We consider the problem of sharing demand information in two competing supply chains, each consisting of one manufacturer and one retailer. A supply chain that engages in information sharing triggers a reaction from the rival chain. Such a reaction may benefit or hurt the first supply chain, depending on whether the retailers compete on quantity or price, and whether the manufacturers are efficient in cost reduction or not. With quantity competition, information sharing occurs when the manufacturers are efficient in cost reduction. It is more likely to occur when either information is less accurate or competition is less intense. With price competition, information sharing occurs when either competition is intense or the manufacturers are efficient in cost reduction. It is more likely to occur when information is more accurate. When information sharing is beneficial to a supply chain, a manufacturer can buy information from a retailer with a side payment, which is zero if the manufacturer is sufficiently efficient in cost reduction. We show that the retailers may be worse off when information becomes more accurate, or they may be better off when competition becomes more intense, if any of these changes induces a change in the information sharing equilibrium.

Key words: Supply chain management, information sharing, cost reduction

1. Introduction

Competition is no longer firm against firm, but rather supply chain against supply chain (Fung et al. 2008). To compete effectively with rival supply chains, firms in the same supply chain have to coordinate their decisions. According to Lee and Whang (2000), an important enabler for supply chain coordination is information sharing. When the firms in a supply chain have aligned incentives and act on global information, the overall performance of the supply chain is expected to dominate one with firms that have misaligned incentives and act on local information. With the advance of information technology, it is increasingly more common for firms in a supply chain to share information via collaborative practices such as vendor-managed inventory and collaborative planning, forecasting and replenishment. Keifer (2010) states that many large retailers such as Costco and
Target have started to offer data-sharing programs to manufacturers, sometimes charging them for the data. Munves (2013) reports that many consumer packaged goods manufacturers such as Pepsi, Johnson & Johnson and Procter & Gamble regularly buy data from their retailers.

It is well known that low cost can be a major source of competitive advantage for a firm (Porter 1998). Not surprisingly, many manufacturing firms regularly engage in production cost reduction activities. One example is target costing. Feil et al. (2004) state that target costing is a cost management tool for reducing the overall cost of a product over its entire life cycle with the help of production, engineering, R&D, marketing and accounting departments. It originated in Japan in the 1960s, and has been adopted by many large companies in North America and Europe. According to the Institute of Management Accountants (1994), target costing has been used by companies such as Toyota, Nissan, NEC, Ford, Chrysler, Compaq and Dell. Okano and Bonzemba (1997) report that in the automobile industry, some manufacturers use target costing to set cost reduction target based on sales forecasts. When the sales volume of a part is expected to increase, a manufacturer will set a higher cost reduction target. Besides target costing, DeHart Consulting, a manufacturing management consulting firm, advocates a cost reduction planning process that is based on demand forecast (http://www.dehartconsulting.com/practice-areas/product-cost-reduction). These practices of demand-driven cost reduction planning are supported by theory. For example, in a multi-period model with cost reduction investment, Bernstein and Kök (2009) show that the optimal dynamic investment policy depends on the demand in each period.

When a manufacturer receives demand information from a retailer, it can make better cost reduction decision that could help the supply chain compete with the rival chains. However, the retailer may not want to share information because of the fear that the manufacturer will abuse the information to gain an advantage in future price negotiations. Li and Zhang (2002) show that, with linear production cost, information sharing indeed benefits the manufacturer but hurts the retailer when the only decision made by the manufacturer is wholesale price. More importantly, information sharing hurts the supply chain by making the double marginalization effect of wholesale price more significant. Li et al. (2014) show that a retailer may have an incentive to share information with a manufacturer even under a linear wholesale price contract when the latter has encroachment capability. For the case of non-linear production cost, Ha et al. (2011) and Shang et al. (2014) show that information sharing can benefit the supply chain due to the order uncertainty effect when production cost is highly non-linear. Despite the popularity of production cost reduction in practice, its impact on the incentive for information sharing has not been explored in the literature. We hope to fill this gap by addressing the following research questions. How does a retailer’s incentive for sharing demand information with a manufacturer depend on production cost reduction in the supply chain? How should a manufacturer make cost reduction decision in response to the
shared information? How do the answers to these questions depend on cost reduction efficiency, information accuracy and supply chain competition?

We consider a model of two supply chains, each with a manufacturer (she) selling to a retailer (he). The retailers compete on price or quantity by selling partially substitutable products to the same market with uncertain demand. The manufacturers face linear production cost, and therefore the order uncertainty effect investigated by Ha et al. (2011) and Shang et al. (2014) plays no role in our model. We develop a multi-stage game to study the firms’ information sharing, wholesale price, cost reduction and retail decisions. In the beginning, before the retailers observe their private demand signals, each manufacturer offers a payment to the retailer in the same chain for the information and each retailer decides whether to accept the offer. Next, each retailer observes his private demand signal and shares it with his manufacturer if there is such an agreement. Then, each manufacturer determines a wholesale price and a production cost reduction level. Finally, each retailer decides on quantity in Cournot competition or price in Bertrand competition.

First consider the case of no competition. If there is information sharing, the manufacturer adjusts her wholesale price and cost reduction level in response to the demand signal received from the retailer. Without cost reduction, Li and Zhang (2002) show that the manufacturer’s wholesale price always responds positively to the demand signal, i.e., it is higher when the signal is higher, and lower when the signal is lower. As a result, double marginalization becomes stronger on average, which hurts the supply chain. With cost reduction, the manufacturer’s cost reduction level always responds positively to the demand signal, which benefits the supply chain. However, her wholesale price may respond positively or negatively to the demand signal. When the manufacturer is not efficient in cost reduction, a higher demand signal induces her to raise the wholesale price to increase profit margin. As a result, her wholesale price responds positively to the signal and double marginalization becomes stronger, which hurts the supply chain. When the manufacturer is efficient in cost reduction, however, a higher demand signal induces her to reduce the production cost so much that she prefers to lower the wholesale price to increase demand without sacrificing profit margin. Consequently, her wholesale price responds negatively to the signal and double marginalization becomes weaker, which benefits the supply chain. When the manufacturer is not efficient in cost reduction, the value of information sharing to the supply chain is negative because the negative double marginalization effect dominates the positive cost reduction effect. As her efficiency in cost reduction increases and becomes high enough, the positive cost reduction effect now dominates the negative double marginalization effect, and the value of information sharing to the supply chain is positive. In this case, the manufacturer can use a side payment to induce the retailer to share information. As her efficiency increases further, the double marginalization effect also becomes positive (when the wholesale price responds negatively to the demand signal). In this
case, not only the value of information sharing to the supply chain is positive, but also the retailer is willing to share information for free (i.e., without a side payment) because he benefits from a weaker double marginalization.

Now consider the case when there is competition. Following Ha et al. (2011), we distinguish three effects of information sharing. The direct effect of information sharing in a supply chain is the impact on itself without accounting for the rival supply chain’s reaction. The competitive effect is the additional impact on the supply chain due to the rival supply chain’s reaction to its information sharing activity. The spillover effect is the impact on the rival supply chain. When the manufacturer is not efficient in cost reduction, information sharing induces her wholesale price to respond positively to the demand signal. This decreases the demand variability faced by the supply chain in Cournot competition but increases it in Bertrand competition. Because a larger demand variability benefits a supply chain that can adjust quantity or price decision in response to the variability, the competitive effect is negative for Cournot competition but positive for Bertrand competition. This also increases the demand variability faced by the rival supply chain, which results in a positive spillover effect regardless of whether competition is Cournot or Bertrand. These results are consistent with those in Ha et al. (2011). However, when the manufacturer is efficient in cost reduction, information sharing induces her wholesale price to respond negatively to the demand signal and the effects on demand variability are reversed. In this case, competitive effect is positive for Cournot competition but negative for Bertrand competition. Spillover effect is negative, again regardless of whether competition is Cournot or Bertrand.

We fully characterize the equilibrium information sharing decisions in our model. In Cournot competition, information sharing occurs when the manufacturer is efficient in cost reduction. It is more likely to occur when either information is less accurate or competition is less intense. Under any of these conditions, either both the direct and competitive effects are positive, or the positive direct effect dominates the negative competitive effect. Therefore the value of information sharing to the supply chain is positive and the manufacturer can use a side payment to induce the retailer to share information. In Bertrand competition, information sharing occurs when either competition is intense or the manufacturer is efficient in cost reduction. It is more likely to occur when information is more accurate. Under any of these conditions, either the positive direct effect dominates the negative competitive effect, or both effects are positive, or the positive competitive effect dominates the negative direct effect. Then the value of information sharing to the supply chain is positive and the manufacturer can use a side payment to induce the retailer to share information. When the manufacturer is sufficiently efficient in cost reduction, not only the value of information sharing to the supply chain is positive, but also the retailer is willing to share information for free, regardless of whether competition is Cournot or Bertrand.
Our analysis yields some counter-intuitive results. In Cournot competition, when a higher information accuracy induces the supply chains to cease information sharing, the retailers are worse off. In Bertrand competition, when a higher competition intensity induces the manufacturers to buy information, the retailers are better off. None of these results holds when the information sharing decisions do not change.

Our study provides novel insights to practices. For instance, many large retailers have started offering data-sharing programs to manufacturers (Keifer 2010). We show that whether a retailer should share data with a manufacturer or not, and if so, whether to charge a fee for the data or not, depend critically on the manufacturer’s efficiency in cost reduction. We also show that, contrary to convention wisdom, retailers may have an incentive to intensify price competition. This is the case when more intense price competition induces the manufacturers to buy information.

To the best of our knowledge, our paper is the first that examines production cost reduction as a driver of the incentive for vertical information sharing in a supply chain. We show how this incentive depends on the efficiency in cost reduction, information accuracy, competition intensity and type of competition. Although Ha et al. (2011) also consider information sharing in two competing supply chains, there are several major differences. First, Ha et al. (2011) assume production diseconomy (i.e., marginal cost is increasing in volume) and identify order uncertainty as the main driver of the incentive for information sharing. Here we assume linear production cost and therefore order uncertainty is not relevant. Second, for Bertrand competition, Ha et al. (2011) can only characterize the equilibrium information sharing decisions for some special cases. In our model, we fully characterize the equilibrium information sharing decisions for both Cournot and Bertrand competition. Our results lead to much sharper insights into the role of competition that have not been fully revealed in the literature. For instance, we show that the competitive effect of information sharing under Cournot competition is completely opposite to that under Bertrand competition. Moreover, the effects of information accuracy and competition intensity on the incentive for information sharing under Cournot competition are also opposite to those under Bertrand competition. Third, with the assumption of production diseconomy in Ha et al. (2011), information sharing always makes double marginalization stronger and, without a side payment, the retailer does not have any incentive to share information. In our model, when a manufacturer is efficient in production cost reduction, information sharing can weaken double marginalization and the retailer is willing to share information for free. Our results offer interesting insights into how information sharing could create a win-win situation in practice.

