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Abstract—We consider a two-user multiple access channel (MAC) with a wireless-powered relay-to-destination (R-D) link, where the relay harvests energy from a radio frequency (RF) signal sent by a dedicated Power Beacon (PB). Each frame is divided into three phases. In the first phase, the relay harvests energy from an RF signal sent by a dedicated PB. The relay then receives information from user nodes in the second phase and forwards it to the destination in the third phase using its harvested energy. We investigate the sum rate maximization problem and characterize the capacity region of such a channel with the relay’s maximum transmit power constraint, under both the amplify-and-forward (AF) and decode-and-forward (DF) relay strategies. Optimal solutions are obtained for both cases. It is interesting to find that the shape of the capacity region is still pentagonal with the wireless-powered relay. And the relay’s maximum transmission power constraint greatly affects the system’s performance. Finally, simulation results demonstrate the correctness of our analysis.

I. INTRODUCTION

Long battery lifetime and low-cost communication capability are two attractive attributes for future portable devices. The recent advance of microwave wireless power transfer (WPT) enables wireless-powered communication networks (WPCNs) to be built, which offer the aforementioned advantages [1].

There are two main differences between conventional networks and WPCNs. First, there is an extra energy transfer phase before the information transmission phases in WPCNs. Second, by replacing some conventional user nodes with wireless-powered nodes, the system’s objective or power constraints can be dramatically changed. This would require re-design of communication protocols and optimal resource allocation for many conventional networks.

In the literature, many problems have been investigated. The authors in [2] considered the optimal time allocations to maximize the sum-throughput of a system in which one hybrid access point (HAP) with constant power supply coordinates the wireless energy/information transmissions to/from a set of distributed users that do not have other energy sources. Optimal resource allocations has also been studied in [3]. It was pointed out in [3] that when the information receiver and the energy transmitter are co-located as a HAP, there exists the “doubly-near-far” problem, which causes issues of user fairness. One way to cope with the fairness issue is to deploy multiple antennas at the HAP such that beamforming techniques can be applied. In [4], a joint design of downlink (DL)-uplink (UL) time allocation, DL energy beamforming, and UL transmit power allocation, as well as receive beamforming was investigated to optimize the users’ throughput and yet guarantee their rate fairness. Except for the case that the energy transmitter and information receiver are co-located, the scenario that the power station and information sink are located separately has also been considered in [5]. Optimal time allocation and beamforming were derived to maximize the system throughput. Another way to tackle user fairness is through user cooperation. This comes naturally from the consideration that user nodes with a larger amount of energy may help those nodes with less energy, to achieve a better system performance [6].

Apart from the problems like beamforming or user cooperation mentioned above, some researchers have been interested in more fundamental properties in wireless powered communication systems. The authors in [7] obtained the capacity region for a multiple access channel (MAC) with transmitters equipped with energy harvesters, while the sum-capacity in a three-user MAC and a two-way channel with bi-directional energy cooperation were investigated in [8]. However, few works have considered a MAC with a relay-to-destination (R-D) link.

In this paper, we consider a two-user MAC with a wireless-powered relay, where the relay harvests energy from a dedicated power beacon and helps forward users’ information to the destination. We investigate the maximum sum rate and capacity regions for both the amplify-and-forward (AF) and decode-and-forward (DF) relay strategies. Optimal solutions are obtained for both cases and simulation results demonstrate the correctness of our theoretical analysis.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a two-user MAC with a wireless-powered relay, where the relay is denoted as R, the destination D and the two user nodes U_1 and U_2, respectively. It is assumed that the relay harvests energy from a dedicated
Power Beacon (PB), which has a maximum transmission power $P_{pb}$, $U_1$ and $U_2$ have a steady power supply and their transmission powers, $p_1$ and $p_2$, are upper bounded by $P_1$ and $P_2$, respectively. All nodes are equipped with a single antenna.

Each frame is divided into three phases as shown in Fig. 2, starting from the first phase in which the relay harvests energy from the PB, to the second phase in which the relay receives information from the users, followed by the last phase in which the relay decodes information and forwards it to the destination. It is assumed the relay operates in half-duplex mode. Without loss of generality, each frame length is taken as $T = 1$. We denote by $h_1$ and $h_2$ the channel gains from the source nodes to the relay, by $h_1$, the channel gain from the relay to the destination and by $h_{pb}$ the channel gain from the PB to the relay. The noise variance of the relay and the destination are denoted as $\sigma^2$ and $\sigma_d^2$, respectively. We let $\gamma_1 = h_1/\sigma^2, \gamma_2 = h_2/\sigma^2$, and $\gamma_r = h_r/\sigma_d^2$ in the rest of this paper.

