Reilly's (1931) deterministic gravity model, through Huff's (1962) probabilistic model, and subsequent spatial interaction models. (For a review, see Ghosh and McLafferty (1987).) When the "attraction" of these models is due to price, then there is some tradeoff between price and distance. Typically these models are empirically tested, and are calibrated to determine some model parameters; these parameters can then be used to evaluate the sales potential of a proposed outlet, considering the existing market and the competition. The models have been quite successful, and also have a strong intuitive appeal. In fact, even without a formal model, a manager can readily look at trading areas and create a reasonable intuitive model of the price-distance tradeoff.

Let us compare the stockpiling shopping model with a simple gravity-type model, where consumers trade off prices and distances only. Suppose that, at the boundary of the trading area between stores, the ratio of distances to the stores depends the inverse of the ratio of prices as follows:
\[
\frac{d_s}{d_l} = \left( \frac{p_l}{p_s} \right)^\lambda
\]  

(39)

where \(d_s\) and \(d_l\) are the distances from the consumer to the small and large store, and \(p_s\) and \(p_l\) are the expected prices at the small and large store. In an empirical application, \(\lambda\) would be estimated. Here, for purposes of illustration, it is simply assumed to be unity. For a 10% price difference, the trading area boundary is shown in Figure 11. For comparison, the stockpiling model, with the same parameters, is shown again in Figure 12.

Imagine Figure 12 represented actual data. The much simple gravity model (Figure 11) would be a good approximation of the trading areas. The analyst, or manager operating intuitively, would be justified in applying Occam’s razor, and accepting equation (39) as a fair model of shopping behaviour. She would have a reasonable tool for predicting trading areas, as long as all competition was within a 10% price range.

![Figure 11: Trading area defined by the gravity model, (42), with a 10% price difference; cf. Figure 32.](image1)

![Figure 12: Trade areas with 10% price difference using the stockpiling model (from Figure 29).](image2)

But what if there is a potential market entrant with 20% price reduction? Applying the model (79) to predict trading areas would give the map in Figure 13. The incumbent
would see the threat of some erosion of market share, but perhaps not enough to elicit a
dramatic response. Unfortunately for the established retailer, consumers can stockpile, and
the resulting trade areas turn out to be those shown in Figure 14. The market area, and hence
share of the established store, has declined dramatically, from 41% to 28% for perishables,
and from 37% to near zero for nonperishables. "Safeway" has been taken completely off-
guard.

If this argument is reasonable, then we should expect to see substantially greater price
differences between RCS and the supermarkets, than among supermarkets. Figure 1 showed
that this was indeed the case, with actual price differences very similar to the above scenario.

8. Conclusions

This research addresses the problem of consumer response to power retailers in the
context of the grocery industry. We show that, for planned grocery shopping behaviour,
consideration of consumer stockpiling, and the fact that some goods are perishable and others
not, is important.
While the retailing and promotions literature recognize that loyalty to one store is low, the possibility of a multistore shopping strategy resulting from the combination of a power retailer (with a large price advantage and few locations) with the fact that some groceries are perishable and others are not, has not been considered. A second interesting result is the market share response function, which suggests a reason why incumbents do not expect entering power retailers to be as dramatically successful as they are.

While the structure of our problem is similar to the multipurpose shopping problem, there are important differences. Multipurpose shopping models seek to explain how stores offering frequently purchased (low-order) goods will agglomerate with stores offering infrequently purchased (high order) goods, due to savings associated with multipurpose shopping, thereby creating a high order centre: an entirely different context than in this paper. While multipurpose shopping may imply multistore shopping, it seems less surprising that consumers will shop at several stores when they offer different goods, than it does when they offer the same goods, and are not agglomerated.

Finally, this research provides another example of a joint-replenishment problem that has a periodic optimal policy.

There are at least four general research directions suggested by this work. First and foremost, there is the need for empirical validation of the model. While actual calibration would be ideal, it would also be a major project. The model, however, makes predictions that could be much more easily tested. On the consumer side, this includes the relation between trip patterns to the different stores, distances travelled, and types of goods purchased. From the stores' perspectives, it suggests the power retailers share of nonperishables should be larger than its share of perishables.

A second direction is strategic. What are the theoretical implications for equilibrium
prices and locations? From the large literature on spatial competition (Hotelling, 1929; de Palma et. al., 1985; Vandenbosch and Weinberg, 1992), it is known that increased separation on spatial dimensions (either geographic space, or product attribute space) decreases price competition. In this model, consumers travel further for nonperishables than perishables, which suggests that price competition should be fiercer for nonperishables. However, the fact that each store carries both goods may create interactions that affect this conclusion. Other strategic issues include how perishables and nonperishables might be treated differently, and how advertising might augment this treatment.

A third direction is model extensions. Additional variables, such as advertising, service, or product quality may be considered. The dynamic effects of short term promotions could be incorporated. This could lead to consumer uncertainty about prices, and require a stochastic approach. Consumers could also be uncertain about their own sensitivities to the various costs in the model; or, perhaps more interestingly from a managerial point of view, these sensitivities might be susceptible to manipulation; in fact, one of Safeway's responses to RCS has been to emphasize convenience of location in their advertising, presumably to increase the consumer's sensitivity to travel costs. For any model extension, an overriding issue is the robustness of the increasing returns effect.

An interesting issue is the structure of the trip costs. In the numerical section, trip costs were assumed to be only travel costs. However, it has long been acknowledged that there is a cost associated with being in a store. Baumol and Ide (1957) suggest that the larger a store becomes, the more time consuming, and hence costly, shopping becomes. Ingene and Ghosh (1990) explicitly state that their trip costs include a fixed store-specific cost. This is intuitively reasonable, and of interest to management because, once location is set, the only possible short term control over trip costs is the in-store costs. The RCS, for example, is not
only large, but often has long check-out lines. Is it worth their while to increase the number of checkouts? One step they have taken is to introduce express checkouts, but that only assists the small-purchase consumer. The effect of making the trip cost to the large store greater by a fixed amount for each customer would be interesting to know. It would certainly reduce the large store’s exclusive area, but would it have a greater effect on the mixed-trip region, or the small store’s exclusive area?

The final issue relates to the dynamic and unpredictable nature of retailing. In this chapter, a specific example is given of how consumers following a relatively simple optimizing model can dramatically alter their shopping behaviour in response to a relatively small change in the price structure of the market, and that while this change may seem perfectly sensible in hindsight, it would be asking much of managers to predict it before the fact. There are many rapid changes in competitive environments, store formats, consumer needs, and regulatory environments that are even less predictable. While this is true of any industry in a market economy, retailing is perhaps more dynamic and uncertain than most. Nonetheless, even if management is unable to exercise much foresight in this environment, it can be very clever in its response. Retailers typically engage in experimentation and imitation of successful strategies, and respond quickly to adverse situations. The question is, then, if static equilibria are unlikely to occur, are there other patterns that may arise? Many authors have long felt that there are patterns in retailing, and have sought to describe them; the evidence, however, tends to be case-based and anecdotal. The issue of dynamic patterns arising from many spatially competing agents with limited foresight, but the ability to react effectively to adversity, is addressed in Krider and Weinberg (1994).
REFERENCES


Kahn, B.E. and D.C. Schmittlein (1992), "The Relationship Between Purchases Made on


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