2. Literature Review
This paper belongs to the literature on incentive for vertical information sharing under different supply chain structures. For the case of one manufacturer selling to one retailer, several papers
investigate issues such as signaling unverifiable information (Cachon and Lariviere 2001), dual distribution channels (Yue and Liu 2006) and bilateral information sharing (Mishra et al. 2009). Özer et al. (2011) study the role of trust in information sharing between a supplier and a manufacturer. Li et al. (2014) consider supply encroachment and their results show that a retailer may have an incentive to share information with a manufacturer who has encroachment capability. Many papers consider the case of one manufacturer selling to several competing retailers. They focus on the incentive for retailers to share demand information with the manufacturer (Li 2002, Li and Zhang 2002, Zhang 2002) and how this incentive depends on confidentiality (Li and Zhang 2008) and the wholesale contracts (Shin and Tunca 2010, Tang and Girotra 2010). Gal-Or et al. (2008) consider a setting where both the manufacturer and the two competing retailers have private demand information. For the case of two competing supply chains, Ha and Tong (2008) examine the role of contract and Ha et al. (2011) analyze the impact of production diseconomy on the incentive for a retailer to share demand information with a manufacturer. For the case of several suppliers selling different components to a manufacturer in a two-echelon assembly system, Zhang (2006) investigates the issue of sharing inventory information among these suppliers. Sošić (2010) considers the stability of information sharing alliances in a three-level supply chain. Zhao et al. (2014) study the issue of information sharing in outsourcing when two suppliers compete for the service contract from a client. Motivated by the popularity of retailers’ data-sharing programs, Shang et al. (2014) investigate the incentive for a common retailer to share demand information with two competing manufacturers. Most of the papers in this literature assume truthful information sharing. One exception is Shamir (2012), which considers the case when information is not verifiable and shows how a signaling mechanism can induce truthful information sharing in a supply chain with one manufacturer selling to multiple retailers.

Our paper is related to the recent literature on the impact of information sharing and dissemination on socially responsible operations in developing economies. Chen et al. (2013a) investigate an innovative business model with a platform that provides market information to farmers and helps them distribute their products. Chen et al. (2013b) examine the incentive for competing farmers to share demand and price information with each other via a voice-based information service. Chen and Tang (2013) study the impact of public and private information on farmers’ production decisions and welfare. They show that public information does not always create value to the farmers when they compete with each other. Tang et al. (2014) consider how a central planner should disseminate agricultural advice and market information to competing farmers. Zhou et al. (2014) analyze how a central planner should selectively provide market information to a group of competing farmers. None of the papers in this literature considers vertical information sharing in a supply chain.
Our paper is also related to the literature on cost reduction in a supply chain. One stream of this body of work considers investment by the upstream firms to reduce production cost and examines issues such as decentralization (Gupta and Loulou 1998), product line design (Heese and Swaminathan 2006), outsourcing (Gilbert et al. 2006), and knowledge spillovers (Gupta 2008). Another stream of work considers investment by the downstream firms to reduce production cost and investigates issues related to wholesale price commitment (Gilbert and Cvsa 2003) and coordination mechanisms (Cho and Gerchak 2005). There are also papers that consider how firms in a supply chain jointly invest in cost reduction. Iida (2012) studies collaborative cost reduction in an assemble-to-order supply chain and Kim and Netessine (2012) analyze collaborative cost reduction under information asymmetry. All the above papers consider static models. Bernstein and Kök (2009) study dynamic cost reduction in an assembly network and show that the optimal investment policy depends on the demand in each period. This body of work does not consider the issue of information sharing.

3. The Model

Consider two competing supply chains, each consisting of one manufacturer (she) selling multiple generations of products through a retailer (he). In each product generation, the retailers engage in either Cournot (i.e., quantity) or Bertrand (i.e., price) competition. For Cournot competition, the inverse demand function of retailer $i$ is given by:

$$p_i = a + \theta - q_i - \gamma C q_j.$$  \hspace{1cm} (1)

For Bertrand competition, the demand function of retailer $i$ is

$$q_i = a + \theta - p_i + \gamma B p_j.$$  \hspace{1cm} (2)

In (1) and (2), $p_i$ and $q_i$ are respectively the retail price and selling quantity of retailer $i$, $\gamma_Z \in [0,1)$ is a measure of competition intensity, where $Z = C$ for Cournot and $Z = B$ for Bertrand, and $\theta$, which represents demand uncertainty, is a random variable with mean zero and variance $\sigma^2$.

In each product generation, manufacturer $i$ exerts effort to reduce her unit production cost $c_i$ by an amount of $x_i$ with cost of effort given by

$$\frac{1}{2} k_i x_i^2.$$ 

Here $k_i$ denotes manufacturer $i$’s efficiency in cost reduction, i.e., a higher $k_i$ means that she has to incur a higher cost of effort for the same cost reduction level $x_i$. The quadratic form of the cost of effort function captures the diminishing return of effort, which has been widely used in the
literature (e.g., Gupta and Loulou 1998, Gilbert and Cvsa 2003, Gilbert et al. 2006, and Gupta 2008). The retailers have constant marginal operating cost, which we normalize to zero.

Each retailer $i$ has access to a demand signal $Y_i$, which is an unbiased estimator of $\theta$ (i.e., $E[Y|\theta] = \theta$), and decides whether to share it with manufacturer $i$ before the demand is observed. We assume linear information structure: the expectation of $\theta$ conditional on the signal(s) is a linear function of the signal(s), and the two signals, conditional on $\theta$, are statistically independent. This information structure has been commonly used in the literature (e.g., Li 2002, Taylor and Xiao 2010, and Wu and Zhang 2014) and includes well-known conjugate pairs like normal-normal, beta-binomial, and gamma-Poisson. Define signal accuracy as $t_i = 1/E[Var(Y_i|\theta)]$, which can be interpreted as retailer $i$’s ability in demand forecasting. It can be shown (Ericson 1969) that

$$E[\theta|Y_i] = E[Y_j|Y_i] = \frac{t_i \sigma^2}{1 + t_i \sigma^2} Y_i.$$

This information structure is common knowledge. For more details of the linear-expectation information structure, refer to Vives (1999), Section 2.7.2.

We consider a multi-stage game with the following sequence of events.

1. Each manufacturer $i$ offers a payment $m_i$ to retailer $i$ for the demand signal $Y_i$. Retailer $i$ decides whether to accept this payment before he observes $Y_i$. We say that supply chain $i$ is communicative if retailer $i$ accepts the payment and agrees to share the signal with manufacturer $i$, and non-communicative otherwise. Firms in supply chain $j$ can observe whether there is information sharing in supply chain $i$ though they cannot observe the payment $m_i$.

2. Each retailer $i$ observes a private demand signal $Y_i$ and shares it truthfully with manufacturer $i$ if supply chain $i$ is communicative. Manufacturer $i$ is said to be informed if supply chain $i$ is communicative and uninformed otherwise.

3. Each manufacturer $i$ determines a wholesale price $w_i$ and exerts effort to reduce her production cost $c_i$ by an amount $x_i$. Neither $w_i$ nor $x_i$ is observable to firms in supply chain $j$.

4. In Cournot competition, each retailer $i$ orders $q_i$ from manufacturer $i$, who then produces to fill the order and the market clearing prices $p_1$ and $p_2$ realize, and firms receive their profits. In Bertrand competition, each retailer $i$ chooses a price $p_i$, market demands $q_1$ and $q_2$ realize, each manufacturer $i$ produces to fill the demand $q_i$ and the firms receive their profits.

We assume that the information sharing decisions in the first stage have longer terms than those in the subsequent stages. If the firms in a supply chain agree to share information, they have to set up systems and business processes for information transmission. These are long-term commitments that cannot be changed frequently. After the information sharing decisions are made, these firms engage repeatedly in wholesale contracting and cost reduction activities, say in each
product generation. Therefore information sharing and wholesale contracting decisions cannot be made simultaneously. We assume whether a supply chain is communicative or not can be observed by the rival firms. This is reasonable because the rival firms can observe information sharing activities such as the setting up of systems for information transfer or know these activities from third parties (e.g., consultants, vendors or employees). We assume that the payment for information sharing is not observable to the rival firms because it is usually determined in closed-door meetings. We also assume that neither the cost reduction level nor the wholesale price is observable to the rival firms. In practice, firms usually do not publicly disclose detailed operational data related to cost reduction. On the unobservability of wholesale price, Coughlan and Wernerfelt (1989) argue that if wholesale price becomes known to the rival firms, firms in a supply chain have an incentive to secretly re-negotiate. Moreover, they cannot credibly make a commitment not to do so. Finally, we assume that information sharing is truthful. This holds when a retailer shares tangible and verifiable data such as point-of-sale (POS) data, or the retailer fears that false reporting of information could jeopardize a long-term relationship with the manufacturer.

4. Single Supply Chain

In this section, we consider the case when there is only one supply chain and therefore the index $i$ is suppressed. Similar to Gupta and Loulou (1998) and Gilbert et al. (2006), we assume $k > a/(4c)$, which means that it is not too cheap to reduce production cost. This guarantees that the optimal cost reduction level cannot exceed the original production cost $c$. We also assume that $k > 1/3$ and $a > c$ so that the equilibrium price, quantity and cost reduction level are non-negative.

From the sequence of events given in Section 3, the firms decide in the first stage whether to share information or not. Let $X$ denotes the information sharing arrangement, where $X = S$ if the firms agree to have information sharing and $X = N$ otherwise. Given $X$, we first derive the equilibrium wholesale price, cost reduction level, and either retail quantity or retail price. Then we compute the firms’ ex-ante profits. Based on these profits for the two information sharing arrangements, we finally analyze the equilibrium information sharing decisions in the first stage.

4.1. Equilibrium Analysis for Different Information Sharing Arrangements

If the retailer’s decision is selling quantity $q$, the inverse demand function is

$$p = a + \theta - q.$$  \hspace{1cm} (3)

Given $w$, the retailer maximizes his profit,

$$(a + E[\theta|Y] - q - w)q,$$
by choosing
\[ \hat{q}(w) = \frac{1}{2} (a + E[\theta|Y] - w). \]

If his decision is retail price \( p \), the demand function is
\[ q = a + \theta - p. \quad (4) \]

Given \( w \), he maximizes his profit,
\[ (p - w)(a + E[\theta|Y] - p), \]
by choosing
\[ \hat{p}(w) = \frac{1}{2} (a + E[\theta|Y] + w), \]
which results in selling quantity
\[ \hat{q}(w) = a + \theta - \frac{1}{2} (a + E[\theta|Y] + w). \]

Since the signal is imperfect, the selling quantity is uncertain at the time when the retail price is determined. No matter whether the retailer makes decision on quantity or price, we have
\[ E[\hat{q}(w)|Y] = \frac{1}{2} (a + E[\theta|Y] - w). \]

For an informed manufacturer, she maximizes her profit,
\[ (w - c + x)E[\hat{q}(w)|Y] - \frac{1}{2} kx^2, \]
by choosing
\[ w^S = \frac{(2k - 1)a + 2kc}{4k - 1} + \frac{2k - 1}{4k - 1} E[\theta|Y] \]
\[ x^S = \frac{a - c}{4k - 1} + \frac{1}{4k - 1} E[\theta|Y] = \frac{a - c}{4k - 1} + \frac{1}{4k - 1} \frac{t\sigma^2 + 1}{t\sigma^2 + 1} Y. \]

Substituting \( w^S \) back to the retailer’s best-response functions \( \hat{q}(w) \) and \( \hat{p}(w) \), the retailer’s equilibrium decision is
\[ q^S = \frac{ka - c}{4k - 1} + \frac{k}{4k - 1} E[\theta|Y] = \frac{ka - c}{4k - 1} + \frac{k}{4k - 1} \frac{t\sigma^2 + 1}{t\sigma^2 + 1} Y \]
if he makes decision on the selling quantity, or
\[ p^S = \frac{(3k - 1)a + kc}{4k - 1} + \frac{3k - 1}{4k - 1} E[\theta|Y] = \frac{(3k - 1)a + kc}{4k - 1} + \frac{3k - 1}{4k - 1} \frac{t\sigma^2 + 1}{t\sigma^2 + 1} Y \]
if he makes decision on the retail price. Here the superscript \( S \) denotes that the supply chain is communicative.
For an uninformed manufacturer, she maximizes her profit,

\[(w - c + x)E[q] - \frac{1}{2}kx^2,\]

by choosing

\[w^N = \frac{(2k - 1)a + 2kc}{4k - 1},\]
\[x^N = \frac{a - c}{4k - 1}.\]

Again, by substituting \(w^N\) back to the retailer’s best-response functions \(\hat{q}(w)\) and \(\hat{p}(w)\), the retailer’s equilibrium decision is

\[q^N = \frac{k(a - c)}{4k - 1} + \frac{1}{2}E[\theta|Y] = \frac{k(a - c)}{4k - 1} + \frac{1}{2} \frac{t\sigma^2}{t\sigma^2 + 1} Y,\]

or

\[p^N = \frac{(3k - 1)a + kc}{4k - 1} + \frac{1}{2}E[\theta|Y] = \frac{(3k - 1)a + kc}{4k - 1} + \frac{1}{2} \frac{t\sigma^2}{t\sigma^2 + 1} Y,\]

where the superscript \(N\) denotes that the supply chain is non-communicative. By considering how the equilibrium wholesale price \(w^S\) and cost reduction level \(x^S\) depend on the demand signal \(Y\), we obtain the following result.