### A. Amplify-and-Forward

For the AF, the durations of the information phases are equal, i.e., $\tau_1 = \tau_2$, and the time causality constraint requires that $\tau_0 + 2\tau_1 \leq 1$.

In phase 1, the PB sends $\sqrt{P_{pb}}x_0$, with $E[|x_0|^2] = 1$. The received signal at the relay is

$$y_{r1} = \sqrt{P_{pb}}h_{pb}x_0 + n_1. \quad (1)$$

It is assumed that $P_{pb}$ is large and the energy harvested from the noise can be neglected. Therefore, the amount of harvested energy at the relay in phase 1 equals

$$E_h = \eta h_{pb}P_{pb}(1 - 2\tau_1), \quad (2)$$

where $\eta$ represents the energy harvesting efficiency at the relay and it is assumed to be a constant for convenience.

In phase 2, the user nodes send signals $u_i = \sqrt{p_i}x_i$, with $E[|x_i|^2] = 1, i = 1, 2$. The relay receives

$$y_{r2} = \sqrt{h_1p_1}x_1 + \sqrt{h_2p_2}x_2 + n_2. \quad (3)$$

where $n_2 \sim CN(0, \sigma^2)$, a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance $\sigma^2$.

The relay then amplifies the signal and sends

$$t_r = \sqrt{ah_1p_1}x_1 + \sqrt{ah_2p_2}x_2 + \sqrt{\alpha}n_2. \quad (4)$$

In phase 3, the destination receives

$$y_d = \sqrt{h_r}a_1p_1x_1 + \sqrt{h_r}a_2p_2x_2 + \sqrt{h_r}\alpha n_2 + n_d. \quad (5)$$

Using successive interference cancellation (SIC) at the destination and assuming user 2’s information is decoded first, the end-to-end rates are given as follows:

$$r_1 = \tau_1 \log_2 \left( 1 + \frac{\gamma_1 a_1 p_1}{\gamma_0 \alpha \sigma^2 + 1} \right) \quad (6)$$

$$r_2 = \tau_1 \log_2 \left( 1 + \frac{\gamma_1 a_2 p_2}{\gamma_0 a_1 p_1 + \gamma_0 \alpha \sigma^2 + 1} \right), \quad (7)$$

where $\alpha$ satisfies the relay’s maximum transmission power constraint $|t_r|^2 \leq \min\{\beta(1 - 2\tau_1), P_R\}$.

The end-to-end sum rate can be expressed as

$$R_{sum} = \tau_1 \log_2 \left( 1 + \frac{\alpha \gamma_1 (a_1 p_1 + a_2 p_2)}{\gamma_0 \alpha \sigma^2 + 1} \right). \quad (8)$$

### B. Decode-and-Forward

For the DF, the first phase is the same as that of the AF. In phase 2, we use SIC at the relay and without loss of generality assume $h_1 > h_2$. Thus the relay decodes user 2’s information first and cancels it from the signal before decoding user 1’s information. The case that $h_1 \leq h_2$ can be solved similarly. The individual rates from the users to the relay under this decoding order are given as

$$r_{11} = \tau_1 \log_2 \left( 1 + \frac{\gamma_1 p_1}{\gamma_0 p_2} \right) \quad (9)$$

$$r_{21} = \tau_1 \log_2 \left( 1 + \frac{\gamma_2 p_2}{1 + \gamma_0 p_1} \right). \quad (10)$$

In phase 3, the relay re-encodes and transmits $x_r$ to the destination. At the end of phase 3, the destination decodes the users’ information and the end-to-end user rates are expressed as $R_1 = \min\{r_{11}, r_{12}\}$ and $R_2 = \min\{r_{21}, r_{22}\}$ with the rate constraint of the relay-to-destination link, which is given by $r_{12} + r_{22} \leq \tau_2 \log_2 (1 + \gamma_r P_r)$ and the relay’s power constraint $P_r \leq \min\{\beta(1 - \tau_1 - \tau_2)/\tau_2, P_R\}$.

The end-to-end sum rate can be expressed as

$$R_{sum} = \min \left\{ \tau_1 \log_2 \left( 1 + \gamma_1 p_1 + \gamma_2 p_2 \right), \tau_2 \log_2 \left( 1 + \gamma_r P_r \right) \right\}. \quad (11)$$

For notation simplicity, in the rest of the paper, we define $S_r = \{(\tau_1, \tau_2) : \tau_1 + \tau_2 < 1, \tau_1, \tau_2 > 0\}$, $S_{pb} = \{p : p_1 \leq P_1, p_2 \leq P_2, p_1, p_2, p_r \geq 0\}$, $S_p = \{p : p_1 \leq P_1, p_2 \leq P_2, p_1, p_2, p_r \geq 0\}$, where $p = [p_1, p_2, p_r]$, $p_s = [p_1, p_2]$. We further let $\beta = \eta h_{pb}P_{pb}, c_1 = \log_2(1 + \gamma_1 P_1 + \gamma_2 P_2), c_2 = \log_2(1 + \gamma_0 P_R), c_3 = \log_2(1 + \gamma_1 P_1), c_4 = \log_2(1 + \gamma_0 P_R), m_1 = \beta \gamma_1, m_2 = \beta \gamma_2(\gamma_1 P_1 + \gamma_2 P_2 + 1), m = \gamma_1 P_1 + \gamma_2 P_2 + 1, \alpha = \gamma_0 \gamma_2 P_2 \beta, b = \gamma_1 P_1 + \gamma_2 P_2 + 1, \text{ and } c = (\gamma_1 P_1 + 1) \gamma_2 \beta.$
III. SUM RATE MAXIMIZATION AND CAPACITY REGION WITH A WIRELESS-POWERED RELAY

In this section, we investigate the problems with a wireless-powered relay. The case with a conventional relay is much easier and thus omitted due to space limitation.

A. Amplify-and-Forward

1) Sum Rate Maximization: For the AF, the problem can be formulated as follows:

$$\max_{\tau_1, \alpha, p_1, p_2 \in S_p} \quad \tau_1 \log_2 \left( 1 + \frac{\alpha h_1 p_1 + h_2 p_2}{\gamma_1 \alpha \sigma^2 + 1} \right) \tag{12}$$

s.t. \hspace{1cm} \alpha h_1 p_1 + p_2 h_2 + \alpha \sigma^2 
\leq \min \left\{ \frac{(1 - 2 \tau_1)}{\tau_1}, P_R \right\} \tag{13}

$$0 < \tau_1 < 1/2. \tag{14}$$

**Theorem 3.1:** The maximum sum rate $R_{sum}^*$ is

$$\max \max_{\tau_1, \alpha, p_1, p_2 \in S_p} \quad \tau_1 \log_2 \left( 1 + \frac{\alpha h_1 p_1 + h_2 p_2}{\gamma_1 \alpha \sigma^2 + 1} \right) \tag{15}$$

s.t. \hspace{1cm} \alpha h_1 p_1 + p_2 h_2 + \alpha \sigma^2 
\leq \min \left\{ \frac{(1 - 2 \tau_1)}{\tau_1}, P_R \right\} \tag{16}

$$0 < \tau_1 < 1/2. \tag{17}$$

First we claim that constraint (16) achieves equality in the optimal solution. When $R$ is small, constraint (15) can be satisfied with $p_1 = P_1$. And $R_2$ is an increasing function of $p_2$. With the increasing of $R_2$, $p_2$ will eventually achieve $P_2$, which corresponds to the case of the maximum sum rate. After that, $p_1$ should be decreased to guarantee user 2’s rate constraint and this would make the rate pair lie strictly inside the capacity region. Thus we do not need to consider the case that $R \geq R_s$ when user 2’s information is decoded first at the destination. Similarly, when user 1’s information is first decoded at the destination, we can derive the other part of the capacity region and the segment between them is achieved by time sharing between these two decoding orders. The result is concluded in the following theorem.

**Theorem 3.2:** For $\overline{R} \in [0, \overline{R}]$, the optimal solution is given in two cases.

Case 1: If $g_2(\tau_1) \leq 0$, $R_0 = R_2 = g_2(\tau_1)$. For $R \leq R_2$, the optimal rate pair is $(R_1^*, R_2)$, where $R_1^* = \arg \max g_1(\tau_1)$ with $\tau_1 \in [\tau_0^*, \tau_1^*]$. With the increasing of $\tau_1$, $g_1(\tau_1)$ would make $R_2$ lie strictly inside the rate region. Similarly, when user 1’s information is first decoded at the destination, we can derive the other part of the capacity region and the segment between them is achieved by time sharing between these two decoding orders. The result is concluded in the following theorem.