**Lemma 1.** \(x^S\) responds positively to the demand signal, while \(w^S\) responds positively if \(k \geq 1/2\) and negatively otherwise.

An informed manufacturer adjusts her cost reduction level in the same direction as the demand signal \(Y\) (i.e., \(x^S\) responds positively to \(Y\)). This is because a larger \(Y\) indicates that it is more likely for the demand to be high, and the value of cost reduction will be high too. When the manufacturer is not efficient in cost reduction, a larger \(Y\) induces her to raise wholesale price (i.e., \(w^S\) responds positively to \(Y\)) to increase profit margin. When the manufacturer is efficient in cost reduction, however, a larger \(Y\) induces her to lower production cost so much that she is better off by lowering wholesale price (i.e., \(w^S\) responds negatively to \(Y\)) to increase sales without sacrificing profit margin.

Based on the equilibrium decisions, we compute the firms’ and the supply chain’s ex-ante profits for the two cases of whether the supply chain is communicative or not.

\[
\Pi^S_R = \frac{k^2 (a - c)^2}{(4k - 1)^2} + \frac{k^2}{(4k - 1)^2} \frac{t\sigma^4}{t\sigma^2 + 1}, \quad \Pi^N_R = \frac{k^2 (a - c)^2}{(4k - 1)^2} + \frac{1}{4} \frac{t\sigma^4}{t\sigma^2 + 1},
\]
\[
\Pi^S_M = \frac{k(a - c)^2}{2(4k - 1)} + \frac{k}{2(4k - 1)} \frac{t\sigma^4}{t\sigma^2 + 1}, \quad \Pi^N_M = \frac{k(a - c)^2}{2(4k - 1)},
\]
\[
\Pi^S = \frac{k(6k - 1)(a - c)^2}{2(4k - 1)^2} + \frac{k(6k - 1)}{2(4k - 1)^2} \frac{t\sigma^4}{t\sigma^2 + 1}, \quad \Pi^N = \frac{k(6k - 1)(a - c)^2}{2(4k - 1)^2} + \frac{1}{4} \frac{t\sigma^4}{t\sigma^2 + 1}.
\]
Here the superscripts $S$ and $N$ denote respectively that the supply chain is communicative and non-communicative, while the subscripts $M$ and $R$ denote respectively the manufacturer and the retailer.

By comparing the ex-ante profits, we evaluate the effect of information sharing on the firms and the supply chain. The results are given in the following proposition.

**Proposition 1.** Information sharing benefits the manufacturer; it benefits the retailer if and only if $k < 1/2$; it benefits the supply chain if and only if $k < (3 + \sqrt{5})/4$.

With linear production cost, Li and Zhang (2002) show that information sharing makes the double marginalization of wholesale price more significant and hurts the retailer as well as the supply chain. With non-linear production cost, Ha et al. (2011) and Shang et al. (2014) show that information sharing could benefit the supply chain, because it allows the manufacturer to lower production cost by adjusting wholesale price to influence the uncertainty of production volume. In all these papers, the manufacturer’s only decision is wholesale price. In our setting, we assume linear production cost but the manufacturer makes both wholesale price and cost reduction decisions. Information sharing allows the manufacturer to adjust both decisions in response to demand fluctuation and therefore it always benefits the manufacturer. From Lemma 1, when $k \geq 1/2$, information sharing induces the manufacturer’s wholesale price to respond positively to the demand signal and makes double marginalization more significant, which hurts the retailer. When $k < 1/2$, information sharing induces the manufacturer’s wholesale price to respond negatively to the demand signal and weakens double marginalization, which benefits the retailer. When $k \geq (3 + \sqrt{5})/4$, information sharing hurts the supply chain because the negative double marginalization effect dominates the positive cost reduction effect. When $k < (3 + \sqrt{5})/4$, it benefits the supply chain because either the positive cost reduction effect dominates the negative double marginalization effect or double marginalization effect also becomes positive (when $k < 1/2$).

We now consider the impact of demand variability on the firms’ profits. This is useful for understanding the role of competition in information sharing. The following proposition shows that a larger demand variability benefits an informed firm, because it can adjust decisions in response to demand fluctuation. It has no impact on an uninformed firm.

**Proposition 2.** When the demand becomes more variable (larger $\sigma^2$), (a) for a communicative supply chain, the manufacturer, the retailer and hence the supply chain are all better off; (b) for a non-communicative supply chain, the manufacturer is indifferent, the retailer is better off and therefore the supply chain is also better off.
4.2. Equilibrium Information Sharing Decisions

In the first stage, the manufacturer decides whether to buy information from the retailer by offering a side payment $m$. The retailer then decides whether to sell information in exchange for the payment $m$. Denote the retailer decision by $X = S$ (share) or $N$ (do not share). Given the payment $m$ offered by the manufacturer, the retailer’s optimal decision is

$$X = \begin{cases} 
S, & \text{if } m \geq \Pi_N^R - \Pi_S^R, \\
N, & \text{otherwise}.
\end{cases}$$

Anticipating the retailer’s response, the manufacturer offers the minimum payment to induce information sharing if it generates a positive net surplus. Note that when $\Pi_N^R \geq \Pi_S^R$, the retailer is willing to share information voluntarily without any side payment. The following proposition, which characterizes the equilibrium information sharing decision $X^*$, follows from Proposition 1.

**Proposition 3.** (a) When $k < 1/2$, the retailer shares information voluntarily with $m = 0$, and $X^* = S$ is the unique equilibrium. (b) When $1/2 < k < (3 + \sqrt{5})/4$, the manufacturer buys information with the payment $m = \Pi_N^R - \Pi_S^R$, and $X^* = S$ is the unique equilibrium. (c) When $k > (3 + \sqrt{5})/4$, $X^* = N$ is the unique equilibrium.

The above result shows that when the manufacturer is efficient in cost reduction, information sharing creates a “win-win” situation. This could explain why some retailers are willing to share information for free (Keifer 2010).

5. Competing Supply Chains

In this section, we assume that the two supply chains are identical. In particular, $c_1 = c_2 = c$, $k_1 = k_2 = k$, and $t_1 = t_2 = t$. As in Section 4, to ensure interior equilibrium solutions, we assume that $k > a/(4c), k > 1/3$ and $a > c$. For the Bertrand model, we further assume that $k > a/[c(4 - 3\gamma_B)]$.

From the sequence of events given in Section 3, the firms in each supply chain decide in the first stage whether to share information or not. For each supply chain $i$, let $X_i$ denote the information sharing arrangement where $X_i = S$ if the firms agree to have information sharing and $X_i = N$ otherwise. Given $(X_1, X_2)$, we first derive the equilibrium wholesale price, cost reduction level and either retail quantity or retail price. Then we compute the firms’ ex-ante profits. Based on these profits for different information sharing arrangements, we finally analyze the equilibrium information sharing decisions in the first stage.

5.1. Equilibrium Analysis for Different Information Sharing Arrangements

Because both manufacturer $i$ and retailer $i$ cannot observe the wholesale price $w_j$ and the cost reduction level $x_j$ in supply chain $j$, they make their decisions based on a common conjecture of
By setting her wholesale price and cost reduction level to $q_j$ (for Cournot competition) or $p_j$ (for Bertrand competition). These conjectures will be fulfilled in an equilibrium.

For Cournot competition, given $w_i$, $x_i$, and a conjecture of $q_j$, retailer $i$ maximizes his profit,

$$ (a + E[\theta|Y_i] - q_i - \gamma C E[q_j|Y_i] - w_i)q_i, $$

with the selling quantity

$$ \hat{q}_i(q_j, w_i, x_i) = \frac{1}{2} \left( (a + E[\theta|Y_i] - w_i) - \gamma C E[q_j|Y_i] \right). $$

If supply chain $i$ is communicative ($X_i = S$), with a conjecture of $q_j$, manufacturer $i$ maximizes her expected profit,

$$ (w_i - c + x_i)q_i(q_j, w_i, x_i) - \frac{1}{2} k x_i^2, $$

by setting her wholesale price and cost reduction level to

$$ \hat{w}_i(q_j) = \frac{2k-1}{4k-1} (a + E[\theta|Y_i] - \gamma C E[q_j|Y_i]) + c \frac{2k-1}{4k-1}, $$

$$ \hat{x}_i(q_j) = \frac{1}{4k-1} (a + E[\theta|Y_i] - \gamma C E[q_j|Y_i] - c). $$

These are manufacturer $i$’s best response functions to $q_j$ when $X_i = S$. By substituting (6) into (5), we have

$$ \hat{q}_i(q_j, \hat{w}_i(q_j), \hat{x}_i(q_j)) = \frac{k}{4k-1} (a + E[\theta|Y_i] - \gamma C E[q_j|Y_i] - c), $$

which is retailer $i$’s best response function to $q_j$ when $X_i = S$. If supply chain $i$ is not communicative ($X_i = N$), manufacturer $i$ maximizes her expected profit,

$$ (w_i - c + x_i)\hat{E}[q_i(q_j, w_i, x_i)] - \frac{1}{2} k x_i^2, $$

by choosing

$$ \hat{w}_i(q_j) = \frac{2k-1}{4k-1} (a - \gamma C E[q_j]) + c \frac{2k-1}{4k-1}, $$

$$ \hat{x}_i(q_j) = \frac{1}{4k-1} (a - \gamma C E[q_j] - c). $$

These are manufacturer $i$’s best response functions to $q_j$ when $X_i = N$. By substituting (8) into (5), we have

$$ \hat{q}_i(q_j, \hat{w}_i(q_j), \hat{x}_i(q_j)) = \frac{k}{4k-1} (a - c) + \frac{1}{2} \left( E[\theta|Y_i] - \gamma C E[q_j|Y_i] + \frac{2k-1}{4k-1} \gamma C E[q_j] \right), $$

which is retailer $i$’s best response function to $q_j$ when $X_i = N$. A Bayesian Nash equilibrium is found by solving $q_i = \hat{q}_i(q_j, \hat{w}_i(q_j), \hat{x}_i(q_j))$ and $q_j = \hat{q}_j(q_i, \hat{w}_j(q_i), \hat{x}_j(q_i))$ simultaneously.

We conduct a similar analysis for Bertrand competition. For any given information sharing arrangement $(X_i, X_j)$, let $w_i^{x_i x_j}, x_i^{x_i x_j}, q_i^{x_i x_j}$ and $p_i^{x_i x_j}$ be the equilibrium decisions, where the subscript $Z = C$ or $B$, $C$ denotes Cournot and $B$ denotes Bertrand.
Lemma 2. (a) For Cournot competition, there exists a unique equilibrium in which an informed firm uses a strategy that is linear in its demand signal:

\[
\begin{align*}
q_i &= \frac{k(a-c)}{(\gamma_C + 4)k - 1} + C_i X_i Y_i, \\
w_{i,C} &= \frac{(2k-1)a + (2 + \gamma)ck}{(\gamma + 4)k - 1} + \alpha_{i,C} Y_i, \\
x_{i,C} &= \frac{a - c}{(\gamma + 4)k - 1} + \beta_{i,C} Y_i,
\end{align*}
\]

where

\[
\begin{align*}
C_i^{SS} &= \frac{kt\sigma^2}{(4t\sigma^2 + 4 + \gamma_C t\sigma^2)k - (t\sigma^2 + 1)}, \\
C_i^{SN} &= \frac{[2 + (2 - \gamma_C) t\sigma^2]kt\sigma^2}{8(1 + t\sigma^2)^2 - \gamma_B^2 t^2 \sigma^4}k - 2(t\sigma^2 + 1)^2, \\
C_i^{NS} &= \frac{[(4 + 4t\sigma^2 - \gamma_C t\sigma^2)k - (t\sigma^2 + 1)]t\sigma^2}{8(t\sigma^2 + 1)^2 - \gamma_B^2 t^2 \sigma^4}k - 2(t\sigma^2 + 1)^2, \\
C_i^{NN} &= \frac{ta^2}{2(t\sigma^2 + 1) + \gamma_C t\sigma^2}, \\
\alpha_{i,C}^{SX} &= \frac{2k - 1}{4k - 1}(1 - \gamma_C C_j^{NS})\frac{t\sigma^2}{1 + t\sigma^2}, \alpha_{i,C}^{NX} = 0, \\
\beta_{i,C}^{SX} &= \frac{1}{4k - 1}(1 - \gamma_C C_j^{NS})\frac{t\sigma^2}{1 + t\sigma^2}, \beta_{i,C}^{NX} = 0.
\end{align*}
\]

(b) For Bertrand competition, there exists a unique equilibrium where an informed firm uses a strategy that is linear in its demand signal:

\[
\begin{align*}
p_i &= \frac{(3k - 1)a + kc}{(4 - 3\gamma_B)k - (1 - \gamma_B)} + B_i^{X_i Y_i}, \\
w_{i,B} &= \frac{(2k - 1)a + (2 - \beta)ck}{(4 - 3\beta)k - (1 - \beta)} + \alpha_{i,B} Y_i, \\
x_{i,B} &= \frac{a - (1 - \beta)c}{(4 - 3\beta)k - (1 - \beta)} + \beta_{i,B} Y_i,
\end{align*}
\]

where

\[
\begin{align*}
B_i^{SS} &= \frac{(3k - 1)t\sigma^2}{(4(t\sigma^2 + 1) - 3\gamma_B t\sigma^2)k - (t\sigma^2 + 1 - \gamma_B t\sigma^2)}, \\
B_i^{NS} &= \frac{[(4(t\sigma^2 + 1) + 3\gamma_B t\sigma^2)k - (t\sigma^2 + 1 + \gamma_B t\sigma^2)]t\sigma^2}{8(1 + t\sigma^2)^2 - 3\gamma_B^2 t^2 \sigma^4}k - 2(t\sigma^2 + 1)^2 + \gamma_B^2 t^2 \sigma^4, \\
B_i^{SN} &= \frac{(3k - 1)(\gamma_B t\sigma^2 + 2(t\sigma^2 + 1))t\sigma^2}{8(1 + t\sigma^2)^2 - 3\gamma_B^2 t^2 \sigma^4}k - 2(t\sigma^2 + 1)^2 + \gamma_B^2 t^2 \sigma^4, \\
B_i^{NN} &= \frac{t\sigma^2}{2(1 + t\sigma^2) - \gamma_B t\sigma^2}.
\end{align*}
\]
In the above lemma, the coefficients \( \alpha_{i,B}^{S_{X_j}} \), \( \beta_{i,B}^{S_{X_j}} \), \( C_i^{X_j} \), and \( B_j^{X_i} \) capture the responsiveness of the firms’ decisions to the demand signal \( Y_i \). The superscript indicates that the supply chain \( i \)'s decision is based on the information sharing arrangement \((X_i, X_j)\). By analyzing the equilibrium decisions given in Lemma 2, we obtain the following result.

**Lemma 3.** (a) For \( Z = C \) or \( B \), \( x_{i,z}^{X_j} \) responds positively to \( Y_i \) if \( k \geq 1/2 \) and negatively otherwise. (b) For Cournot competition, information sharing in supply chain \( i \) makes \( q_i \) less responsive to \( Y_i \) and \( q_j \) more responsive to \( Y_j \) if \( k \geq 1/2 \), and \( q_i \) less responsive to \( Y_i \) and \( q_j \) more responsive to \( Y_j \) otherwise. (c) For Bertrand competition, information sharing in supply chain \( i \) makes both \( p_i \) and \( p_j \) more responsive to respectively \( Y_i \) and \( Y_j \) if \( k \geq 1/2 \), and both \( p_i \) and \( p_j \) less responsive to respectively \( Y_i \) and \( Y_j \) otherwise.

For part (a), an informed manufacturer adjusts her cost reduction level to respond positively \( (\beta_{i,z}^{S_{X_j}} \geq 0) \) to the demand signal. She adjusts her wholesale price to respond positively \( (\alpha_{i,z}^{S_{X_j}} \geq 0) \) to the demand signal if she is not efficient in cost reduction, and negatively \( (\alpha_{i,z}^{S_{X_j}} < 0) \) otherwise. These are consistent with the case of single supply chain. For part (b), note that \( q_i \) responds positively to \( Y_i \) and retailer \( i \) adjusts \( q_i \) in the opposite direction of the adjustment in \( w_i \). When \( k \geq 1/2 \), \( w_i \) responds positively (negatively) to \( Y_i \) and it makes \( q_i \) less (more) responsive. This in turn makes \( q_j \) more (less) responsive because they are strategic substitutes. For part (c), note that \( p_i \) responds positively to \( Y_i \) and retailer \( i \) adjusts \( p_i \) in the same direction of the adjustment in \( w_i \). When \( k \geq 1/2 \), \( w_i \) responds positively (negatively) to \( Y_i \) and it makes \( p_i \) more (less) responsive. This in turn makes \( p_j \) more (less) responsive because they are strategic complements. Note that two decisions are strategic substitutes (complements) if a change in one decision induces the other to change in the opposite (same) direction.

Now we are ready to derive the firms’ ex-ante profits. First, consider Cournot competition. Given the quantity decision of supply chain \( j \), \( q_j = k(a - c) / [((\gamma C + 4)k - 1)] + C_j Y_j \), where \( C_j \) represents how \( q_j \) responds to \( Y_j \), the inverse demand function for supply chain \( i \) can be written as

\[
p_i = a_i + \theta_i = q_i,
\]

where \( a_i = a - \gamma C k(a - c) / [((\gamma C + 4)k - 1)] \) and \( \theta_i = \gamma C C_j Y_j \). It is easy to show that

\[
E[(E[\theta_i | Y_i])^2] = (1 - \gamma C C_j)^2 \frac{t \sigma^4}{t \sigma^2 + 1}.
\]
Let \( \pi_{R_i,C}(C_j) \), \( \pi_{M_i,C}(C_j) \) and \( \pi_{i,C}(C_j) \) be respectively the profits of retailer \( i \), manufacturer \( i \) and supply chain \( i \). When supply chain \( j \)'s strategy is \( q_j = k(a - c) / ((\gamma_C + 4)k - 1) + C_j Y_j \), by applying the results from the single supply chain analysis with (3) replaced by (10), we obtain the following ex-ante profits in supply chain \( i \).

\[
\begin{align*}
\pi_{R_i,C}^{S}(C_j) &= \frac{k^2(a - c)^2}{(4k + k\gamma_C - 1)^2} + \frac{k^2}{(4k - 1)^2}\left(1 - \gamma_C C_j\right)^2 \frac{t\sigma^4}{t\sigma^2 + 1}, \\
\pi_{M_i,C}^{S}(C_j) &= \frac{k(4k - 1)(a - c)^2}{2(4k + k\gamma_C - 1)^2} + \frac{k}{2(4k - 1)}\left(1 - \gamma_C C_j\right)^2 \frac{t\sigma^4}{t\sigma^2 + 1}, \\
\pi_{i,C}^{S}(C_j) &= \frac{k(6k - 1)(a - c)^2}{2(4k + k\gamma_C - 1)^2} + \frac{k(6k - 1)}{2(4k - 1)^2}\left(1 - \gamma_C C_j\right)^2 \frac{t\sigma^4}{t\sigma^2 + 1}, \\
\pi_{R_i,C}^{N}(C_j) &= \frac{k^2(a - c)^2}{(4k + k\gamma_C - 1)^2} + \frac{1}{4}\left(1 - \gamma_C C_j\right)^2 \frac{t\sigma^4}{t\sigma^2 + 1}, \\
\pi_{M_i,C}^{N}(C_j) &= \frac{k(4k - 1)(a - c)^2}{2(4k + k\gamma_C - 1)^2}, \\
\pi_{i,C}^{N}(C_j) &= \frac{k(6k - 1)(a - c)^2}{2(4k + k\gamma_C - 1)^2} + \frac{1}{4}\left(1 - \gamma_C C_j\right)^2 \frac{t\sigma^4}{t\sigma^2 + 1}.
\end{align*}
\]

Given an information sharing arrangement \((X_i, X_j)\), denote the equilibrium profits for manufacturer \( i \), retailer \( i \), and supply chain \( i \) as \( \Pi_{M_i,C}^{X_i,X_j} \), \( \Pi_{R_i,C}^{X_i,X_j} \) and \( \Pi_{i,C}^{X_i,X_j} \), respectively. We can derive the ex-ante profits in equilibrium which can be expressed as

\[
\Pi_{M_i,C}^{X_i,X_j} = \pi_{M_i,C}(C_j^{X_i,X_j}), \Pi_{R_i,C}^{X_i,X_j} = \pi_{R_i,C}(C_j^{X_i,X_j}), \Pi_{i,C}^{X_i,X_j} = \pi_{i,C}(C_j^{X_i,X_j}).
\]

For Bertrand competition, we follow the same approach to derive the firms’ ex-ante profits. Given the price decision of supply chain \( j \), \( p_j = [(3k - 1)a + kc] / [(4 - 3\gamma_B)k - (1 - \gamma_B)] + B_j Y_j \), where \( B_j \) represents how \( p_j \) responds to \( Y_j \), the demand function for supply chain \( i \) can be written as

\[
q_i = a_i + \theta_i - p_i, \quad (11)
\]

where \( a_i = a + \gamma_B [(3k - 1)a + kc] / [(4 - 3\gamma_B)k - (1 - \gamma_B)] \) and \( \theta_i = \theta + \gamma_B B_j Y_j \). It is easy to show that

\[
E[(E[\theta_i | Y_i])]^2 = (1 + \gamma_B B_j)^2 \frac{t\sigma^4}{t\sigma^2 + 1}.
\]

Let \( \pi_{R_i,B}(B_j) \), \( \pi_{M_i,B}(B_j) \) and \( \pi_{i,B}(B_j) \) be respectively the profits of retailer \( i \), manufacturer \( i \) and supply chain \( i \). When supply chain \( j \)'s strategy is \( p_j = [(3k - 1)a + kc] / [(4 - 3\gamma_B)k - (1 - \gamma_B)] + B_j Y_j \), by applying the results from the single supply chain analysis with (4) replaced by (11), we obtain the following ex-ante profits in supply chain \( i \).

\[
\pi_{R_i,B}^{S}(B_j) = \frac{k^2(a - c + c\gamma_B)^2}{(4 - 3\gamma_B)k - (1 - \gamma_B))^2} + \frac{k}{2(4k - 1)}(1 + \gamma_B B_j)^2 \frac{t\sigma^4}{t\sigma^2 + 1}.
\]
Given an information sharing arrangement \((X_i, X_j)\), denote the equilibrium profits for manufacturer \(i\), retailer \(i\), and supply chain \(i\) as \(\Pi^X_{M_i,B}\), \(\Pi^X_{R_i,B}\) and \(\Pi^X_{i,B}\), respectively. We can derive the ex-ante profits in equilibrium which can be expressed as

\[
\Pi^X_{M_i,B} = \pi^S_{M_i,B}(B_j^i, X_i^j), \quad \Pi^X_{R_i,B} = \pi^N_{R_i,B}(B_j^i, X_i^j), \quad \Pi^X_{i,B} = \pi^S_{i,B}(B_j^i, X_i^j).
\]

### 5.2. Effects of Information Sharing

We first study how information sharing in a supply chain impacts the firms in the rival supply chain, which is called the spillover effect.