B. Decode-and-Forward

1) Sum Rate Maximization: The problem can be formulated as follows:

$$\max_{\tau_1, \alpha, p_1, p_2 \in S_p} \quad \min \left\{ \tau_1 \log_2 (1 + \gamma_1 p_1 + \gamma_2 p_2), g_2 \log_2 \left( 1 + \frac{\alpha h_1 p_1 + h_2 p_2}{\gamma_1 \alpha \sigma^2 + 1} \right) \right\} \tag{18}$$

s.t. \hspace{1cm} p_1 \leq \min \left\{ \frac{(1 - 2 \tau_1 - \tau_2)}{\tau_1}, P_R \right\} \tag{19}

$$0 < \tau_1 < 1/2. \tag{20}$$

First we claim that constraint (16) achieves equality in the optimal solution. When $R$ is small, constraint (15) can be satisfied with $p_1 = P_1$. And $R_2$ is an increasing function of $p_2$. With the increasing of $R_2$, $p_2$ will eventually achieve $P_2$, which corresponds to the case of the maximum sum rate. After that, $p_1$ should be decreased to guarantee user 2’s rate constraint and this would make the rate pair lie strictly inside the capacity region. Thus we do not need to consider the case that $R \geq R_s$ when user 2’s information is decoded first at the destination. Similarly, when user 1’s information is first decoded at the destination, we can derive the other part of the capacity region and the segment between them is achieved by time sharing between these two decoding orders. The result is concluded in the following theorem.
\[ \tau^*_1 = \begin{cases} \max(\tau_1^1, \tau_2^1), & \text{if } p_{r^*}^r \leq P_R, \\ \tau_1^2, & \text{otherwise}, \end{cases} \] 

where

\[ \tau_1^1 = \frac{\beta \log_2(1 + \gamma_1 p_{r^*}^r)}{\beta \log_2(1 + \gamma_1 p_{r^*}^r) + \log_2(1 + \gamma_2 p_{r^*}^r)}, \quad \tau_2^2 = \frac{\beta \log_2(1 + \gamma_2 p_{r^*}^r)}{\beta \log_2(1 + \gamma_2 p_{r^*}^r) + \log_2(1 + \gamma_1 p_{r^*}^r)}, \]

and \( p_{r^*}^r \) is the unique solution of equation \( \log(1 + \gamma_r p_{r^*}^r) - \gamma_r (\beta + p_{r^*}^r) = 0 \).

**Proof:** We can prove this by KKT conditions and the details are omitted due to limited space.

2) **Capacity Region:** The problem can be formulated as

\[
\begin{aligned}
\max_{R_1, R_2, \tau \in S_+} & \quad R_1, R_2, \\
\text{s.t.} & \quad R_1 \leq \tau_1 \log_2(1 + \gamma_1 p_1), \\
& \quad R_2 \leq \tau_2 \log_2(1 + \gamma_2 p_2), \\
& \quad R_1 + R_2 \leq \tau_1 \log_2(1 + \gamma_1 p_1) + \tau_2 \log_2(1 + \gamma_2 p_2), \\
& \quad p_1, p_2 \leq \min\left\{ \frac{\beta (1 - \tau_1 - \tau_2)}{\tau_1 \theta_1}, \frac{\beta (1 - \tau_1 - \tau_2)}{\tau_2 \theta_2}, \frac{\beta (1 - \tau_1 - \tau_2)}{\tau_1 \theta_2}, \frac{\beta (1 - \tau_1 - \tau_2)}{\tau_2 \theta_1} \right\}.
\end{aligned}
\]

The key observation is that constraint (29) can be dropped after the discussion of \( R \) and it is again a convex optimization problem. The result is concluded in the following theorem.

**Theorem 3.4:** For given \( R \in [0, \max(R^{c_1}, R^{c_2})] \), the rate pair \((R_1, R_2)\) on the capacity region corresponds to \((\tau_1^*, \tau_2^*), R\)

\[
\tau_1^* = \begin{cases} \frac{f(p_{r^*}^r)}{f(p_{r^*}^r)}, & \text{if } p_{r^*}^r \leq P_R, \text{ and } f(p_{r^*}^r) \geq f(R), \\
\text{or } R \in [R^{c_1}, R^{c_2}], & f(p_{r^*}^r), \text{ otherwise}, \\
\end{cases}
\]

\[
f(x) = \frac{\beta \log_2(1 + \gamma_1 p_1) - \beta \log_2(1 + \gamma_2 p_2)}{\beta \log_2(1 + \gamma_1 p_1) + \beta \log_2(1 + \gamma_2 p_2)}, \quad g(p_{r^*}^r) = g(R), \quad g(p_{r^*}^r) = g(p_{r^*}^r),
\]

\[
f(x) = \frac{\beta \log_2(1 + \gamma_1 p_1) - \beta \log_2(1 + \gamma_2 p_2)}{\beta \log_2(1 + \gamma_1 p_1) + \beta \log_2(1 + \gamma_2 p_2)}, \quad g(p_{r^*}^r) = g(R), \quad g(p_{r^*}^r) = g(p_{r^*}^r),
\]

and \( p_{r^*}^r \) is the same as that in theorem 3.3.