**Proposition 4.** For both types of competition, when information is shared in supply chain \(i\), (a) if supply chain \(j\) is communicative, both manufacturer \(j\) and retailer \(j\) are worse off when \(k < 1/2\) and better off otherwise; (b) if supply chain \(j\) is non-communicative, manufacturer \(j\) is indifferent, but retailer \(j\) is worse off when \(k < 1/2\) and better off otherwise.

From Lemma 3, for Cournot competition, information sharing in supply chain \(i\) makes \(q_i\) more (less) responsive when \(k < 1/2\) (\(k > 1/2\)), which in turn makes the demand intercept of supply chain \(j\), \(a + \theta - \gamma_C q_i\), less (more) variable. Similarly, for Bertrand competition, it makes \(p_i\) less (more) responsive when \(k < 1/2\) (\(k > 1/2\)), which makes the demand intercept for supply chain \(j\), \(a + \theta + \gamma_B p_i\), less (more) variable. The results then follow from Proposition 2.

Now we consider how information sharing in a supply chain impacts its own profit. Following Ha et al. (2011), we decompose the value of information sharing into direct and competitive effects.

Given the information sharing arrangement \(X_j\) in supply chain \(j\), we define the value of information sharing to supply chain \(i\) as

\[
\Pi^X_{i,B} = \Pi^X_{i,B} - \Pi^X_{i,B}.
\]

For Cournot competition, \(\Pi^X_{i,C}\) and \(\Pi^X_{i,B}\) are functions of supply chain \(j\)’s strategy \(q_j^X X_i = \tilde{q}_j + C_j^X X_i Y_j\). We can decompose \(\Pi^X_{i,B}\) into the sum of the direct effect and the competitive effect,

\[
\left\{ \pi^S_{i,C}(C_j^X) - \pi^N_{i,C}(C_j^X) \right\} + \left\{ \pi^S_{i,C}(C_j^X) - \pi^S_{i,C}(C_j^X) \right\}.
\]
where the direct effect (first bracket term) ignores supply chain \( j \)'s reaction (i.e., \( q_j \) remains as \( k(a-c)/[(\gamma_C+4)k-1]+C_j^{Xj}Y_j \)) and the competitive effect (second bracket term) accounts for such change (i.e., \( q_j \) changes to \( k(a-c)/[(\gamma_C+4)k-1]+C_j^{Xj}Y_j \)). Similarly, for Bertrand competition, we decompose \( V_i^i,B \) into the sum of the direct effect and the competitive effect,

\[
\left\{ \pi_i^S(B_j^{Xj,N}) - \pi_i^N(B_j^{Xj,N}) \right\} + \left\{ \pi_i^S(B_j^{Xj,S}) - \pi_i^S(B_j^{Xj,N}) \right\}
\]

where the direct effect (first bracket term) ignores supply chain \( j \)'s reaction (i.e., \( p_j \) remains as \( [(3k-1)a+kc]/[(4-3\gamma_B)k-(1-\gamma_B)]+B_j^{Xj,N}Y_j \)) and the competitive effect (second bracket term) accounts for such change (i.e., \( p_j \) changes to \( [(3k-1)a+kc]/[(4-3\gamma_B)k-(1-\gamma_B)]+B_j^{Xj,S}Y_j \)).

**Proposition 5.** (a) For both types of competition, the direct effect of information sharing is positive if \( k<(3+\sqrt{5})/4 \) and negative otherwise. (b) For Cournot competition, the competitive effect is positive if \( k<1/2 \) and negative otherwise. (c) For Bertrand competition, the competitive effect is negative if \( k<1/2 \) and positive otherwise.

Without considering the reaction of the competing supply chain, the direct effect is positive if \( k<(3+\sqrt{5})/4 \), which follows directly from the case of the single supply chain. For the competitive effect, we need to look at the impact of the competitive reaction from supply chain \( j \). From Lemma 3, for Cournot competition, information sharing in supply chain \( i \) makes \( q_j \) less (more) responsive when \( k<1/2 \) (\( k>1/2 \)), which in turn makes the demand intercept of supply chain \( i \), \( a+\theta-\gamma_Cq_j \), more (less) variable. Similarly, for Bertrand competition, it makes \( p_j \) less (more) responsive when \( k<1/2 \) (\( k>1/2 \)), which makes the demand intercept for supply chain \( i \), \( a+\theta+\gamma_Bp_j \), less (more) variable. Parts (b) and (c) then follow from Proposition 2.

**Proposition 6.** (a) For Cournot competition, there exist \( k_C^S \) and \( k_C^N \) such that (1) \( V_i^S,C > 0 \) iff \( k < k_C^C \) and \( V_i^N,C > 0 \) iff \( k < k_C^N \); (2) \( 1/2 < k_C^N < k_C^C < (3+\sqrt{5})/4 \); (3) \( k_C^S \) and \( k_C^N \) are decreasing in \( t \) and \( \gamma_C \).

(b) For Bertrand competition, there exist \( k_B^S \), \( k_B^N \), and \( \hat{\gamma}_B \) such that (1) when \( \gamma_B > \hat{\gamma}_B \), \( V_i^S,B > 0 \); when \( \gamma_B < \hat{\gamma}_B \), \( V_i^S,B > 0 \) iff \( k < k_B^S \); when \( \gamma_B > \sqrt{2/3}\hat{\gamma}_B \), \( V_i^N,B > 0 \); when \( \gamma_B < \sqrt{2/3}\hat{\gamma}_B \), \( V_i^N,B > 0 \) iff \( k < k_B^N \); (2) \( (3+\sqrt{5})/4 < k_B^N < k_B^S \); (3) \( k_B^S \) and \( k_B^N \) are increasing in \( t \) and \( \gamma_B \).

The above results can be explained by the direct and competitive effects of information sharing. From Proposition 1, a supply chain that is more efficient in cost reduction benefits from information sharing. For Cournot competition, Part 1(a) shows that the value of information sharing is positive when \( k \) is less than \( k_C^N \) (\( k_C^S \)) if the rival supply chain is non-communicative (communicative). For Part 1(b), \( k_C^N \) and \( k_C^S \) are larger than than 1/2 because both the direct and competitive effects are
positive when \( k < 1/2 \). They are smaller than \((3 + \sqrt{5})/4\) because both the direct and competitive effects are negative when \( k > (3 + \sqrt{5})/4 \). Moreover, for supply chain \( i \) to benefit from information sharing, a lower efficiency in cost reduction is allowed if supply chain \( j \) is communicative. This is because when \( k > 1/2 \), information sharing in supply chain \( j \) makes \( q_j \) less responsive (Lemma 3). A less responsive \( q_j \) weakens the negative competitive effect from supply chain \( j \). Thus \( k^N_C < k^S_C \).

Part 1(c) shows how information accuracy \( t \) and competition intensity \( \gamma_C \) impact the thresholds \( k^N_C \) and \( k^S_C \). If the demand signal is more accurate or the competition is more intense, the negative competitive effect becomes stronger when \( k > 1/2 \), which requires a smaller \( k \) to induce a more positive direct effect to offset it so that the value of information sharing is positive. Thus \( k^N_C \) and \( k^S_C \) are decreasing in \( t \) and \( \gamma_C \).

For Bertrand competition, the direct effect is positive when \( k < (3 + \sqrt{5})/4 \) and the competitive effect is positive when \( k > 1/2 \). For Part 2 (a) and (b), when \( k < 1/2 \), the positive direct effect due to a high efficiency in cost reduction dominates the negative competitive effect, no matter how intense the competition is. When \( 1/2 < k < (3 + \sqrt{5})/4 \), both the direct and competitive effects are positive. Thus \( k^N_B \) and \( k^S_B \) are larger than \((3 + \sqrt{5})/4\). When \( k > (3 + \sqrt{5})/4 \), we need to consider the relative magnitudes of the negative direct effect and the positive competitive effect. If competition is very intense, the positive competitive effect is so strong that the value of information sharing is always positive. If competition is not very intense, the value of information sharing is positive if and only if \( k \) is less than \( k^S_B \) or \( k^N_B \), which means that the direct effect is not too negative and therefore dominated by the positive competitive effect. For Part 2(c), when \( k > 1/2 \), if the signal is more accurate or the competition is more intense, the retail prices are more responsive and the positive competitive effect becomes stronger. Thus \( k^N_C \) and \( k^S_C \) are increasing in \( t \) and \( \gamma_B \).

**5.3. Equilibrium Information Sharing Decisions**

In the first stage, before the retailers observe their private demand signals, manufacturer \( i \) decides whether to buy information from retailer \( i \) by offering a side payment \( m_i \), and retailer \( i \) then decides whether to share information in exchange for the payment. The information sharing arrangement in supply chain \( i \) is given by \( X_i \), where \( X_i = S \) (share) or \( N \) (not share).

In making their decisions, because manufacturer \( i \) and retailer \( i \) cannot observe the information sharing arrangement \( X_j \) in supply chain \( j \), they form a common conjecture about \( X_j \). Under this common conjecture, retailer \( i \)'s profit is \( \Pi^{SX_j}_{R_i,Z} + m_i \) if \( X_i = S \), and \( \Pi^{NX_j}_{R_i,Z} \) if \( X_i = N \). Manufacturer \( i \)'s profit is \( \Pi^{SX_j}_{M_i,Z} - m_i \) if \( X_i = S \), and \( \Pi^{NX_j}_{M_i,Z} \) if \( X_i = N \).

We distinguish two cases depending on whether \( k \leq 1/2 \) or not. When \( k \leq 1/2 \), we can show that \( \Pi^{NX_j}_{R_i,Z} \leq \Pi^{SX_j}_{R_i,Z} \), regardless of whether \( X_j = S \) or \( N \). Then retailer \( i \) is willing to share information without any side payment, and it is optimal for manufacturer \( i \) to offer \( m_i = 0 \) to increase his profit.
from $\Pi_{M_i^X,Z}^{NX}$ to $\Pi_{M_i^X,Z}^{SX}$. The dominant strategy in each supply chain is to have information sharing and $(X_i,X_j) = (S,S)$ is the unique equilibrium. In this case, a manufacturer cannot extract all the surplus created by information sharing in the supply chain.