**Proof:** When \( R \) is small, constraint (29) will not be a problem. However, we have to find out how large \( R \) can be before we need to consider this constraint. For the same reason as in the AF case, we only need to consider the critical situation that \( p_2 = P_2 \) when deriving the capacity region. Luckily, by first looking into the problem without considering constraint (29), we find that the right hand side of constraint (29) is always a decreasing function of \( R \). Thus it is straightforward to say there indeed exists a critical \( R^c \) such that for \( R \leq R^c \), constraint (29) always holds for the optimal case. Therefore, we solve the original problem by the discussion of \( R \) and by checking the KKT conditions. Specifically, we solve the two sub-problems individually and the optimal solution corresponds to the case with a larger \( R_1 \).

Case 1: \( \beta (1 - \tau_1 - \tau_2)/\tau_2 \leq P_R \). By using variable substitutions \( E_1 = P_1 \tau_1 \) and \( E_r = P_R \tau_2 \), the Lagrangian of the reduced problem is

\[
\mathcal{L}(R_1, E_1, E_r, \tau) = R_1 - \lambda_1 [R_1 - \tau_1 \log_2(1 + \gamma_1 E_1/\tau_1)] - \lambda_2 [R_1 + \tau_2 \log_2(1 + \gamma_2 E_r/\tau_2)] - \lambda_3 (E_1 - P_1 \tau_1) - \lambda_4 (E_r - P_R \tau_2) - \mu [E_1 - \beta (1 - \tau_1 - \tau_2)].
\]
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Fig. 4: The capacity regions vs $P_R$, for conventional and wireless-powered AF with $\gamma_1 = 1$, and $\gamma_2 = 1$.

Fig. 5: The capacity regions vs $P_R$, for wireless-powered DF with $\gamma_1 = 0.1$, $\gamma_2 = 1$, $\gamma_r = 2$, and $\beta = 2.8$.

degradation, which is our expectation. However, a larger $P_{pb}$ or a better PB to the relay channel would help decrease the gap, as we will show in Fig. 6 in the DF case.

Fig. 5 shows the capacity region under different $P_R$ for the wireless-powered DF R-D link. It is shown that there exists a $P_R$ such that after $P_R > P_R^*$, increasing $P_R$ no longer enlarges the capacity region. This is because a large $P_R$ results in a small $\tau_2^*$ in case 2 and thus the optimal solution falls to case 1, which does not change with $P_R$. Moreover, it is found that $P_R^* = \tau_1^*$, which depends on the channel condition of the R-D link and conditions of the wireless power transfer phase. The capacity region in the DF case is still a pentagon, as shown in theorem 3.4. The corner points of the capacity region for the wireless-powered relay lie on two lines, with the slopes given by $l_1 = \frac{\log_2((1+\tau_1)P_1)}{\log_2((1+\tau_2)P_2/(1+\tau_1)P_1)}$ and $l_2 = \frac{\log_2((1+\tau_1)P_1/(1+\tau_2)P_2)}{\log_2((1+\tau_1)P_1/(1+\tau_2)P_2)}$, respectively. And we can characterize the angle $\theta = \arctan\frac{l_1}{1+l_2}$ further. The slope of part AB, i.e., the characteristic of user 1’s rate with respect to user 2’s rate, is not affected by user 2’s channel condition $h_2$, as shown in this figure. Similarly, the slope of part CD would not be affected by $h_1$ either.

Finally, Fig. 6 illustrates the impact of $P_{pb}$ on the capacity region with the wireless-powered DF R-D link under a given $P_R$. For a given $P_R$, the capacity region of the conventional relay is shown as the outer bound, and we can see that with the increasing of $P_{pb}$, the capacity region of the wireless-powered relay eventually converges to the conventional relay case. The fact that the corner points lie on lines also holds for the impact of $P_{pb}$. Moreover, the slopes and thus the angle are the same as the different $P_R$ case since they depend on $l_1$ and $l_2$, which are not affected by $P_R$ or $P_{pb}$.

V. CONCLUSION

In this paper, we have investigated a MAC with a wireless-powered R-D link. We have considered sum rate maximization and derived the capacity regions of the systems, for both the AF and DF relay strategies. Simulation results show that DF outperforms AF and both are greatly affected by the relay’s maximum transmit power constraint. Unlike conventional relaying, the capacity region in wireless-powered relaying is upper bounded even with $P_R$ continuously increasing. In addition, a better PB to the relay channel helps decrease the gap in the capacity regions between the conventional relaying and wireless-powered relaying.

REFERENCES