Now consider the case when $k > 1/2$. We can show that $\Pi_{M_i^X,Z}^{NX} > \Pi_{M_i^X,Z}^{SX}$, and the retailer is not willing to share information without a side payment. Given the payment $m_i$ offered by manufacturer $i$, retailer $i$’s optimal decision in response to $X_j$ is

$$X_i(X_j,m_i) = \begin{cases} S, & \text{if } m_i \geq \Pi_{R_i,Z}^{NX_j} - \Pi_{R_i,Z}^{SX_j}, \\ N, & \text{otherwise}. \end{cases}$$

The minimum payment $m_i$ to induce retailer $i$ to share information is $m_i = \Pi_{R_i,Z}^{NX_j} - \Pi_{R_i,Z}^{SX_j} > 0$. Information sharing benefits manufacturer $i$ if and only if $\Pi_{M_i^X,Z}^{SX_j} - m_i \geq \Pi_{M_i^X,Z}^{NX_j}$, which is equivalent to $V_i^{X_j} \geq 0$. In response to $X_j$, manufacturer $i$’s optimal offer to retailer $i$ is

$$\hat{m}_i^{X_j} = \begin{cases} \Pi_{R_i,Z}^{NX_j} - \Pi_{R_i,Z}^{SX_j}, & \text{if } \Pi_{R_i,Z}^{NX_j} > \Pi_{R_i,Z}^{SX_j}, \\ 0, & \text{otherwise}, \end{cases}$$

which induces retailer $i$ to make the following best-response:

$$X_i(X_j,\hat{m}_i^{X_j}) = \begin{cases} S, & \text{if } \Pi_{R_i,Z}^{SX_j} + \hat{m}_i^{X_j} \geq \Pi_{R_i,Z}^{NX_j}, \\ N, & \text{otherwise}. \end{cases}$$

Manufacturer $i$ essentially makes the information sharing decision for her supply chain and earns all the surplus, if any, due to that decision. We can simplify the first stage to a game having only manufacturers as players and a payoff matrix given by Table 1.

<table>
<thead>
<tr>
<th>Manufacturer 1 \ Manufacturer 2</th>
<th>Share ($X_2 = S$)</th>
<th>Not share ($X_2 = N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share ($X_1 = S$)</td>
<td>$(\Pi_{M_1^X,Z}^{NX_1} + V_{1,Z}^{SN}, \Pi_{M_2^X,Z}^{NX_2} + V_{2,Z}^{SN})$</td>
<td>$(\Pi_{M_1^X,Z}^{NX_1}, \Pi_{M_2^X,Z}^{NX_2})$</td>
</tr>
<tr>
<td>Not share ($X_1 = N$)</td>
<td>$(\Pi_{M_1^X,Z}^{NX_1}, \Pi_{M_2^X,Z}^{NX_2} + V_{2,Z}^{SN})$</td>
<td>$(\Pi_{M_1^X,Z}^{NX_1} + V_{1,Z}^{SN}, \Pi_{M_2^X,Z}^{NX_2})$</td>
</tr>
</tbody>
</table>

Table 1 Payoff Matrix of the Stage-One Game

The following proposition characterizes the information sharing equilibrium $(X_1^*,X_2^*)$.

**PROPOSITION 7.** (a) For Cournot competition, if $k < k_B^N$, $(S,S)$ is the unique equilibrium; if $k_B^N < k < k_B^S$, $(S,S)$ and $(N,N)$ are two possible equilibria, and $(S,S)$ is Pareto optimal; otherwise, $(N,N)$ is the unique equilibrium.

(b) For Bertrand competition, (1) when $\gamma_B \geq \hat{\gamma}_B$, $(S,S)$ is the unique equilibrium; (2) when $\sqrt{2/3}\gamma_B < \gamma_B < \hat{\gamma}_B$, if $k < k_B^N$, $(S,S)$ is the unique equilibrium; if $k > k_B^N$, $(S,S)$ and $(N,N)$ are two possible equilibria, and $(S,S)$ is Pareto optimal; (3) when $\gamma_B < \sqrt{2/3}\gamma_B$, if $k < k_B^N$, $(S,S)$ is the unique equilibrium; if $k_B^N < k < k_B^S$, $(S,S)$ and $(N,N)$ are two possible equilibria, and $(S,S)$ is Pareto optimal; if $k > k_B^S$, $(N,N)$ is the unique equilibrium.

(c) For both types of competition, if $k < 1/2$, $m_1 = m_2 = 0$ and $(S,S)$ is the unique equilibrium.
Proposition 7 follows from Proposition 6. Part (c) says that when the manufacturers are sufficiently efficient in cost reduction, the retailers are willing to share information for free.

Next we consider how information accuracy ($t$) and competition intensity ($\gamma_B$) impact the retailers’ profits. If both ($S,S$) and ($N,N$) are equilibria, we select ($S,S$) as the outcome of the game because it is Pareto-optimal. Let $\tilde{\Pi}^Z_{R_i}$ be the total ex-ante profit of retailer $i$ after accounting for the side payment for information sharing. Let $t^S_C$ and $\gamma^S_B$ be the inverse functions of respectively $k^S_C(t)$ and $k^S_B(\gamma_B)$.

**Proposition 8.** (a) For Cournot competition, there exists $\delta_C > 0$ such that $\tilde{\Pi}^Z_{R_i}(t^S_C - \delta_C) \geq \tilde{\Pi}^Z_{R_i}(t^S_C + \delta_C)$. (b) For Bertrand competition, there exists $\delta_B > 0$ such that $\tilde{\Pi}^Z_{R_i}(\gamma^S_B - \delta_B) \leq \tilde{\Pi}^Z_{R_i}(\gamma^S_B + \delta_B)$.

For Cournot competition, more accurate information (larger $t$) always benefits the retailers if the information sharing equilibrium doesn’t change. However, if more accurate information induces the equilibrium to change from ($S,S$) to ($N,N$), it leads to a downward jump for a retailer’s profit because of the loss of the side payment for information sharing. Part (a) implies that when the forecasting accuracy is improved for both retailers, for instance, due to technological advance, such an improvement could make them worse off if it induces both supply chains to cease information sharing. For Bertrand competition, more intense competition (larger $\gamma_B$) always makes the retailers worse off if the information sharing equilibrium doesn’t change. However, if more intense competition induces the equilibrium to change from ($N,N$) to ($S,S$), it leads to an upward jump for a retailer’s profit because of the side payment for information sharing. Part (b) implies that under suitable conditions, retailers might want to take actions to intensify price competition so that the manufacturers are incentivized to buy information from them.

**6. Numerical Study: Non-Identical Supply Chains**

In the previous sections, we assume that the two supply chains are identical. In this section, we allow them to have different efficiencies in production cost reduction. Without loss of generality, we assume $k_1 \geq k_2$. Because the model becomes analytically intractable, we conduct an extensive numerical study to investigate the equilibrium information sharing decisions.

For Cournot competition, we consider $\gamma_C$ to be from 0.1 to 0.9 with an increment of 0.1, and $t\sigma^2$ to be in the set \{0.1, 1, 10, 20, 50, 100, 500\}. Thus, there are 63 cases in total. For each case with a given pair of $\gamma_C$ and $t\sigma^2$, we can numerically establish functions $k_i^{X_j}(k_j)$ such that $V_i^{X_j} > 0$ if and only if $k_i < k_i^{X_j}(k_j)$, where $i = 1$ or 2, $i \neq j$, and $X_j = S$ or $N$. Then based on the payoff matrix given by Table 1, we characterize the equilibrium information sharing decisions $(X_1^*, X_2^*)$ in the space of $\{(k_1, k_2) | k_1 \geq k_2\}$. We find that there is only one equilibrium structure, which is
illustrated in Figure 1. To improve the presentation, we use a logarithmic scale for both vertical and horizontal axes. Along the 45-degree line with \( k_1 = k_2 \), the information sharing equilibrium is symmetric and consistent with the results in Proposition 7. From Figure 1, we observe that it is possible to have partial information sharing when \( k_1 > k_1^S \) and \( k_2 < k_2^N \). In this case, information is shared only in supply chain 2, which is more efficient in cost reduction when compared with supply chain 1.

![Figure 1 Information Sharing Decisions under Cournot Competition (\( a = 200, t\sigma^2 = 100, \gamma_C = 0.7 \))](image)

For Bertrand competition, we consider \( \gamma_B \) to be from 0.1 to 0.9 with an increment of 0.1, as well as \( \gamma_B = 0.93 \). We also consider \( t\sigma^2 \) to be in the set \{0.1, 1, 10, 20, 50, 100, 500\}. Thus, there are 70 cases in total. For each case with a given pair of \( \gamma_B \) and \( t\sigma^2 \), as before, we can numerically establish functions \( k_i^{X_j}(k_j) \) such that \( V_i^{X_j} > 0 \) if and only if \( k_i < k_i^{X_j}(k_j) \), where \( i = 1 \) or 2, \( i \neq j \), and \( X_j = S \) or \( N \). Then based on the payoff matrix given by Table 1, we characterize the equilibrium information sharing decisions \( (X_1^*, X_2^*) \) in the space of \{\((k_1, k_2) | k_1 \geq k_2\}\}. We find that depending on the value of \( \gamma_B \), there are three possible equilibrium structures, which are illustrated in Figure 2. Similar to Figure 1, we use a logarithmic scale for both vertical and horizontal axes. From Figure 2, we observe that when \( \gamma_B \) is small, the equilibrium structure in Figure 2(a) is similar to that of Figure 1. However, as \( \gamma_B \) becomes larger, the firms have more incentive to share information and therefore the regions of \((N, N)\) and \((N, S)\) become smaller or even disappear. This is because the positive competitive effect becomes stronger and the value of information sharing to a supply chain is more likely to be positive. Moreover, similar to Cournot competition, it is possible to have partial information sharing when supply chain 2 is sufficiently more efficient in cost reduction when compared with supply chain 1.

For both Cournot and Bertrand competition, if \( k_2 > 1/2 \), a larger \( k_2 \) or a smaller \( k_1 \) always makes supply chain 2 worse off if the information sharing equilibrium does not change. However, when
a larger $k_2$ or a smaller $k_1$ induces the information sharing equilibrium to change from $(N, S)$ to $(S, S)$, supply chain 2 is straightly better off because of the positive spillover effect of information sharing in supply chain 1.

### 7. Conclusion

In this paper, we investigate production cost reduction as a driver of the incentive for information sharing in a supply chain, and how this incentive depends on efficiency in production cost reduction, information accuracy, competition intensity and type of competition. Our analysis reveals novel insights that are of interest to the practitioners.

There are several limitations to this study. First, we assume identical supply chains in the main model and examine different cost reduction efficiencies in the numerical study. An interesting extension of our model is to investigate the case when the supply chains have different cost reduction efficiencies and information accuracies. Second, we consider two competing supply chains, each of which is characterized by an exclusive manufacturer-retailer relationship. It would be interesting to consider other supply chain settings, for instance, when there is a common manufacturer selling to competing retailers, or there are competing manufacturers selling to a common retailer. Because a complete and tractable analysis of these extensions requires different model setups and modes of analysis, we leave these issues for future research.

### Acknowledgments

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Appendix

Proof of Lemma 1

It is straightforward to show that the coefficient of $Y$ in $x^S$ is positive, and that in $w^S$ is positive if $k \geq \frac{1}{2}$ and negative otherwise. The result follows.

Proof of Proposition 1

We can show that

$$
\Pi^S_R - \Pi^N_R = \frac{k \sigma^4 (4k-1)(1-2k)}{4(4k-1)^2 (\sigma^2+1)} > 0 \text{ if and only if } k < \frac{1}{2},
$$

$$
\Pi^S_M - \Pi^N_M = \frac{k}{2(4k-1)} \frac{\sigma^4}{\sigma^2+1} > 0, \text{ and}
$$

$$
\Pi^S - \Pi^N = \frac{(6k-4k^2-1)\sigma^4}{4(4k-1)^2 (\sigma^2+1)} > 0 \text{ if and only if } k < \frac{3+\sqrt{5}}{4}.
$$

The results follow.

Proof of Proposition 2

It is easy to show that $\frac{d\Pi^S_R}{d\sigma^2} > 0$, $\frac{d\Pi^N_R}{d\sigma^2} > 0$, $\frac{d\Pi^S_M}{d\sigma^2} > 0$, $\frac{d\Pi^N_M}{d\sigma^2} = 0$, $\frac{d\Pi^S}{d\sigma^2} > 0$ and $\frac{d\Pi^N}{d\sigma^2} > 0$. Hence the results.

Proof of Proposition 3
(a) When \( k < 1/2 \), we have \( \Pi_R^N > \Pi_R^N \) and \( \Pi_M^S > \Pi_M^N \). Thus, the retailer is willing to share information without a side payment (i.e., \( m = 0 \)) and \( X^* = S \) is the unique equilibrium. (b) When \( 1/2 < k < (3 + \sqrt{5})/4 \), we have \( \Pi_R^N > \Pi_R^N \) and \( \Pi_M^S - (\Pi_R^N - \Pi_R^N) \geq \Pi_M^N \). Thus, the manufacturer can offer \( m = \Pi_R^N - \Pi_R^N \) to the retailer for sharing information and \( X^* = S \) is the unique equilibrium. (c) When \( k > (3 + \sqrt{5})/4 \), we have \( \Pi_R^N < \Pi_R^N \) and \( \Pi_M^S - (\Pi_R^N - \Pi_R^N) < \Pi_M^N \). Thus, the manufacturer cannot offer enough side payment to offset the retailer’s loss due to information sharing and \( X^* = N \) is the unique equilibrium.

Proof of Lemma 2

For part (a), for Cournot competition, given the information structure \((X_1, X_2)\), we can verify that \( q_i^{X_iX_j} = \hat{q}_i(q_j^{X_jX_i}) \), where \( \hat{q}_i(q_j) \) is given by (7), \( i = 1 \) or 2, and \( i \neq j \). We can also verify that \( w_i^{X_iX_j} = \hat{w}_i(q_j^{X_jX_i}) \) and \( x_i^{X_iX_j} = \hat{x}_i(q_j^{X_jX_i}) \), where \( \hat{w}_i(q_j) \) and \( \hat{x}_i(q_j) \) are given by (6). Thus \((q_1^{X_1X_2}, w_1^{X_1X_2}, x_1^{X_1X_2}, x_2^{X_1X_2}, q_2^{X_2X_1}, w_2^{X_2X_1}, x_2^{X_2X_1}, x_2^{X_2X_1})\) is an equilibrium. The proof of the uniqueness follows the proof of Lemma 1 in Ha et al. (2011).

For part (b), for Bertrand competition, given the information structure \((X_1, X_2)\), we can similarly show that \((p_1^{X_1X_2}, w_1^{X_1X_2}, x_1^{X_1X_2}, p_2^{X_2X_1}, w_2^{X_2X_1}, x_2^{X_2X_1})\) is a unique equilibrium.

Proof of Lemma 3

For part (a), for Cournot competition, we can verify that \( C_j^{X_jX_i} < 1 \). Thus, \( \beta_{i,C}^{X_jX_i} > 0 \) and \( \alpha_{i,C}^{X_jX_i} > 0 \) if and only if \( k > \frac{1}{2} \). The results follow.

For parts (b) and (c), we can show that

\[
\begin{align*}
(1) \quad C_{i}^{SS} - C_{i}^{NS} &= \frac{\sigma^2(t^2+1)^2(4k-1)(1-2k)}{(4\sigma^2+4+\gamma_C t^2\sigma^2)k^2-2(t^2+1)^2} > 0 \text{ if and only if } k < \frac{1}{2}, \\
(2) \quad C_{i}^{SN} - C_{i}^{NN} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(2\sigma^2+2t^2\sigma^2+2)(8(1+t+2)^2-\gamma_C \sigma^2)(k-2(t^2+1)^2)} > 0 \text{ if and only if } k < \frac{1}{2}, \\
(3) \quad C_{j}^{SS} - C_{j}^{NS} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(4t^2+4+\gamma_C t^2\sigma^2)k^2-2(t^2+1)^2} > 0 \text{ if and only if } k > \frac{1}{2}, \\
(4) \quad C_{j}^{SN} - C_{j}^{NN} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(8(1+t+2)^2-\gamma_C \sigma^2)(k-2(t^2+1)^2)} > 0 \text{ if and only if } k > \frac{1}{2}, \\
(5) \quad B_{i}^{SS} - B_{i}^{NS} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(4(1+t+2)^2-3\gamma_B \sigma^2)(k-2(t^2+1)^2)\sigma^2} > 0 \text{ if and only if } k > \frac{1}{2}, \\
(6) \quad B_{i}^{SN} - B_{i}^{NN} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(8(1+t+2)^2-3\gamma_B \sigma^2)(k-2(t^2+1)^2)\sigma^2} > 0 \text{ if and only if } k > \frac{1}{2}, \\
(7) \quad B_{j}^{SS} - B_{j}^{SN} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(4(1+t+2)^2-3\gamma_B \sigma^2)(k-2(t^2+1)^2)\sigma^2} > 0 \text{ if and only if } k > \frac{1}{2}, \\
(8) \quad B_{j}^{SN} - B_{j}^{NN} &= \frac{2t^2(2\sigma^2+2t^2\sigma^2)}{(8(1+t+2)^2-3\gamma_B \sigma^2)(k-2(t^2+1)^2)\sigma^2} > 0 \text{ if and only if } k > \frac{1}{2}.
\end{align*}
\]

For part (b), when information is shared in supply chain \( i \), (1) and (2) imply that \( q_i \) becomes less responsive to \( Y_i \), while (3) and (4) imply that \( q_j \) becomes more responsive to \( Y_j \). Similarly, the results of part (c) follow from (5) to (8).
Proof of Proposition 4

For Cournot competition, we can show that

1. \( \pi^S_{M_i,C}(C_i^{SS}) - \pi^S_{M_i,C}(C_i^{NS}) = \frac{k^3\gamma_C^2 (2-\gamma_C)\gamma_C^2}{4(k-1)(k^2+1)}(C_i^{NS} - C_i^{SS}) > 0 \) if and only if \( k > \frac{1}{2} \),
2. \( \pi^S_{R_j,C}(C_i^{SS}) - \pi^S_{R_j,C}(C_i^{NS}) = \frac{k^2 t^4 (2-\gamma_C^2\gamma_C^2)}{4(k-1)^2(k^2+1)}(C_i^{NS} - C_i^{SS}) > 0 \) if and only if \( k > \frac{1}{2} \),
3. \( \pi^S_{M_i,C}(C_i^{SN}) - \pi^S_{M_i,C}(C_i^{NN}) = 0 \),
4. \( \pi^S_{R_j,C}(C_i^{SN}) - \pi^S_{R_j,C}(C_i^{NN}) = \frac{k^3\gamma_C^2 (2-\gamma_C^2\gamma_C^2)}{4(k-1)^2(k^2+1)}(C_i^{SN} - C_i^{NN}) > 0 \) if and only if \( k > \frac{1}{2} \).

Part (a) follows from (1) and (2) while part (b) follows from (3) and (4). For Bertrand competition, the results can be proved in a similar way.

Proof of Proposition 5

We can show that

1. \( \pi^S_{i,C}(C_j^{X_j N}) - \pi^S_{i,C}(C_j^{X_j N}) = \frac{t^4 (1-\gamma_C)\gamma_C^2}{4(k-1)^2(t^2+1)}(6k - 4k^2 - 1) > 0 \) if and only if \( k < \frac{3+\sqrt{5}}{4} \),
2. \( \pi^S_{i,B}(B_j^{X_j N}) - \pi^S_{i,B}(B_j^{X_j N}) = \frac{t^4 (\gamma_B B_j)^2}{4(k-1)^2(t^2+1)}(6k - 4k^2 - 1) > 0 \) if and only if \( k < \frac{3+\sqrt{5}}{4} \),
3. \( \pi^S_{i,C}(C_j^{X_j S}) - \pi^S_{i,C}(C_j^{X_j N}) = \frac{k^2 t^4 (2-\gamma_C^2\gamma_C^2)}{4(k-1)^2(t^2+1)}(C_j^{X_j N} - C_j^{X_j S}) > 0 \) if and only if \( k < \frac{1}{2} \),
4. \( \pi^S_{i,B}(B_j^{X_j S}) - \pi^S_{i,B}(B_j^{X_j N}) = \frac{k^4 t^4 (2-\gamma_C^2\gamma_C^2)}{2(k-1)^2(t^2+1)}(B_j^{X_j S} - B_j^{X_j N}) > 0 \) if and only if \( k > \frac{1}{2} \).

Part (a) follows from (1) and (2), and parts (b) and (c) follow respectively from (3) and (4).

Proof of Proposition 6

For part (a), it is easy to check \( V^S_{i,C} > 0 \) if and only if \( g > 0 \) where

\[
g = k^3 \frac{t^4 \gamma_C^2}{(t^2+1)^2} - 8k^3 \frac{t^4 \gamma_C^2}{(t^2+1)^2} - 2(6k - 4k^2 + 1)
\]

Let \( \xi = \frac{t^4 \gamma_C^2}{(t^2+1)^2} \). We have \( 0 < \xi < 1 \), and \( \xi \) is increasing in \( \gamma_C \) and \( t \). Now we can write \( g \) as

\[
g(\xi) = k^3 \xi^2 - 8k^3 \xi - 2(6k - 4k^2 + 1),
\]

\( g(\xi) \) has two roots

\[
\xi_1(k) = \frac{1}{k^3} \left( -\sqrt{2} \sqrt{k^3 (6k-1)} + 4k^3 - 2 \sqrt{2k \sqrt{k^3 (6k-1)}} \right),
\]

\[
\xi_2(k) = \frac{1}{k^3} \left( \sqrt{2} \sqrt{k^3 (6k-1)} + 4k^3 + 2 \sqrt{2k \sqrt{k^3 (6k-1)}} \right).
\]

We can verify that when \( k > \frac{1}{2} \), \( \xi_2(k) > 1 \), \( \xi_1(k) \) is decreasing in \( k \), \( \xi_1(1) = (\sqrt{6} + 4) \) and \( \xi_1(2) < 0 \).

So we don’t need to consider \( \xi_2(k) \) since \( 0 < \xi < 1 \) if \( k < 1 \). Given \( t \) and \( \gamma_C \), we have a unique \( k_C^* \) such that

\[
\xi_1(k_C^*) = \frac{t^4 \gamma_C^2}{(t^2+1)^2}.
\]

We can conclude that \( g > 0 \) if and only if \( k < k_C^* \). Note that \( \xi \) is increasing in \( \gamma_C \) and \( t \) and \( \xi_1(k) \) is decreasing in \( k \). Thus, \( k^*_C \) is decreasing in \( \gamma_C \) and \( t \). Since \( \xi_1(k) > 1 \) when \( k < \frac{1}{2} \), and \( \xi_1(k) > 0 \) if and only if \( k < \frac{\sqrt{7} + 3}{4} \), we can get that \( \frac{1}{2} < k_C^* < \frac{\sqrt{7} + 3}{4} \).
Similarly, $V_{i,C}^S > 0$ if and only if $h > 0$, where

$$h = k(4k - 1)\frac{4^4\gamma_C^4}{(\sigma^2 + 1)^4} - 16k^2\frac{4^2\gamma_C^2}{(\sigma^2 + 1)^2} - 8(-6k + 4k^2 + 1).$$

Let $\rho = \frac{4^2\gamma_C^2}{(\sigma^2 + 1)^2}$. We have $0 < \rho < 1$, and $\rho$ is increasing in $\gamma_C$ and $t$. Now we can write $h$ as

$$h(\rho) = k(4k - 1)\rho^2 - 16k^2\rho - 8(-6k + 4k^2 + 1).$$

$h(\rho)$ has two roots

$$\rho_1(k) = \frac{1}{4k^2 - k}(2\sqrt{2}\sqrt{k(6k - 1)} + 8k^2 + 4\sqrt{2}\sqrt{k(6k - 1)}),$$

$$\rho_2(k) = \frac{1}{4k^2 - k}(2\sqrt{2}\sqrt{k(6k - 1)} + 8k^2 - 4\sqrt{2}\sqrt{k(6k - 1)}).$$

We can verify that when $k > \frac{1}{3}$, $\rho_1(k) > 1$, $\rho_2(k)$ is decreasing in $k$, $\rho_2(\frac{1}{3}) = (2\sqrt{6} + 8)$ and $\rho_2(2) < 0$. So we don’t need to consider $\rho_1(k)$ since $0 < \rho < 1$. Given $t$ and $\gamma_C$, we have a unique $k_C^N$ such that

$$\rho_2(k_C^N) = \frac{4^2\gamma_C^2}{(\sigma^2 + 1)^2}.$$ We can conclude that $h > 0$ if and only if $k < k_C^N$. Note that $\rho$ is increasing in $\gamma_C$ and $t$, and $\rho_2(k)$ is decreasing in $k$. Thus, $k_C^N$ is decreasing in $\gamma_C$ and $t$. Since $\rho_2(k) > 1$ when $k < \frac{1}{2}$, and $\rho_2(k) > 0$ if and only if $k < \frac{\sqrt{\pi} + 3}{4}$, we can get that $\frac{1}{2} < k_C^N < \frac{\sqrt{\pi} + 3}{4}$. $k_C^N < k_C^S$ because $\xi_1(k) > \rho_2(k)$ if $\frac{1}{2} < k < \frac{\sqrt{\pi} + 3}{4}$.

For Part (b), it is easy to check $V_{i,B}^S > 0$ if and only if

$$f = \frac{4^4\gamma_B^4}{(\sigma^2 + 1)^4} (3k - 2)(3k - 1)^2 - 4\frac{4^2\gamma_B^2}{(\sigma^2 + 1)^2} (3k - 1)(-6k + 6k^2 + 1) + 2(4k - 1)(-6k + 4k^2 + 1) < 0.$$

Let $\eta = \frac{4^4\gamma_B^4}{(\sigma^2 + 1)^4}$. We have $0 < \eta < 1$, and $\eta$ is increasing in $\gamma_B^4$ and $t$. Now we can write $f$ as $f(\eta) = \eta^2(3k - 2)(3k - 1)^2 - 4\eta(-6k + 6k^2 + 1)(3k - 1) + 2(4k - 1)(-6k + 4k^2 + 1)$. If $k = \frac{2}{3}$, we have $V_{i,B}^S > 0$ since $f(\eta) = \frac{2}{27}(18\eta - 55) < 0$. When $k \neq \frac{2}{3}$, $f(\eta) = 0$ have the following two roots:

$$\eta_1(k) = \frac{1}{(3k - 2)(3k - 1)}(-12k - 2\sqrt{2}\sqrt{k(6k - 1)} + 12k^2 + 2\sqrt{2}k\sqrt{k(6k - 1)} + 2),$$

$$\eta_2(k) = \frac{1}{(3k - 2)(3k - 1)}(-12k + 2\sqrt{2}\sqrt{k(6k - 1)} + 12k^2 - 2\sqrt{2}k\sqrt{k(6k - 1)} + 2).$$

We can verify that $\eta_1(k) > 1$ as long as $k > \frac{1}{3}$; if $\frac{1}{3} < k < \frac{2}{3}$, $\eta_2(k) > 1$; if $\frac{2}{3} < k < \frac{\sqrt{\pi} + 3}{4}$, $\eta_2(k) < 0$; if $k > \frac{\sqrt{\pi} + 3}{4}$, $0 < \eta_2(k) < 1$. Note that $f(\eta)$ is concave when $\frac{1}{3} < k < \frac{2}{3}$ and convex when $k > \frac{2}{3}$. Thus, as long as $k < \frac{\sqrt{\pi} + 3}{4}$, $V_{i,B}^S > 0$ since $f(\eta) < 0$. Now, we focus on $k > \frac{\sqrt{\pi} + 3}{4}$. If $k > \frac{\sqrt{\pi} + 3}{4}$, $\frac{d\eta(k)}{dk} > 0$ and $\eta_2(k)\big|_{k \to \infty} = \frac{12 - 4\sqrt{\pi}}{9}(\approx 0.5635)$, which means $\eta_2(k)$ is increasing with the upper bound $\frac{12 - 4\sqrt{\pi}}{9}$. We can conclude that given $t$, if there exists $\gamma_B^t$ such that $\frac{4^4\gamma_B^4}{(\sigma^2 + 1)^4} = \frac{12 - 4\sqrt{\pi}}{9}$, then for any $\gamma_B > \gamma_B^t$, $f(\eta) < 0$; if $\gamma_B < \gamma_B^t$, we have $f(\eta) < 0$ if $\eta > \eta_2(k)$. Denote $k_B^\eta = \eta_2^{-1}(\gamma_B,t)$. We have that $V_{i,B}^S > 0$ if $k < k_B^\eta$. And $k_B^\eta$ is increasing in $t$ and $\gamma_B$ because that $\eta$ is increasing in $t$ and $\gamma_B$. Since $\eta_2(k) > 0$ if and only if $k > \frac{\sqrt{\pi} + 3}{4}$, we have $k_B^\eta > \frac{3\sqrt{\pi}}{4}$.
Note that $V_i^N > 0$ if and only if $o < 0$, where

$$o = (4k-1)(3k-2) \frac{t^4 \sigma^8 \gamma_B^4}{(t \sigma^2 + 1)^4} - 8 \left(-6k + 6k^2 + 1\right) \frac{t^2 \sigma^4 \gamma_B^2}{(t \sigma^2 + 1)^2} + 8 \left(-6k + 4k^2 + 1\right).$$

Let $\lambda = \frac{t^4 \sigma^8 \gamma_B^4}{(t \sigma^2 + 1)^4}$. We have $0 < \lambda < 1$, and $\lambda$ is increasing in $\gamma_B^4$ and $t$. Now we can write $o$ as $o(\lambda) = (4k-1)(3k-2) \lambda^2 - 8(-6k+6k^2+1) \lambda + 8(-6k+4k^2+1)$. If $k = \frac{2}{3}$, we have $V_{i,B}^S > 0$ since $o(\lambda) = \frac{8}{3} (3 \lambda - 11) < 0$. When $k \neq \frac{2}{3}$, $o(\lambda) = 0$ have the following two roots:

$$\lambda_1(k) = \frac{1}{-11k + 12k^2 + 2 \left(-24k + 24k^2 + \sqrt{-k + 6k^2 \left(k - \frac{1}{2}\right) + 4}\right)}$$

$$\lambda_2(k) = \frac{1}{-11k + 12k^2 + 2 \left(-24k + 24k^2 - \sqrt{-k + 6k^2 \left(k - \frac{1}{2}\right) + 4}\right)}$$

We can verify that $\lambda_1(k) > 1$ as long as $k > \frac{1}{3}$; if $\frac{2}{3} < k < \frac{2}{3}$, $\lambda_2(k) > 1$; if $\frac{2}{3} < k < \frac{\sqrt{7}+3}{4}$, $\eta_2(k) < 0$; if $k > \frac{\sqrt{7}+3}{4}$, $0 < \lambda_3(k) < 1$. Note that $f(\eta)$ is concave when $\frac{1}{3} < k < \frac{2}{3}$ and convex when $k > \frac{2}{3}$. Thus, as long as $k < \frac{\sqrt{7}+3}{4}$, $V_{i,B}^S > 0$ since $o(\lambda) < 0$. Now, we focus on $k > \frac{\sqrt{7}+3}{4}$. If $k > \frac{\sqrt{7}+3}{4}$, $\frac{d\lambda_2(k)}{dk} > 0$ and $\lambda_2(k)|_{k=\infty} = \frac{6-2\sqrt{7}}{3} (\approx 0.8453)$, which means $\eta_2(k)$ is increasing with the upper bound $\frac{6-2\sqrt{7}}{3}$. We can conclude that given $t$, if there exists $\gamma_B$ such that $\frac{t^4 \sigma^8 \gamma_B^4}{(t \sigma^2 + 1)^4} = \frac{6-2\sqrt{7}}{3}$, then for any $\gamma_B > \gamma_B$, $o(\lambda) < 0$; if $\gamma_B < \gamma_B$, we have $o(\lambda) < 0$ iff $\gamma > \lambda_2(k)$. Denote $k_B^N = \lambda_2^{-1}(\gamma_B, t)$. We have that $V_{i,B}^N > 0$ if and only if $k < k_B^N$. And $k_B^N$ is increasing in $t$ and $\gamma_B$ because $\eta$ is increasing in $t$ and $\gamma_B$. Since $\lambda_2(k) > 0$ if and only if $k > \frac{3+\sqrt{7}}{4}$, we have $k_B^N < \frac{3+\sqrt{7}}{4}$. $k_B^N < k_B^S$ because $\eta_2(k) < \lambda_3(k)$ if $k > \frac{\sqrt{7}+3}{4}$.

**Proof of Proposition 7**

For part (a), if $k < \frac{1}{2}$, we have $\Pi_{R_i,C}^{NX_i} \leq \Pi_{R_i,C}^{SX_i}$. So the retailer is willing to share information for free. We also have $\Pi_{M_i,C}^{NS} > \Pi_{M_i,C}^{NN}$. Thus, $(S, S)$ is the unique equilibrium. If $\frac{1}{2} < k < k_N^N$, we know $V_{i,C}^S > 0$ for $i = 1, 2$, so the dominant strategy for either manufacturer is $S$ and $(S, S)$ is the unique equilibrium. If $k_N^N < k < k_C^C$, we have $V_{i,C}^N > 0$ and $V_{i,C}^S > 0$ for $i = 1, 2$. Hence $(S, S)$ and $(N, N)$ are two possible equilibria. It can be proven that $\Pi_{M_i,C}^{NS} + V_{i,C}^S > \Pi_{M_i,C}^{NN}$ for $i = 1, 2$. Hence $(S, S)$ is Pareto optimal. If $k > k_C^S$, we have $V_{i,C}^N < 0$ and $V_{i,C}^S < 0$. So the dominant strategy for both manufacturers is $N$ and $(N, N)$ is the unique equilibrium. Part (b) can be proved in a similar way and details are omitted. For part (c), if $k \leq \frac{1}{2}$, $\Pi_{R_i,C}^{NX_i} \leq \Pi_{R_i,C}^{SX_i}$ and $\Pi_{R_i,B}^{NX_i} \leq \Pi_{R_i,B}^{SX_i}$. The results follow. 

**Proof of Proposition 8**

For part (a), given an infinitesimal and positive number $\delta_C$, when $t = t_C^S - \delta_C$, the information sharing decision is $(S, S)$. We have $\hat{\Pi}_{R_i}^C(t_C^S - \delta_C) = \Pi_{R_i,C}^{SS}(t_C^S - \delta_C) + m_i = \Pi_{R_i,C}^{NS}(t_C^S - \delta_C)$. When $t = t_C^S + \delta_C$, the information sharing decision is $(N, N)$. We have $\hat{\Pi}_{R_i}^C(t_C^S + \delta_C) = \Pi_{R_i,C}^{NN}(t_C^S + \delta_C)$. We can show that $(\Pi_{R_i}^{NN}(t_C^S + \delta_C) - \Pi_{R_i}^{NS}(t_C^S - \delta_C))|_{\delta_C \to 0} < 0$ and the result follows. Similarly, we can prove part (b).